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Quantum Backaction Evading Measurement of Collective Mechanical Modes

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The standard quantum limit constrains the precision of an oscillator position measurement. It arises from a balance between the imprecision and the quantum backaction of the measurement. However, a measurement of only a single quadrature of the oscillator can evade the backaction and be made with arbitrary precision. Here we demonstrate quantum backaction evading measurements of a collective quadrature of two mechanical oscillators, both coupled to a common microwave cavity. The work allows for quantum state tomography of two mechanical oscillators, and provides a foundation for macroscopic mechanical entanglement and force sensing beyond conventional quantum limits.

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The interplay of measurement imprecision and quantum backaction limits the sensitivity with which the position of an oscillator can be continuously monitored, to be at best equal to the oscillator’s zero-point fluctuations [1,2]. This is known as the standard quantum limit (SQL). However, any single quadrature of the motion can, in principle, be measured without limit, provided that the measurement backaction is shunted to the orthogonal quadrature. Such backaction evading measurements of a collective quadrature of two mechanical oscillators, both coupled to a common microwave cavity. The work allows for quantum state tomography of two mechanical oscillators, and provides a foundation for macroscopic mechanical entanglement and force sensing beyond conventional quantum limits.

Our system is shown in Fig. 1(a). It consists of a microwave cavity resonator and two mechanical oscillators that have no direct coupling. Previously, optomechanical systems containing more than one mechanical oscillator have been experimentally studied both in the optical [22–26] and microwave [27,28] regimes. Our cavity is a superconducting on-chip LC resonator, with frequency ω0, decay rate κ, and mode operator a. The mechanical oscillators are realized as aluminum drumheads with mode operators b1, frequencies ω1, and decay rates γ1 (i = 1, 2). Each mechanical oscillator is individually coupled to the cavity via the radiation-pressure interaction \( H_i = g_i a^i a(b_1^i + b_i) \), realizing a three-mode cavity optomechanical system [29]. The single-photon coupling rates \( g_i \) arise from a position-dependent capacitance between the cavity and each mechanical oscillator. The total Hamiltonian is hence

\[
H = \omega_0 a^d a + \sum_i (\omega_1 b_1^i b_1^i + H_i).
\]

We describe the system in a reference frame set by the cavity frequency and the average of the two mechanical frequencies [30]. In this frame we define the position quadrature \( X_\pm = (X_1 X_2) / \sqrt{2} \) and momentum quadrature \( P_\mp = (P_1 \mp P_2) / \sqrt{2} \). Note that these quadratures are not QND variables, but have (as we will see) useful dynamics. When each oscillator is in equilibrium with a bath of temperature \( T_i \), it has the thermal occupation \( n_i^T = k_B T_i / \hbar \omega_i \). In such a thermal state, the variances of all collective quadratures are equal: \( \langle \Delta X_\pm^2 \rangle = \langle \Delta P_\mp^2 \rangle = (n_1^T + n_2^T + 1) / 2 \), as depicted in Fig. 1(c).

In order to achieve two-mode BAE using only two tones, the system is simultaneously pumped at the frequencies below the zero-point fluctuations and an evasion of quantum backaction caused by microwave shot noise (below the backaction arising in a continuous position measurement). The canonically conjugate quadrature is heated predominantly by the quantum backaction.
shown in Fig. 1(b) [20]. Assuming the system is in the resolved-sideband limit measured by observing the cavity output. The cavity has the frequency \( \Omega \), previously observed in an atomic system[19]. Only the reduced concept of a \( \omega \) representation of our setup. Two micromechanical oscillators (ports 1 and 2) are capacitively coupled to a superconducting microwave resonator \( \omega_o \). Pump tones are injected to port 1, and the output spectrum of port 2 is measured on a signal analyzer. (b) Two strong microwave pump tones (frequencies \( \omega_o \) and \( \omega_o \)) realize a BAE measurement. The mechanical spectrum appears as sidebands (pink peaks) on the thermal cavity spectrum (green line). The backaction can be probed by a set of weak probe tones (short arrows), slightly detuned from the pump tones. (c) In thermal equilibrium, the fluctuations (gray circle) of all collective quadratures are equal to \( \Delta X^T \). (d) The backaction of the BAE measurement heats up the \( P_+ \) quadrature, whereas \( X_+ \) remains unaffected. The red axes represent the projection on a basis defined by the probes.

\[
\omega_\pm = \omega_o \pm (\omega_1 + \omega_2)/2 \text{ with equal (real) amplitudes } \Omega, \text{ as shown in Fig. 1(b) [20]. Assuming the system is in the resolved-sideband limit } \omega_o \gg \kappa, \text{ we neglect terms oscillating at } \pm 2\omega_o. \text{ Further assuming that } g_1 \approx g_2, \text{ the effective Hamiltonian is}
\]

\[
H = \Omega (X_+X_- + P_+P_-) + 2G(a_1^\dagger + a)X_+ \quad (1)
\]

where \( \Omega = (\omega_1 - \omega_2)/2 \) is the effective mechanical oscillator frequency, and the effective optomechanical coupling is \( G = (g_1 + g_2)\sigma/2 \) [30]. Under this Hamiltonian, \( X_+ \) and \( P_- \) act dynamically like the position and momentum of a single harmonic oscillator, even though they commute with one another [18,20,38]. It is an example of the recently introduced concept of a “quantum-mechanics-free subsystem” [18], previously observed in an atomic system [19]. Only the \( X_+ \) collective quadrature couples to the cavity, and is measured by observing the cavity output.

The spectra of the mechanical collective quadrature \( X_+ \) and its conjugate collective quadrature \( P_+ \) are given by [30]

\[
S_{X_+}[\omega] = \frac{1}{2}(n_T^2 + n_T^2 + 1)S_0[\omega],
\]

\[
S_{P_+}[\omega] = \frac{1}{2}(n_T^2 + n_T^2 + 1 + 2n_{BA})S_0[\omega],
\]

where \( S_0[\omega] = \sum_{\sigma=\pm} 2\gamma/\sigma^2 + 4(\omega + \sigma\Omega)^2 \) and

\[
n_{BA} = 2C \frac{k^2}{k^2 + 4\Omega^2}(2n_T^2 + 1) \quad (3)
\]

is the total measurement backaction, with \( n_T^2 \) being the thermal occupation of the cavity. The cooperativity is introduced as \( C = 4G^2/(\kappa\gamma) \) with \( \gamma = (\gamma_1 + \gamma_2)/2 \). The backaction can be divided into classical and quantum contributions: \( n_{BA} = n_{BA,c} + n_{BA,q} \) corresponding to, respectively, the terms \( 2n_T^2 \) and +1 inside the parentheses in Eq. (3). We define the occupation number by \( n_{X_+} = (\Delta X^T)^2 = (2\pi)^{-1} \int S_{X_+}[\omega]d\omega, \) and similarly for \( n_{P_+}. \) As is evident from Eq. (2), only the \( P_+ \) quadrature is heated by backaction, while the measured quantity \( X_+ \) remains unaffected. Experimentally, there can be additional technical backaction via an increase in \( n_T^2 \) from equilibrium values due to pump current heating.

The mechanical spectrum \( S_{X_+}[\omega] \) is faithfully reproduced in the cavity output spectrum \( S_{out}[\omega] \). As shown in Fig. 1(b), it appears as sidebands at frequencies \( \omega_o \pm |\Omega| \), on top of a noise floor due to experimental contributions [30]. This allows for experimental reconstruction of the \( X_+ \) quadrature by a spectral analysis of the scattered pump microwave light.

We stress that the symmetry inherent in Eq. (1) that protects \( X_+ \) from the backaction heating of \( P_+ \) requires closely matched single-photon optomechanical couplings. This thus introduces an additional experimental complication compared to the single-mode optomechanical BAE measurement. It should be noted that the coupling asymmetry cannot be compensated by tuning the pump power ratio. In practice, one needs to have \( g_1 \approx g_2 \) within about 10% accuracy, which is experimentally challenging. A theoretical description including deviations from the ideal case is thoroughly discussed in Ref. [20].

As schematically depicted in Fig. 1(a), the mechanical oscillators [39] are connected to opposite ends of a transmission line cavity resonator, nearly 1 mm apart from each other. The experiments are carried out in a dilution refrigerator at a temperature of 27 mK (unless stated otherwise). The cavity is probed using a transmission measurement. The cavity has the frequency \( \omega_o = 2\pi \times 5.5 \text{ GHz} \), total linewidth \( \kappa = 2\pi \times 1.22 \text{ MHz} \) dominated by coupling to the output line with the rate \( \kappa_{EO} = 2\pi \times 0.98 \text{ MHz} \). The coupling at the input side line is \( \kappa_{EI} = 2\pi \times 60 \text{ kHz} \), and the internal decay rate is \( \kappa_I = 2\pi \times 180 \text{ kHz} \). The mechanical oscillators have the frequencies \( \omega_1 = 2\pi \times 10.0 \text{ MHz} \) and \( \omega_2 = 2\pi \times 11.3 \text{ MHz} \), and linewidths \( \gamma_1 = 2\pi \times 130 \text{ Hz} \) and \( \gamma_2 = 2\pi \times 150 \text{ Hz} \), respectively. For this sample we obtained \( g_1/g_2 = 0.94 \pm 0.02 \), which allows for a nearly ideal two-mode BAE measurement [30]. The two pump tones need to be equal in amplitude as well, at the highest cooperativities presented here at 0.1% accuracy. We can calibrate the pump amplitude ratio with about 5% accuracy, and then use the pump power ratio as an adjustable parameter within the calibrated window.
Figure 2 shows the BAE measurement results. In Fig. 2(a) the measured cavity output spectra are shown for different measurement strengths (cooperativities). Figure 2(b) shows the integrated area of the spectra (purple dots) as a function of measurement strength given by the cooperativity. The solid line is expected for perfect BAE; the dotted line includes technical heating. The measured effective thermal area for cooling (arbitrary units). (e) and (f) Effective thermal linewidth of the spectral peaks corresponding to the data in (b). (d) BAE thermometry (purple dots, \( n_{X+} \)) for \( C = 0.2 \) as a function of cryostat temperature. The solid line is a fit to the data, used to calibrate the mechanical signal. Red squares show the integrated area for cooling (arbitrary units). (e) and (f) Effective thermal occupation of the (e) mechanical oscillators and (f) cavity, measured with a single red-sideband pump. Mechanical oscillator 2 (blue diamonds) shows technical heating. Dashed lines correspond to an empirical model.

For cooperativity up to \( C \approx 5 \), the data are in excellent agreement with the expected linear behavior, indicating that the \( X_+ \) quadrature is not perturbed by measurement backaction. At the largest measurement strengths available in our experiment (limited by the technical requirement of equal pump tone amplitudes), a small increase in \( n_{X+} \) is observed. This is in good agreement with independently measured technical heating of oscillator 2; see Fig. 2(e).
The effective occupation can be written as

\[ n_T \]

where \( n_T \) is the initial thermal occupation to be measured. \( n_{\text{amp}} \) is the residual backaction including technical heating, and \( n_{\text{imp}} \) is the imprecision noise. The latter, assuming the cavity output is measured with a high-gain phase-insensitive amplifier, is [30]

\[ \omega_2 - \omega_1 \quad (i = 1, 2) \]

For the two-tone measurement the heating is given by the total pump power. We typically observe heating in varying amounts in different samples [13, 27], likely related to the presence of surface two-level systems [43].

An important benchmark for a sensitive measurement is that the total measurement imprecision is below the fundamental quantum limit for continuous position detection. This has been demonstrated in single-oscillator optomechanical systems [8, 44-46]. The effective quadrature occupation can be written as

\[ n_{X_+} = n_{X_+}^T + n_{X_+}^{BA} + n_{X_+}^{imp} \]

where \( n_{X_+}^T = (n_1^T + n_2^T) / 2 \) is the initial occupation to be measured, \( n_{X_+}^{BA} \) is the residual backaction including technical heating, and \( n_{X_+}^{imp} \) is the imprecision noise. The latter, assuming the cavity output is measured with a high-gain phase-insensitive amplifier, is [30]

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where \( n_{X_+}^T = (n_1^T + n_2^T) / 2 \) is the initial occupation to be measured, \( n_{X_+}^{BA} \) is the residual backaction including technical heating, and \( n_{X_+}^{imp} \) is the imprecision noise. The latter, assuming the cavity output is measured with a high-gain phase-insensitive amplifier, is [30]
Furthermore, the total backaction and imprecision of $X_\alpha$ is below the quantum backaction by $3.2 \pm 1.0$ dB, demonstrating that our system outperforms a perfect phase-insensitive position measurement. The stated uncertainty is dominated by the statistical uncertainty (95% confidence), calculated from the spread of the data in Fig. 3(b), but also includes the calibration uncertainty of $\pm 0.3$ dB.

In summary, we have performed sensitive measurements of the collective motion of two mechanical oscillators without disturbance from the quantum backaction of the measurement. The measurement sensitivity exceeds the standard quantum limit, leading to a strong backaction observed in the canonically conjugate observable. The residual backaction is dominated by technical heating of one mechanical oscillator, which does not correspond to a fundamental limitation. Neglecting technical heating while keeping other parameters unchanged, the backaction reduces to a few quanta, closely approaching the SQL. Further, with an improved imprecision $n_{\text{amp}} \sim 1$, this can be reached at small pump power, $C \sim 1$. The two-mode backaction evading measurement can be used for quantum state tomography and for measuring both quadratures of a force without a quantum limit. This work also provides a foundation for the preparation and detection of macroscopic mechanical entanglement.

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