
(Affiliations can be found after the references)

Received 21 March 2013 / Accepted 22 August 2014

ABSTRACT

We discuss the methods employed to photometrically calibrate the data acquired by the Low Frequency Instrument on Planck. Our calibration is based on a combination of the orbital dipole plus the solar dipole, caused respectively by the motion of the Planck spacecraft with respect to the Sun and by motion of the solar system with respect to the cosmic microwave background (CMB) rest frame. The latter provides a signal of a few mK with the same spectrum as the CMB anisotropies and is visible throughout the mission. In this data release we rely on the characterization of the solar dipole as measured by WMAP. We also present preliminary results (at 44 GHz only) on the study of the Orbital Dipole, which agrees with the WMAP value of the solar system speed within our uncertainties. We compute the calibration constant for each radiometer roughly once per hour, in order to keep track of changes in the detectors’ gain. Since non-idealities in the optical response of the beams proved to be important, we implemented a fast convolution algorithm which considers the full beam response in estimating the signal generated by the dipole. Moreover, in order to further reduce the impact of residual systematics due to sidelobes, we estimated time variations in the calibration constant of the 30 GHz radiometers (the ones with the largest sidelobes) using the signal of an internal reference load at 4 K instead of the CMB dipole. We have estimated the accuracy of the LFI calibration following two strategies: (1) we have run a set of simulations to assess the impact of statistical errors and systematic effects in the instrument and in the calibration procedure; and (2) we have performed a number of internal consistency checks on the data and on the brightness temperature of Jupiter. Errors in the calibration of this Planck/LFI data release are expected to be about 0.6% at 44 and 70 GHz, and 0.8% at 30 GHz. Both these preliminary results at low and high f are consistent with WMAP results within uncertainties and comparison of power spectra indicates good consistency in the absolute calibration with WMAP (0.3%) and a 1.4r discrepancy with WMAP (0.9%).

Key words. cosmic background radiation – instrumentation: polarimeters – methods: data analysis

1. Introduction

This paper, one of a set associated with the 2013 release of data from the Planck mission (Planck Collaboration I 2014), describes the techniques we employed to calibrate the voltages measured by the LFI radiometers into a set of thermodynamic temperatures (photometric calibration). We also discuss the quality of our calibration in terms of the required accuracy needed to achieve Planck’s final science goals. This paper is part of the Planck 2013 results, Special feature project (http://www.esa.int/Planck) of the European Space Agency (ESA) with instruments provided by two scientific consortia funded by ESA member states (in particular the lead countries France and Italy), with contributions from NASA (USA) and telescope providers, sponsored by a collaboration between ESA and a scientific consortium led and funded by Denmark.
of a larger set of articles (Planck Collaboration II 2014; Planck Collaboration III 2014; Planck Collaboration IV 2014) which explain the methodology used to produce maps from raw LFI data which have been issued in the Planck 2013 data release. A similar paper, Planck Collaboration VIII (2014), describes the approach used by HFI. Planck Collaboration XXXI (2014) contains a comparison of the LFI/HFI approaches and assess the consistency of Planck’s maps and power spectra.

By “calibration” here we mean the process that converts each voltage measured by an analogue-to-digital converter (ADC) into a thermodynamic temperature. (We describe the incoming flux as a thermodynamic temperature because the cosmic microwave background (CMB) signal is a nearly perfect blackbody, and thus the temperature is a more physically significant quantity to measure.) The process can be modelled by the following equation:

\[ V_{\text{out}}(t) = G(t) \times \left[ B \ast (T_{\text{sky}} + D) \right]_{\text{int/lo},t} + M, \]

where \( x(t) \) is the direction of the beam axis at time \( t \). This relates the voltage \( V_{\text{out}} \) measured by the ADC with the sum of three terms: (1) the convolution \(^2\) between the brightness temperature \( T_{\text{sky}} \) of the sky (CMB and galactic/extragalactic foregrounds) and the beam response \( B \) of the instrument (\( \int \Delta B \, \Delta \Omega = 1 \)) at a given time; (2) the convolution between \( B \) and the CMB dipole \( D \) (including the solar and orbital terms, as well as their associated kinematic quadrupoles); and (3) an offset term \( M \) (monopole, including instrumental offsets), which is of little importance for differential instruments \(^3\) like LFI. (The dependence of the \( B \ast (T_{\text{sky}} + D) \) term on both \( x \) and \( t \), expressed in Eq. (1) by a subscript, is due to the fact that \( D \) depends on the velocity of Planck, which is a time-dependent quantity.) The transfer function \( G \) represents the overall “gain” of the instrument. We are primarily interested in \( K = G^{-1} \), as the purpose of the calibration is to convert \( V \) back into a temperature. Since a number of environmental factors influence the value of \( K \), we expect it to change with time. The output of a calibration procedure is therefore a time series of values \( K_t \), which sample the unknown function \( K(t) \) at a reasonable frequency (i.e., higher than the frequency of the expected instrumental fluctuations in \( K \)) and which allow us to reconstruct the value of \( B \ast T_{\text{sky}} \) in Eq. (1) with good accuracy (for LFI, this accuracy is between 0.6 and 0.8%, with 0.25% of uncertainty coming from the error bars on the characterization of the CMB dipole provided by WMAP).

Proper relative calibration (i.e., precise tracking of the variations of each radiometer’s gain throughout the mission) is a necessary condition for a self-consistent map-making. In addition, accurate determination of the absolute calibration (i.e., translation of the observed voltages into physical units, in terms of antenna temperature), together with proper reconstruction of the beam window function, is crucial for any scientific exploitation of the maps and power spectra. The LFI calibrated maps are used extensively in the Planck data analysis. They are a fundamental input to the component separation process (Planck Collaboration XII 2014), which leads to Planck’s full-sky CMB map. This map is the basis for the extraction of the Planck power spectrum in the low-multipole regime (Planck Collaboration XV 2014), and for all analyses on non-Gaussianity (Planck Collaboration XXIV 2014), isotropy, and second-order statistics (Planck Collaboration XXIII 2014). Furthermore, the LFI power spectrum provides a unique consistency check internal to Planck, particularly in the comparison between the LFI 70 GHz and the HFI 100 GHz channels (Planck Collaboration XXXI 2014). The LFI beams and window functions are discussed in detail in Planck Collaboration IV (2014). Here we give a detailed account of the LFI absolute and relative calibration. Earlier accounts of the calibration procedure for LFI were given by Villa et al. (2010) and Mennella et al. (2010), which present the results of the LFI on-ground calibration campaign, and by Mennella et al. (2011), which describes the LFI calibration procedure used for producing the Planck Early Results (Planck Collaboration I 2011).

The structure of this paper is the following. In Sect. 2 we introduce a number of important ideas that are going to be used in this work, namely the time scales of variations in \( K \) (Sect. 2.1), the treatment of beam sidelobes in the calibration and their impact on subsequent analyses of LFI’s calibrated data (Sect. 2.2), and an updated list of colour corrections (Sect 2.3). Then, in Sect. 3, we explain the methods we have developed to calibrate the data acquired by the LFI radiometers. We discuss the type of systematic effects affecting the calibration procedure in Sect. 4. In Sect. 5 we estimate the accuracy of our calibration, and we include the most relevant results from Planck Collaboration XXXI (2014), which compares LFI results to HFI as well as WMAP. Finally, in Sect. 6 we summarize our results and propose a number of improvements to be implemented for the future releases of LFI data.

2. Calibration philosophy and conventions

2.1. Time scale of gain variations

In this section we establish the time scale over which we expect significant variations in the gain of the LFI radiometers. This quantity drives the design of the calibration algorithms we then discuss in the next sections.

Changes in the gain of the LFI radiometers are mainly triggered by changes in the thermal environment of the LFI instrument, particularly in the front-end and back-end modules (Bersanelli et al. 2010). The time scale of gain changes can therefore be estimated either by considering the rate of change of temperature sampled near the radiometer amplifiers, or by using the radiometer to continuously measure the temperature of a load kept at a stable temperature. The latter solution is viable for LFI, because each radiometer continuously observes a stable 4 K load mounted on the external shield of HFI (Valenziano et al. 2009; Lamarre et al. 2010); the temperature of each 4 K load drifts by less than one mK per year (so that the ratio of a yearly drift to the system temperature, \( T_{\text{sys}} \approx 20 \, \text{K} \), is \( \leq 0.01\% \)), and the 4 K signal entering LFI radiometers goes through the same chain as the signal coming from the telescope. It can therefore be used to assess the rate of change in the gain with good accuracy.

Despite the fact that Planck orbits around the Sun-Earth \( L_2 \) point, which grants a stable thermal environment

\(^2\) In this work we use the following notation for convolution:

\[ (A * B)(\theta, \phi) = \int_{\Omega} A(\theta', \phi') B(\theta - \theta', \phi - \phi') \, d\Omega'. \]

\(^3\) Although LFI directly measures \( V_{\text{out}} \), it can be considered a differential instrument (Bersanelli et al. 2010) as the analysis is performed on the value \( V_{\text{out}} - rV_{\text{ref}} \), where \( V_{\text{ref}} \) is the measurement of the temperature of a stable 4 K heat load (Sect. 3.2) and \( r \) is a coefficient which removes the contribution of \( M \) from both \( V_{\text{out}} \) and \( V_{\text{ref}} \), so that the time average of \( V_{\text{out}} - rV_{\text{ref}} \) is zero. Refer to Planck Collaboration II (2014) for further details.
which has the unit of time and quantifies the typical time required to induce a change of level \( \varepsilon \) in \( f \) at time \( t \). Using \( \varepsilon = 0.01 \) (i.e., we are looking for 1% changes), we find the following timescales:

1. Fluctuations in the temperature \( T_{fp} \) of the focal plane happen on timescales of the order of weeks.
2. The time scale for fluctuations in the temperature \( T_{be} \) of the back-end are faster during the first survey, as \( (\tau_{be}) \) is of the order of tens of hours. After the first survey\(^5\) the transponder was left on continuously, and this value increases to roughly one week.
3. Results similar to those for the back-end are found when considering the total-power voltage \( V_{ref} \), which measures the temperature of the 4 K loads (i.e., setting \( f \) equal to \( V_{ref} \) instead of \( T_{ip} \) or \( T_{be} \)).

These results motivate the need to re-calibrate each radiometer more than once per day. The most natural length of time for \( Planck \) is the duration of one pointing period, i.e., the interval between two consecutive repointings of the spacecraft, which happens roughly once per hour (Dupac & Tauber 2005) and is short enough to detect any significant change in the gains. At the same time, the interval is long enough to sample the dipole signal (our main calibration source) with good signal-to-noise ratio (S/N), since during one pointing period the telescope scans the same circle in the sky tens of times (the median value is 39 times, and 50% of the pointings fall in the 36–42 range).

For this data release we chose not to explicitly consider effects due to the aging of the radiometric components and variations in the emissivity of the telescope. Such phenomena can lead to variations in the gain, noise temperatures, or an increase in the instabilities of the instrument, but we have had no clear evidence that they are significant on the relatively short time span covered by this data release (one year and a half). In September-October 2013 we have run a number of End-of-Life tests on the instrument, with the purpose of quantitatively assessing such effects: we will present the results of our analysis in a future \( Planck \) data release.

### 2.2. Handling beam efficiency

As described by Bersanelli et al. (1997), Cappellini et al. (2003), and Mennella et al. (2011), the calibration of LFI is referenced to the dipole signal, which is a nearly ideal calibrating source. \( Planck \) observes the sky by spinning around the Sun-Earth axis with a speed of 1 rpm (Tauber et al. 2010), and since the main beams are located at \( \sim 85^\circ \) from the spinning axis, the dipole induces a sinusoidal fluctuation in the time ordered data with frequency 1/60 Hz and varying amplitude\(^6\) which can be used for the calibration. For this release, our reference dipole \( D_{ref} \) is the combination of the solar dipole as given by the WMAP values (Jarosik et al. 2011), and the orbital dipole, derived from the known velocity of the \( Planck \) spacecraft relative to the barycentre of the solar system. We now describe the calibration procedure for LFI and, in particular, we discuss how non-ideal beams affect the process. This discussion is similar to the one carried on in \( Planck \) Collaboration XXXI (2014). However, \( 5 \) The number of sky surveys in this \( Planck \) data release is two and a half.

\( 6 \) The scanning strategy has been designed such that this amplitude never vanishes, see Dupac & Tauber (2005). However, as the scan axis changes, variations in the observed amplitude do occur and they affect the accuracy of the calibration, as we discuss in Sect. 3.1.
Here we provide a more rigorous treatment which involves a description of the relevant quantities as a function of time rather than as a function of the observing direction.

The starting point of our discussion is Eq. (1). In the LFI pipeline (Planck Collaboration II 2014) we remove the $T_{\text{sky}}$ term from the data, so that the equation reduces to the following:

$$V_{\text{out}}(t) = G(B \ast D)(x(t), t) + M,$$

where we indicate with $V_{\text{out}}$ the signal $V_{\text{out}}$ without the $T_{\text{sky}}$ component. This signal is compared with a model of the beam-convolved dipole, based on the beam response $B_{\text{model}}$ and on the reference dipole $D_{\text{ref}}$.

$$V_{\text{out}}(t) = \tilde{G}(B_{\text{model}} \ast D_{\text{ref}})(x(t), t) + \tilde{M}.$$  (4)

The LFI beams are discussed in Planck Collaboration IV (2014, main beams and window functions) and Planck Collaboration III (2014, sidelobes); Fig. 2 shows three typical beam profiles in the $[{-5^\circ}, {5^\circ}]$ range. The unknown parameters $G$ and $M$ in Eq. (4) can be found by means of a least square fit between $V_{\text{out}}$ and the right hand side. (Such fit is done once per each pointing period, i.e., the period between two consecutive repointings of the spacecraft, which happens roughly once per hour.) From the point of view of the LFI calibration, the $\tilde{G}$ factor is the only important parameter to estimate. To estimate it, we equate Eqs. (3) and (4) and take the time derivative $\partial_t$ of the two sides:

$$\tilde{G} = G \frac{\partial_t(B \ast D)}{\partial_t(B_{\text{model}} \ast D_{\text{ref}})},$$  (5)

A common approach to the use of this equation is to approximate the beam with a pencil beam, $B_{\text{model}} \approx B_{\text{pencil}}$, as adopted by HFI (Planck Collaboration VIII 2014) and WMAP (Jarosik et al. 2007). For LFI we have computed the full 4\pi beams for all detectors and we developed a fast convolution routine to estimate $B_{\text{model}} \ast D_{\text{ref}}$. This was motivated by the wish to fully control dipole-coupling to the sidelobes, particularly at 30\,GHz where sidelobes are larger. The convolution of a generic beam with a dipole produces a smearing effect (due to the fact that not all the power is in the main beam) and a slight tilt in the dipole axis (resulting from asymmetries in the beam). As shown in detail in Sect. 4.2 and Appendix A, these effects can be quantified by the length and direction, respectively, of the vector $S$ defined there.

We verified a posteriori that the net effect of the convolution does not produce a significant improvement over a pencil beam model at the present stage of the analysis (besides the tilt effect, which is $<0.01\%$). For this data release, therefore, in the convolution routine we rescaled the length of vector $S$ to unity, which is effectively equivalent to modelling the beam as a pencil beam. The advantage of this approach is that it matches the convention on the normalization of the beam that has been assumed in the calibration of HFI data. In the following description of map calibration and associated uncertainties, therefore, we assume $B_{\text{model}} \approx B_{\text{pencil}}$. Full use of the 4\π convolver, including integrated frequency-dependent sidelobes within the LFI radio metric bands, will be applied for later analyses, including polarization calibration.

We can write the true beam $B$ as the sum of two terms, $B = B_{\text{main}} + B_{\text{side}}$, where $B_{\text{main}}$ represents the contribution of what we define as the main beam (defined as the portion of the beam within $5^\circ$ of the beam centre), and $B_{\text{side}}$ represents the much smaller remaining part (the sidelobes). Equation (5) becomes:

$$\tilde{G} = \frac{\partial_t(B_{\text{main}} \ast D_{\text{ref}}) + \partial_t(B_{\text{side}} \ast D_{\text{ref}})}{\partial_t(B_{\text{pencil}} \ast D_{\text{ref}})}.$$  (6)

The convolution of the main beam with the signal from the sky, $T = D_{\text{sky}}$, is nearly identical to an ideal pencil beam except that only a fraction $1 - f_{\text{side}}$ of the antenna gain is contained in the main beam:

$$B_{\text{main}} \ast T = (1 - f_{\text{side}})B_{\text{pencil}} \ast T \approx (1 - f_{\text{side}})T.$$  (7)

So we have:

$$\tilde{G} = G \frac{(1 - f_{\text{side}})\partial_t(B_{\text{pencil}} \ast D) + \partial_t(B_{\text{side}} \ast D)}{\partial_t(B_{\text{pencil}} \ast D_{\text{ref}}) + \partial_t(B_{\text{side}} \ast D_{\text{ref}})} = G(1 - f_{\text{side}})(1 + \phi_d),$$  (8)

where

$$\phi_d = \frac{\partial_t(B_{\text{side}} \ast D) - \partial_t(B_{\text{pencil}} \ast D)}{\partial_t(B_{\text{main}} \ast D)}.$$  (9)

7 This step is based on an algorithm which starts from the approximation $T_{\text{sky}} \ast D \approx D$ and then refines the solution iteratively. The real algorithm is more complex than this, as it removes $T_{\text{sky}}$ at the same time as the least-square fit discussed later in this section; the details of the algorithm are presented in Sect. 3.1.

8 Although the equation is mathematically correct, we note that it is of little use for a numerical implementation, as the denominator is a sinusoid which periodically goes to zero, thus making the quantity diverge to infinity.

9 In the context of this paper, a pencil beam $B_{\text{pencil}}$ is a Dirac delta function centered on the beam axis $\hat{e}_b$:

$$B_{\text{pencil}}(\hat{x}) = \delta(\hat{x} - \hat{e}_b).$$

10 The quantity $1 - f_{\text{side}}$ is approximately equal to $|S|$. However, as explained in Appendix A, the vector $S$ models the coupling of the full beam (including the main beam, which causes a tiny smearing effect because of its finite FWHM) with the dipole and only uses the first directional moments of the beam shape.
Fig. 3. Results of a simulation which shows how \( \phi_D \) is computed. We assume to observe a 1 K peak-to-peak dipole in the sky for 5 minutes with a scanning strategy very similar to the one used for Planck's 30 GHz radiometers, i.e., the sky is scanned in circles of high amplitude (\( \sim 85^\circ \)) with a rotation frequency \( \nu = 1/60 \) Hz and a sampling frequency of 32.5 Hz (so that 10 000 temperature samples are generated for each data stream). We observe the dipole using a realistic 30 GHz beam \( B = B_{\text{main}} + B_{\text{side}} \) with FWHM 0.5. Panel A): plot of the \( (B_{\text{main}} \ast D)(t) \) term, which oscillates as a sinusoid with amplitude \( \leq 0.5 \) K; the term \( (B_{\text{side}} \ast D)(t) \) is negligible (see panel C for a close-up). Panel B): plot of the \( \partial_t(B_{\text{main}} \ast D)(t) \) term, used in the definition of \( \phi_D \) (Eq. (9)). Panel C): Close-up of the \( (B_{\text{side}} \ast D)(t) \) term shown in panel A). Panel D): close-up of the \( \partial_t(B_{\text{side}} \ast D)(t) \) shown in panel B). Panel E): value of \( \phi_D \) as a function of time, calculated using the definition in Eq. 9. Panel F): distribution of the 10 000 values of \( \phi_D \) plotted in panel E). Half of the values fall within the 0.19–0.34% range.

Figures 3 shows the result of a simulation which illustrates how \( \phi_D \) can be computed from the TODs.

Consider now a properly calibrated timeline \( T_{\text{sky}} \) where the dipole and the monopole terms in Eq. (1) have been removed, so that \( T_{\text{sky}} = G(B \ast T_{\text{sky}}) \). Within each pointing period of constant \( G \), we can write the relationship between the measured sky temperature \( T_{\text{sky}} \) and the level of the true sky temperature \( T_{\text{sky}} \) as:

\[
\hat{T}_{\text{sky}}(x(t)) = \frac{G(B \ast T_{\text{sky}})(x(t))}{G} = \frac{(B \ast T_{\text{sky}})(x(t))}{(1 - f_{\text{side}})(1 + \phi_D)}.
\]

Solving for the true temperature \( T_{\text{sky}} \), we find:

\[
T_{\text{sky}} \approx \hat{T}_{\text{sky}}(1 - \phi_{\text{sky}} + \phi_D),
\]

where

\[
\phi_{\text{sky}} = \frac{B_{\text{side}} \ast T_{\text{sky}}}{B_{\text{main}} \ast T_{\text{sky}}} \cdot \frac{T_{\text{sky}}}{\bar{T}_{\text{sky}}}
\]

is a time-dependent quantity which quantifies how much of the sky signal enters the beam through its sidelobes.

A precise evaluation of the correction terms \( \phi_D \) and \( \phi_{\text{sky}} \) in Eq. (11) requires detailed simulations. We have performed such calculations for \( \phi_D \) by computing the convolution of the sidelobes with the dipole from sample GRASP full beams, leading to \( \phi_D \approx 0.15\% \) (see Fig. 4). The term \( \phi_{\text{sky}} \) requires a full convolution of a sky model with the full beam, and it is frequency dependent. We performed a simulation for a 70 GHz channel, projected the values of \( \phi_{\text{sky}} \) on a map and found values ranging...
observing foreground emission with intensity power-law index $\alpha$, then $T_{\text{RJ}}(v) = T_0 (v/v_0)^{\alpha - 2}$, and

$$\tilde{T} = T_0 \int g(\nu) (v/v_0)^{\alpha - 2} \, d\nu \over \int g(\nu) \eta_{\text{RJ}}(\nu) \, d\nu.$$

(16)

The LFI colour correction is defined as:

$$C(\alpha) = \int g(\nu) \eta_{\text{RJ}}(\nu) \, d\nu \over \eta_{\text{RJ}}(v_0) \int g(\nu) (v/v_0)^{\alpha - 2} \, d\nu.$$

(17)

Hence the colour-corrected temperature is:

$$C(\alpha) \tilde{T} = T_0 / \eta_{\text{RJ}}(v_0).$$

(18)

This gives the thermodynamic brightness differential temperature (units “K$^\text{RJ}$”) at the reference frequency $v_0$. To get to the standard (Rayleigh-Jeans) brightness temperature at the same frequency we use $\eta_{\text{RJ}}(v_0)$ (see Eq. (14)):

$$T_0 [\text{K}_\text{RJ}] = \tilde{T} [\text{K}_\text{RJ}] \eta_{\text{RJ}}(v_0) C(\alpha).$$

(19)

For the present series of Planck papers, the reference frequencies $v_0$ are defined as precisely 28.4, 44.1, and 70.4 GHz for the three LFI bands; in the Planck Early Release papers, slightly different values were used.

Values for $C(\alpha)$ for the expected range of foreground spectral indices are listed in Table 1. Note that this definition of $C$ is inverted relative to the colour correction $C_{\text{ER}}(\alpha)$ used in the Planck Early Release. We note additionally that $C(\alpha)$ quoted here are not exactly equal to $1/C_{\text{ER}}(\alpha)$ from Mennella et al. (2011) and Zacchei et al. (2011), due both to the change in reference frequency and also to a small error in the earlier estimates.

Colour corrections at intermediate spectral indices may be derived accurately from a quadratic fit to the values in Table 1. In addition, the data release includes the Unit conversion and Colour Correction (UcCC) IDL package (Planck Collaboration IX 2014) which calculates colour corrections and unit conversions using the band-averaged bandpass information stored in the Reduced Instrument MOdel (RIMO, see Planck Collaboration 2013) file also included in the data release.

From Eq. (17) it appears that the absolute scaling of the bandpass is irrelevant for the colour correction, although Eqs. (13)–(15) require that:

$$\int g(\nu) \eta_{\text{RJ}}(\nu) \, d\nu = 1,$$

(20)

for consistency with the definition of gain, $G$, and calibration factor, $K$, elsewhere in this paper. In practice, all the colour corrections listed in Table 1 are derived from averages across two or more bandpasses: values for individual radiometric chain assemblies (RCA) require averaging the main and side radiometers in each RCA, and the response of each radiometer is the average of the two independent detectors (Zonca et al. 2009).

For consistency, the weighting for these averages must duplicate the procedure used to average the data, as described in Planck Collaboration II (2014). We recall here the procedure: (i) calibrate the individual data streams to give equal response to the CMB; (ii) combine the raw data for the detectors for each radiometer using fixed weights based on the inverse variance measured early in the mission, incorporating fixed calibration values; (iii) recompute the data using time-dependent factors as described in the present paper; (iv) combine the two radiometers in each RCA with equal weights to minimize polarization leakage; and (v) combine the data from individual RCAs with

---


Fig. 4. Estimated value of $\phi_0$ averaged over the first and second surveys. The central frequencies of the radiometers are 70 GHz for 18M ... 28S, 44 GHz for 24M ... 26S, and 30 GHz for 27M ... 28M. In order to reduce the amount of data to consider, the time-dependent quantity $\phi_0$ was projected on a Healpix map (with NSIDE equal to 64) and binned. The error bars represent the first and third quartiles of the values of the 49 152 pixels in the map, while the middle point is the median.
<table>
<thead>
<tr>
<th>RCA</th>
<th>-2.0</th>
<th>-1.5</th>
<th>-1.0</th>
<th>-0.5</th>
<th>0</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
<th>3.5</th>
<th>4.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>LFI-18</td>
<td>0.948</td>
<td>0.961</td>
<td>0.972</td>
<td>0.981</td>
<td>0.988</td>
<td>0.994</td>
<td>0.997</td>
<td>0.998</td>
<td>0.997</td>
<td>0.995</td>
<td>0.990</td>
<td>0.983</td>
<td>0.975</td>
</tr>
<tr>
<td>LFI-19</td>
<td>0.856</td>
<td>0.878</td>
<td>0.899</td>
<td>0.919</td>
<td>0.939</td>
<td>0.957</td>
<td>0.975</td>
<td>0.991</td>
<td>1.006</td>
<td>1.020</td>
<td>1.032</td>
<td>1.043</td>
<td>1.053</td>
</tr>
<tr>
<td>LFI-20</td>
<td>0.889</td>
<td>0.908</td>
<td>0.925</td>
<td>0.941</td>
<td>0.956</td>
<td>0.970</td>
<td>0.983</td>
<td>0.994</td>
<td>1.003</td>
<td>1.011</td>
<td>1.018</td>
<td>1.023</td>
<td>1.027</td>
</tr>
<tr>
<td>LFI-21</td>
<td>0.917</td>
<td>0.933</td>
<td>0.947</td>
<td>0.960</td>
<td>0.971</td>
<td>0.981</td>
<td>0.989</td>
<td>0.996</td>
<td>1.001</td>
<td>1.004</td>
<td>1.006</td>
<td>1.006</td>
<td>1.004</td>
</tr>
<tr>
<td>LFI-22</td>
<td>1.024</td>
<td>1.026</td>
<td>1.027</td>
<td>1.026</td>
<td>1.023</td>
<td>1.018</td>
<td>1.011</td>
<td>1.003</td>
<td>0.993</td>
<td>0.982</td>
<td>0.969</td>
<td>0.955</td>
<td>0.940</td>
</tr>
<tr>
<td>LFI-23</td>
<td>0.985</td>
<td>0.991</td>
<td>0.996</td>
<td>0.999</td>
<td>1.001</td>
<td>1.002</td>
<td>1.002</td>
<td>1.000</td>
<td>0.997</td>
<td>0.993</td>
<td>0.988</td>
<td>0.982</td>
<td>0.975</td>
</tr>
<tr>
<td><strong>70 GHz</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.938</td>
<td>0.951</td>
<td>0.963</td>
<td>0.973</td>
<td>0.982</td>
<td>0.988</td>
<td>0.994</td>
<td>0.997</td>
</tr>
<tr>
<td>LFI-24</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.978</td>
<td>0.984</td>
<td>0.988</td>
<td>0.993</td>
<td>0.996</td>
<td>0.998</td>
<td>0.999</td>
<td>0.998</td>
</tr>
<tr>
<td>LFI-25</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.967</td>
<td>0.974</td>
<td>0.980</td>
<td>0.985</td>
<td>0.990</td>
<td>0.994</td>
<td>0.996</td>
<td>0.999</td>
</tr>
<tr>
<td>LFI-26</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.957</td>
<td>0.966</td>
<td>0.973</td>
<td>0.980</td>
<td>0.985</td>
<td>0.990</td>
<td>0.995</td>
<td>0.998</td>
</tr>
<tr>
<td><strong>44 GHz</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.968</td>
<td>0.975</td>
<td>0.981</td>
<td>0.986</td>
<td>0.990</td>
<td>0.994</td>
<td>0.997</td>
<td>0.999</td>
</tr>
<tr>
<td>LFI-27</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.948</td>
<td>0.959</td>
<td>0.969</td>
<td>0.978</td>
<td>0.985</td>
<td>0.991</td>
<td>0.995</td>
<td>0.998</td>
</tr>
<tr>
<td>LFI-28</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.946</td>
<td>0.958</td>
<td>0.968</td>
<td>0.977</td>
<td>0.985</td>
<td>0.991</td>
<td>0.996</td>
<td>0.998</td>
</tr>
<tr>
<td><strong>30 GHz</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.947</td>
<td>0.959</td>
<td>0.969</td>
<td>0.977</td>
<td>0.985</td>
<td>0.991</td>
<td>0.995</td>
<td>0.998</td>
</tr>
</tbody>
</table>

Table 1. Multiplicative colour corrections $C(\alpha)$ for individual LFI radiometer chain assemblies and for the band average maps.

intradimensional weights (fixed for the whole mission). When combining the bandpasses, the initial calibration step is equivalent to normalizing each component bandpass according to Eq. (20). The band-averaged bandpass stored in the RIMO, however, normalized the bandpasses using $\int g(\nu) d\nu = 1$. The difference is minor, since $\eta_{\text{eff}}$ varies by only a very small amount within any of the LFI bands, but it accounts for a small (<0.1%) difference between the $C(\alpha)$ values listed here and those derivable from the RIMO bandpasses.

Our best estimate of the uncertainty in the values of $C(\alpha)$, dominated by bandpass uncertainty (Zonca et al. 2009), comes from an indirect method, as follows. The two radiometers in each RCA, known as the main- and side-arms, are sensitive to orthogonal polarizations. The bandpasses for the two arms differ, leading to different colour corrections. The polarization signal is derived from the difference of the calibrated signals from the two arms (Leahy et al. 2010); unpolarized foreground emission does not precisely cancel due to the differential colour factors. This is the “bandpass leakage” effect. This leakage can be estimated from the flight data, as described in Planck Collaboration II (2014), but it can also be estimated from the pre-launch bandpass models that we use here to calculate $C(\alpha)$. To a good approximation, we can write the leakage factor as

$$C_\alpha(\alpha) - C_M(\alpha))/2 = (\beta - \beta_{\text{CMB}}) \alpha,$$

where the $\alpha$-factors depend solely on the bandpass profile, $\beta = \alpha - 2$ is the temperature spectral index, and $\beta_{\text{CMB}}$ is the in-band spectral index of the CMB, which we can take as zero for present purposes. The flight measurements of the $\alpha$-factors are demonstrably more accurate than the pre-launch (“QUCS”) estimates, and so

$$(\beta - \beta_{\text{CMB}})(\alpha_{\text{QUCS}} - \alpha_{\text{flight}}) = (\delta C_\alpha(\alpha) - \delta C_M(\alpha))/2,$$

where $\delta C_\alpha$ is the colour correction error. Statistically

$$\left\langle (\delta C_\alpha(\alpha) - \delta C_M(\alpha))^2 \right\rangle = \left\langle (\delta C_\alpha(\alpha) + \delta C_M(\alpha))^2 \right\rangle = \sigma_{C,\alpha}^2,$$

where $\sigma_{C,\alpha}$ is the colour correction error for one radiometer pair, i.e. one RCA. Hence for individual RCAs, $\sigma_{C,\alpha} \approx |\beta| \sigma_\alpha$. There are too few pairs for this approach to give an accurate value for $\sigma_{C,\alpha}$, especially as the error sources for the bandpasses in each band are different; therefore results from the three bands cannot be combined. Our rough estimates for $\sigma_\alpha$ are 0.14%, 0.46%, and 0.51% in the 30, 44, and 70 GHz bands, respectively. For the band-averaged maps, the errors in the colour corrections are reduced by $\sqrt{N_{\text{RCA}}}$, giving overall rough uncertainties of 0.1% in the 40, 30, and 20 GHz bands, respectively.

3. Calibration techniques for LFI

There is not a unique way to produce an estimate of the calibration constant $K = G^{-1}$ (see Eq. (1)). In picking the method to use, one has to consider a number of elements:

1. the calibration should be as accurate as possible;
2. calibrated data should also be self-consistent; for example, with the same beam and the same pointing direction and orientation in the sky, the value of $T_{\text{sky}}$ at any frequency should not depend on when the measurement was done, nor on the detector (under the hypothesis that everything else, e.g., the bandpass, can be assumed to be the same);
3. the estimated shape of $K(t)$ should be motivated by a physically meaningful model of the radiometer.

To help in the classification of the calibration methods discussed in this work, we write:

$$K(t) = K_0(1 + \xi(t)),$$

thus decomposing $K$ into a constant term $K_0$ and a unitless time-varying quantity $\xi(t)$. Typically $\xi(t)$ varies by a few percent per year. This allows us to categorize calibration methods into the following families: (1) absolute methods produce an estimate for $K_0$; (2) relative methods estimate how $\xi$ changes with time; and (3) some methods are able to estimate $K(t)$ directly and can therefore be considered both absolute and relative.

We implemented two calibration schemes for the LFI radiometers:

- The OSG dipole calibration is an improved version of the pipeline used to calibrate the data used in the Planck Early
The chief reason why we decided to use two different calibration schemes was to improve the self-consistency of the maps.

12 Note however that, even if the calibration constants $K_i$ are not affected by uncertainties in beam shapes, this is not necessarily true for the calibrated data. After having applied the calibration factors, we must remove the dipole signal convolved with the beam from the data, and therefore any uncertainty in the beam shape will lead to a systematic error in the calibrated data, even if the 4 K calibration is used.

We were not able to derive a calibration method that was robust against both radiometric non-idealities and optical effects in time for this data release. Since optical effects are most significant in the 30 GHz channels, we decided to apply the 4 K calibration to these radiometers instead of the other one (which was the base-line for the Early and Intermediate Planck/LFI papers). We provide a summary of the calibration methods used for each LFI frequency in Table 2. In Sect. 4.1 we discuss the level of consistency between the two calibration methods in the context of the treatment of optical systematics.

The outline of this section is as follows. The OSG dipole calibration and the 4 K calibration are explained in Sects. 3.1 and 3.2. In Sect. 3.3 we present the current status of our efforts to use the orbital dipole signal for calibrating data. Finally, Sect. 3.4 deals with the task of setting the zero-level of Planck’s LFI maps.

3.1. OSG dipole calibration

The combination of the motions of the spacecraft, the Earth, and the solar system with respect to the CMB produce a dipole signal with an amplitude of $(3.355 \pm 0.008) \, \text{mK}$ (Hinshaw et al. 2009) on the full sky. Like COBE (Kogut et al. 1996) and WMAP (Jarosik et al. 2011), we use this signal to estimate the value of $K = G^{-1}$ in Eq. (1). We indicate the gain estimates produced using this method with $K_{dip}$. To avoid the contamination of the dipole signal with residual Galactic emission, we employ an iterative technique that removes this sky signal from the calibration process.

Fig. 5. Variation in time of a few quantities relevant for calibration, for radiometer LFI21M (70 GHz, left) and LFI27M (30 GHz, right). Vertical dashed lines mark boundaries between complete sky surveys. All temperatures are thermodynamic. Panel A): calibration constant $K$ estimated using the expected amplitude of the CMB dipole. Note that the uncertainty associated with the estimate changes with time, according to the amplitude of the dipole as seen in each ring. Panel B): expected peak-to-peak difference of the dipole signal. The shape of the curve depends on the scanning strategy of Planck (see also Fig. 8), and it is strongly correlated with the uncertainty in the gain constant (see panel A)). The two dots indicate at which time the data used to produce the plots in Fig. 9 were taken. Panel C): 4 K total-power output voltages $V$. Note that the noise level for $V$ is much more stable than it is for $K$, as it does not depend on the dipole amplitude. (All the large jumps seen in this figure occurred because of some change in the operational state of the spacecraft; e.g., 456 days after launch we switched off the primary 20 K sorption cooler and turned on the redundant unit.) Panel D): The calibration constants $K$ used to actually calibrate the data for this Planck data release are either derived by applying a smoothing filter to the raw gains in panel A) (70 and 44 GHz, e.g., LFI21M, left), as described in Sect. 3.1, or through Eq. (32) and the 4 K total-power voltages in panel C (30 GHz, e.g., LFI27M, right).
Table 2. Methods used to calibrate the radiometers.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Calibration</th>
<th>Absolute</th>
<th>Relative</th>
<th>Primary systematics</th>
<th>Relevant paper sections</th>
</tr>
</thead>
<tbody>
<tr>
<td>30 GHz</td>
<td>CMB dipole</td>
<td>4 K</td>
<td></td>
<td>ADC non-linearities, variations of $T_{\text{noise}}$ with time</td>
<td>Sect. 3.1, 3.2, 4.3</td>
</tr>
<tr>
<td>44 GHz</td>
<td>CMB dipole</td>
<td>CMB dipole</td>
<td></td>
<td>Optical systematics, bandpass response of the radiometers</td>
<td>Sect. 3.1, 4.1</td>
</tr>
<tr>
<td>70 GHz</td>
<td>CMB dipole</td>
<td>CMB dipole</td>
<td>Same as for 44 GHz</td>
<td></td>
<td>Sect. 3.1, 4.1</td>
</tr>
</tbody>
</table>

Fig. 6. Schematics of a LFI radiometer. The radiation entering the feed horns in the 20 K front end is split by an OrthoMode transducer (OMT) into its two linearly polarized components. The two signals feed two twin radiometers. (Only one of them is shown in this figure.) The 2.7 K signal is mixed with the signal emitted by a reference blackbody at 4.5 K by an hybrid coupler before being amplified by a Low Noise Amplifier (LNA). A 4096 Hz phase switch induces an alternating 0°/180° phase shift, so that the outputs of the second hybrid are two sequences of phase-switched signals which both propagate to the warm back-end. Here they are further amplified and measured, before being compressed into packets and sent to Earth. Refer to Mennella et al. (2010) for more details.

Table 3. Parameters used in the model of the dipole signal.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>CMB monopole</td>
<td>$T_{\text{CMB}}$</td>
<td>$2.725 \pm 0.002$ K</td>
</tr>
<tr>
<td>Solar speed$^a$</td>
<td>$v_{\text{Sun}}$</td>
<td>$263.99 \pm 0.14$ km s$^{-1}$</td>
</tr>
<tr>
<td>$l$</td>
<td>$263.99 \pm 0.14$ km s$^{-1}$</td>
<td>Jarosik et al. (2011)</td>
</tr>
<tr>
<td>$b$</td>
<td>$48.26 \pm 0.03$ km s$^{-1}$</td>
<td>Jarosik et al. (2011)</td>
</tr>
<tr>
<td>$v_{\text{Planck}}$</td>
<td>Velocity of the spacecraft$^b$</td>
<td>$369.0 \pm 0.9$ km s$^{-1}$</td>
</tr>
<tr>
<td>$(v_{\text{Planck}})$</td>
<td></td>
<td>$30.0 \pm 0.4$ km s$^{-1}$</td>
</tr>
<tr>
<td>min $v_{\text{Planck}}$</td>
<td></td>
<td>$29.39$ km s$^{-1}$</td>
</tr>
<tr>
<td>max $v_{\text{Planck}}$</td>
<td></td>
<td>$30.60$ km s$^{-1}$</td>
</tr>
</tbody>
</table>

Notes. $^a$ Relative to the CMB rest frame. $^b$ The values reported here are representative of the overall speed of the spacecraft. The calibration code uses the full timestream of velocity components ($v_i$, $v_j$, $v_k$), sampled once every minute.

The basic steps of this calibration procedure are as follows:

1. We combine the velocity of the spacecraft with respect to the Sun $v_{\text{Pla}}$ and the velocity of the Sun with respect to the CMB $v_{\text{Sun}}$ to estimate the amplitude and alignment of the dipole in the sky in a given direction $x$:

$$D(x, t) = T_{\text{CMB}} \left( \frac{1}{\gamma(t)(1 - \beta(t) \cdot x)} - 1 \right).$$

where $T_{\text{CMB}}$ is the temperature of the CMB monopole, $\beta = (v_{\text{Pla}} + v_{\text{Planck}})/c$, $\gamma = (1 - \beta^2)^{-1/2}$, $v_{\text{Sun}}$ is the velocity of the solar system with respect to the CMB rest frame, and $v_{\text{Pla}}$ is the spacecraft’s velocity with respect to the solar system’s barycentre. Table 3 reports the numerical values used in the pipeline.

2. We produce discrete time-ordered data (TOD) of the expected overall dipole signal (thermodynamic temperature) at time $t_i$:

$$D_i = D(x(t_i), t_i).$$

where $i$ ranges from 1 to $N \sim 10^5$ (the number of samples in a pointing period). As explained in Sect. 2.2, the computation of the expected dipole signal takes the shape of the beams into account, following a method described in Sect. 4.2.

3. Using pointing information, we project both $V_i$ (the voltage $V$ at time $i$, as used in Eq. (1)) and $D_i$ on a HEALPix map (Górski et al. 2005) with $N_{\text{side}} = 256$. Multiple hits on the same pixels are averaged in both cases. The result is a pair of maps, $V^\text{map}_k$ and $D^\text{map}_k$, with $k$ being the pixel index$^{13}$.

$^{13}$ Most of the pixels in the maps contain no data, since during one pointing period the beam paints a thin circle in the sky. We assume hereafter that the index $k$ runs only through the pixels which have been hit at least once.
4. We use weighted least squares to estimate $K$ in Eq. (1) from the correlation between the signal in volt $V_{\text{map}}$ with $D_{\text{map}}$:

$$V_k = K_{\text{dip}} D_k + \epsilon,$$

where $K_{\text{dip}}$ and $\epsilon$ are the parameters used in the fit. Each sample $k$ is weighted according to the number of hits per pixel.

In computing the fit, we use a frequency-dependent mask to avoid those pixels where a strong non-Gaussian signal other than the dipole is expected. Such masks are the union of a Galactic mask (CG80, see Planck Collaboration XII 2014) and a point source mask (conceptually similar to the point source masks used in component separation, but the masking radius is kept fixed at 32'). See Fig. 7 for an example (30 GHz). In Sect. 5.3.1 we explore the effect of changing the mask on the calibration.

5. The fit described in the previous step suffers from the presence of noise in the sky signal, $V_{\text{map}}$ (mainly due to the CMB and to Galactic emission that was not properly masked). Therefore we apply an iterative algorithm, named Mademoiselle, that uses destriping techniques to iteratively improve the calibration. The algorithm subtracts the convolution between the dipole signal and the 4$\pi$ beam from $V_{\text{map}}$, thus obtaining a set of maps (one per each pointing period $m$) which estimate the sky signal alone, $T_{\text{sky}}$:

$$T_k = (V_{\text{map}} - K_{\text{dip}} D_k - \epsilon) K_{\text{dip}},$$

with $k$ being the pixel index. (Here again each map covers only a tiny fraction of the whole sky, i.e., the circle covered during each pointing period.) We apply a destriping algorithm to the set of $m$ maps of $T_{\text{sky}}$ and use the result to apply a correction to $K_{\text{dip}}$ and $\epsilon$. We iterate this process until a convergence criterion is satisfied. The result of Mademoiselle is a new set of gains, $K_i$ (again, one per pointing period), and offsets, $\epsilon_i$. See also the section about TOD processing in the Planck explanatory supplement (Planck Collaboration 2013).

6. The gains produced by this procedure need to be further processed in order to reduce the statistical noise. We applied an adaptive smoothing filter based on wavelets, which smooths more around dipole minima and less around maxima (see Planck Collaboration II 2014). In those cases where the noise is too high for the filter to produce meaningful results (i.e., near dipole minima), we substituted the gains with a straight line. This can be seen in the left panel D of Fig. 5, near 100, 300, and 500 days after launch.

7. Once a set of $K_i$ gains (one per pointing period) is produced, the pipeline calibrates the timelines and subtracts the dipole signal, so that in the calibrated time series the Galactic signal and the CMB fluctuations are the only relevant astrophysical components.

Details of the code implementation are given in Planck Collaboration II (2014).

The accuracy in gain reconstruction depends critically on the orientation of the spacecraft with respect to the dipole axis, i.e. the value of the product $(\theta_{\text{Sun}} + \phi_{\text{Planck}}) \cdot x$. We speak of a dipole minimum when the spacecraft’s orientation is such that the value of the scalar product reaches a minimum, and a dipole maximum when reaches a maximum. Figure 5 shows this idea for two LFI radiometers, LFI21M (left) and LFI27M (right): in panel A we show the values of $D_{\text{dip}}$ as a function of time, while in panel B we show the expected amplitude of the dipole $\Delta T_{\text{dip}}$ as seen in the circle in the sky (ring) which is scanned during each pointing period. It is clear that the noise in $D_{\text{dip}}$ is mainly due to the variation of $\Delta T_{\text{dip}}$. The reason why the peak-to-peak difference of the measured dipole signal varies is represented in Fig. 8, which shows which parts of the sky are observed by LFI27M during two pointing periods, one (4800) near a dipole maximum, and another (6636) near a dipole minimum. Finally, Fig. 9 shows the data used to compute the weighted linear regression presented in item 4 above for the same pointing periods.

3.2. 4 K calibration

Gain changes in a LFI radiometer can be modelled using the emission of the internal 4 K reference load as measured by the radiometer itself (Valenziano et al. 2009).

We indicate with $V_{\text{ref}}$ the output voltage which measures the temperature of the 4 K reference load. Equation (1) changes via the transformation $B \rightarrow (T_{\text{sky}} + D \rightarrow T_{\text{ref}}$:

$$V_{\text{ref}} = G(T_{\text{ref}} + T_{\text{noise}} + \epsilon_{\text{ref}}(B \cdot T_{\text{sky}} + D + T_{\text{sky}}))$$

$$= G(T_{\text{ref}} + T_{\text{noise}} + \epsilon_{\text{ref}} \Sigma_{\text{sky}}),$$

where the term $T_{\text{noise}}$ is analogous to the monopole term $M$ in Eq. (1), and we include a possible leakage from the sky signal.

Fig. 8. Top: dipole signal due to the motion of the solar system only (in Ecliptic coordinates). The points observed by LFI27M during pointing periods 4800 (250 days after launch) and 6636 (300 days) are shown with black and blue lines, respectively. (In Fig. 5, panel B on the right, we indicate the times when these two periods occurred with black dots.) Bottom: dipole signal along the same two pointing periods as a function of the North phase angle. Note the difference in the amplitude of the two sinusoidal waves. Such differences are due to the Planck’s scanning strategy and determine the statistical error in the estimation of the calibration constant $K = G^{-1}$ (Eq. (1)). Refer also to Fig. 9 for details about how $K$ is computed for these two pointing periods.
where \( \Delta T_{\text{dip}} \) seen by the radiometer during the \( i \)-th pointing, that is, \( w_i \propto \Delta T_{\text{dip}} \). According to our classification based on Eq. (24), this calibration scheme is relative (with \( \xi(t) = (V_{\text{ref},0} - V_{\text{ref}}(t))/V_{\text{ref},0} \)) but not absolute, as we need \( K_{\text{dip}}(t) \) in order to estimate \( K_0 \).

As an example, see Fig. 5, panel D (right), which shows the values of \( K_{\text{dip}} \) calculated for LFI27M (30 GHz radiometer). Unlike the case for LFI21M (panel D, left), which required a smoothing filter to be applied to the raw gains in panel A, here we use the total-power voltages without any filtering, as their error does not depend on the amplitude of the observed dipole in the sky, but only on the stability of the 4 K reference load, which is better by orders of magnitude.

When we presented Eq. (32), we began with the assumption that variations in time of the three terms \( T_{\text{ref}} \), \( \epsilon_{\text{iso}}\Sigma_{\text{sky}} \), and \( T_{\text{noise}} \) were negligible, in order not to induce significant systematic effects in the determination of the gain. We now quantify how much they are expected to change during the mission. (Refer to Sect. 2.1 for a general discussion of time scales and LFI calibration.) If we write \( V_{\text{ref}}(t) - V_{\text{ref},0} \) as \( \delta V_{\text{ref}} \), and similarly for \( \delta T_{\text{ref}} \),

\[ V_{\text{ref}}^{-1} = V_{\text{ref},0}^{-1} \left(1 + \frac{V_{\text{ref}} - V_{\text{ref},0}}{V_{\text{ref},0}}\right)^{-1} \]

\[ \approx V_{\text{ref},0}^{-1} \left(1 - \frac{V_{\text{ref}} - V_{\text{ref},0}}{V_{\text{ref},0}}\right) \]

\[ = V_{\text{ref},0}^{-1} \left(2 - \frac{V_{\text{ref}}}{V_{\text{ref},0}}\right) \]

where we applied a first-order Taylor expansion\(^{14}\) for \( V_{\text{ref}} \) around the value \( V_{\text{ref},0} \). We substitute it into Eq. (30) to derive the following equation:

\[ K_{4K}(t) \approx \frac{T_{\text{ref},0} + \epsilon_{\text{iso}}\Sigma_{\text{sky},0} + T_{\text{noise},0}}{V_{\text{ref},0}} \left(2 - \frac{V_{\text{ref}}(t)}{V_{\text{ref},0}}\right) \]

\[ (31) \]

\[ K_{\text{dip}}(t) = K_0 \times \left(2 - \frac{V_{\text{ref}}(t)}{V_{\text{ref},0}}\right) \]

\[ (33) \]

(\( \xi(0) \) represents the noise accuracy of the first-order approximation. Depending on what we mean \( V_{\text{ref},0} \), \( \epsilon_{\text{iso}}\Sigma_{\text{sky}} \), and \( T_{\text{noise}} \) should be negligible; we quantify their variation later.) In principle, Eq. (32) would be enough to estimate \( K \) at any time, if one knew all the terms in the right side of the equation with the desired accuracy (e.g., 0.1%). Since this is not our case, to find the unknown factor in the equation we use the estimate of \( K(t) \) from the dipole (see Sect. 3.1) to perform a weighted linear least squares from which we estimate \( K_0 \):

\[ \begin{align*}
\Sigma_{\text{sky}} &= B \times T_{\text{sky}} + D \times T_{\text{sky}}^{\text{noise}} \\
&\text{(the term } T_{\text{sky}}^{\text{noise}} \text{ represents the noise temperature associated with the measurement of the sky signal). Solving for } K = G^{-1} \text{ yields a formula that can be used to estimate } K \text{ from the ratio between the sum of temperatures and the 4 K reference voltage:}
\end{align*} \]

\[ K_{4K}(t) = \frac{T_{\text{ref}} + \epsilon_{\text{iso}}\Sigma_{\text{sky}} + T_{\text{noise}}}{V_{\text{ref}}} \approx \frac{T_{\text{tot}}}{V_{\text{ref}}}, \]

\[ (30) \]

where \( \epsilon_{\text{iso}} \) is the isolation (for its definition see Villa et al. 2010) and the term \( \epsilon_{\text{iso}}\Sigma_{\text{sky}} \) represents the amount of signal coming from the sky and leaking into the reference load signal. We express the term \( V_{\text{ref}}^{-1} \) as:

\[ \begin{align*}
V_{\text{ref}}^{-1} &= V_{\text{ref},0}^{-1} \left(1 + \frac{V_{\text{ref}} - V_{\text{ref},0}}{V_{\text{ref},0}}\right)^{-1} \\
&\approx V_{\text{ref},0}^{-1} \left(1 - \frac{V_{\text{ref}} - V_{\text{ref},0}}{V_{\text{ref},0}}\right) \\
&= V_{\text{ref},0}^{-1} \left(2 - \frac{V_{\text{ref}}}{V_{\text{ref},0}}\right)
\end{align*} \]

\[ (31) \]

\[ \begin{align*}
K_{\text{dip}}(t) &= K_0 \times \left(2 - \frac{V_{\text{ref}}(t)}{V_{\text{ref},0}}\right)
\end{align*} \]

\[ (33) \]

(\( \xi(0) \) represents the noise accuracy of the first-order approximation. Depending on what we mean \( V_{\text{ref},0} \), \( \epsilon_{\text{iso}}\Sigma_{\text{sky}} \), and \( T_{\text{noise}} \) should be negligible; we quantify their variation later.) In principle, Eq. (32) would be enough to estimate \( K \) at any time, if one knew all the terms in the right side of the equation with the desired accuracy (e.g., 0.1%). Since this is not our case, to find the unknown factor in the equation we use the estimate of \( K(t) \) from the dipole (see Sect. 3.1) to perform a weighted linear least squares from which we estimate \( K_0 \):

\[ K_{\text{dip}}(t) = K_0 \times \left(2 - \frac{V_{\text{ref}}(t)}{V_{\text{ref},0}}\right) \]

\[ (33) \]
...dipoles, then fit for a robust approach is to create a map of each section independently, apply the 4 K calibration model separately for each section of the sky maps. The time-ordered data are binned into rings for the 5-year and 7-year analyses (Hinshaw et al. 2009; Jarosik et al. 2011) an adequate solution for relative calibration was obtained using a simple parametrized fit based on detector voltages and hot and cold stage temperatures. For the LFI radiometers, which are cryogenically cooled to 20 K and use blackbody reference loads cooled to 4 K, a more complex thermal model is needed for a high precision parametric solution based on housekeeping information. In this analysis, the approximate method described in Sect. 3.2 is implemented, together with the OSG approach reported in Sect. 3.1.

Our starting point for using the orbital dipole is the time-ordered-data, since the direction of the induced dipole is in the direction the satellite is traveling, and it is therefore time-dependent. Furthermore, one must use TODs which have not been processed with the calibration module, since that module removes both dipoles from the data. As most of the pixels on the sky are observed with two observations separated by approximately six months, where the velocity is in opposite directions, the orbital dipole signal cancels out to first order on the full mission sky maps. The time-ordered data are binned into rings for each pointing period, based on the rotation phase angle of the satellite. In this way all the relevant information is stored in a more compact and accessible form, with the advantage that the orbital dipole appears as a near sinusoid variation with the same phase in each ring.

A good relative calibration is required to remove the gain differences between the rings. For this we use the gains used in the final analysis as these are the best available. For the 70 and 44 GHz channels this may appear to be potentially circular as they are already calibrated on the solar dipole, but tests introducing a scaling factor into the relative gains shows that the...
Table 4. Dipole residuals and correction factors applied to the 4 K calibration at 30 GHz in response to major thermal events.

<table>
<thead>
<tr>
<th>Time range [days]</th>
<th>Dipole LFI27 [µK]</th>
<th>Dipole LFI28 [µK]</th>
<th>Correction LFI27 [%]</th>
<th>Correction LFI28 [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>91-257</td>
<td>−17.03</td>
<td>−8.91</td>
<td>0.58</td>
<td>0.30</td>
</tr>
<tr>
<td>258-454</td>
<td>−17.62</td>
<td>−13.06</td>
<td>0.60</td>
<td>0.44</td>
</tr>
<tr>
<td>455-563</td>
<td>20.66</td>
<td>−6.71</td>
<td>−0.70</td>
<td>0.23</td>
</tr>
</tbody>
</table>

Table 5. Preliminary estimates of the CMB dipole in the 44 GHz channels.

<table>
<thead>
<tr>
<th>Horn</th>
<th>Arm</th>
<th>Correction</th>
<th>$v_{\text{sh}}$ [km s$^{-1}$]</th>
<th>$l$</th>
<th>$b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>M</td>
<td>1.0005</td>
<td>367.76</td>
<td>263.96</td>
<td>48.54</td>
</tr>
<tr>
<td>S</td>
<td>1.0000</td>
<td>368.03</td>
<td>263.96</td>
<td>48.50</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>M</td>
<td>1.0006</td>
<td>367.97</td>
<td>263.93</td>
<td>48.45</td>
</tr>
<tr>
<td>S</td>
<td>0.9967</td>
<td>369.35</td>
<td>264.01</td>
<td>48.28</td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>M</td>
<td>1.0025</td>
<td>367.44</td>
<td>263.87</td>
<td>48.42</td>
</tr>
<tr>
<td>S</td>
<td>1.0004</td>
<td>368.33</td>
<td>263.92</td>
<td>48.41</td>
<td></td>
</tr>
</tbody>
</table>

Notes. The ratio of expected orbital dipole to measured dipole.

The other approach we implemented is similar to the one adopted by WMAP and described by Bennett et al. (2003, 2013). The procedure is the following: (1) we smooth the maps at 1° angular resolution; (2) we subtract the estimated CMB signal, again smoothed at the common resolution; and (3) we fit the observed variation with Galactic latitude assuming a plane-parallel model for the Galactic emission. We consider only the behaviour on the southern hemisphere (in the range $-90^\circ < b < -15^\circ$) in order to avoid possible contamination from the high-galactic latitude structures present in the Galactic emission such as the North Polar Spur. We divide the map into stripes of constant latitude 2.5° wide and we evaluate the mean level of CMB subtracted signal. Then, this is fitted with a cosecant model of the form $T = A \csc b + B$. The offset is the value by which the intercept $B$ is zero.

The results yielded by the two methods are in agreement. We set the zero levels of the maps released by the Planck collaboration to the estimates obtained using the second method. The numerical values are reported in Table 8.

4. Systematic effects that affect calibration

Raw gains from the nominal pipeline are sensitive to various effects related both to previous data processing steps and to instrument characteristics. A full treatment of systematic effects in the current release of LFI data is provided by Planck Collaboration III (2014); here we concentrate only on those systems that have a significant impact on the calibration of the radiometers.

4.1. Impact of the optics on calibration estimation

The optical response of each radiometer, including the effect caused by the presence of the telescope and the baffles, impacts the estimation of the calibration constants. Apart from the effects due to the coupling of the CMB dipole with the full beam response described in Sect. 2.2, an important effect (particularly significant at 30 GHz) is the fact that the brightness of the Galactic plane produces a non-trivial spurious signal which biases the fit with the dipole.

Consequently, for the current release, we have decided to calibrate the 30 GHz channels using the 4 K calibration, as described in Sect. 3.2. This procedure avoids the spurious signals induced by the sidelobes (see Fig. 11). In a future release, we plan to use the fully modelled sidelobe patterns to correct the timelines in the iterative calibration sequence, thereby using the same calibration scheme for all three frequency bands; we also treat polarization calibration.

Note however that the two methods produce maps that show a remarkable agreement: their peak-to-peak difference, if we neglect a few outlying pixels, is of the order of a few tens of µK, thus of the same order of magnitude as the rms of the survey difference maps discussed in Sect. 5.3.2.
the sky, under the simplifying hypothesis that only depends on the beam shape. The maximum deviation of the e
calculate the data. We describe the mathematical details of the model in Appendix A; here we provide a
In Sect. 2.2 we mentioned that the procedure is based on a convolution of a reference dipole with the beam shape. This procedure
can be time-consuming, as it must be done once for each sample measured by each radiometer. Fortunately, the spatial symmetry of the dipole signal allows us to derive a fast and elegant method to properly take the shape of the full beams into account when using the dipole to calibrate the data. We describe the mathematical details of the model in Appendix A; here we provide a physical interpretation.

The model applies a convolution with the full beam to Eq. (25), and it takes advantage of the high level of symmetry in the dipole to greatly simplify the calculation. The result of the convolution is still a dipole signal, but its direction and amplitude are slightly different from the dipole before the convolution, for the following reasons:

1. the amplitude changes because of all the power entering the beam from directions other than the beam axis;
2. the direction changes because of asymmetries in the beam shape.

Such changes are characterized by a constant vector $S$, which only depends on the beam shape. The maximum difference between the amplitudes of the convolved dipole and of the dipole in the sky, under the simplifying hypothesis that $\theta_{\text{Sun}} + \theta_{\text{Planck}} \approx \theta_{\text{Sun}}$, is:

$$\Delta T_{\text{sc}} = T_{\text{CMB}} \left| \frac{\theta_{\text{Sun}}}{c} \right| (1 - |S|).$$

(36)
The direction of $S$ in the beam reference frame represents the deviation of the effective axis of the dipole signal with respect to the axis of the dipole in the sky. A convenient measure of the overall deflection is represented by the deflection angle

$$\delta_z = \arccos(e_z \cdot S/|S|).$$

(37)

where $e_z$ is the nominal direction of the beam.

Figure 12 plots the corresponding $\delta_z$ versus $\Delta T_{\text{sc}}$ for each detector, and the numerical values are tabulated in Table 6. There is an evident linear correlation between the two quantities, with the best fit being

$$\delta_z = 1.2368 \Delta T_{\text{sc}} + 0.2329.$$  (38)

The deflection angles are between $1/3$ and $12/6$, so that the largest correction is $13 \mu K$. It is interesting to note the slight level of asymmetry between the side and main arms of each feed-horn. This might lead to differences in corrections nearly as large as $1 \mu K$. An example is given by the two arms of LFI28. In contrast, the two arms of LFI21 show no asymmetries.

As we already stated in Sect. 2.2, for this data release we have neglected $\Delta T_{\text{sc}}$ (thus setting $|S| = 1$), but we have considered the tilt effect given by $\delta_z$. The tilt was considered not only when using Eq. (25) (i.e., fitting times-streams with the expected dipole signal), but also when we cleaned the calibrated data of the dipole itself (see point 7 in Sect. 3.1).

### 4.3. Imperfect behaviour of the ADC

As stated in Sect. 3, the calibration process also includes the conversion of the analogue detector voltages by the ADCs. This should be absorbed into the final radiometer calibration, but the ADCs can suffer a differential linearity problem, where the voltage step between binary levels varies slightly. The ADC employed use a digital to analogue circuit (DAC) and a successive approximation algorithm to match the voltages, so an imperfect step associated with a particular binary bit will lead to a recurring non-linear glitch when that bit changes. This can lead to a small but sharp deviation in the response curve as shown in Fig. 10 of Planck Collaboration III (2014). Since the slope of

<table>
<thead>
<tr>
<th>Radiometer</th>
<th>$\delta_z$</th>
<th>$\Delta T_{\text{sc}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LFI18M</td>
<td>5.94</td>
<td>7.19</td>
</tr>
<tr>
<td>LFI18S</td>
<td>7.35</td>
<td>9.10</td>
</tr>
<tr>
<td>LFI19M</td>
<td>6.83</td>
<td>8.49</td>
</tr>
<tr>
<td>LFI19S</td>
<td>7.92</td>
<td>10.09</td>
</tr>
<tr>
<td>LFI20M</td>
<td>9.43</td>
<td>11.56</td>
</tr>
<tr>
<td>LFI20S</td>
<td>10.10</td>
<td>12.65</td>
</tr>
<tr>
<td>LFI21M</td>
<td>9.46</td>
<td>11.59</td>
</tr>
<tr>
<td>LFI21S</td>
<td>9.78</td>
<td>12.31</td>
</tr>
<tr>
<td>LFI22M</td>
<td>6.96</td>
<td>8.62</td>
</tr>
<tr>
<td>LFI22S</td>
<td>8.20</td>
<td>10.38</td>
</tr>
<tr>
<td>LFI23M</td>
<td>5.68</td>
<td>6.90</td>
</tr>
<tr>
<td>LFI23S</td>
<td>7.23</td>
<td>8.98</td>
</tr>
<tr>
<td>70 GHz (mean)</td>
<td>7.91</td>
<td>9.82</td>
</tr>
<tr>
<td>LFI24M</td>
<td>2.37</td>
<td>2.95</td>
</tr>
<tr>
<td>LFI24S</td>
<td>1.43</td>
<td>1.98</td>
</tr>
<tr>
<td>LFI25M</td>
<td>1.31</td>
<td>1.95</td>
</tr>
<tr>
<td>LFI25S</td>
<td>0.88</td>
<td>1.49</td>
</tr>
<tr>
<td>LFI26M</td>
<td>1.31</td>
<td>1.91</td>
</tr>
<tr>
<td>LFI26S</td>
<td>0.90</td>
<td>1.44</td>
</tr>
<tr>
<td>44 GHz (mean)</td>
<td>1.37</td>
<td>1.95</td>
</tr>
<tr>
<td>LFI27M</td>
<td>8.54</td>
<td>11.11</td>
</tr>
<tr>
<td>LFI27S</td>
<td>7.92</td>
<td>10.78</td>
</tr>
<tr>
<td>LFI28M</td>
<td>8.66</td>
<td>11.25</td>
</tr>
<tr>
<td>LFI28S</td>
<td>7.92</td>
<td>10.77</td>
</tr>
<tr>
<td>30 GHz (mean)</td>
<td>8.26</td>
<td>10.98</td>
</tr>
</tbody>
</table>

Notes. (a) Deflection in the polar axis of the dipole, in arcmin (Eq. (36)). (b) Correction to the amplitude of the dipole, in $\mu K_{\text{CMB}}$ (Eq. (37)).
the response curve determines the effective calibration, a large
gradient change can happen over a small range of voltage, po-
tentially becoming the dominant source of gain uncertainty.
Such an effect is seen in the LFI radiometers for some chan-
nels, especially those with low back-end gain, where the quanti-
zation levels will be more evident. For this reason the radiometers
of 44 GHz are the most affected, as shown in the upper
panel of Fig. 13 for the worst affected LFI25S-11 channel (for
more information, refer to Planck Collaboration III 2014).
The correction for this effect requires the reconstruction of the
response curves, which can be inferred from the related differ-
ce in behaviour between white noise and detector voltages.
The reason is that the white noise, like the dipole and sky tem-
perature fluctuations, is sensitive to the gradient of the response
curve, whereas the detector voltages are just determined by the
point on the curve. This is explained in Appendix A of Planck
Collaboration III (2014) and an example for LFI25S-11 is shown
in that paper together with estimates of ADC residual errors
in terms of systematic effects at the map level. The implementa-
tion in the pipeline uses splines18, fitted to pre-calculated template
curves of the non-linearities. The result of correcting the TOD
data is shown in the lower panel of Fig. 13, where the dipole
gains and inverse sky voltage and white noise are much more
consistent.
In the case where calibration used the level of the 4 K load,
the uncorrected data would produce uncertainties directly in the
calibration of the order of 5% leading to stripes of about 150 μK,
due to the solar dipole being incorrectly removed. Using a dipole
based calibration would correct the ADC non-linearity, but the
need to low-pass filter it for an acceptable S/N would mean
that gain variations shorter than the filter timescale were not
corrected. This in turn would result in fine striping at the level of
20–30 μK. Further, since the ADC non-linearity can differ be-
tween the two arms of a radiometer, there is also a direct impact
on polarization through differential calibration errors. This is il-
ustrated in Fig. 14, where we show the difference between maps
made from the main and side arms of radiometer LFI25.

It should be pointed out that the LFI and HFI ADC non-
linearity problems are completely different, due to the diffe-
rent implementations of fast switching schemes. LFI use phase
switching to swap sky and reference load signals, whereas HFI
use a square wave modulation and AC coupling to avoid 1/f
contributions from the back-end electronics. More details about
the ADCs used by HFI are found in Planck Collaboration VIII
(2014).

18 We use the spline functions provided by the GNU Scientific Library
(Galassi et al. 2003).
Fig. 14. Three degree smoothed map of the difference between the main and side arm of LFI25 before (top) and after (bottom) ADC non-linearity correction. Stripes can be seen which are stronger in the region where the solar dipole peaks, as a result of residuals from calibration uncertainties. This relative uncertainty between radiometer arms directly affects polarization as shown here, since the main and side arm difference represents a pseudo-polarization.

5. Accuracy of the calibration

In this section we present results of consistency checks sensitive to calibration inaccuracies, as well as simulations which quantify the estimated level of calibration systematics in the data. Table 7 shows an overview of the elements that have been taken into account in estimating the calibration accuracy for LFI maps. Table 8 quantifies the impact of each element in the overall accuracy budget for the calibration.

5.1. Statistical uncertainty

5.1.1. Propagation of statistical errors from WMAP’s dipole estimate

As stated in Sect. 2.2, we have calibrated LFI data using the CMB dipole as measured by WMAP (Jarosik et al. 2011). In our calibration we used both the speed of the solar system in the CMB rest frame ($v_{\text{Sun}}$ in Table 3) as well as the direction of motion ($l$, $b$). In this section we estimate how the quoted errors on these parameters propagate through the calculation of the calibration constant $K_{\text{dip}}$ in Eq. (27).

We ran the LFI calibration pipeline a number of times for a subset\(^\text{19}\) of radiometers; for each run we changed either $v_{\text{Sun}}$ or $l$ from the nominal value listed in Table 3, and we compared the new average level $\langle K'_{\text{dip}} \rangle$ of the calibration constant with respect to the level $\langle K_{\text{dip}} \rangle$ of the nominal calibration. We also ran these tests without applying the iterative calibration code that cures the bias induced by the CMB and Galactic signal in dipole fitting (Mademoiselle; see point 5 in Sect. 3.1), in order to quantify the improvement in the stability of the calibration achieved by the iterative code.

Figure 15 shows the results of our simulations in the case of LFI27M (results for the other radiometers are in nearly perfect agreement with this). Mademoiselle’s ability to reduce the discrepancy $\langle K'_{\text{dip}} \rangle-\langle K_{\text{dip}} \rangle$ is clear if one compares the error on $K_{\text{dip}}$ with and without using Mademoiselle. Considering WMAP’s uncertainties on $v_{\text{Sun}}$ and $l$ and the results of our simulations, we

\(^{19}\)Specifically, we considered LFI18M, LFI18S (70 GHz), LFI24M, LFI24S (44 GHz), LFI27M, and LFI28M (30 GHz).
can estimate that the relative error $\sigma_{K_{\text{dip}}} / K_{\text{dip}}$ due to the uncertainties in $r_{\text{Sun}}$, $l$, and $b$ is

$$\sigma_{K_{\text{dip}}} / K_{\text{dip}} = 0.19\% \left( r_{\text{Sun}} \right) + 0.05\% \left( l \right) + 0.01\% \left( b \right) = 0.25\%,$$

where the error for $b$ has been estimated by simple proportionality using the error on $l$. (These numbers are the same for all the radiometers we analysed.)

**5.1.2. Simulation of noise in dipole fitting**

We have run a set of simulations to assess how much the statistical noise in the radiometric signal affects gain reconstruction using the CMB dipole signal. We first decalibrate the input map or dipole with a fiducial gain, the $4\,\text{K}$ calibration, and then we simulate how the dipole calibration pipeline reconstructs those gains, i.e., we add the statistical noise, cut the low-dipole regions and then apply a smoothing filter.

We simulated separately the impact of the statistical noise acting on the modelled dipole and the Galaxy/CMB contribution; we model the input dipole using the WMAP direction and velocity for the orbital dipole. Moreover, we used Planck full frequency maps as the input Galactic and dipole-removed CMB signal.

We use a fiducial gain in order to make our simulation as realistic as possible, including the impact of the calibration pipeline on the underlying “true” gain. The $4\,\text{K}$ calibration (see Sect. 3.2) offers a good choice, because it also includes realistic fluctuations on shorter timescales, which are expected to be smoothed out by the dipole calibration algorithm.

After decalibrating the input signal with the fiducial gain, we add a realization of the expected statistical gain error coming from the dipole fit due to white noise and to Galaxy masking.
We model the statistical gain error according to the following equation:

\[ \sigma_{G_i} = \frac{\sqrt{2}\sigma_0}{D_i \sqrt{\eta_{\text{mask},i}}} \sqrt{1 + \left( \frac{f_0}{f_{\text{knee}}} \right)^\alpha}, \]  

(40)

where \( i \) is an index identifying a pointing period, \( \sigma_{G_i}/G_i \) is the relative error on the gain \( G_i \), \( \sigma_0 \) is the noise-equivalent temperature of the radiometer, \( D_i \) is the amplitude of the dipole signal during pointing period \( i \) (in Rayleigh-Jeans units), \( \tau_i \) is the integration time of pointing period \( i \), \( 0 \leq \eta_{\text{mask},i} \leq 1 \) is a dimensionless coefficient that takes into account the loss of integration time due to Galaxy masking within pointing period \( i \), \( f_0 \) is the frequency of the dipole sinusoid seen by Planck’s scanning strategy (\( f_0 = 16.7 \text{ mHz} \)), and \( f_{\text{knee}} \) and \( \alpha \) are the constants that characterize the 1/f noise for a given radiometer. The output of this step is equivalent to the “raw” dipole calibration as computed by the Mademoiselle iterative calibration software (see Sect. 3.1, point 5, and the Planck Collaboration II 2014).

The last step is to cut the low-dipole regions, replace them with a straight line, and then heavily smooth the data with a moving average filter of about 200 pointing periods. This is a simplified version of the wavelet-based filtering in the pipeline (Sect. 3.1, point 6). We run destripping on the output timelines and produce surveys and nominal mission maps of the noise for each frequency. Figure 16 shows the power spectrum of the artefacts produced by this simulation for the same timespan covered by this Planck data release.

5.2. Impact of known systematics

5.2.1. ADC non-linearities

We have run a set of simulations on the non-linearity of the ADCs (see Sect. 4.3) in order to quantify its impact on the calibration of LFI. The simulations are described by Planck Collaboration III (2014), but we briefly recall the procedure here.

After we characterized the non-linearity of the ADCs to estimate the response curve \( R(V') \) of the ADC for the 44 converters, we produced time-ordered data by summing Galactic, dipole, and CMB signals, as well as 1/f noise. To do this we considered Planck’s scanning strategy and the measured characteristics of each LFI detector. We de-calibrated the TODs using the same gains used to calibrate flight data. We then produced two sets of maps:

1. the first set uses these TODs and produces maps, which are immune from the ADC effects;
2. the second set filters the TODs using the inverse of the response function \( R \) in order to simulate the ADC non-linearity, and then uses the same procedure used for flight data to estimate a new response function, \( R' \) (obviously, \( R \approx R' \)); finally the ADC correction code is applied to the data, and new maps are produced.

We considered the difference between the two sets of maps to be a reasonable estimate of the level of errors due to the uncertainties in the procedure for estimating ADC non-linearities. These differences reveal that such uncertainties introduce an error in the estimation of the calibration constant \( K \) (Eq. (1)) which is of the order of 0.1%.

5.2.2. Beam uncertainties

In Table 8 we quantify the error due to the fact that we considered monochromatic beams when estimating the vector \( S \) in Eq. (A.5). We have estimated the beam response of LFI’s radiometers (see Sect. 4.2) over their bandwidth using the GRASP software (see Planck Collaboration III 2014, for more information about this) and we have computed a new set of vectors \( S'(\nu) \), which of course depend on the frequency \( \nu \). The errors we report under “Beam uncertainties” are equal to the average of the value \( ||S|| - ||S'(\nu)|| \) for the 22 radiometers, grouped according to their centre frequency.

5.3. Impact of unknown systematics

To estimate the impact of all those systematics which affect the calibration, but for which we do not have simulations or direct measurements at hand, we relied on internal consistency checks, namely inter-channel comparisons and null tests. We explain the logic behind each analysis as well as the results in the next paragraphs.

5.3.1. Inter-channel calibration consistency

A sensitive way to test the relative calibration among the different LFI radiometers is to make independent maps and compare their angular power spectra in the vicinity of the CMB acoustic peak. This has the advantage of testing the calibration on a source with the relevant amplitude and spectrum for our cosmology goals. To reduce the effects of noise bias on the power spectrum estimation, we intercompare the radiometers using cross-spectra; since the noise is quite uncorrelated among the radiometers, we obtain clean spectra this way. In order to compare any pair of radiometers, we first mask the Galaxy, point sources, and all unobserved pixels. We then calculate the cross-spectrum of maps made by each radiometer with maps of a third radiometer, and take the ratio. By comparing all possible combinations we average over any small impact from the choice of the third map. For example:

\[ A_{\text{LFI18S,LFI19M}} = \frac{C_{\text{LFI18S,LFI19M}}}{C_{\text{LFI18S,LFI18M}}} \]

In this case, “other” runs over LFI18M, LFI18S, and both radiometers of horns 20, 21, 22, and 23.
We show the results of this in Fig. 17, which captures the relative gain deviation among all the radiometers, based on the first acoustic peak. The horizontal axis is the horn number, with points representing the main and side arm radiometers placed just left and just right of the corresponding horn number. Figure 17 also includes the gain deviations averaged over the full frequency, labelled as $f_{30}$, $f_{44}$, and $f_{70}$. LFI 70 GHz full frequency at 70% sky coverage is taken as reference.

Figure 18 shows the results of a similar comparison between surveys, in this case comparing the full frequency estimates of the acoustic peak for surveys 1, 2, and 3. The full frequency comparison is even more striking, with each band remaining within ±0.15% over the three independent surveys. Of course, many systematics which plague calibration may be common to all channels, particularly within a frequency cohort; however, $1/f$ noise, receiver degradation, and certain other sources of systematic uncertainties in the temperature calibration seem to be controlled to below 0.15%.

5.3.2. Null tests

We have developed a number of tests to highlight possible issues related to instrumental systematic effects which are not properly corrected within the pipeline, or are related to known changes in the operating conditions of the instrument or to intrinsic instrument properties coupled with the sky, like stray-light from sidelobes. Such tests are also discussed by Planck Collaboration III (2014); here we concentrate on how those tests can be used to estimate the impact of calibration errors in LFI maps.

The idea behind the many tests we ran for this Planck data release is to compare pairs of distinct estimates of the sky signal, that should ideally be the same, such as:

- comparing the sky signal measured during two surveys (not necessarily consecutive), we call this kind of test survey-difference null tests;
- comparing the sky signal measured by two radiometers, we call this a radiometer-difference null test;
- comparing the map produced using only data taken in the first half of each pointing period with data taken in the other half. This is a half-ring null test.

Clearly, it is not trivial to determine the cause of each discrepancy in each comparison. The following criteria are useful in interpreting the results of these null tests:

1. The results of radiometer-difference null tests can reveal artefacts due to imbalances in the frequency response, as well as to their different optical systematics. Such optical effects can be caused by an error in pointing reconstruction (which is therefore unrelated to calibration), but also by two different ways of picking up the astrophysical signal (including the dipole) through the sidelobes. The latter effect can be critical for calibration, as the subtraction of an improper dipole signal leads to unwanted residuals in the map.

2. Survey-difference null tests are not affected by frequency response mismatches. However, they can be affected by the optical effects described above. Because of the scanning strategy employed by the Planck spacecraft, such systematics will be present only if an odd number survey is compared with an even survey, or vice versa, since odd surveys share the same scanning strategy, and the same applies for even

---

20 These masks are the same used for component separation (Planck Collaboration XII 2014).
surveys. (This is true for the first four surveys, including those considered here, but not for subsequent surveys, where the precession phase angle was changed.)

3. Half-ring null tests are able to spot systematics at high frequency (more than 1/60 Hz, as this is the spinning frequency of the \textit{Planck} spacecraft). Thus, calibration errors are unlikely to affect this kind of test, since we expect the gain of the radiometers to change on much longer time scales (days or weeks). In fact, we expect half-ring null test to be sensitive only to the level of unavoidable white noise in the data.

For all the reasons stated above, we believe that a fair upper limit of the calibration errors that are unrelated to optical effects can be deduced by studying odd-odd and even-even survey-difference null tests, as they are not affected by pointing errors, imbalance in frequency response, or “fast” systematics. Even-odd and odd-even differences are affected by systematics caused by optics as well. Therefore, these should be larger than the former and can also be used to produce upper limits for the calibration accuracy. Figure 19 shows two examples.

We have used odd-odd survey difference maps to produce the numbers reported in the row “Unknown systematics” in Table 8. These numbers are the rms value of the pixels, after the maps have been degraded to a resolution which roughly coincides with the FWHM of the LFI beams. We subtracted from these values the part due to statistical noise, estimated from half-ring difference maps using the procedure described by \textit{Planck Collaboration III} (2014). As noted above, these rms values represent upper limits for calibration inaccuracy, because some of the systematics affecting survey difference maps are unrelated to calibration. Values under the “Total” group have been calculated from odd-even survey difference maps; as noted above, they include optical systematics as well.

We show the power spectra of the even-odd and odd-odd null tests in Fig. 20, where we compare them with the estimated statistical errors of the gains (Sect. 5.1) and the spectra of half-ring maps. The latter provide an estimate of the level of statistical noise in the maps and generally agree with survey difference spectra for $\ell > 20$. This means that for a wide range of multipoles we do not expect any significant contribution of calibration systematics on the maps. For $\ell < 20$ there is some disagreement for the 30 GHz maps. This might indicate the presence of residual optical systematics in the data. In fact, the chief reason that induced us to use a different calibration for 30 GHz channels (using the 4 K total-power output, see Sect. 3.2) was to minimize the impact of optical systematics on the calibration of these channels (which suffer from a relatively high level of sidelobes).

5.4. Checks with external calibrations

5.4.1. Using bright point sources for relative calibration

In order to perform an independent check of the relative calibration between different RCAs and also to test both the colour corrections and central frequencies of each LFI band, we look at three bright radio sources in the LFI maps: Orion A (M 42); Taurus A; and Cassiopeia A\footnote{Orion A and Tau A are not expected to vary significantly over the time scales relevant here. Cas A has a secular decrease of $-0.6\%$ at high frequencies (O’Sullivan & Green 1999), which over the five year time difference between the averaged \textit{WMAP} and \textit{Planck} datasets will add to $3\%$. This has not been taken into account in this analysis; however, it has a negligible effect on the measured spectral index of the source that is well within the uncertainties.}. The emission in these three sources is due to synchrotron or free-free emission, such that they have very different spectra from the CMB. This means that colour corrections are important at the level of about 3–10\% depending on the RCA, which is in excess of the uncertainties on the flux densities of these sources.

Comparisons between ERCSC flux densities (\textit{Planck Collaboration VII} 2011) at 30, 44, and 70 GHz and interpolated or extrapolated flux densities from the Karl G. Jansky Very Large Array (VLA) follow-up observations have previously shown good statistical consistency at the 3\% level (\textit{Planck Collaboration XIV} 2011; Kurinsky et al. 2013; \textit{Planck Collaboration XXVIII} 2014). A comparison has also been made between the \textit{Planck}-ATCA coeval observations (PACO) and PCCS catalogues in \textit{Planck Collaboration XXVIII} (2014), which finds consistency at the level of around 4\%. However, we ideally want to test the calibration much more accurately than this, which can be accomplished comparing different LFI measurements and by comparing these results to \textit{WMAP}. Additionally,
the sources that we examine here are significantly less variable than the typical source in the VLA and PACO tests.

We use the Planck single detector maps for each survey. We first average the main and side maps to create individual RCA maps per survey, which has the effect of cancelling out the polarization such that the maps are of total intensity only. We look at the individual survey maps, and the average of surveys 1–3. We also make use of the WMAP-9 deconvolved Stokes I sky maps (Bennett et al. 2013).

We measure source emission from the maps using aperture photometry, as described in Planck Collaboration XX (2011), with an aperture of 70′ radius and a background annulus of 80′–100′. By keeping the aperture and annuli sizes constant rather than changing them to reflect the resolution of the instrument, we ensure that we are measuring the same source extent and subtracting the same background emission at all frequencies, since the background may vary with distance due to the structure of the extended source, or background emission surrounding it. A correction factor for the source flux that falls outside of the aperture is applied as per Planck Collaboration XXVIII (2014); this factor is non-negligible for the WMAP data points. The measured emission, in $T_{\text{CMB}}$, is converted to $T_B$, and then into Jansky at the nominal frequency (28.4, 44.1 or 70.4 GHz as appropriate). The uncertainty in the flux densities is calculated using the rms of the pixels in the background annulus.

The properties of the sources are summarized in Table 9. The flux densities for the sources in the first section of the table are the APERFLUX flux densities from PCCS. We only use these here for consistency checks, however, as the PCCS flux densities are the average across all radiometers of a given frequency rather than the flux densities as measured by each radiometer. We note that there is a difference in reported flux densities from WMAP between Page et al. (2007) and Weiland et al. (2011) of up to around 8%, due to the method used: Page et al. (2007) used aperture photometry, and Weiland et al. (2011) fitted the beams to the sources. As such, care needs to be taken when comparing the numbers from the different analyses.

In order to calculate the necessary colour corrections, we iteratively calculate the amplitude (at a reference frequency of 33 GHz) and spectral index of the source based on the Planck and WMAP aperture photometry flux densities, the values for which are quoted in Table 9. We then use this spectral index to colour-correct the flux densities per RCA using quadratic fits to the colour correction values given in Sect. 2.3. The average flux densities, and the standard deviation calculated between the RCAs, both before and after colour correction, are given in Table 9. The percentage of the standard deviation to the mean is also given. When the colour corrections are applied, there are a few significant improvements in the standard deviations and no relevant degradation, giving confidence that the colour corrections are improving the consistency between the RCAs.

Figure 21 shows the spectra of the three sources (left-hand column) and the flux densities from individual survey maps for the different RCAs (right-hand column). For Orion A, two sets of
Fig. 21. Spectra (left) and per-survey and per-RCA measurements (right) for Orion A (top), Cas A (middle) and Tau A (bottom). For the spectra, the flux densities measured using aperture photometry are shown for LFI and WMAP, as well as the best-fitting power-law to the measurements (solid line) and a ±1% range (dotted line). The fit and data points from Weiland et al. (2011) are shown for Cas A and Orion A, and the Planck Catalogue of Compact Sources (PCCS) flux densities for all three sources are shown. For the per-RCA measurements, RCAs 18–23 are at 70 GHz, 24–26 at 44 GHz and 27–28 at 30 GHz. For each RCA, the average from the three surveys is shown, followed by the measurements from each survey.

points differ significantly from the spectrum from aperture photometry. At WMAP 93.5 GHz, the measured flux density is much higher as it includes dust emission, which is not present at the LFI frequencies and as such is not considered here. Additionally, the PCCS aperflux is much steeper ($\alpha = -0.21$) than the best fit to the aperture photometry here; this is most likely due to the different sized apertures used, which depend on frequency for PCCS, meaning that it will systematically exclude the more diffuse emission from the source than is included in this analysis. This is particularly relevant for Orion A due to its more complex morphology. For Cas A, we find very good agreement between the results. For Tau A, we find a slightly flatter spectrum ($-0.287 \pm 0.013$) than the PCCS and Weiland et al. (2011) ($-0.302 \pm 0.005$), and other analyses also report steeper spectra, e.g. Hafez et al. (2008) find $-0.32 \pm 0.01$ at 30 GHz, and Macías-Pérez et al. (2010) find $-0.296 \pm 0.006$; this is again likely due to the different aperture sizes considered here.

Although aperture size affects the comparison of the absolute flux densities, it does not affect the results between the different Planck RCAs at the same frequencies, or between the different survey maps for the same RCAs. There is general consistency for all RCAs between the surveys, within the uncertainties. Note that some of the sources were not observed in every survey due to the survey definitions, for example Cas A was not observed by
Table 9. Properties of three compact calibrators used to check the calibration: Orion A (M 42); Cas A; and Tau A (M 1).

<table>
<thead>
<tr>
<th>Property</th>
<th>Orion A</th>
<th>Cas A</th>
<th>Tau A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position (l, b)</td>
<td>(209°01, −19°38)</td>
<td>(111°74, −02°14)</td>
<td>(184°56, −5°78)</td>
</tr>
<tr>
<td>( S_{28.4 \text{GHz}} )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{PCCS} )</td>
<td>( [\text{Jy}]^b )</td>
<td>365 ± 2</td>
<td>192.0 ± 1.4</td>
</tr>
<tr>
<td>( \text{PCCS} )</td>
<td>( [\text{Jy}]^c )</td>
<td>341 ± 3</td>
<td>137.0 ± 1.2</td>
</tr>
<tr>
<td>( \text{PCCS} )</td>
<td>( [\text{Jy}]^c )</td>
<td>301 ± 3</td>
<td>102.3 ± 0.5</td>
</tr>
<tr>
<td>( A^b )</td>
<td>370.5 ± 1.7</td>
<td>179.9 ± 0.8</td>
<td>319.1 ± 1.0</td>
</tr>
<tr>
<td>( \alpha^b )</td>
<td>−0.026 ± 0.014</td>
<td>−0.668 ± 0.014</td>
<td>−0.287 ± 0.013</td>
</tr>
</tbody>
</table>

Notes. (a) Flux densities from PCCS, without colour correction. (b) Coefficients for the fitted power law, \( S = A(\nu/33 \text{GHz})^b \). (c) Average and standard deviation of the flux densities measured by photometry within a fixed aperture. (d) Average and standard deviation of the colour-corrected aperture photometry flux densities.

Table 10. Average ratio of flux densities for RCA pairs.

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Discrepancya</th>
<th>( \delta C_{\alpha = -1} )</th>
<th>( \delta C_{\alpha = 0} )</th>
<th>Corrected discrepancyb</th>
</tr>
</thead>
<tbody>
<tr>
<td>70 GHz</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \delta_{\text{RCA15}/\text{RCA18}} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \delta_{\text{RCA20}/\text{RCA18}} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \delta_{\text{RCA21}/\text{RCA18}} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \delta_{\text{RCA22}/\text{RCA18}} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \delta_{\text{RCA23}/\text{RCA18}} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \delta_{\text{RCA26}/\text{RCA18}} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \delta_{\text{RCA27}/\text{RCA18}} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>44 GHz</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \delta_{\text{RCA24}/\text{RCA26}} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \delta_{\text{RCA25}/\text{RCA26}} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30 GHz</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \delta_{\text{RCA27}/\text{RCA28}} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes. (a) Value of \( S_A/S_B = 1 \), calculated over the whole sky. (b) Value of the column “Discrepancy” plus the difference of colour corrections for \( \alpha = -1 \) (see Table 1).

In order to better characterize these differences, we have studied how much the ratio \( S_A/S_B \), the average ratio between compact source flux densities calculated using RCAs A and B, differs from unity. In addition to aperture photometry, we have estimated this ratio using two other methods:

1. We used the template fitting code from Davies et al. (2006), run in a simplified way (i.e., without making use of a covariance matrix for noise or the CMB, such that it essentially does a least-squares fit), to compare “template” maps of RCA 28 (30 GHz), RCA 26 (44 GHz) and RCA 18 (70 GHz) with the other maps at the same frequencies. The maps have been smoothed to \( 1^\circ \), and a constant offset is used as a second template to remove any differences in the offsets between the template and the map. This provides an estimate of the ratio \( S_A/S_B \) for the all-sky coefficients between RCAs. We have run this analysis on the full, unmasked sky and the Galactic plane (\( |b| < 10^\circ \)), which yields consistent results with those obtained through aperture photometry; in addition we have used the WMAP KQ75 mask on 3°-smoothed maps and find consistent results at 30 GHz, but better agreement at 44 and 70 GHz, where the maps are more dominated by the CMB.

2. We combined pairs of RCA maps according to the formula \( m_A - \gamma m_B \), with \( m_A \) and \( m_B \) the arrays of pixels for the two maps, and visually compared the three sources considered above. We found those values of \( \gamma \) that either made asymmetries in the residual beam patterns around point sources disappear (for 30 and 44 GHz), or that produced a source difference with the noise uncertainty (70 GHz). We then assumed that \( \gamma \approx S_A/S_B \). Although this method is rather qualitative, the values we obtained differ from aperture photometry estimates by only about 1%.

Table 10 shows a comparison of the average fractional difference between source densities calculated using two RCAs, e.g., RCA 18 vs. RCA 19. The table reports an estimate of the percentage discrepancy between the channels (calculated simply by averaging the discrepancies found by aperture photometry, template fitting and visual inspection of sources), as well as the discrepancy minus the expected difference in colour corrections between the RCAs. We initially assume \( \alpha = -1 \) and tabulate the difference in colour corrections as \( \delta C \) in Table 10. The “corrected discrepancy” in column 5 is the sum of \( \delta C \) and the raw discrepancy. We also give \( \delta C \) for \( \alpha = 0 \) for comparison; these two values of \( \alpha \) span the range of spectral indices of the sources considered. The expected uncertainties on the colour corrections for an RCA for \( \alpha = -1 \) are 0.42%, 1.38% and 1.53% for 30, 44, and 70 GHz, respectively, such that the effects shown in Table 10 are 4\( \sigma \) at 30 and 70 GHz.

As the consistency between the RCAs is seen to be significantly better when comparing CMB emission between the RCAs (e.g., Sect. 5.3.1), we conclude that the effects seen here are likely due to differences in bandpasses or the intermediate beams between the RCAs, although neither of these is expected to be sufficiently large to explain the differences seen here. These effects will be investigated further prior to the next release of Planck data.

5.4.2. Planet flux densities

Planets can be used as a source for calibration cross-checks. In particular, Jupiter has several advantages: its signal is up to fifty times higher than the receiver noise per scan; it is visible to
Table 11. Visibility times of planets.

<table>
<thead>
<tr>
<th></th>
<th>30 GHz</th>
<th>44 GHz</th>
<th>70 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mars</td>
<td>09/10/18–21</td>
<td>09/10/18–21</td>
<td>09/10/20–24</td>
</tr>
<tr>
<td></td>
<td>10/04/17–20</td>
<td>10/04/19–03</td>
<td>10/04/15–18</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10/04/09–12</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>10/04/18–20</td>
<td></td>
</tr>
<tr>
<td>Jupiter</td>
<td>09/01/29–11/02</td>
<td>10/09/29–11/02</td>
<td>10/09/29–11/02</td>
</tr>
<tr>
<td>Saturn</td>
<td>09/01/07–11/02</td>
<td>10/09/07–11/02</td>
<td>10/09/07–11/02</td>
</tr>
<tr>
<td></td>
<td>09/09/07–11/02</td>
<td>10/09/07–11/02</td>
<td>10/09/07–11/02</td>
</tr>
<tr>
<td>Uranus</td>
<td>09/01/07–11/02</td>
<td>10/09/07–11/02</td>
<td>10/09/07–11/02</td>
</tr>
<tr>
<td></td>
<td>09/09/07–11/02</td>
<td>10/09/07–11/02</td>
<td>10/09/07–11/02</td>
</tr>
<tr>
<td>Neptune</td>
<td>09/01/07–11/02</td>
<td>10/09/07–11/02</td>
<td>10/09/07–11/02</td>
</tr>
<tr>
<td></td>
<td>09/09/07–11/02</td>
<td>10/09/07–11/02</td>
<td>10/09/07–11/02</td>
</tr>
</tbody>
</table>

Notes. Dates are in the format YY/MM/DD. Only the times between 2009 Aug. 13 and 2010 Nov. 28 are reported here.

Table 12. Averaged brightness temperatures for planets*. We have applied no correction for planetary disk obscuration.

<table>
<thead>
<tr>
<th></th>
<th>30 GHz</th>
<th>44 GHz</th>
<th>70 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mars</td>
<td>183 ± 1</td>
<td>187 ± 10</td>
<td>183 ± 3.5</td>
</tr>
<tr>
<td>Jupiter</td>
<td>134 ± 0.5</td>
<td>148 ± 0.5</td>
<td>157 ± 0.5</td>
</tr>
<tr>
<td>Saturn</td>
<td>121 ± 3.5</td>
<td>128 ± 3.0</td>
<td>131 ± 2.5</td>
</tr>
<tr>
<td>Uranus</td>
<td>190 ± 133</td>
<td>230 ± 10.5</td>
<td>138 ± 7.5</td>
</tr>
<tr>
<td>Neptune</td>
<td>79 ± 5</td>
<td>74 ± 22</td>
<td>101 ± 11</td>
</tr>
</tbody>
</table>

Notes. *Values are in Kelvin. The first error represents the intra-survey difference, the second one the confusion noise.

Samples for each timeline are accepted as valid data of the fit if their instantaneous angular distance from the planet is smaller than 5° at the epoch of observation. The background is assumed to be constant over the acceptance disc and is left as a free parameter. In order to properly destripe the data, we have used the baselines calculated by Madam (Keihänen et al. 2010), the standard map-making tool used to produce the LFI maps.

The histogram of the data in the timelines can be approximated as the sum of two Gaussian distributions, one due to the signal and one due to the background. The histogram of differences of timelines and the fitted model overlap very well, one of the peaks assessing proper separation of the planetary signal. The width of the residual peak is an estimate of the confusion noise, dominated by white noise and CMB fluctuations. Table 12 reports the measurements of the brightness temperature for a number of planets. The two quoted uncertainties are the maximum intra-survey difference with respect to the median of the measures and the confusion noise measured on the fitted data. The central wavelengths assumed for each of the three bands are 10.56 mm, 6.80 mm, and 4.26 mm (taken from LFI instrument model, see Planck Collaboration 2013).

Jupiter is the only planet bright enough to be detected in the timeline without averaging. This is the reason why the random uncertainty is much smaller than the maximal survey-by-survey variation. Such variation can be ascribed to a number of small systematics not completely understood yet. One of them is the fact that planets have been extracted as soon as they were observed, so that different observations have had slightly different calibration procedures. Despite having a slightly worse S/N, the same is true for Saturn. However, we have not introduced any correction for the effect of Saturn’s rings yet. In the same manner the observations of Mars are not corrected for diurnal variability due to the rotation of the planet. Uranus and Neptune have quite low S/N, in particular at 30 GHz, and therefore they are very sensitive to the confusion noise, background, uncertainties in beam orientations, and aperture corrections.

Figure 23 compares the Jupiter spectrum from LFI averaged through the surveys with the WMAP measurements (Weiland et al. 2011). It is possible to see the good agreement between the two datasets.

5.5. Consistency with HFI and WMAP

A key aspect of Planck’s data analysis is the internal consistency of calibrated maps and power spectra obtained by LFI and HFI detectors. Because the two instruments employ highly independent technologies, inter-instrument comparisons provide useful indications on systematic effects of instrumental origin. An even more independent calibration cross-check can be made
Fig. 22. Timeline of planet crossings. Each line shows the times when the planet fell within the beam of at least one of the radiometers at 30, 44, or 70 GHz. Grey bands at the edges mark the start of the survey and the end of the period considered in the 2013 data release. Vertical grey lines mark the separation between sky surveys.

by comparing maps and power spectra obtained by LFI and WMAP in similar frequency bands. Because of the importance and complexity of these comparisons, a full paper of this 2013 release is dedicated to these issues, Planck Collaboration XXXI (2014).

The LFI 70 GHz and HFI 100 GHz power spectra in the region of the first CMB acoustic peak (multipole range 70–390) are consistent within 0.8%, corresponding to a map calibration consistency of ~0.4%. These values, which include a small correction for unresolved sources, are within the LFI and HFI calibration uncertainties discussed in this paper and in Planck Collaboration VIII (2014). In Planck Collaboration XXXI (2014) we also show that an improved model of HFI near sidelobes, which were not fully accounted for initially, leads to a further improvement in the LFI-HFI consistency by a factor of 2. Such good agreement between LFI and HFI, while not a guarantee, is an indication of lack of major undetected systematics and of good calibration consistency.

A larger deviation, of order 1.8% in the power spectrum (~0.9% in calibrated maps), is found between LFI 70 GHz and WMAP V-band in the region of the first peak (the former measuring less power than the latter), a 1.4σ tension suggesting some undetected effect in LFI, in WMAP, or in both. These could be due to a combination of errors in the calibration process or beam window function, issues in the data processing, or imprecise estimation of the solar dipole. Our estimates of the solar dipole based on LFI 44 GHz orbital dipole calibration (Table 5) suggest good consistency with WMAP. Our results, however, are very preliminary, and more redundancy and cross-checks are needed before an assessment of inter-calibration at sub-percent level can be performed. We note that a ~1% calibration discrepancy would result in a noticeable offset in the Jupiter temperature as measured by LFI and by WMAP (Fig. 23, bottom panel). Taken at face value, the combination of the good agreement of these two sets of measurements and the 1.8% discrepancy of the power spectra at degree scales (first peak of the CMB) might suggest a beam or window function issue, either in LFI or WMAP. However, these deviations are (marginally) within uncertainties. The 2014 Planck release will greatly increase the LFI data redundancy compared to the present 2013 release (from 2.5 to 8 surveys, and from 2 to 7 Jupiter measurements), which we expect will help to settle the issue.

6. Conclusions and next steps

In this paper we have described the procedures used to calibrate Planck/LFI data and the ways we used to assess the accuracy of our calibration. We have shown that the level of consistency in the maps is quite good, since discrepancies are roughly 0.6% at 44 and 70 GHz, and 0.8% at 30 GHz.

However, we believe there are areas where we can significantly improve our understanding of the instrument and therefore produce a better calibration, e.g., modelling of far sidelobes (see Sects. 4.1 and 4.2) and proper modelling of variations in the noise temperature of the radiometers (Sect. 3.2). We therefore need to concentrate our efforts in improving our algorithms and analysis methods for the next Planck data delivery, in order also to provide a sound polarization calibration.

An important point which is still missing is the production of a full characterization of the dipole that is independent of WMAP. We have shown that the level of consistency in the maps is quite good, since discrepancies are roughly 0.6% at 44 and 70 GHz, and 0.8% at 30 GHz.

However, we believe there are areas where we can significantly improve our understanding of the instrument and therefore produce a better calibration, e.g., modelling of far sidelobes (see Sects. 4.1 and 4.2) and proper modelling of variations in the noise temperature of the radiometers (Sect. 3.2). We therefore need to concentrate our efforts in improving our algorithms and analysis methods for the next Planck data delivery, in order also to provide a sound polarization calibration.

An important point which is still missing is the production of a full characterization of the dipole that is independent of WMAP. We have shown that the level of consistency in the maps is quite good, since discrepancies are roughly 0.6% at 44 and 70 GHz, and 0.8% at 30 GHz.

A5, page 25 of 29
Appendix A: Fast convolution of the beam with the dipole

In this section we provide the details of the model used to convolve the dipole signal used in the calibration with the beam response over the full sky (Sect. 4.2).

A.1. Analytical model

Our starting point is Eq. (25), which relates the dipole signal (as measured by a pencil beam) with the combined velocity of the spacecraft and of the solar system with respect to the CMB rest frame. Following Peebles & Wilkinson (1968), we rewrite Eq. (25) as

$$\frac{\Delta T(t_i)}{T_{\text{CMB}}} = x(t_i) \cdot D_E + (x \cdot D_E)^2 - \frac{1}{2} |D_E|^2 + O(|D_E|^3).$$  \tag{A.1}$$

where $\Delta T$ is the cosmological kinematic signal as measured by a pencil beam, $x$ is the pointing direction in the Ecliptic reference frame and $D_E = (\Omega_{\text{Sun}} + \Omega_{\text{Planck}})$ in the same reference frame. The equation accounts\(^{22}\) for both the CMB dipole and the relativistic transverse Doppler effect, since the latter can be considered as a correction to the quadrupole proportional to $T_{\text{CMB}}|\theta_{\text{Sun}}|^2/c^2 \approx 3.8 \mu K$. (We can safely discard $-\frac{1}{2} |D_E|^2$, since it is a constant and plays no role in the calibration.) We denote with $x_0 = (x_0, y_0, z_0)$ the observing direction in the reference frame of the beam. Then the convolved dipole signal is

$$\frac{\Delta T(t)}{T_{\text{CMB}}} = N \int_{4\pi} d\Omega B(x_0) x_0 \cdot U(t) D_E$$

$$+ N \int_{4\pi} d\Omega B(x_0) (x_0 \cdot U(t) D_E)^2,$$  \tag{A.2}$$

where $U(t)$ is the matrix which transforms vectors from the Ecliptic reference frame to the beam reference frame, and $N$ is a normalization constant:

$$N^{-1} = \int_{4\pi} d\Omega B(x_0).$$  \tag{A.3}$$

The directional moments of the beam are defined as

$$M_{l,m,n} = N \int_{4\pi} d\Omega B(x_0) x_0^l y_0^m z_0^n.$$  \tag{A.4}$$

They depend only on the shape of the beam and their value does not change during the mission. If we set

$$S = (M_{l,0,0}, M_{l,1,0}, M_{l,0,1}) \quad \text{(a vector in the beam reference frame)}$$  \tag{A.5}$$

and

$$A = \begin{pmatrix} M_{l,0,0} & M_{l,1,0} & M_{l,0,1} \\ M_{l,1,0} & M_{l,0,1} & M_{l,0,2} \end{pmatrix} \quad \text{(a symmetric matrix)},$$  \tag{A.6}$$

then Eq. (A.1) reduces to the bilinear form

$$\frac{\Delta T(t)}{T_{\text{CMB}}} = D^T_E U(t)^T S + D^T_E U(t)^T A U(t) D_E.$$  \tag{A.7}$$

Because of a bug in the code which implemented Eq. (A.1), the value of $(x \cdot D_E)^2$ was wrongly divided by two before the subtraction. Therefore, the LFI maps in the Planck 2013 data release still have half of the kinetic quadrupole. This residual quadrupole has an amplitude of $T_{\text{CMB}}|\theta_{\text{Sun}}|^2/c^2 \approx 1.9 \mu K$, and it will be properly removed in the next Planck data release.

\(^{22}\) Higher-order terms result in corrections at least a thousand times smaller and are therefore neglected.
The computation of the components of $A$ (Eq. (A.5)) and $A$ (Eq. (A.6)) was based on the integration of separate GRASP maps for both sidelobes and main beam, plus intermediated beams computed for each radiometer at its nominal central frequency, i.e., 30 GHz, 44 GHz, and 70 GHz. In this context, sidelobes are defined as the part of the beam pattern outside 5° from the beam optical axis. The inner part of the beam pattern defines the intermediate beam and main beam maps.

Each map takes the small differences in the optical path for each radiometer into account, given its polarization and location in the focal-plane. For sidelobes, we used GRASP maps with a resolution of 0.5° along the azimuthal angle and 0.5° along the zenith angle. For intermediate and main beams, the resolution is 0.5°(azimuth) and 1° (zenith). We integrated Eq. (A.4) separately for the intermediate beam and the sidelobes, and then we summed them together. (The same was done for the normalization constant $N$, Eq. (A.3)) To compute the integral, we converted GRASP maps into HEALPix maps (Górski et al. 2005) by bilinear interpolation and applied a simple midpoint algorithm. We chose the resolution of the HEALPix maps in order to have a resolution much smaller than that of the corresponding GRASP maps. Sidelobes were gridded onto an HEALPix map with $N_{\text{side}} = 1024$, and intermediate beams onto a map with resolution equivalent to $N_{\text{side}} = 65536$.

In order to check the convergence, we computed the value of $M_{\text{int},n},$ a number of times for increasing values of $N_{\text{side}}$. All the integrals showed a regular convergence curve. The largest $N_{\text{side}}$ was fixed by having $M_{\text{int},n}(N_{\text{side}})/M_{\text{int},n}(N_{\text{side}})/2 < 10^{-3}$ for the moment with the slower convergence (including $N$).

The beam pattern changes over the bandpass of each radiometer, causing the moments $M_{\text{int},n}$ to change slightly with frequency. However, the change mainly affects the sidelobes, which contribute less than 0.5% of the full-sky average. For temperature maps it is therefore enough to use a monochromatic correction.

References


Planck Collaboration 2013, The Explanatory Supplement to the Planck 2013 results. V11 title=Main Page (ESA)
Planck Collaboration 2013, The Explanatory Supplement to the Planck 2013 results. V11 title=Main Page (ESA)

\footnote{1} APC, AstroParticule et Cosmologie, Université Paris Diderot, CNRS/IN2P3, CEA/Irfu, Observatoire de Paris, Sorbonne Paris Cité, 10 rue Alice Domon et Léonie Duquet, 75205 Paris Cedex 13, France

\footnote{2} Aalto University Metsähovi Radio Observatory, Metsähoviinniemi 114, 02540 Kylmälä, Finland

\footnote{3} African Institute for Mathematical Sciences, 6-8 Melrose Road, Muizenberg, Cape Town, South Africa

\footnote{4} Agenzia Spaziale Italiana Science Data Center, via del Policentrico snc, 00133 Roma, Italy

\footnote{5} Agenzia Spaziale Italiana, Viale Liegi 26, Roma, Italy

\footnote{6} Astrophysics Group, Cavendish Laboratory, University of Cambridge, J J Thomson Avenue, Cambridge CB3 0HE, UK

\footnote{7} CITA, University of Toronto, 60 St. George St., Toronto, ON M5S 3H8, Canada

\footnote{8} CNRS, IRAP, 9 Av. colonial Roche, BP 44346, 31028 Toulouse Cedex 4, France

\footnote{9} California Institute of Technology, Pasadena, California, USA

\footnote{10} Centre for Theoretical Cosmology, DAMTP, University of Cambridge, Wilberforce Road, Cambridge CB3 0WA, UK

\footnote{11} Centro de Estudios de Física del Cosmos de Aragón (CEFCA), Plaza San Juan 1, planta 2, 44001 Teruel, Spain