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Angular response of anomalous reflectors: analysis and design prospects

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Abstract—Metasurfaces for controlling wavefronts are typically designed for plane-wave illumination from a specific direction. However, the behaviour of these metasurfaces for arbitrary illuminations has not been analyzed so far. In this presentation, we study the angular response of anomalous reflectors for arbitrary illumination angles. We will compare the angular response of anomalous reflectors designed with different design strategies. In particular, we explain the dependence of the scattered power for arbitrary illumination angles in phase-gradient metasurfaces, strongly nonlocal metasurfaces, and power flow-conformal metasurfaces. This study will allow us to complete understanding of the scattering properties of anomalous reflectors and to discern the way to engineer their angular response.

Index Terms—anomalous reflectors, metasurface, angular response.

I. INTRODUCTION

During the last few years, metasurfaces have shown great potential for manipulating wave fronts. One of the most studied functionalities provided by metasurfaces is anomalous reflection. In an anomalous reflector, a plane wave is reflected breaking the reflection law, i.e., the reflection angle θ_{rd} is not equal to the incidence angle θ_{id} . The design of anomalous reflectors was initially approached using the phased-array antenna principle: each meta-atoms of the metasurface introduced a change in the phase of the reflection coefficient [1]. The physical principles behind these designs have been meticulously analyzed and it has been shown that their efficiency drops considerably with large transformations of the wave propagation direction due to the impedance mismatch between the incident and reflected waves [2]. Alternative, theoretically perfect solutions have been found using nonlocal structures [2], [3], diffraction gratings [4], auxiliary evanescent fields [5], or power-flow-conformal surfaces [6].

Most of the studies in the literature only consider the scattering properties of metasurfaces under the design conditions. In this work, we characterize the response of anomalous reflectors for angles of incidence that are different from the design angle ($\theta_i \neq \theta_{id}$). In particular, we will consider phase-gradient metasurfaces, strongly non-local metasurfaces, and power flow-conformal metasurfaces.

II. LINEAR PHASE GRADIENT ANOMALOUS REFLECTORS

Following the conventional design approach of phased arrays, metasurfaces for manipulating the direction of reflected waves can be implemented by a variation of the reflection phase along the metasurface. If we consider an incident wave

Fig. 1. Analysis of the scattering properties of a flat infinite anomalous reflector designed for $\theta_{id} = 0^\circ$ and $\theta_{rd} = 70^\circ$ using a local phase gradient. (a) For a periodicity given by $D = \lambda / |\sin \theta_{id} - \sin \theta_{rd}|$, relation between the angle of incidence θ_i and the angle of reflection of the diffracted modes θ_r . (b) Energy distribution between the reflected propagating harmonics for different incident angles. The power is normalized to the amplitude of the normal component of the incident power flow.

propagating in θ_{id} direction and a reflected wave propagating in θ_{rd} direction, the corresponding local reflection coefficient, defined as the ratio between tangential electric fields of the incident and reflected waves at each point of the metasurface, can be written as $\Gamma(x) = \exp[j(\sin \theta_{id} - \sin \theta_{rd})kx]$. Here, $k = \omega\sqrt{\mu\varepsilon}$ is the wavenumber in the background medium. Knowing that the reflection coefficient can be defined by the local surface impedance of the metasurface by $\Gamma(x) = [Z_s(x) - Z_{w0}]/[Z_s(x) + Z_{w0}]$, the surface impedance of anomalous reflectors can be written as

$$Z_s(x) = jZ_{w0} \cot[(\sin \theta_{id} - \sin \theta_{rd})kx/2] \quad (1)$$

where Z_{w0} is the characteristic impedance of the incident plane wave – the ratio between the tangential to the reflector plane components of the electric and magnetic fields of the incident wave.

The periodicity of the metasurface, dictated by the surface impedance variations ($D = \lambda/|\sin \theta_{id} - \sin \theta_{rd}|$), will lead to existence of infinitely many diffracted modes (Floquet harmonics). In the absence of losses, all the incident energy will be coupled to the propagating diffracted modes and scattered into different directions. The direction of propagation for the diffracted modes can be written as

$$\sin \theta_{rn} = \sin \theta_i + \frac{n2\pi}{kD} \quad (2)$$

where n represents the order of the diffracted mode. Figure 1(a) shows the direction of propagation of the diffracted modes for anomalous reflectors designed for $\theta_{id} = 0^\circ$ and $\theta_{rd} = 70^\circ$. Notice that $n = 0$ corresponds to specular reflection.

To analyze the scattering properties of these metasurfaces, we need to calculate the amount of power that is coupled to each diffracted mode. If we assume that the surface impedance remains constant for any illumination angle, we can determine the amplitude of each Floquet harmonic using the method described in [7]. Knowing the amplitudes of all the reflected waves, one can easily calculate the conversion efficiency for each propagating plane-wave mode, defined as the ratio between the normal component of the reflected power carried by a certain mode and the normal component of the incident power. Figure 1(b) presents the normalized energy distribution between the reflected propagating harmonics for different incident angles when $\theta_{id} = 0^\circ$ and $\theta_{rd} = 70^\circ$. For the normal illumination, most of the incident power (76%) goes to the first diffracted mode $n = 1$ approximately realizing the desired anomalous reflection. The rest of the incident power is scattered toward $\theta_r = -70^\circ$ direction (mode $n = 1$) and in the normal direction (mode $n = 0$).

From the energy distribution for different incident angles, we can see that when this metasurface is illuminated from $\theta_i = -28^\circ$ (see squared symbols) all the energy is efficiently transferred to the diffracted mode $n = 1$, and the direction of propagation of this modes is $\theta_{rn} = 28^\circ$. Under this illumination, the anomalous reflector behaves as a perfect retroreflector. It is also interesting to see that the same behaviour is also found at $\theta_i = 28^\circ$.

Fig. 2. Analysis of the scattering properties of perfect anomalous reflectors designed for $\theta_{id} = 0^\circ$ and $\theta_{rd} = 70^\circ$. The upper picture shows two different topologies for implementing perfect anomalous reflectors. The bottom picture represents the energy distribution between the reflected propagating harmonics for different incident angles. The power is normalized to the amplitude of the normal component of the incident power flow. Solid line: non-local realization; dashed line: power flow-conformal realization.

III. PERFECT ANOMALOUS REFLECTORS

There are different possibilities to overcome the fundamental limitations of anomalous reflectors based on a linear phase gradient principle. Because the physics behind each solution is different, one can expect different angular responses for different solutions. Here, we compare two cases: anomalous reflectors based on strongly non-local metasurfaces and anomalous reflectors realized as power flow-conformal metasurfaces.

To understand strongly non-local metasurfaces, let us consider the same anomalous reflector as in the previous section, designed for $\theta_{id} = 0^\circ$ and $\theta_{rd} = 70^\circ$. A simple calculation of the ratio of the tangential electric and magnetic fields of the desired field distribution gives the required surface impedance [2], which is complex-valued. Even though the surface-averaged normal component of the Poynting vector is zero, and the surface is overall lossless, the real part (surface resistance) takes both positive (loss) and negative (gain) values. In paper [2], we proposed to realize such non-local gain-loss property by designing an inhomogeneous leaky-wave surface as an array of metal patches over a ground plane (see the top-right picture in Fig. 2).

In this case, because we do not have an equivalent model that allows fast calculation of the scattering properties, we have used HFSS to calculate the angular response. The normalized energy distribution between the propagating harmonics for different incident angles is shown in Fig. 2. We can see that, for the normal incidence, all the energy is coupled into the first diffracted mode that corresponds to reflection into 70° . It is also important to notice that, due to reciprocity, the energy is also coupled to the first diffracted mode when $\theta_i = -70^\circ$ sending the energy into the normal direction. When the metasurface is illuminated from $\theta_i = 70^\circ$, the energy is sent back into the same direction, realizing a perfect retroreflector. Finally, if we compare the overall response with the response of an anomalous reflector designed with a linear phase-gradient, we find a completely different behaviour.

The last case that we would like to consider is an anomalous reflector implemented using power-flow conformal metasurfaces. This implementation allows avoiding the inherent non-locality of perfect anomalous reflectors by tailoring not only the surface impedance of the metasurface, but also the surface profile [6]. Locality of the power-flow conformal metasurfaces provides an opportunity to implement the metasurface using small phase shifters. In this work, we will consider a simple example where the metasurface is implemented by an array of metallic grooves with different depths (see the top-left picture in Fig. 2). The angular response of the structure is calculated using HFSS and represented in Fig. 2 (dashed line).

If we compare the response of the two topologies implementing perfect anomalous reflectors, we can see that the response is only equal for the design conditions and the incident angles dictated by reciprocity. However, in general, the responses are different.

IV. CONCLUSION

In this paper, we have analyzed the angular response of anomalous reflectors implemented using different methodologies. The results show that the angular response varies depending on the methodology. The first difference to be noticed is that anomalous reflectors based on linear phase gradient and the so-called perfect anomalous reflectors have fundamentally different angular responses. In fact, an anomalous reflector designed with a linear phase gradient of local reflection coefficient can be seen as an equivalent perfect retroreflector for a different angle. The second important difference is that different perfect anomalous reflectors have different responses if the operation principle is different. From our results, we can see that local designs where only propagating modes are involved and non-local design where many evanescent modes are excited exhibit different distributions of the scattered energy among propagating Floquet modes. These results highlight the importance of reactive fields in the angular response and open up a possibility to use them as a design and control tool. During our talk, we will extend the analysis and discuss possible ways to control the angular response.

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