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New Time Variant Control System Design for Networked Control Systems*

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1 Introduction

A recent trend in industrial automation and control systems is to use different kind of asynchronous networks for data transfer. A typical networked control system is described in Fig. 1. The main problem in their design is caused by the timevarying and often unknown network delays between the controlled system (process) and the controller [3]. Even in the case where the delays can be measured the time variance makes the control design difficult. For instance, there is no rigorous frequency domain methodology suitable for time-varying systems, but the only possibility is to use time domain models and methods. The second difficulty is the non-commutativity of subsystems in serial interconnections which makes the consideration of interconnections much more complicated than in the time-invariant case.

The consideration of delayed systems in continuous time with delay-differential models is complicated and mathematically difficult. In particular, this holds for time-varying systems. Instead, in discrete time the models can be presented, at least as a good approximation, with time-varying difference equations. This results in a simpler methodology, even though the lack of continuity of signals and parameters causes some difficulties. In spite of these, the discrete-time models are used in what follows. Both state space and input-output methods are, in principle, applicable but their combination, so-called polynomial systems theory, seems to offer theoretically sound and practically realizable tools for analysis and design of time-varying systems.

In this paper the design methodology based on time-varying difference systems is presented and applied to the observer and controller design. First some basic concepts and definitions of time-varying difference systems and their interconnections as presented in [11], [5] are given. The mathematical descriptions are based on linear equations over skew polynomials in a unit delay or prediction operator. Then the pole placement designs of observers and feedback controllers are considered [1], [5], [6]. The methodology is applied to the networked control system in Fig. 1. First, the overall model of the controlled system with input and output delays is constructed. Then the observers for prediction of the process output are considered and a Smith predictor type predictor is taken as an example. Finally, the feedbacks of the predicted output are parameterized and a numerical example is presented.

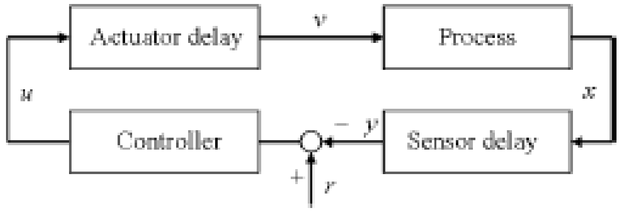

Fig. 1. Control system with network delays

1.1 Description of systems

Time-varying linear discrete time input-output systems are usually described by difference equations of the form

$$\sum_{i=0}^{n} a_i(k)y(k-i) = \sum_{i=0}^{m} b_i(k)u(k-i) \tag{1}$$

where $k \in \boldsymbol{Z} \triangleq \textit{time set}$, $u, y \in X \triangleq \textit{signal space} \subset \boldsymbol{C}^{\boldsymbol{Z}}$ and $a_i, b_i \in K \triangleq \textit{coefficient space} \subset \boldsymbol{C}^{\boldsymbol{Z}}$. **C**, **Z** above denote the complex numbers and the integers, respectively, and $\boldsymbol{C}^{\boldsymbol{Z}}$ is the set of infinite bisequences over **C**. Provided that the signal space is closed with respect to the *(unit) delay operator r*

$$(rx)(k) = x(k-1) \tag{2}$$

and to the pointwise multiplication by coefficients the equation (1) can be written as an operator equation

$$\underbrace{\sum a_i r^i}_{a(r)} y = \underbrace{\sum b_i r^i}_{b(r)} u \tag{3}$$

Alternatively, the *(unit) prediction operator*

$$(qx)(k) = x(k+1) \tag{4}$$

can be used leading to the model

$$\tilde{a}(q)y = \tilde{b}(q)u \tag{5}$$

Note that in the case $\boldsymbol{X} = \boldsymbol{C}^{\boldsymbol{Z}}$ the operators r and q are invertible and $q^{-1} = r$. The sum of operators $a(r) = \sum a_i r^i$ and $b(r) = \sum b_i r^i$ can be written as

$$\sum a_i r^i + \sum b_i r^i = \sum (a_i + b_i) r^i \tag{6}$$

The product (= the composition of operators) is more complicated. It must satisfy

$$\begin{aligned} \big((rb)x\big)(k) = \big(r(bx)\big)(k) = (bx)(k-1) = b(k-1)x(k-1) = \\ (r_K b)(k)(rx)(k) = ((r_K b)(rx))(k) \end{aligned} \tag{7}$$

where $b \in K$, $r_K \triangleq$ is the unit delay operator on K, i.e.

$$rb = r_K(b) \tag{8}$$

Thus the product $a(r)b(r) = c(r)$ can be constructed using the property 8 repeatedly.

Thus under the assumptions that the signal space and the coefficient space are closed with respect to the operations above, the set of operators *a(r)* constitute the (noncommutative) ring $K[r;\ r_K,\ 0_K]$ of *skew polynomials* (or *skew polynomial forms*) [2]. Here $0_K \triangleq$ the zero operator on K. Similarly, the use of the (unit) prediction operator gives the skew polynomial ring $K[q;\ q_K,\ 0_K]$.

Most of the concepts and properties of ordinary polynomials can be applied to skew polynomials. Let X be 'sufficiently rich' to make the powers $r^0, r^1, r^2, \ldots$ *linearly independent* over K. Then the representation of a skew polynomial *a(r)* is unique and its *degree* $deg\ a(r)$ is well-defined. The choice $X = C^Z$ obviously guarantees this. The same choice for coefficients *K*, unfortunately, leads to such kind of weak algebraic structures which do not offer any methodology and tools for consideration of system models above.

For instance, the *division algorithms*, e.g. the (*right*) *division algorithm (RDA)*

$$a(r) = b(r)c(r) + d(r), \deg d(r) < \deg b(r) \quad (9)$$

are satisfied uniquely for *all* $a(r), b(r) \neq 0$ if and only if the coefficient ring *K* is a field. This is important because the division algorithms are needed for manipulation of skew polynomial matrices used in descriptions of multivariable systems. Usual coefficient rings are not fields, but often they can be *extended* to their *fields of fractions* and the signals to corresponding rational signals [11)].

Note that a skew polynomial can be invertible as a skew polynomial only if it is of degree zero but there can exist skew polynomials of higher degree which are invertible as mappings (e.g. *r* and *q*).

A matrix with skew polynomial entries, i.e. a skew polynomial matrix is *unimodular* if it is invertible as a skew polynomial matrix. Note that for skew polynomial matrices there is no determinant which could be used for testing the unimodularity. Furthermore, a skew polynomial matrix can be invertible as a mapping even though it is not unimodular.

Two skew polynomial matrices $A(r), B(r)$ are *row (column) equivalent* if there is a unimodular matrix $P(r)$ such that $A(r) = P(r)B(r)$ ($A(r) = B(r)P(r)$). Skew polynomial matrices can be brought to *row or column* equivalent forms e.g. to an upper triangular form using the *elementary operations*. These are: (i) add a row (column) multiplied from the left (right) by a skew polynomial to another row (column), (ii) interchange of two rows (columns), (iii) multiply a row (column) from the left (right) by an invertible skew polynomial [11)].

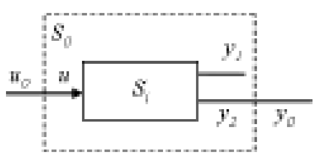

Fig. 2. General composition

1.2 Systems and compositions

A set of linear, time-varying difference equations can be written as matrix equations

$$A(r)y = B(r)u \Leftrightarrow [A(r) \vdots -B(r)] \begin{bmatrix} y \\ u \end{bmatrix} = 0 \quad (10)$$

where $u \in X^r, y \in X^s$ and $A(r), B(r)$ are skew polynomial matrices. Then the multivariable *input-output (IO) relation* generated by (10) is defined as the set

$$S = \{(u, y) | A(r)y = b(r)u\} \quad (11)$$

The matrix $[A(r) \vdots -B(r)]$ is called a *generator* for *S*. Generators for the same input-output relation are *input-output (IO-) equivalent*. Obviously, two row equivalent generators are IO-equivalent, but also multiplication by a matrix invertible as a mapping gives an IO-equivalent generator.

A *composition* of input-output relations consists of a set of input-output relations ('subsystems') or their generators, and a description of the interconnections between the (signals of the) subsystems. Every composition can be brought to the general form of Fig. 2, where S_i is the *internal IO-relation* and S_o the *overall IO-relation* generated by the composition. Conversely, the composition is then said to be a *decomposition* of S_o. Decompositions of the same IO-relation are *input-output (IO-) equivalent*. For the internal IO-relation S_i it is always possible to construct a generator from the generators of the subsystems and the interconnections

$$\begin{bmatrix} A_1(r) & A_2(r) & \vdots & -B_1(r) \\ A_3(r) & A_4(r) & \vdots & -B_2(r) \end{bmatrix} \tag{12}$$

Instead, for the overall IO-relation

$$S_o = \{(u_o, y_o) | \exists y_1 [(u_o, (y_1, y_o)) \in S_i]\} \tag{13}$$

the construction of a generator is a more complicated task.

2 Analysis of systems and compositions

2.1 Systems

Realizability of a system model requires that the output of the model can be solved uniquely whenever the input and a sufficient, finite number of initial values of output are given. Then the model is said to *regular*. If only past and present values of the input are needed for solving the output, then the model is *nonanticipative* or *causal*.

For instance a single-input-single-output (SISO) system (5) is causal, if the degree of $\tilde{a}(q)$ is not lower than the degree of $\tilde{b}(q)$ (i.e. the system as well as its generator are *proper*) and the leading coefficient (the coefficient of the highest power of q) is invertible. An IO-relation S generated by$[A(r) \vdots -B(r)]$ is said to be *stable* if every solution y to $A(r)y = 0$ approaches 0 when the time t approaches the infinity.

It should be noted that in general the stability cannot be tested from the '*pointwise*' roots of $\det A(k)(r)$, where $A(k)(r)$ denotes the ordinary polynomial matrix obtained from $A(r)$ by replacing the coefficients by their values at time k. Let S be generated by

$$[A(r) \vdots -B(r)] = L(r)[A_1(r) \vdots -B_1(r)] \tag{14}$$

Now, if $L(r)$ is not invertible as a mapping, S contains modes related to $L(r)$ which cannot be affected by the input u. This means that S is not *controllable* [5].

2.2 Compositions

Consider the composition of Fig. 2 and suppose that the composition is regular, i.e. the internal IO-relation is regular. The generator (12) can be brought to upper triangular form

$$\begin{bmatrix} \tilde{A}_1(r) & \tilde{A}_2(r) & \vdots & -\tilde{B}_1(r) \\ 0 & \tilde{A}_4(r) & \vdots & -\tilde{B}_2(r) \end{bmatrix} \tag{15}$$

Now if for each (u_o, y_o) satisfying the equation

$$\tilde{A}_4(r)y_o = -\tilde{B}_2(r)u_o \tag{16}$$

there exists a y_1 such that $(u_o, (y_1, y_o))$ satisfies the equation

$$\tilde{A}_1(r)y_1 = -\tilde{A}_2(r)y_o + \tilde{B}_1(r)u_o \tag{17}$$

then the overall IO-relation S_o is generated by the equation (16) or by the generator $[\tilde{A}_4(r) \vdots -\tilde{B}_2(r)]$. If $\tilde{A}_1(r)$ is invertible as a mapping, then the y_1 satisfying (17) must be unique. In this case the composition is *observable*. If the system is causal, then it is always possible to take $\tilde{A}_1(r) = I$. Consider again the composition of Fig.2. If the generator of S_i can be brought to the form

$$\begin{bmatrix} \hat{A}_1(r) & 0 & \vdots & -\hat{B}_1(r) \\ \hat{A}_3(r) & I & \vdots & -\hat{B}_2(r) \end{bmatrix} \tag{18}$$

the composition is called a *generalized state space decomposition* of S_o, and y_1 is the corresponding *generalized state*.

3 System with Time-varying Delays

3.1 Time-varying delays

Return to the system in Fig.1. Provided that the time-varying input delay is a multiple of the sampling interval, it can be presented as

$$v(k) = u(k - \theta(k)) = (r^{\theta(.)}u)(k) \tag{19}$$

This is not a skew polynomial representation but it can be written as a skew polynomial equation

$$v = (d_0 + d_1 r + \cdots + d_n r^n)u = d(r)u \tag{20}$$

where the coefficients are zero otherwise but

$$d_{\theta(k)}(k) = 1 \tag{21}$$

Another way to describe the time-varying delay is to use prediction

$$v(k + \vartheta(k)) = (q^{\vartheta(.)}v)(k) = u(k) \tag{22}$$

which can be written as a skew polynomial equation

$$\tilde{c}_0 + \tilde{c}_1 q + \cdots + \tilde{c}_n q^n)v = \tilde{c}(q)v = u \tag{23}$$

If the delay θ and the prediction interval ϑ are related to each other by

$$\vartheta(k) = \theta(k + \vartheta(k)) \tag{24}$$

then

$$q^{\vartheta(.)}r^{\theta(.)}u = u \tag{25}$$

for all $u \in X$. On the other hand, if

$$\theta(k) = \vartheta(k - \theta(k)) \tag{26}$$

then

$$r^{\theta(.)}q^{\vartheta(.)}v = v \tag{27}$$

but only if v belongs to the range of $r^{\theta(.)}$. Thus $r^{\theta(.)}$and $q^{\theta(.)}$as well as the corresponding skew polynomials are invertible mappings only if this range is the whole X which means that each value $u(k - \theta(k))$ appears only once in the values $v(k)$.

Similarly to the input delay above, the output delay can be written in two ways

$$y = (f_0 + f_1 r + \cdots + f_n r^n)x = f(r)x \tag{28}$$
$$(\tilde{e}_0 + \tilde{e}_1 q + \cdots + \tilde{e}_n q^n)y = \tilde{e}(q)y = x \tag{29}$$

3.2 Overall system

Now, if the process is described by $a(r)x = b(r)v$, the actuator delay by $v = d(r)u$, the sensor delay by $y = f(r)x$, the controlled system is a series composition of $S_p = \{(v,x)|a(r)x = b(r)v\}$, $S_a = \{(u,v)|v = d(r)u\}$, and $S_s = \{(x,y)|y = f(r)x\}$ with internal model generated by

$$\left[\begin{array}{ccc:c} 1 & 0 & 0 & -d(r) \\ -b(r) & a(r) & 0 & 0 \\ 0 & -f(r) & 1 & 0 \end{array}\right] \tag{30}$$

with variables (v, x, y, u) (see Fig.3). Using elementary row operations this can be brought to the form

$$\left[\begin{array}{ccc:c} 1 & 0 & 0 & -d(r) \\ 0 & a(r) & 0 & -b(r)d(r) \\ 0 & -f(r) & 1 & 0 \end{array}\right] \tag{31}$$

so that v can be uniquely eliminated. Then the remaining model is written using the operator q by multiplying by the highest order of $d(r)$

$$\left[\begin{array}{cc:c} \tilde{a}(q) & 0 & -\tilde{b}_d(q) \\ -1 & \tilde{e}(q) & 0 \end{array}\right] \tag{32}$$

Using again the elementary row operations gives

$$\begin{bmatrix} 1 & -\tilde{e}(q) & \vdots & 0 \\ 0 & \tilde{a}(q)\tilde{e}(q) & \vdots & -\tilde{b}_d(q) \end{bmatrix} \tag{33}$$

Thus x can be uniquely eliminated and the overall system S_0 is generated by

$$\begin{bmatrix} \tilde{a}(q)\tilde{e}(q) & \vdots & -\tilde{b}_d(q) \end{bmatrix} \tag{34}$$

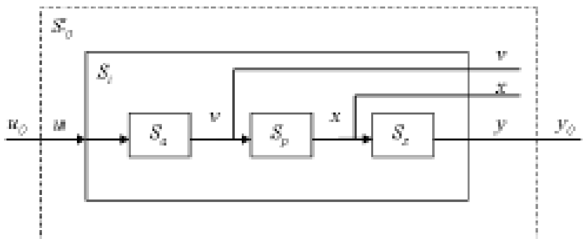

Fig. 3. Series composition of IO-relations

4 Observer Design

4.1 General observer

Consider the composition of Fig. 2 and suppose that only the overall input $u_0 = u$ and output $y_0 = y_2$ are measured. The problem is to design a dynamic system, a so-called *observer* for continuous estimation of the internal output y_1, so that the estimation error $\tilde{y}_1 = y_1 - \hat{y}_1$ behaves in a satisfactory way.

Let the internal IO-relation S_i be generated by the generator (15) of the upper triangular form and the observer $\hat{S}$ to be designed by the generator $[C(r) \vdots -D1(r) \ -D2(r)]$. In what follows, ($r$) (or ($q$)) is in some places omitted in order to shorten the notations. If the observer is chosen to satisfy

$$\begin{bmatrix} C & -D_1 & -D_2 \\ 0 & \widetilde{A}_4 & -\widetilde{B}_2 \end{bmatrix} = \begin{bmatrix} T_1 & T_2 \\ 0 & I \end{bmatrix} \begin{bmatrix} \widetilde{A}_1 & \widetilde{A}_2 & -\widetilde{B}_1 \\ 0 & \widetilde{A}_4 & -\widetilde{B}_2 \end{bmatrix} \tag{35}$$

for some T_1, T_2 then the error is generated by

$$C\tilde{y} = T_1\tilde{A}_1\tilde{y}_1 = 0 \tag{36}$$

[5, 6]. Thus the design problem has been changed to the construction of the matrices T_1, T_2. The matrix T_1 affects the stability of the estimation error and after T1 of order high enough has been chosen the matrix T_2 is used to achieve a causal (proper, if q is used) observer. Both matrices can be constructed sequentially using the elementary row operations.

4.2 Smith predictor

The well-known Smith predictor is often used for compensating delays in control loops [10] [4]. The idea is to predict the non-delayed output using the process model and to correct the prediction by the difference of measured and simulated outputs. Then the controller can be tuned using the non-delayed model. It is easy to show that the dynamics of the the final prediction error is the same as the process dynamics. In the time-invariant case the design is simple but for time-varying systems more complicated.

Consider the delayed system (30) and construct an observer for x using the input u and output y.

The design is started from (33). Multiplication of (33) by

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{a}(q) & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \tilde{a}(q) & 1 \\ 0 & 1 \end{bmatrix} \qquad (37)$$

results in the *open observer*

$$\begin{bmatrix} \tilde{a}(q) & 0 & \vdots & -\tilde{b}_d(q) \\ 0 & \tilde{a}(q)\tilde{e}(q) & \vdots & -\tilde{b}_d(q) \end{bmatrix} \qquad (38)$$

Furthermore, multiplication of (33) by

$$\begin{bmatrix} \tilde{e}(q)\tilde{a}(q) & \tilde{e}(q) - 1 \\ 0 & 1 \end{bmatrix} \qquad (39)$$

gives the Smith predictor

$$\begin{bmatrix} \tilde{e}(q)\tilde{a}(q) & -\tilde{a}(q)\tilde{e}(q) & \vdots & -(\tilde{e}(q) - 1)\tilde{b}_d(q) \\ 0 & \tilde{a}(q)\tilde{e}(q) & \vdots & -\tilde{b}_d(q) \end{bmatrix} \qquad (40)$$

5 Feedback compensator design

5.1 General feedback

Consider the feedback composition in Fig.4 consisting of an IO-relation S to be compensated and a feedback compensator S_2 to be designed so that the resulting composition is stable, robust, realizable etc. Let S be controllable and generated by $[A \vdots -B]$ and the feedback IO-relation S_2 be generated by $[C \vdots -D]$. Then the feedback composition is generated by

$$\begin{bmatrix} A & -B \\ -D & C \end{bmatrix} = \begin{bmatrix} I & 0 \\ T_3 & T_4 \end{bmatrix} \underbrace{\begin{bmatrix} A & -B \\ Q_3 & Q_4 \end{bmatrix}}_{Q} \qquad (41)$$

where Q is invertible and can be constructed by elementary column operations and T_3, T_4 are appropriate matrices [6, 5]. The dynamic behaviour of the system depends on T_4. Thus the feedback compensator can be designed starting from a suitable T_4 and constructing then T_3 so that the resulting feedback compensator is causal and the whole composition is robust against the parameter variations. The construction can be carried out step by step using elementary row operations.

The generalized state representations (18) can be controlled by state feedback. Instead of the state y_1 the corresponding estimate $\hat{y}_1$ determined by the observer (35) can be used for feedback control.

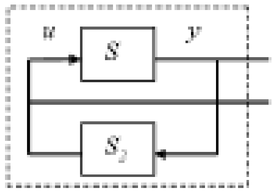

Fig. 4. General feedback control composition

5.2 Control of system with time-varying delays

Consider again the system of Fig.1 and suppose that the non-delayed output x is available for feedback. Hence the design can be started from (31) or from the generator $[a(r) \vdots -b(r)d(r)]$. Suppose further that the system is controllable and the time-varying delay such that $d(r)$ is an invertible mapping. Let the feedback controller be generated by $[l(r) \vdots -m(r)]$. Then the feedback-composition is generated by

$$\begin{bmatrix} a & -bd \\ -m & l \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ t_3 & t_4 \end{bmatrix} \begin{bmatrix} a & -bd \\ q_3 & q_4 d \end{bmatrix} \tag{42}$$

Thus all possible feedback controllers can be generated by

$$[l \vdots -m] = [-t_3 bd + t_4 q_4 d \vdots t_3 a + t_4 q_3] \tag{43}$$

varying the parameters $t_3(r)$ and $t_4(r)$.

Example. Consider a second order system described by (3) with

$$a(r) = 1 - 1.95r + 0.9493r^2 \tag{44}$$
$$b(r) = 0.983r + 0.966r^2$$

and with time-varying input delay $\theta_i = k_d$ and output prediction $\vartheta = k_c$ presented in Fig. 5. The system is controlled by a PI controller

$$(1 - r)u = (0.055 - 0.0098r)(y_{ref} - y) \tag{45}$$

where the controller parameters were obtained by minimizing the integral of the square errors (ISE) for the system with the network delay in an actuator and sensor paths of 1 ms.

Using (40) and again noting that $f(r) = r^{\theta(.)}$, $\tilde{e}(q) = q^{v(.)}$, $\tilde{e}(q) = (q^2 - 1.95q + 0.9493)$, $\tilde{b}_d(q) = \tilde{b}(q) = 0.983q^2 + 0.966q$ gives the following expression for Smith predictor:

$$(q^2 - 1.95q + 0.9493)\hat{x} = r^{\theta(.)}(q^2 - 1.95q + 0.9493)q^{v(.)}y - r^{\theta(.)}(q^{v(.)} - 1)$$
$$(0.983q^2 + 0.966q)u \tag{46}$$

and a Smith predictor expression used in simulations to compensate the output delay is:

$$\hat{x}(k) = 1.95\hat{x}(k-1) - 0.9493\hat{x}(k-2) + y(k-\theta(k-2)+ \nu(k-\theta(k-2))) - 1.95y(k-1-\theta(k-2)+\nu(k-1- \theta(k-2))) + 0.9493y(k-2-\theta(k-2)+\nu(k-2-\theta(k-2)))- (0.983u(k-2-\theta(k-2)+\nu(k-2-\theta(k-2))+2)0.966u(k+1- \theta(k-2)+\nu(k-2-\theta(k-2))) - 0.983u(k-\theta(k-2))- 0.966u(k-1-\theta(k-2))) \quad (47)$$

A simulated response of the closed loop system is presented in Fig.6. Fig.7 shows respectively the prediction error evolution during the examination period. From the figure it can be concluded that with the performance of the overall system is improved significantly with the implementation of the Smith predictor and the system becomes insensitive to the delay variations.

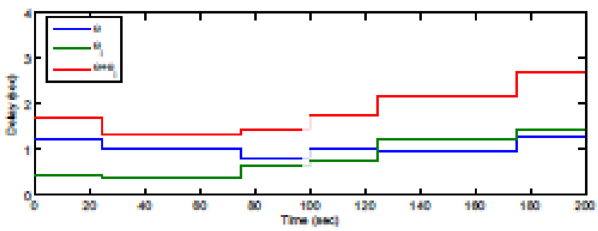

Fig. 5. Input delay k_d and output prediction k_c

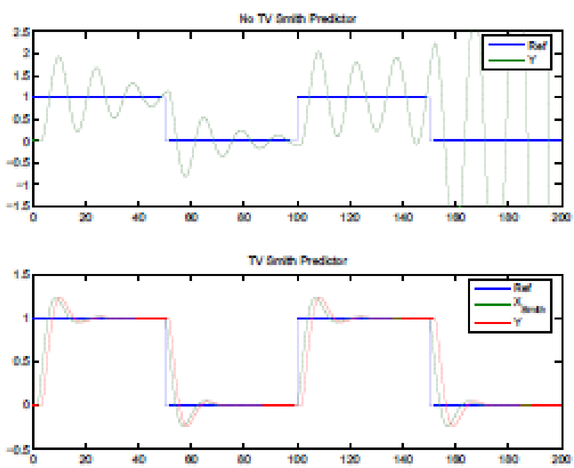

Fig. 6. Response of PI controlled system

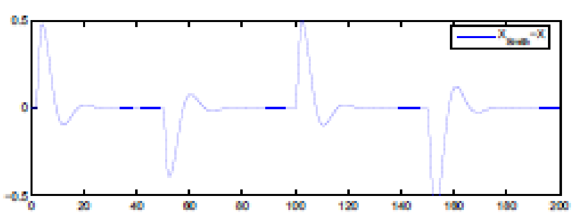

Fig. 7. The evolution of the prediction error

6 Concluding Remarks

The time-varying polynomial systems theory gives tools for the analysis and design of linear estimators and feedback controllers. In this paper the methodology has been applied to design of predictors and controllers for networked control with time-varying but measurable delays. The main problems in the design are related to complicated symbolic calculation of skew polynomials. For multivariable systems the calculation must be done using special software for symbolic mathematics.

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