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Effects of large-scale photovoltaic power integration on electricity distribution networks

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ABSTRACT
The public support in PV technologies and increasing markets have resulted in extensive applications of grid-connected PV, in particular in the consumer side and electricity distribution grid. In this paper the effects of a high level of grid connected PV in the middle voltage distribution network have been analyzed. The emphasis is put on static phenomena, including voltage drop, network losses and grid benefits. A multi-purpose modeling tool is used for PV analysis in Lisbon and Helsinki climates. All network types studied can handle PV without problems with an amount of PV equaling at least up to the load (1kWp/household). The comb-type network showed the best performance. The PV is unable to shave the domestic load peak in the early evening hours but through orientating the PV panels both to east and west, the noon peak from PV can be reduced by 30%. PV integration reduces network losses positively up to a 1kWp/hh (100% of annual domestic load) level. For 2 kWp/hh all but the comb-type networks demonstrate clear over-voltage situations and the annual network losses are much higher than without PV.

KEYWORDS
Photovoltaics, network effects, distributed power generation, simulation
INTRODUCTION

Photovoltaic (PV) power generation has been steadily growing supported by large deployment campaigns in some countries [1]. Most of the new PV capacity is assigned to grid connected distributed power generation [2]. This raises new engineering challenges at individual household (hh) level due to the intermittency of the solar resource.

Advances in PV technology have been fast in recent years making this technology technically viable for local power production in small scale although the cost-effectiveness is not yet adequate. The state of art and prospects of PV technology have recently been reviewed by Green [3] and van der Zwaan [4]. The planning tools of individual photovoltaic systems are also well available for evaluating sizing and system economy. [5,6].

When large scale applications of PV in the grid are considered little references are, however, found on PV in the individual customer line. Most of the work on large-scale PV is reported either on large scale PV power generation in general e.g. in scenarios or on central PV power schemes [7]. Some experimental facilities on central PV power stations were erected already in the 80's [8].

In the present paper large scale implementation of distributed photovoltaic power generation in the end-use side is studied. The individual PV systems are scattered into the consumer nodes and connected to the distribution network. We have used a distribution network simulation tool to provide over 150 different case studies to get a detailed view of the effects from high photovoltaic penetration. These case studies include two geographical locations with a total of three different domestic electricity consumption profiles that were created using a load method published earlier by the authors [9]. Five different penetration levels and four different strategies for orientating the solar panels were included. The main parameters used for evaluating the impacts of PV on the distribution network are the voltage balance, system losses, and peak load compensation. Our results show, that the medium voltage distribution network can be straightforwardly modeled to evaluate the effects of high PV penetration levels.

METHODOLOGY

The power distribution system in this paper is modeled with the DESIGEN simulation tool (Decentralized system simulation tool for optimized generation) developed at Helsinki University of Technology. Figure 1 shows the flow chart of DESIGEN and the iterative processes used to solve the power flows and balances. The program combines power flow calculations, custom load data and models for distributed power generation and simulates the static power flow effects with a given time interval, usually using an 1 hour time step.

To compare the system performance in a temperate and cold climate, the load and photovoltaic (PV) generation data included two climatic zones: Helsinki (60°N) and Lisbon (39°N).
The type of energy system modeled is sketched in Figure 2 showing a hypothetical network, loads and distributed generation units. In the simulation runs, the PV systems are distributed evenly, i.e. each node in the network includes PV. As the low voltage network is not simulated per se, one medium voltage node represents typically a group of houses, or about 40 households each.

**Power distribution network**

DESIGEN employs two different network calculation methods. The first one calculates radial networks using a simple five step iterative method similar to that published by Jenkins et al.[10]. The method is described in detail in Appendix A.

The second network calculation method solves looped networks using the traditional polar power-mismatch version of the iterative Newton-Raphson algorithm [11,12], designed for solving a set of non-linear equations simultaneously. There, network flow equations form a so-called Jacobian-matrix equation

\[
\begin{bmatrix}
\Delta P_i \\
\Delta Q_i \\
\Delta \delta_i \\
\Delta U_i
\end{bmatrix} =
\begin{bmatrix}
H & N & \Delta \delta_i \\
M & L & \Delta U_i
\end{bmatrix}
\begin{bmatrix}
\Delta P_i \\
\Delta Q_i \\
\Delta \delta_i \\
\Delta U_i
\end{bmatrix}
\]

(1)

which is iteratively solved until the power values \( P_{r+1} = P_i + \Delta P_i \) (real power) and \( Q_{r+1} = Q_i + \Delta Q_i \) (reactive power) converge. The matrixes H, N, M and L above are Jacobian matrixes \( H_{ki} = \partial P_k / \partial \delta_i \), \( N_{ki} = \partial P_k / \partial U_i \), \( M_{ki} = \partial Q_k / \partial \delta_i \) and \( L_{ki} = \partial Q_k / \partial U_i \), correspondingly. \( \Delta \delta_i \) is the phase angle and \( \Delta U_i \) the voltage step size. This traditional Newton-Raphson method was chosen due to its simplicity, although faster decoupled power flow method exists [13,14]. The modeled network can include transformers and all kinds of load and generation components, including reactive current compensators. The whole procedure for simulating looped networks is available at Appendix B.

The transmission losses of both the radial and looped networks can be calculated for each time step as

\[
S_{ki} = (U_k - U_i) J_{ki} = Y_{ki} [U_k - U_i]^2
\]

(2)

where loss \( S_{ki} \) on line between adjacent nodes k and i is computed using voltage \( U_k \) for node k and \( U_i \) for node i, correspondingly. \( Y_{ki} \) is the admittance between the nodes i and k. \( J_{ki} \) is the current.

DESIGEN can also be applied to model transformers in the network, but it is out of the scope here, as only one distribution network level is considered.
**Consumer load**

DESIGN reads the end-use load for each network node in the power distribution network from a file. In practice, the load data is simulated separately using a statistical approach to handle a large load data set.

The load model is described in detail elsewhere [9]. The key input data needed include firstly the mean electricity consumption rate and the season induced weekly changes in mean load on an annual level. Secondly, the appliances to be included into the model and their mean daily consumption profiles, preferably separately for weekdays and weekend days, need to be defined. Next the cycle length, power levels and standby power level of each appliance type used need to be defined. Reactive power can also be included. Finally, the mean saturation levels per household (hh) and average usage per day of the appliances is defined.

Based on the above data and statistics, hourly load profiles are created stochastically for each consumer node individually. For each time point the usage (on/off) of the appliances is determined. The turning-on criterion of an appliance is checked with a random number RN (0,1) using the following criterion:

\[
P_{\text{start}}(A,w,\Delta t_{\text{comp}},\sigma_{\text{flat}},h,d) = \begin{cases} 
\text{ON} & \text{if } RN \leq P_{\text{start}} \\
\text{OFF} & \text{else}
\end{cases}
\]

\[
P_{\text{start}}(A,w,\Delta t_{\text{comp}},\sigma_{\text{flat}},h,d) = P_{\text{season}}(A,w) \cdot P_{\text{hour}}(A,h,d) \cdot f(A,d) \cdot P_{\text{step}}(\Delta t_{\text{comp}}) \cdot P_{\text{social}}(\sigma_{\text{flat}})
\]

where

- \(P_{\text{season}}\) = seasonal probability factor describing the season dependent changes;
- \(P_{\text{hour}}\) = hourly probability factor describing the activity levels during the day;
- \(P_{\text{step}}\) = scaling factor, scales the probabilities in relation to the time step used \(\Delta t_{\text{comp}}\);
- \(P_{\text{social}}\) = social random factor, models behavior influenced by weather and social factors;
- \(f\) = mean daily starting frequency, models the mean frequency of use for an appliance [1/day];
- \(A\) = type of appliance or group of appliances;
- \(h\) = hour of the day; \(d\) = day of the week; \(w\) = week of the year;
- \(\Delta t_{\text{comp}}\) = computational time step [here 60 minutes];
- \(\sigma_{\text{flat}}\) = standard deviation for \(P_{\text{social}}\).

The procedure described results in a consumption pattern with a realistic statistical behavior [9].

**Power generation models**

In addition to customer loads, distributed generation technologies can be included into the distribution network modeled by DESIGN. The power generation units form their own nodes in the distribution network for modeling reasons. In present paper only local photovoltaic (PV) generation is described, but the DESIGN includes also other technology options.
For the PV modules, a temperature dependent PV model has been applied. The output from the photovoltaic array $P_{PV}$ is calculated as follows:

$$P_{PV} = A_c \times I \times \eta$$  \hspace{1cm} (4)

where $A_c$ is total array area, $I$ is incident solar insolation, and $\eta$ is system conversion efficiency of the photovoltaic system. The solar radiation on the PV surface is calculated using standard incidence angle formulas [15]. The time step for the calculation is one hour.

The system efficiency $\eta$ is composed mainly of three factors: the solar module efficiency ($\eta_0$) and its temperature dependence and the DC to AC conversion efficiency ($\eta_{inv}$) including losses in cabling. This can be presented for silicon solar cells in the following way:

$$\eta = \eta_0 \times \left[ 1 - 0.0042 \times \left( \frac{I}{18} + T_a - 20 \right) \right] \times \eta_{inv}$$  \hspace{1cm} (5)

where $T_a$ is the ambient temperature. Typically $\eta_{inv}=0.85$ and $\eta_0=0.15$. The temperature dependence of the solar module conversion efficiency is described by the correlation inside the brackets and it may influence the efficiency by tens of percents. PV temperature elevation reduces the solar electricity conversion efficiency typically by 0.4-0.5%/°C.

To account for possible obstructions or shadings at low solar elevation in urban environments, the Equation (4) is subject to a threshold value, or $P_{PV} = 0$ if $I<50$ W/m$^2$.

**INPUT DATA**

The aim of the case studies is to determine the weak and critical points in the middle voltage distribution network when a large amount of photovoltaic (PV) power generation is integrated into the network. A total of 153 case studies was analyzed to high-light these issues. The input data sets comprised the following:

- 3 sets of household load profiles in 2 climates, 2 in Portugal and 1 in Finland;
- 2 radial and 1 looped network set-ups;
- 4 climate specific orientation strategies of the PV panels;
- 4 four different penetration levels of PV power generation.

This yields $3 \times 3 \times 4 \times 4 = 144$ cases to be simulated by DESIGEN. 9 reference case studies without PV panels were also included which yields then a total of 153 simulations.

**Network Set-ups**

The first network geometry studied in the medium voltage distribution network set-ups is a multi-branched tree-type network, which corresponds mainly to an urban distribution network. Due to its symmetrical shape, some of the branches were reduced
to one node in order to improve computational speed. The final network includes 118 nodal points with a total of 9840 hhs, including one high voltage supply double node, shown as node 1 in Figure 3a. About 40% of the network was reduced due to symmetry as shown in Figure 3a. Nodes number 23, 26 and 93 represent the reduced symmetrical network branches through a single node representing each a 33 node branch. Some minor reductions in less significant parts of network have been made, namely nodes 22, 24, 25, 27, 91 and 93, respectively. The numbers followed by a small "k" indicate the number of households in the reduced branches.

The shape of the second network is a comb-type network with two linear and symmetrical branches with a short side branch in-between. The network has a high voltage feeding node in one end of the branches (node number 1). This geometry includes a particularly strong main line and weak side branches. The network includes a total of 236 nodal points and 9360 households. Due to symmetry only 50% of the network needs to be computed. Figure 3b shows the computational network excluding the symmetrical branch.

The third network is shaped as two opposite loops resembling number eight. The loops have short side branches. The looped form allows the main line to be triple the length of the linear network using weaker main line cables. Also, due to the loops the network was made of a simplified structure that allows faster computation. This network includes a total of 113 nodal points and 8960 households, while only 50% of the network needed to be computed. Figure 3c shows the computational network excluding the symmetrical branch.

The networks shown in Figure 3 employ cable types presented in Table 1. The tree-network uses Ibis-type cables between node 1 and nodes 4, 14 and 82. The thicker branches (main cables) are Raven-type while the rest are Sparrow-type. In the loop-network the main loop is Ibis-type while the side branches are Sparrow-type. The Condor-type cable is used only in the comb-network, where it goes from node 1 to 104 while the side branches are Ibis-type and the sub branches Raven-type.

The voltage of the network should preferably stay within 0.975 and 1.025 p.u. when PV is connected. These limits were chosen as a compromise when the rural (5-15% of nominal voltage) and urban (0.5-2%) voltage drop recommendations given by ABB were considered [16]. The unit p.u. stands for Per Unit which corresponds to the voltage relative to the nominal value which is 20 kV in the medium voltage network used here. The described choice of network cables was necessary to achieve a minimum voltage of 0.975 p.u. in each network with all the different load sets. An additional limit to the network was the feeding node that was limited to a voltage 1.0 p.u. in order to accommodate the PV integration. As a result, the comb-type network became very strong, able to handle some 10,000 households. As strongest network it would be least affected by the PV integration shown later in the analysis chapter.

For all the three networks, a set of simulations were performed. As a reference, simulations with no PV power generation were first executed for each network and load set. This totaled nine runs, with three different load sets per network. The amounts of consumers have been selected in such a way, that the annual minimum of the hourly
voltage is between 0.975 and 0.974 p.u. in normalized units which is an often used design limit.

**Household load profiles**

The Finnish household data set applied in this work is based on the data presented in [9] and the Portuguese data sets were created by the same method. The data profiles are based on publicly available statistics and data. The load data represents apartment houses.

The first consumption profile is based on experimental data from Finnish blocs of flats. This data has been analyzed and modeled in reference [9]. In addition, the reactive consumption of households is also modeled here.

The second consumption profile is generated with the same model as above, but using domestic consumption data from Portugal and Lisbon in particular. Three data sources were mainly used there: the total domestic electricity consumption in Lisbon 2003 [17], estimated population data of the same year [18,19], and mean saturation levels for domestic appliances in Portugal [19-21]. The weekly annual load curve was compiled based on unpublished test reference year weather and insolation data from Lisbon, compiled in the same manner as the CEC 1985 data [22]. Public weather data from Lisbon is available at Russian Weather Archive [23]. As the annual changes in Lisbon weather are rather small, slightly convex consumption patterns were generated except cold appliances that follow a hump-like model presented in [19].

The third consumption profile is from most parts similar on the second one. The difference in this model is that 50% of the hhs are assumed to be air-conditioned based on the equipment profiles measured in reference [19].

The main characteristics of the three applied datasets are shown in the Table 2. Daily mean loads over one year are shown in Figure 4. The load profiles differ significantly in their shapes and magnitude as well as in the amount of reactive load. Very important for the PV applications is the shape of annual load curve and clearly the case of Lisbon with air conditioning provides a promising shape with a lot of summer time consumption. The mean hourly consumption is shown later in the Analysis chapter along with some results. In the Helsinki dataset, the weekend and week day load curves are modeled separately.

**PV set-ups**

The PV options considered included 4 different levels of penetration of PV in the network and 4 orientation cases of the panels. The orientation of the PV modules has an important effect on the temporal behavior of the PV output and on its matching with the load. The share of the PV of household electricity was sized at 5, 50, 100 and 200% of households having a 1 kWp PV system. The 200% corresponds to a 2 kWp PV capacity per each household.
The inclination angle of the PV panels on the house roofs was chosen optimally for the two locations: 30° for Lisbon and 45° for Helsinki, respectively. The orientation of the panels was varied to cover different town planning cases as follows:

- all panels pointing to south (azimuth angle $\alpha = 0^\circ$);
- 50% of the panels to south, 25% to east ($\alpha = -90^\circ$) and 25% to west ($\alpha = 90^\circ$);
- 50% of the panels to south and 50% to west;
- 50% of the panels to east and 50% to west.

ANALYSIS
In the next, the results from the simulation runs using the above described input data sets are described in more detail.

Effect of climate and household types
The local climate affects mainly the total PV electricity produced through the available solar radiation. Thus Helsinki representing a northern climate gives typically 50-60% less PV output than in Lisbon on an annual base. The Helsinki weather caused considerable fluctuation in PV output over time whereas the Lisbon was quite stable. This is well demonstrated in Figure 5 for a summer week. The power peaks were of same magnitude in both climates and the maximum hourly PV output was found in early spring due to sunny but cool weather.

The household load profiles also differ quite much. In Helsinki the consumption peak is in the late autumn and a high consumption level continues through all the winter. On the other hand the Lisbon consumption peak occurs at the end of July and beginning of August and most of the summer is high consumption season. This is found both with and without the air conditioning which only emphasizes the weight of summer consumption.

The summer peak found in Lisbon may suggest that PV could match ideally the consumption of electricity. Unfortunately, the main energy consumption takes place both in Lisbon and Helsinki in the evening. There is a 7-8 hour difference between the PV and load peaks and the PV production in Lisbon is almost at zero when the evening peak begins. In Helsinki the situation is slightly better as the evening peak is smoother and wider. The mean PV production in summer in Helsinki therefore cuts slightly the evening peak, but not the main peak to influence the low voltage levels during peak load hours. Mean summer and winter time PV production and consumption curves for both locations are shown in Figure 6.

Effect of PV panel orientation
The 4 PV azimuth angle cases provided a good view how orientation could affect the load and PV matching. Figure 7 illustrates the summer time average output for the different cases. The “East-West”-case has the most flat mean power, but the total energy production is reduced. On the other hand, the “All South”-case gives the highest total
energy output as expected, but the shape of the PV curve is steepest of all cases. The “South-West” and “South-East-West” cases lie in between the two previous curves.

Figure 8 illustrates the system dynamics on an hourly scale for different orientation cases in Helsinki and Lisbon during a clear summer day. The power generation peak during the PV operation is smoother when the PV panels are orientated to east and west than to the south only, or 10% smoother in Lisbon and 30% in Helsinki, respectively. Accordingly, network over-voltage is reduced 25% in Lisbon and 46% in Helsinki. The total solar yields from the panels would not differ much between the two cases for the day shown but on an annual bases the East-West case would give a 16% lower total yield in Lisbon and 23% in Helsinki, respectively.

**Effect of shading**

The question of possible shading of PV modules deserves attention as the number of households in the urban setting considered here is high, or some 10,000 households. Previous studies indicate that shading losses could reduce average PV output by 4-7% [25, 26] but the instantaneous drop on a module level could be even much higher [27]. In practice it is almost impossible to determine accurately the influence of shading on the PV output as this would necessitate knowing the exact topology of the buildings and the city plan. As the shading effects are worst during low zenith angles, a virtual horizon was used, i.e. the direct solar radiation under 10 degrees solar zenith was cut off, which would lead to a small decrease in the yearly PV yield. Around noon time when the solar insolation has its maximum and the PV electricity feed into the distribution network is at highest, the effects of the shading will be at minimum due to the high zenith angle. Assuming that most of the PV panels would be on the roofs or on the top floor façades, the effects of the shading on the load and PV matching may therefore remain small.

**Effect of PV penetration levels**

The different PV penetration levels cause nonlinear effect in the network. The lowest penetration level, 5% of all the nearly 10,000 households having the 1 kW$_p$ PV system (=50W$_p$/hh) caused in all cases positive effects to the network, i.e. smaller network losses. All the generated energy was consumed at local level and the grid only experienced reduced consumption level at daytimes. The second penetration level 50% (=0.5kW$_p$/hh) caused minor current flow to the network making modest voltage rise at some of the network tail sections, but in none of the cases more than 1.007 p.u which is well within the limits of fluctuations accepted. In the comb-type network no over-voltages (>1.0 p.u.) were observed.

The third case, or 100% (1kW$_p$/hh), caused noticeable voltage rises in all the network types. With south-oriented PV panels in tree-type network (Figure 3a) the voltage went close to the 1.025 p.u. voltage limit, while the comb-type network experienced voltages around 1.01 p.u. When the fourth 200% or 2kW$_p$/hh PV penetration level was applied, most of the cases topped the 1.025 p.u. voltage limit except for the comb-type network which remained under 1.020 p.u. The “East-West”-orientating case with the looped network also stayed within the voltage limits. When the PV power generation is
compared with the induced network losses, the south orientated PV panels with 1kWp PV per household is closest to the optimum.

The key results from the case studies have been collected to Table 3 but excluding the lowest PV cases (not real effects) and the cases (mainly 2 kWp/hh) exceeding the voltage limit 1.025 p.u. For comparison, the maximum voltage levels for 2 kWp/hh representing the worst case are also shown in the last column. In practice, only the comb-type network could absorb the highest PV penetration rate, in other cases half of this could be used when staying within the voltage limits. The 1kWp/hh PV use caused negligible voltage rise in the comb case. The highest voltage rise was experienced with tree-type networks.

Feeding PV into the electric network affects not only the voltage but also the losses in the network. The network losses without PV would be 0.65-0.83% of the total load in Helsinki and 0.67-0.94% in Lisbon, respectively. Adding PV up to 1 kWp/hh typically reduces the electric losses, but a 2 kWp/hh level increases the losses in all cases. The losses would be then from 1.5 to 1.9 % in Helsinki and 1.6 to 2.2% in Lisbon, respectively, the highest being with a tree-type network. With the 0.5 kWp/hh level, the losses typically decrease by 20%.

The “East-West”-orientated PV case with a looped network was also included in Table 3. This case demonstrates that when the 2.5% voltage rise limit was slightly exceeded when the panels were directed to south, the redirection to East-West would reduce the voltage rise and keep it within the limits. This is caused by lower energy production and slightly different hourly distribution, which reduces the mid-day voltage peaks.

**Other observations**

Transformers connected to medium- and low-voltage distribution networks may also have a significant influence to the ability of the network to accommodate distributed generation. If the transformer cannot dynamically adapt to voltage changes caused by the distributed generation, which is a typical situation with conventional transformers, only a moderate level of variation can be allowed to the distributed PV generation. On the other hand, if a dynamic response in the range of ±2.5% voltage fluctuation is allowed, larger penetration levels are possible, or in our case 100% PV of total load.

It also appears that a single linear network that is strong enough to supply a large amount of households is also good for PV-integration. A loop-type network proved to be slightly weaker than the linear comb-network. An individual loop could be extended to a longer distance than an open-ended comb-type network with a significantly stronger cable type. The comb-type network overdid by far the other networks, when it comes to the PV integration, due to the strong main line needed in this type of a network and topological effects. Our observations indicate that the larger the share of the customers that are far away from the supplying transformer (e.g. large tree-type network), the poorer is the network’s capability to accommodate PV generation capacity.

In Table 3, the maximum voltage levels in the whole network were shown. In practice the voltage along the line varies, the variation increasing when moving away from the
transformer (node 1). In Figure 9, an example of the voltage variation is shown for Lisbon during 12-13 hours on a bright spring day in a tree-type network for all PV penetration levels. Both the adverse and positive effects of PV are clearly demonstrated.

CONCLUSIONS
The simulation results demonstrated, that high penetration levels of photovoltaic (PV) power generation may cause voltage problems in the electrical network but also depend on the network type. The comb-type network showed the best performance. On the other hand all network types studied can handle PV without significant problems both in a northern and southern climate with an amount of PV equaling up to the load (here 1kW_p/hh, case 100%).

The results also demonstrate that the mismatch between PV production and domestic electricity consumption cannot be fully compensated through orientation of the PV panels. Thus the PV is unable to shave the domestic load peak appearing in our cases in the early evening hours. An east-west orientation of the PV panels reduces, however, the PV peak in the noon by 30 % in Helsinki and 10% in Lisbon, respectively.

PV integration into the network influences both the voltage and network losses positively at least up to a 1kW_p/hh (100% of annual domestic load) when using the voltage design limits. If no increase in voltage from the non-PV case is allowed, the 0.5 kW_p/hh (50%) would be the limit for PV. When going for 2 kW_p/hh (200% of annual load) all but the comb-type networks demonstrate clear over voltage situations. Also, the annual network losses are more than doubled in this case compared to a neutral network without PV.

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Nomenclature

- $A$: type of appliance or group of appliances;
- $A_c$: total photovoltaic array area;
- $d$: day of the week;
- $f$: mean daily starting frequency, models the mean frequency of use for an appliance;
- $h$: hour of the day;
- $hh$: household;
- $H$: Jacobian matrix where $H_{ki} = \frac{\partial P_k}{\partial \delta_i}$;
$i$ node index, used as subscript;
$I$ incident solar insolation;
$j$ vector of current values;
$j^\ast$ complex unit;
$J$ current in a line or node;
$k$ node index, used as subscript;
$kW_p$ power of photovoltaic unit by peak performance rating.
$L$ Jacobian matrix where $L_{ij} = \frac{\partial Q_i}{\partial U_j}$;
$M$ Jacobian matrix where $M_{ki} = \frac{\partial Q_i}{\partial \delta_j}$;
$n$ number of network nodes;
$N$ Jacobian matrix where $N_{ki} = \frac{\partial P_k}{\partial U_i}$;
$p_{\text{season}}$ seasonal probability factor describing the season dependent changes;
$p_{\text{hour}}$ hourly probability factor describing the activity levels during the day;
$p_{\text{step}}$ scaling factor, scales the probabilities in relation to the time step used $\Delta t_{\text{comp}}$;
$p_{\text{social}}$ social random factor, models behavior influenced by weather and social factors;
$P$ power through a node;
$P_{\text{PV}}$ power a photovoltaic panel / system generates;
$Q$ reactive power through a node;
$RN$ random number;
$S$ power loss on a network line;
$t$ time index, used as subscript or superscript;
$t_{\text{comp}}$ computational time;
$u$ vector of voltage values;
$U$ voltage at a node;
$w$ week of the year;
$Y$ conductance of a line;
$Y_0$ self inductance of a network node;
$Y'$ matrix element of conductance matrix $Y$. If $k \neq i$, then $Y'_{ki} = -Y_{ki}$, else

$$Y'_{kk} = Y_{k0} + \sum_{k=1,k \neq i}^{n} Y_{ki}$$

$Y$ matrix of inductance values;

$Z$ impedance of a line;

Greek letters

$\alpha$ solar panel azimuth angle

$\delta$ voltage phase angle at a node;

$\Delta$ a step in a variable (difference);

$\varepsilon$ convergence limit of an iteration;

$\eta$ system conversion efficiency of the photovoltaic system;

$\eta_0$ solar module efficiency;

$\eta_{inv}$ photovoltaic system DC to AC conversion efficiency including losses in cabling;

$\sigma_{flat}$ standard deviation for $P_{social}$;
Appendix A

Calculation procedure for a radial network

Step 1. Initial values are given to voltages of network nodes \( U_k^0 \) (typically nominal voltage).

Step 2. Network node currents from loads and generation are computed with \( I_k^i = P_k^i / U_k^i \) and then summed following Kirchoff’s first law [24].

Step 3. Node voltages are computed with \( U_k^{i+1} = U_k^{i+1} - Z_k J_k^{i} \). Here \( U_k^{i} \) is the node from where node \( U_k^{i} \) gets its current.

Step 4. If \( |U_k^{i+1} - U_k^{i}| > \varepsilon \), where \( \varepsilon \) is the predefined tolerance, the iteration has not converged. Then \( t = t + 1 \) and go to step 2.

Step 5. When the iteration has converged, the values \( U_k^{i} \) and \( J_k^{i} \) are the result.

\( t \) refers to the computation round and \( k \) to the network node number. \( P_k^i \) refers to production or consumption in node \( k \). In step 3 the nodes that have positive total production that give current to other nodes have to be computed first.
Appendix B

Calculation procedure for a looped network

Step 1. When omitting the inductance between network nodes and earth, the current flow for each network node $k$ can be written by Kirchoff's law as

$$ J_k = Y_{ki}U_k + \sum_{j=i+1}^{n} Y_{kj}(U_k - U_j), \quad (B1) $$

where $y_{kj}$ is the admittance for line between nodes $k$ and $I$ and $y_{i0}$ is the admittance between node $k$ and earth. Equation (B1) can be expressed in matrix form as $j = Yu$ and reopened as

$$ J_k = \sum_{i=1}^{n} Y_{ki}U_i. \quad (B2) $$

Step 2. The power flow into any node $k$ is computed as follows

$$ P_k - jQ_k = U_k^* J_k = U_k^* \sum_{i=1}^{n} Y_{ki}^* U_i. \quad (B3) $$

Using a polar form for voltage ($U_k = U_k e^{j\delta_k}$, $Y_{ki}^* = Y_{ki} e^{-j\delta_k}$) equation (B2) can be divided into two separate problems:

$$ P_k = \sum_{i=1}^{n} U_k Y_{ki}^* U_i \cos(\theta_{ki} - \delta_i + \delta_k) \quad (B4) $$

$$ Q_k = \sum_{i=1}^{n} U_k Y_{ki}^* U_i \sin(\theta_{ki} - \delta_i + \delta_k) \quad (B5) $$

$k = 1...n$

Step 3. The Newton-Raphson method is employed to solve voltage values $U_k$ and phase angles $\delta_k$. The equation to be solved is of the following form (see also Equation (1)):

$$ \begin{bmatrix} \Delta P_1 \\ \vdots \\ \Delta P_{n-1} \\ \Delta Q_1 \\ \vdots \\ \Delta Q_{n-1} \end{bmatrix} = \begin{bmatrix} H & N \\ \vdots & \vdots \\ M & L \end{bmatrix} \begin{bmatrix} \Delta \delta_1 \\ \vdots \\ \Delta \delta_{n-1} \\ \Delta U_1 \\ \vdots \\ \Delta U_{n-1} \end{bmatrix}, \quad (B6) $$

where the Jacobean matrices $H$, $N$, $M$, and $L$ are

$$ H_{ki} = \frac{\partial P_k}{\partial \delta_i}, N_{ki} = \frac{\partial P_k}{\partial U_i}, M_{ki} = \frac{\partial Q_k}{\partial \delta_i}, L_{ki} = \frac{\partial Q_k}{\partial U_i}. \quad (B7) $$
The equation (B5) is solved iteratively until an adequate precision is achieved. For the next simulation time step, the computation is started from Step 1.
References


[17] DGGE-DE. Table "Consumption of electric energy per activity sector". Portugal: Direcção Geral de Geologia e Energia, Divisão de Estatística; 2004. (in Portuguese)


Figures captions

Figure 1. Structure of DESIGEN simulation tool. Left: flow diagram for the computational structure. Right: main elements of the program.

Figure 2. Schematic illustration of a distributed energy system with network, load and generation units.

Figure 3. Network topologies used in reduced form. Numbers in figures correspond to the nodes.

a) tree-type network, where some of the symmetrical or unimportant network branches have been reduced to single node. Letter ‘k’ next to the node indicates the number of end nodes in the reduced branch.

b) loop-type network. 2 such branches are used in the simulation. Node 1 has been showed in double for the sake of clarity.

c) comb-type network. 2 such branches are used in the simulation.

Figure 4. Daily energy demand levels for the different household types on an annual level. ‘Lisbon +AC’ refers to the Lisbon load with air conditioning.

Figure 5. Illustration of hourly summer-time load and PV power levels for a) Lisbon and b) Helsinki. The PV penetration level is 1kWp per household, azimuth angle is 0°.

Figure 6. Summer and winter mean consumption and PV generation. a) Lisbon summer, b) Lisbon winter, c) Helsinki summer, d) Helsinki winter. The PV penetration level is 1kWp per household, azimuth angle is 0°.

Figure 7. Summer time mean consumption and PV generation. 50% of the PV panels are orientated to the west and 50% to the east. a) Lisbon summer, b) Helsinki summer. Dotted line corresponds to all panels orientated to the south. The PV penetration level is 1kW per household.

Figure 8. Example of hourly dynamics of the distributed energy system with PV on a summer day.

Figure 9. Example of voltage fluctuation along the electric network for a single hour (noon) in Lisbon with different PV penetration levels.
Table captions

Table 1. Cable types used in the electrical networks.
Table 2. Key figures of the household data sets.
Table 3. Summary results from the case studies.
Figure 1.

Start

- Initializing data structure
  - Input data
  - Defining loads & power generation
    - Influence of weather
    - Initialization
  - Iteration of power flow
    - Iterative process
  - Data output & New time step
    - Results & NW losses

End

Network
- Electricity Load
- Distributed Generation
Figure 2.
Figure 4.
Figure 5.
Figure 6.
Figure 7.
Figure 8.
Figure 9.
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* as percentage from annual consumption
Author’s Corrigendum to the final version of the publication:


DOI: 10.1016/j.renene.2006.01.005

Date of Corrigendum: 29.9.2009

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