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Published in:
IEEE Transactions on Wireless Communications

DOI:
[10.1109/TWC.2021.3087778](https://doi.org/10.1109/TWC.2021.3087778)

Published: 01/11/2021

Document Version
Peer reviewed version

Please cite the original version:
Ilter, M., Wichman, R., & Hämäläinen, J. (2021). Adaptive Irregular Constellations for Full-Duplex Relaying with Residual Self-Interference. *IEEE Transactions on Wireless Communications*, 20(11), 7682-7693.
<https://doi.org/10.1109/TWC.2021.3087778>

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Adaptive Irregular Constellations for Full-Duplex Relaying with Residual Self-Interference

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Abstract—We propose the use of adaptive optimized irregular constellation to enhance the performance of full duplex (FD) wireless radio systems which suffer from residual self-interference (RSI). Specifically, an error performance expression which takes into account the RSI level and signal-to-noise ratio is first derived for a single FD relay link. During the search for optimal constellation for a given parameter set, there are no predefined assumptions on symbol point locations. The optimization framework yields adaptive irregular constellations depending on RSI levels and power budget values. It is shown that the proposed adaptive transmission along with optimized irregular constellations achieves better error performance than the conventional M -QAM constellations and other M -ary improper Gaussian signaling (IGS) constellations.

I. INTRODUCTION

The current growth of mobile data traffic enforces wireless systems to fully utilize their time and frequency resources for maximizing the throughput and improve network coverage. Half-duplex (HD) relaying is a conventional technology where a relay node transmits and receives information in different time-slots so the coupling effect between the transmitter and the receiver does not exist. Due to time duplexing the HD relaying suffers from spectral efficiency loss, but with the advantage of recent developments in hardware technology, full-duplex (FD) relaying has become an attractive candidate for 5G systems and beyond. In a FD relaying scenario, the relay node is capable of simultaneously transmitting and receiving the information thus potentially doubling the spectral efficiency and reducing the end-to-end delay [1].

The improved spectral efficiency from FD relaying comes with the expense of self-interference between the transmitter and receiver chains. Even though an advanced interference cancellation is used in the relay node, the residual self-interference (RSI) persists. In practice, the RSI cannot be fully canceled due to nonlinear hardware components, channel estimation errors for loop-back channels, and suboptimal cancellation techniques. Thus, increasing the relay's transmit power does not necessarily improve the end-to-end performance either. In fact, it may increase the interference level in the source-relay link [2].

For HD communication, it was recently shown in [3] that signal-to-noise ratio (SNR)-adaptive transmission, where multiple irregular constellations are deployed, enhances the performance of the coded communication. Similar

SNR-adaptive transmission scheme for cognitive radio networks was presented in [4]. Interestingly, employing non-equally spaced constellations, as improper Gaussian signaling (IGS), over interference channel was first proposed in [5], and several studies, [6]–[8], have addressed the use of different constellation types in FD systems. For instance, IGS was applied into FD relaying systems for single hop [6] and multi-hop cases [7] in order to mitigate the deterrent effect of RSI, respectively. It was shown that IGS outperforms proper Gaussian signaling (PGS) in FD relay link in terms of the outage probability. However, constellation design approach for a given signal to noise ratio and RSI level was only presented in [8] for LDPC-coded FD communication. Besides, the use of nonuniform constellations has already emerged in some standards, such that ATSC 3.0 that is the first major broadcasting standard which utilizes non-uniform constellations [9].

As mentioned above, using a set of non-equally spaced constellations can enhance the performance of a given system in different aspects but it comes with some implementation complexity. Specifically, it was shown in [10], [11] that employing non-uniform constellations introduces a bottleneck in real receiver implementations where demappers are supposed to calculate the distance to all constellation points. In addition to the demapping complexity, the use of multiple constellations requires look-up table (LUT)-based structures where the constellation points are stored in them along with suitable labels. The LUT-based approaches were proposed in beamforming techniques [12], optimum input distribution for signaling [13] as well as adaptive irregular constellations over coded scenarios [3]. The complexity analysis and storage requirements of LUT-based systems can be found in [14].

Motivated by these facts, we first derive the exact end-to-end error expression (i.e. symbol error rate (SER)) over Rayleigh fading source-relay and relay-destination channels for arbitrary symbol constellations (located at any point in the 2D space) for a full-duplex relaying system which suffers from residual self-interference. Advanced optimization approaches have become increasingly feasible in the physical layer during the last two decades, so constellation design can be performed without having regular grid and lattice structure in constellation diagrams. Motivated by this, the derived error expression is used to find adaptive optimized irregular constellations with respect to system parameters such that EVM and SNR values. The simulation results show that irregular optimized

M -ary constellations can bring performance gain compared to conventional M -QAM and improper Gaussian signaling constellations, which were already proposed for FD relaying systems in [6]. In order to address the related cost of deploying adaptive optimized irregular constellations, the demapping complexity and storage requirements of look-up-tables are also discussed in Section IV.C.

The rest of the paper is organized as follows. The system model for the FD relaying with RSI is described in Section II. In Section III, the exact E2E SER expression for a given system model is derived. Then, the problem formulations of individual constellation search under equal power allocation (EPA) between the nodes and of joint optimization of constellation and optimal power allocation (OPA) are given in Section IV, respectively and the implementation issues of employing adaptive optimize irregular constellations are also discussed therein. The error performance of optimized irregular constellations along with different levels of RSI and signal-to-noise ratio values are presented in Section V and the proposed irregular optimized constellations are compared with the conventional M -QAM and M -ary IGS constellations. Section VI concludes the paper.

II. SYSTEM MODEL FOR FULL DUPLEX RELAYING WITH RESIDUAL SELF-INTERFERENCE

We consider a setting where a FD relay (R) node helps a source node (S) to communicate with a destination node (D) as shown in Fig. 1. It is assumed that the S and R nodes employ one transmit and one receive antenna, and the direct transmission between S and D does not exist.

The information bits of any given frame $\mathbf{b} = [b_1, \dots, b_{N_b}]$ are fed to the bit-to-symbol mapper in which transmitting symbols with length N_s , $\mathbf{x}_s = [x_{s,1}, \dots, x_{s,N_s}]$, are assigned from the source node's constellation, χ_S . Then, the relay node simultaneously decodes the received signals and forwards them by assigning the decoded bits into symbols $\mathbf{x}_R = [x_{R,1}, \dots, x_{R,N_s}]$ according to the relay node's constellation χ_R .

The received signals at the R and D nodes can be written as

$$y_R = \sqrt{E_S} h_{SR} x_S + \sqrt{E_R} h_{RR} (x_R + m_R) + n_R \quad (1)$$

$$y_D = \sqrt{E_R} h_{RD} (x_R + m_R) + n_D \quad (2)$$

where E_S and E_R denote the source and relay symbol transmission power levels. The channel between the source and relay is denoted by $h_{SR} \in \mathbb{C}$, the channel between relay and destination node is described by h_{RD} and h_{RR} corresponds to the loop-back channel and they are modeled as Rayleigh distributed variables with zero mean and variances, Ω_{SR} , Ω_{RR} and Ω_{RD} , respectively. The transmit side noise of the source node is assumed to be zero, because its effect is negligible compared to the self-interference in the relay [15]. Hardware impairments at the relay are modeled by the factor m_R , and n_R and n_D denote thermal noise at the relay and the destination, respectively. Following the model of [15] we assume that

$$m_R \sim \mathcal{CN}(0, \varsigma), n_R \sim \mathcal{CN}(0, 1), n_D \sim \mathcal{CN}(0, 1). \quad (3)$$

Fig. 1: Full-duplex relaying system model with residual interference.

Herein, ς corresponds to the variance of the transmitter impairment noise, commonly measured as Error Vector Magnitude (EVM). For example, in LTE, maximum allowed EVM levels in the base station transmitter are between -15 and -22 dB depending on the modulation [16].

We assume perfect channel state information in the R and D nodes such that the relay is able to subtract the known part of the self-interference from the received signal y_R . Thus, the residual self-interference term becomes $h_{RR} \sqrt{E_R} m_R$. Throughout the search for optimized irregular constellations, the average fading power values are fixed such that $\Omega_{SR} = \Omega_{RD} = 1$ and $\Omega_{RR} = \{1, 10^{-4}\}$. The case of $\Omega_{RR} = 1$ together with EVM corresponds to typical RSI in case of single-antenna transceiver and $\Omega_{RR} = 10^{-4}$ corresponds to RSI values when the transceiver has spatially isolated transmit and receive antennas.

III. ERROR PERFORMANCE ANALYSIS

When χ_S and χ_R are used in the source and relay nodes, the average end-to-end error probability of FD relay link, $\bar{P}_{e,SRD}$, can be formulated as

$$\bar{P}_{e,SRD}(\bar{\gamma}, \varsigma, \chi_S, \chi_R) = 1 - (1 - \bar{P}_{e,SR}(\bar{\gamma}, \varsigma, \chi_S)) (1 - \bar{P}_{e,RD}(\bar{\gamma}, \varsigma, \chi_R)), \quad (4)$$

for a given ς and $\bar{\gamma}$ values where $\bar{\gamma}$ corresponds to signal-to-noise ratios without EVM and $\text{EVM} = 10^{\frac{-10}{10}}$. Here, $\bar{P}_{e,SR}(\bar{\gamma}, \varsigma, \chi_S)$ and $\bar{P}_{e,RD}(\bar{\gamma}, \varsigma, \chi_R)$ correspond to the average error probabilities for the direct links between S - R and R - D and their derivations will be shown in the following subsections, respectively.

A. Calculation of $\bar{P}_{e,SR}$

In the direct link between S and R , S transmits the symbols selected from an M -ary constellation, χ_s , to R with the transmit power E_S . After subtracting the RSI caused by transmit-noise in (1)-(2), the received instantaneous SINR at R node can be calculated as $\gamma_R = E_S |h_{SR}|^2 / (|n_R|^2 + E_R |h_{RR}|^2 |m_R|^2)$. Then, the instantaneous error probability expression at R can

be written as a function of γ_R :

$$P_{e,SR}(\gamma_R) = \sum_{k=0}^{M-1} \sum_{\substack{l=0 \\ l \neq k}}^{M-1} P_k \Pr[y_R \in D_{\chi_S(l)}(\gamma_R) | x_S = \chi_S(k)], \quad (5)$$

where P_k is the probability of transmitting the k -th symbol, $\chi_S(k)$ is the k -th symbol of the source constellation, and $D_{\chi_S(l)}(\gamma_R)$ is the decision region of the l -th symbol for a given γ_R . The calculation of (5) consists of all possibilities that the actual transmitted symbol, $\chi_S(k)$, falls into a decision region, denoted by $D_{\chi_S(l)}$.

Corollary 1: For equiprobable signaling case, $P_k = 1/M$ and $D_{\chi_S(l)}$ can be expressed as [17]

$$D_{\chi_S(l)}(\gamma_R) = \Pr\left[\mathcal{L}_{k,l}\left(\frac{y_R}{h_{SR}}\right) < 0\right], l \neq k, \quad (6)$$

where $l = 0, \dots, M-1$ and

$$\mathcal{L}_{k,l}(y_R) = \Re[y_R c_{k,l}^*] + d_{k,l}, \quad (7a)$$

$$c_{k,l} = \chi_S(l) - \chi_S(k), \quad (7b)$$

$$d_{k,l} = \frac{|\chi_S(k)|^2}{2} - \frac{|\chi_S(l)|^2}{2}. \quad (7c)$$

Herein, $\Re[x]$ corresponds to the real part of x . In the upcoming steps, $\mathcal{L}_{k,l}(y_R)$ is replaced with $\bar{\mathcal{L}}_{k,l}(y_R)$ which is normalized with respect to $|c_{k,l}|$. By utilizing the geometric trajectory on 2-D space shown in [18], $\Pr[y_R \in D_{\chi_S(l)}(\gamma_R) | x_S = \chi_S(k)]$ can be formulated as

$$\begin{aligned} & \Pr[y_R \in D_{\chi_S(l)}(\gamma_R) | x_S(k)] = \\ & \sum_{t=1}^{T_l} \pm \mathcal{Q}\left(\pm \mathcal{L}_{l,p_t}(\chi_S(k)) \sqrt{2\gamma_R}, \pm \mathcal{L}_{l,p_{t+1}}(\chi_S(k)) \sqrt{2\gamma_R}; \right. \\ & \left. \pm \Re[c_{l,p_t}, c_{l,p_t}^*]\right), \end{aligned} \quad (8)$$

where $\mathcal{Q}(\cdot, \cdot; \cdot)$ denotes the complementary cumulative density function (CCDF) of a bivariate Gaussian variable [19]. In (8), T_l refers to the lines bounding the decision region $D_{\chi_S(l)}$, the neighbor decision regions of the symbol $\chi_S(l)$ are expressed by $D_{\chi_S(p_t)}$ and $D_{\chi_S(p_{t+1})}$ where p_t and p_{t+1} are neighbor symbols of $\chi_S(l)$. The detailed information about the rule of the signs $\pm$ and summation terms can be found in [20].

After utilizing (8) from *Corollary 1*, $\bar{P}_{e,SR}$ can be obtained by taking the average of (5) over γ_R as

$$\begin{aligned} \bar{P}_{e,SR} &= \mathbb{E}_{\gamma_R}[P_{e,SR}(\gamma_R)] \\ &= \sum_{k=0}^{M-1} \sum_{\substack{l=0 \\ l \neq k}}^{M-1} \sum_{t=1}^{T_l} \pm P_k \int_0^\infty \mathcal{Q}(a, b; \rho) f_{\gamma_R}(\gamma_R) d\gamma_R, \end{aligned} \quad (9)$$

where $\mathbb{E}[\cdot]$ denotes the expectation operator, $f_{\gamma_R}(\gamma_R)$ denotes the PDF of γ_R and

$$\begin{cases} a = \pm \bar{\mathcal{L}}_{l,p_t}(\chi_S(k)) \sqrt{2\gamma_R}, \\ b = \pm \bar{\mathcal{L}}_{l,p_{t+1}}(\chi_S(k)) \sqrt{2\gamma_R}, \\ \rho = \pm \Re[c_{l,p_t}, c_{l,p_t}^*]. \end{cases} \quad (10)$$

Proposition 1: The unconditional $\bar{P}_{e,SR}$ is derived as

$$\begin{aligned} \bar{P}_{e,SR} &= \frac{1}{2\pi} \sum \sum \sum \\ & v_1 - \sum_{n=0}^\infty e^{\frac{E_S \Omega_{SR} + 1}{E_R \Omega_{RR\zeta}} \tan(v_1)} \left(\frac{E_R \Omega_{RR\zeta}}{E_S \Omega_{SR}}\right)^{\frac{n-1}{2}} \\ & \quad \times \Gamma\left(\frac{n+1}{2}, \frac{E_S \Omega_{SR} + 1}{E_R \Omega_{RR\zeta}}\right) \\ & + v_2 - \sum_{n=0}^\infty e^{\frac{E_S \Omega_{SR} + 1}{E_R \Omega_{RR\zeta}} \tan(v_2)} \left(\frac{E_R \Omega_{RR\zeta}}{E_S \Omega_{SR}}\right)^{\frac{n-1}{2}} \\ & \quad \times \Gamma\left(\frac{n+1}{2}, \frac{E_S \Omega_{SR} + 1}{E_R \Omega_{RR\zeta}}\right), \end{aligned} \quad (11)$$

where $v_1 = (\alpha_1, \alpha_2, \rho)$, $v_2 = (\alpha_2, \alpha_1, \rho)$ and $\Gamma(\cdot, \cdot)$ is the incomplete Gamma function [21]. *Proof.* Let define an another variable such that $\mathcal{B} = E_R |h_{RR}|^2 |m_R|^2$ and then for a given $\mathcal{B}$, $f_{\gamma_R}(\gamma_R | \mathcal{B})$ can be formulated as

$$f_{\gamma_R}(\gamma_R | \mathcal{B}) = \frac{1}{\bar{\gamma}_R} \exp\left(-\frac{\gamma_R}{\bar{\gamma}_R}\right), \quad (12)$$

where $\bar{\gamma}_R = E_S \Omega_{SR} / (N_0 + \mathcal{B})$. Then, substituting (12) into (9) and utilizing equation (32) of [22], the resulting $\bar{P}_{e,SR}$ for a given $\mathcal{B}$ can be expressed as

$$\begin{aligned} \bar{P}_{e,SR|\mathcal{B}} &= \frac{1}{2\pi} \sum_{k=0}^{M-1} \sum_{\substack{l=0 \\ l \neq k}}^{M-1} \sum_{t=1}^{T_l} \\ & v(\alpha_1, \alpha_2, \rho) - \frac{\alpha_1 \arctan\left(\sqrt{\frac{\gamma_R \alpha_1^2 + 1}{\alpha_1^2 \bar{\gamma}_R}} \tan(v(\alpha_1, \alpha_2, \rho))\right)}{\sqrt{\frac{\gamma_R \alpha_1^2 + 1}{\bar{\gamma}_R}}} \\ & + v(\alpha_2, \alpha_1, \rho) - \frac{\alpha_2 \arctan\left(\sqrt{\frac{\gamma_R \alpha_2^2 + 1}{\alpha_2^2 \bar{\gamma}_R}} \tan(v(\alpha_2, \alpha_1, \rho))\right)}{\sqrt{\frac{\gamma_R \alpha_2^2 + 1}{\bar{\gamma}_R}}}. \end{aligned} \quad (13)$$

In (13) $\arctan(\cdot)$ represents the inverse of the tangent function, $\alpha_1 = \sqrt{2} \bar{\mathcal{L}}_{l,p_t}(\chi_S(k))$, $\alpha_2 = \sqrt{2} \bar{\mathcal{L}}_{l,p_{t+1}}(\chi_S(k))$ and $v(\alpha_1, \alpha_2, \rho)$ is defined by

$$v(\alpha_1, \alpha_2, \rho) = \begin{cases} \arctan\left(\frac{\alpha_1 \sqrt{1-\rho^2}}{\alpha_2 - \rho \alpha_1}\right), & \rho \alpha_1 \leq \alpha_2, \\ \arctan\left(\frac{\alpha_1 \sqrt{1-\rho^2}}{\alpha_2 - \rho \alpha_1}\right) + \pi, & \alpha_2 < \rho \alpha_1, \\ \arctan\left(\frac{1+\rho}{1-\rho}\right), & \alpha_1 = 0, \alpha_2 = 0. \end{cases} \quad (14)$$

obtained from

$$\bar{P}_{e,SR} = \int_0^\infty \bar{P}_{e,SR|\mathcal{B}} f_{\mathcal{B}}(\mathcal{B}) d\mathcal{B}, \quad (15)$$

where $\mathcal{B} \sim \exp(E_R \Omega_{RR\zeta})$. After utilizing Gregory's series for

$\arctan(\cdot)$ [23] into (13), which is,

$$\arctan(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}. \quad (16)$$

(13) is substituted into (15) and utilizing [Eq. (3.382.4), [21]] results in (11). $\square$

Note that we derived the symbol error rate (SER) expression but it can be modified to the bit error rate (BER) expression by adding a multiplication term of $(2 \log_2 M)^{-1} d_H(\chi_S(l), \chi_S(k))$ into (5), where $d_H(\cdot)$ gives the Hamming distance between two corresponding binary codewords resulting from output symbols, $\chi_S(k)$ and $\chi_S(l)$.

B. Calculation of $\bar{P}_{e, RD}$

For R - D link, instantaneous signal to noise and impairment ratio from (2) can be expressed as

$$\gamma_D = \frac{\gamma_{RD}}{\varsigma \gamma_{RD} + 1}, \quad (17)$$

where $\gamma_{RD} = E_R |h_{RD}|^2$.

Proposition 2: The PDF of γ_D can be formulated as

$$f_{\gamma_D}(z) = \frac{1}{\bar{\gamma}_{RD}(\varsigma z - 1)^2} \exp\left(-\frac{z}{\gamma_{RD}(1 - \varsigma z)}\right), \quad (18)$$

for $0 \leq z \leq 1$.

Proof. Considering the Rayleigh fading channel for relay-destination link, the CDF expression for γ_D is defined as

$$F_{\gamma_D}(z) = \Pr\left[\frac{\gamma_{RD}}{\varsigma \gamma_{RD} + 1} \leq z\right], \quad (19)$$

and it can also be represented as $\Pr[(1 - \varsigma z) \gamma_{RD} \leq z]$ where the PDF of $(1 - \varsigma z) \gamma_{RD}$ is formulated as

$$\frac{1}{\bar{\gamma}_{RD}(\varsigma z - 1)} \exp\left(-\frac{\gamma}{\bar{\gamma}_{RD}(1 - \varsigma z)}\right). \quad (20)$$

Thus, $F_{\gamma_D}(z)$ is derived as

$$\begin{aligned} F_{\gamma_D}(z) &= \int_0^z \frac{1}{\bar{\gamma}_{RD}(\varsigma z - 1)^2} \exp\left(-\frac{\gamma}{\bar{\gamma}_{RD}(1 - \varsigma z)}\right) dz \\ &= 1 - \exp\left(\frac{z}{\bar{\gamma}_{RD}(1 - \varsigma z)}\right), \end{aligned} \quad (21)$$

and the derivative of (21) gives (18). $\square$

Then, $\bar{P}_{e, RD}$ can be obtained from

$$\bar{P}_{e, RD} = \sum_{k=0}^{M-1} \sum_{l=0}^{M-1} \sum_{t=1}^{T_1} \pm P_k \int_0^{1/\varsigma} \mathcal{Q}(a, b; \rho) f_{\gamma_D}(z) dz, \quad (22)$$

where

$$\begin{cases} a = \pm \bar{\mathcal{L}}_{l, p_t}(\chi_R(k)) \sqrt{2z}, \\ b = \pm \bar{\mathcal{L}}_{l, p_{t+1}}(\chi_R(k)) \sqrt{2z}, \\ \rho = \pm \Re[c_{l, p_t}, c_{l, p_t}^*]. \end{cases} \quad (23)$$

Following the same manner as in the calculation of S - R link, the calculation of (22) also consists of all possibilities that the actual transmitted symbol, $\chi_R(k)$, falls into a decision region, denoted by $D_{\chi_R(l)}$. Having finite limits in (22) makes it possible to use the n^{th} -order Legendre orthogonal polynomial $P_n(t)$ (see [24], equations (25.4.29 – 30)) to obtain an accurate numerical integration. After substituting (18) into (22) and defining the integral limits via proper parametrization, $\bar{P}_{e, RD}$ can be evaluated from

$$\begin{aligned} \bar{P}_{e, RD} &= \\ &= \sum \frac{1}{2\pi} \int_0^{v(a, b, \rho)} \int_{-1}^1 g(t, a) dt d\theta + \int_0^{v(b, a, \rho)} \int_{-1}^1 g(t, b) dt d\theta \\ &= \sum \frac{1}{2\pi} \int_0^{v(a, b, \rho)} \sum_{k=1}^n \omega_k g(t_k, a) d\theta + \int_0^{v(b, a, \rho)} \sum_{k=1}^n \omega_k g(t_k, b) d\theta + \mathcal{R}_n \end{aligned} \quad (24)$$

where t_k 's are the zeros of the n^{th} -order Legendre polynomial, $\mathcal{R}_n$ is the residual term which gives the approximation error (see [24], equation (25.4.30)) and ω_k 's are weight coefficients which can be calculated from the following formula (see [24], equation (25.4.29))

$$\omega_k = \frac{2}{(1 - t_k^2) P_N^{(1)}(t_k)^2}. \quad (25)$$

The function $g(t_k, x)$ is given as

$$g(t_k, x) = \frac{2 \exp\left(-\frac{(t_k+1)(x^2 \bar{\gamma}_{RD}(t_k-1) \csc^2(\theta)-4)}{4 \bar{\gamma}_{RD}(t_k-1)}\right)}{\bar{\gamma}_{RD}(t_k-1)^2}, \quad (26)$$

where $x \in \{a, b\}$. Since substituting (26) into (24) does not give a closed-form solution, θ in (26) can be replaced by $v(a, b, \rho)$ and $v(b, a, \rho)$ of (14) in order to approximate $\bar{P}_{e, RD}$.

C. Extension to single-stream MIMO

Suppose the relay has $N \times N$ antennas and the source transmits a single data stream. When $N > 1$ the relay can apply null-space projection [15] to completely suppress the self-interference such that EVM=0. Assuming that all channels are i.i.d. the S - R and R - D channels become Nakagami fading due to additional diversity by multiple transmit and receive antennas. In case relay applies antenna selection [15] to mitigate self-interference, the S - R and R - D are Rayleigh channels while some EVM remains. In general, multiple antennas in the relay and spatial self-interface mitigation techniques change channels statistics and EVM levels and the constellation design techniques proposed here can be applied once these changes are taken into account.

IV. ADAPTIVE IRREGULAR OPTIMIZED CONSTELLATIONS

The end-to-end average error performance calculation for the FD relay link of the previous section enables us to construct a constellation design problem where the optimized symbol points for a given modulation order can be searched without any predefined assumption on their locations. Using

this methodology we can build an adaptive transmission model that applies look-up tables in which optimized irregular constellations are stored with the labels including design parameters along with the level of RSI.

A. Optimization over (4)

Since finding an analytical solution for minimizing (4) is intractable, the sequential quadratic programming (SQP) is utilized since it is a powerful iterative method for constrained optimization problems. The symbol point locations of squared M -QAM constellations are fed into SQP algorithm as the initial point and the goal is to show that promising performance gains are possible considering optimization of constellation points based on (4) for a given ς and $\bar{\gamma}$ values.

Each constellation point for those values need to be stored along with a label for future use in the transmission. However; in some cases, employing different irregular constellations both at the source and relay nodes might not be feasible due to system complexity constraints (look-up tables might be impractical, low quantization resolution in the transmitter/receiver, and high demapping complexity in maximum likelihood (ML) decoding). Then, (4) is utilized for different optimization problems, **P0-P3**, for different system configurations, where **P0** and **P2** assume the same constellation used in the nodes and **P1** and **P3** can give different irregular constellation in each node, respectively. Equal power allocation (EPA) cases between source and relay are also considered as reference in addition to optimal power allocation (OPA) cases. For optimal power allocation, a variable β is introduced such that $E_S = \beta E_T$ and $E_R = (1 - \beta) E_T$. The total transmission power, $E_T = E_S + E_R$, is restricted by $E_{\max}$.

P0 assumes that the source and the relay nodes employ the same constellation along with EPA. Explicitly, **P0** can be formulated as

$$\min_{\chi} \quad \bar{P}_{e,SRD}(\bar{\gamma}, \varsigma, \chi_S, \chi_R, \beta) |_{\varsigma=\varsigma_o, \bar{\gamma}=\bar{\gamma}_o, \chi_S=\chi_R=\chi, \beta=0.5} \quad (27a)$$

$$\text{s.t.} \quad 0 < E_i \leq E_i^{\max}, \quad i \in \{S, R\}, \quad (27b)$$

for a given values such that $\varsigma = \varsigma_o, \bar{\gamma} = \bar{\gamma}_o$. In case of relaxing the constraint of using the same constellations at the nodes along with EPA scheme, **P1** can be formulated as:

$$\min_{\chi_S, \chi_R} \quad \bar{P}_{e,SRD}(\bar{\gamma}, \varsigma, \chi_S, \chi_R, \beta) |_{\varsigma=\varsigma_o, \bar{\gamma}=\bar{\gamma}_o, \beta=0.5} \quad (28a)$$

$$\text{s.t.} \quad 0 < E_i \leq E_i^{\max}, \quad i \in \{S, R\}. \quad (28b)$$

After representing minimization of the derived end-to-end error probability along with EPA schemes, now constellation search is combined with the problem of power allocation between the source and relay nodes for different ς and $\bar{\gamma}$ values. **P2** assumes the similar constellation at the nodes and it can be formulated by,

$$\min_{\chi, \beta} \quad \bar{P}_{e,SRD}(\bar{\gamma}, \varsigma, \chi_S, \chi_R, \beta) |_{\varsigma=\varsigma_o, \bar{\gamma}=\bar{\gamma}_o, \chi_S=\chi_R=\chi} \quad (29a)$$

$$\text{s.t.} \quad 0 < E_i \leq E_i^{\max}, \quad i \in \{S, R\}, \quad (29b)$$

$$0 \leq \beta \leq 1. \quad (29c)$$

Now, two different constellations at the source and relay nodes along with the OPA scheme are considered in **P3**, which is

$$\min_{\chi_S, \chi_R, \beta} \quad \bar{P}_{e,SRD}(\bar{\gamma}, \varsigma, \chi_S, \chi_R, \beta) |_{\varsigma=\varsigma_o, \bar{\gamma}=\bar{\gamma}_o} \quad (30a)$$

$$\text{s.t.} \quad 0 < E_i \leq E_i^{\max}, \quad i \in \{S, R\}, \quad (30b)$$

$$0 \leq \beta \leq 1. \quad (30c)$$

Note that only β constraints are linear, and the objective function in (4) is non-convex so the optimization problems in **P0-P3** are non-convex. The SQP algorithm makes quadratic approximations of $\bar{P}_{e,SRD}(\bar{\gamma}, \varsigma, \chi_S, \chi_R, \beta)$ as $\bar{P}_{e,SRD}(\cdot)$ and linear approximation on $\hat{E}_i$, where the original E_i constraint is given as

$$E_i = \frac{1}{M} \sum_{l=0}^{M_i-1} |\chi_i(l)|^2 < E_i^{\max}, \quad \chi_i(l) \in \chi_i, \quad |\chi_i| = M_i, \quad (31)$$

where $i \in \{S, R\}$ and M_i denotes the cardinalities of the source and relay constellations. Then, the optimization problem is transformed into the following

$$\min_{\chi, \beta} \quad \widetilde{\bar{P}_{e,SRD}}(\bar{\gamma}, \varsigma, \chi_S, \chi_R, \beta) + \sum_i \hat{E}_i \quad (32a)$$

$$\text{s.t.} \quad \sum |\chi_i^k - \chi_i^{k+1}| < \delta, \quad (32b)$$

where δ is the size of trust region and χ_i^k corresponds to M_i -ary symbol set for k th iteration. If k th iteration symbol set improves the error performance, trust-region size was expanded by the SQP algorithm. After several iterations, the SQP algorithm converges to a local optimum solution. Note that different modulation orders can be considered in source and relay nodes during the search for optimized constellations so M_i refers to the corresponding modulation order in source and relay nodes. It can be assumed that the relay is able to buffer the received information such that different transmitted data rates instantaneously in source and relay are possible.

B. Finding irregular constellations

In order to compare the complexity of the SQP algorithm in **P0-P3**, the execution times of one iteration of SQP optimizer are illustrated in Fig. 2. To do so, different modulation order, n such that $n = \log_2(M)$ is considered in process of seeking out optimal locations of M symbols such that $M_i = M$ for $i \in \{S, R\}$. As it can be seen from the figure, the execution time increases exponentially with respect to modulation order and the increase in execution time after combining power allocation problem into constellation search as in **P1** and **P3** is negligible for all M -ary cases.

C. Deploying irregular constellations

The degree of freedom (DoF) is important metric to determine the complexity resulting from a constellation choice. Since the proposed optimization framework does limit the position of symbol points for a given design parameter set, optimized irregular constellations are characterized by the

Fig. 2: The execution time of one iteration of the SQP optimizer in the process of constellation search for different modulation orders.

complete freedom of movement, which was also referred as 2D-nonuniform constellations (2D-NUCs) [10]. As being different from 2D-NUCs, 1D-NUCs can be decomposed into two non-uniform pulse amplitude modulation (PAM) signals so their DoFs are $\frac{\sqrt{M}}{2}$ since they can be split into two nonuniform PAM constellations due to quadrant symmetry [11]. Therefore, $\sqrt{M}/2$ discrete values are enough in the symbol demapper. Since 2-D irregular constellations cannot be decomposed into any sub-constellations, the DOF is increased to $\frac{M}{2}$ and the total number of distance computations at the demapping stage is $2M$ whereas QAM and 1-D NUCs require to $2\sqrt{M}$ distance calculation.

In order to examine the effects of irregular constellations from more practical aspect, the demapper complexity should be related with the implementation complexity by including the number of operations in the demapping and decoding process per symbol. Similar to [25], the flop count (floating point operation) is considered herein where each complex addition and multiplication is counted as two and six flops, respectively, where logarithmic and exponential operations correspond to 20 flops, respectively. After receiving y_R and y_D as in (1) and (2), the symbol demapper calculates the ML probabilities and makes the symbol decision as follows

$$\Pr(y_R|x_S) \propto \max_{x_S \in \chi_S} \exp(|\tilde{y}_{SR} - h_{SR}x_S|^2), \quad (33a)$$

$$\Pr(y_D|x_S) \propto \max_{x_R \in \chi_R} \exp(|y_D - h_{SR}x_R|^2), \quad (33b)$$

where $\tilde{y}_{SR}$ is the received signal after subtracting the known part of the self-interference. The calculation of $\Pr(y_R|x_S)$

Fig. 3: Total number of flops in demapping of one symbol in the relay and destination nodes over different M -ary signalling paired with 4-ary, 16-ary and 64-ary cases such that i th and j th nodes $\{i, j\} \in \{S, R\}$ such that $i \neq j$.

requires a total of 28 flops per calculation. Then, the total number of operations are illustrated in Fig. 3 and showing that deploying M -ary irregular constellations in both nodes increase the implementation complexity considerably. On the other hand, deploying irregular constellations only at one node results in much lower implementation complexity compared to 1-D NUC cases.

In addition to DoF and demapping complexity analysis discussed above, irregular constellations are also required to be stored in a Look-up Table (LUT) with a suitable indexing in S , R and D nodes. The higher is the transistor packing density and the more regular are the memory based structures, the less is the dynamic power power consumption and the lower is the latency in the implementation [14]. Therefore, LUT-based approaches have gained popularity in many modern wireless systems [12], [13], [26]. For storing constellation points, LUT-based transmission can be feasible up to 256-ary constellations in real-time applications [27].

D. Mismatch in SNR and EVM values

Since the receiver needs to know the constellation in use, we address in the following the practical implementation of the method. The block diagram of the proposed model is presented in Fig. 4. To select the optimal irregular constellation for transmission, the receivers need to first estimate the received SNR, i.e. the power of the received signal of interest and the joint noise power due to thermal noise and EVM. This can be done using standard channel estimation techniques.

The estimated channel quality information is signaled to the transmitter using a feedback channel as depicted in Fig. 4. In case there are errors in the feedback channel, the transmitter

Fig. 4: The system model of EVM and E_T/N_0 based irregular optimized constellation usage for FD scenario.

can choose a different constellation than anticipated, and the receiver can either estimate the actual constellation in use, or the constellation can be signaled in a separate control channel using a fixed modulation and coding scheme. The latter approach is used, e.g., in LTE downlink where the modulation is signaled within downlink control information (DCI) on physical downlink control channel [28]. In the former case, there are several blind modulation classification techniques [29] available. The clustering based constellation estimation was proposed for short packet communications in [30] and for MIMO transmissions in [31]. More interestingly, low complexity simultaneous determination of modulation types and signal-to-noise ratios was developed in [32], where average estimation error of SNR was as low as 0.79 dB.

V. NUMERICAL RESULTS

In the following, we apply the proposed adaptive transmission schemes to full-duplex relay links with different RSI levels. Throughout simulations, the average fading power values are fixed such that $\Omega_{SR} = \Omega_{RD} = 1$ and $\Omega_{RR} \in \{1, 10^{-4}\}$ and without loss of generality, the modulation orders are the same, $M_i = M$. Before presenting the end-to-end optimization results, we shown in Fig. 5, the $\bar{P}_{e,SR}$ and $\bar{P}_{e,RD}$ curves obtained from the simulations and from (13) and (22) along with different EVM values. The results verify the proposed average exact SER expression for a wide range of $\bar{\gamma} = E_T/N_0$. In the computation of the analytical curves, we employ 10th-order Legendre orthogonal polynomial, $P_n(t)$ and the values of weights and zeros, ω_k and t_k , can be found in [33]. In the calculation of (16), the infinite summation term seen in (16) is truncated after the 5th term for all calculations. Table I shows the convergence of the summation for different number of summands and $\bar{\gamma}$ values.

Now, optimized irregular constellations obtained from **P0-P3**, are evaluated over different ς and $\bar{\gamma}$ values. To do so, the constellations obtained from **P0** for two different EVM values

Fig. 5: Analytical and simulated $\bar{P}_{e,SR}$ and $\bar{P}_{e,RD}$ values for 16-QAM signaling case over different EVM values.

TABLE I: Convergence of (16) with the number of summands N_c and mean SNR, $\bar{\gamma}$.

N_c	$\bar{\gamma} = 10$ dB	$\bar{\gamma} = 25$ dB	$\bar{\gamma} = 40$ dB
1	0.864843	0.599167	0.200007
2	0.757032	0.752047	0.198674
3	0.761064	0.744831	0.198676
4	0.760992	0.744993	0.198676
5	0.760993	0.744991	0.198676
6	0.760993	0.744991	0.198676
100	0.760993	0.744991	0.198676

and ideal case $EVM = -\infty$ dB are first evaluated. As it can be seen in Fig. 6, optimized irregular constellations outperform 16-QAM and provide around 1.5 dB SNR gain in end-to-end performance. The list of the exact coordinates of the symbol point locations used in these simulations can be found in [34].

To consider the practical concerns for the proposed scheme, the effects of the EVM estimation error ($\epsilon = EVM(\text{dB}) - EVM_{\text{estimated}}(\text{dB})$) are also investigated through the simulations. According to the simulation results given in Fig. 6, there is a minor performance loss in cases of when the true EVM = -18 dB and $\epsilon = 2$ dB, and true EVM = -23 dB and $\epsilon = 3$ dB.

In order to see the variations on symbol point locations with respect to different EVM levels, the optimized irregular constellations from **P0** for $\bar{\gamma} = 25$ dB along with different EVM values are represented together in Fig. 7. The results in Fig. 8 show the same trend. It is observed that for a fixed EVM value, symbol point locations are varying less with increasing $\bar{\gamma}$. Note that the index of each constellation point, (from 1 to M for a

Fig. 6: The performance comparison between optimized irregular constellations obtained from **P0** and 16-QAM for different $\bar{\gamma}$ and EVM values.

Fig. 8: Optimized irregular constellations from **P0** for EVM = -30 dB over different $\bar{\gamma}$ values.

Fig. 7: Optimized irregular constellations from **P0** at $\bar{\gamma} = 25$ dB for different EVM cases.

Fig. 9: Optimized irregular constellations from **P0** for no impairment and EVM = -15 dB for $\bar{\gamma} = 10$ dB.

M -ary constellation), is just pinned next to its exact location in the constellation figures for each considered scenario. In order to illustrate how the symbol point locations for lower $\bar{\gamma}$ values, optimized irregular constellations for $\bar{\gamma} = 10$ dB are plotted in Fig. 9 along with two different EVM cases. Aligned with

[10], the optimized irregular constellations in both cases tend to have lower modulation order so a cluster of overlapping symbol points are observed.

Fig. 10: The performance comparison between optimized irregular constellations obtained from **P0**, **P1**, **P3** and 16-QAM for EVM = -20 dB.

Now, the improvement of the error performance due to the joint optimization of symbol constellations and transmit powers is investigated. To do so, EVM is set to -20 dB and the performance of the relay link optimized using **P0**, **P1**, and **P3** and 16-QAM is illustrated in Fig. 10. The power allocation values for **P2** are depicted in the figure as well. It can be seen that the error-floor due to EVM disappears in this SNR range after joint optimization of the constellations and transmit powers using **P2**. When the power budget increases, more power is allocated to the source to balance the increasing self-interference in the relay. Also, we illustrate the case when the self-interference channel is attenuated due to spatial isolation w.r.t. $S - R$ and $R - D$ links. We set $\Omega_{RR} = 10^{-4}$ and EVM = -15 dB such that the residual self-interference level is 55 dB below the transmitted signal level. Another interesting observation is that the power allocation in the optimization formulation triggers the change of symbol point locations for the same EVM values which can be seen in Fig. 12.

It can be seen in Fig. 11 that constellation design still provides end-to-end error performance enhancement but different from $\Omega_{RR} = 1$ cases, joint power allocation and constellation design only makes a slight difference in the performance. In contrast to Fig. 10, now the transmit power in the relay increases with increasing power budget as it can be seen from β_{opt} values in both figures. The $R - D$ link limits the end-to-end performance, because the effect of EVM in the relay is attenuated while the maximum achievable SNR in the destination is dictated by the EVM = -15 dB.

In addition to comparing the optimized irregular 16-ary constellations with M -QAM constellations, improper Gaussian signaling (IGS) constellations are considered as well. In order to generate the IGS constellations for the performance comparison, any given symbol point in IGS constellation can be obtained from any given symbol point from M -QAM as follows [eq.(16),

[35]]

$$x^{IGS} = \sqrt{\frac{1}{2}(1+\xi)}x^{M-QAM} + \sqrt{\frac{1}{2}(1-\xi)}e^{i\phi}x^{M-QAM*}, \quad (34)$$

where $\xi = \sqrt{1-\kappa^2}$ and κ is referred as a circularity coefficient $0 \leq \kappa \leq 1$. Also, the optimal value of ϕ^* which maximizes the minimum distance in IGS constellation was given as [36]

$$\phi^* = \begin{cases} \frac{\pi}{2}, & 0 \leq \kappa \leq 0.5 \\ \arccos\left(\frac{-1+2\sqrt{5\kappa^2-1}}{5\kappa}\right), & 0.5 < \kappa \leq \frac{2}{\sqrt{5}} \end{cases} \quad (35)$$

and the value of ϕ^* can be found from an iterative algorithm for $2/\sqrt{5} < \kappa \leq 1$. In order to make a fair comparison, the simulation parameters are selected as in [6], $\Omega_{SR} = \Omega_{RD} = 20$ dB and $\Omega_{RR} = \{0, 10\}$ dB since IGS constellations were proposed for very similar FD relaying transmission with RSI existence.

In Fig. 13, IGS constellations with different κ values are first plotted and the error performance comparison with optimized irregular 16-ary obtained from **P1** for the mentioned scenario is presented in Fig. 14. From this point of view, the proposed the use of optimized adaptive irregular constellations also brings performance benefits compared to other M -ary schemes for ideal and EVM = -25 dB values. Note that better performance can be obtained by considering **P3** for 16-ary cases while it was already shown in [6] that joint design of constellation and power allocation scheme yields to nearly similar error performance. In the comparison of the structure of optimized IGS and the

Fig. 11: The performance comparison between optimized irregular constellations obtained from **P1**, **P3** and 16-QAM for different $\bar{\gamma}$ values for EVM = -15 dB.

Fig. 12: Optimized irregular constellations for $\text{EVM} = -20$ dB: (a) the source constellation from **P1**. (b) the relay constellation from **P1**. (c) the source constellation from **P3**. (d) the relay constellation from **P3**.

proposed constellations, it should be noted that the design framework in the IGS scenarios implies to find an optimum κ value in (34) for a given design parameter. Therefore, IGS-deploying scenarios leads parallelograms in the optimized constellations. Even there is no predefined grid structure in IGS cases, there are still some implicit constraints between constellation points forcing them in different parallelogram and it is not possible to cover all possible symbol point distribution

with κ . The irregular constellations plotted in Fig. 7-Fig. 9 are the outcomes of fully-relaxed design frameworks on the symbol point locations and those constellations cannot be covered by (34). From this point of view, the proposed optimization framework can lead those types of irregular constellations based on the derived error performance expression, $\bar{P}_{e,\text{SRD}}$. The complete list of the coordinates of the symbol point locations for different proposed optimized irregular constellations can be

Fig. 13: Some IGS constellation examples proposed in [6] with different circularity values (κ).

Fig. 14: The performance comparison between optimized irregular constellations and IGS constellations in [6].

found in [34].

VI. CONCLUSION

We proposed an adaptive transmission scheme for the full-duplex relay link with residual self-interference. We first derived the end-to-end error performance expressions for the source-relay and the relay-destination links. Then, based on those expressions the end-to-end error performance was improved by finding optimal constellations without constraining on its symbol point locations. In addition, joint optimization of constellation and power allocation was also considered, and the simulation results showed that the proposed adaptive

transmission methods yield more reliable communication than the conventional M -QAM constellation and IGS constellation already considered for FD relaying schemes.

Moreover, optimized irregular constellations outperform 16-QAM provided that the EVM estimation error is less than 3 dB. The implementation issues related with deploying adaptive optimized irregular constellations such as the demapping complexity and the look-up-tables were discussed as well.

ACKNOWLEDGMENT

This work was supported by the Academy of Finland (grant numbers 334000 and 339446).

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