This is an electronic reprint of the original article.
This reprint may differ from the original in pagination and typographic detail.

Tcvetkova, Svetlana; Martini, Enrica; Tretyakov, Sergei; Maci, Stefano

Efficient Conversion of TM Surface Wave into TM Leaky Wave by Metasurface

Published in:
15th European Conference on Antennas and Propagation, EuCAP 2021

DOI:
[10.23919/EuCAP51087.2021.9411277](https://doi.org/10.23919/EuCAP51087.2021.9411277)

Published: 22/03/2021

Document Version
Publisher's PDF, also known as Version of record

Please cite the original version:
Tcvetkova, S., Martini, E., Tretyakov, S., & Maci, S. (2021). Efficient Conversion of TM Surface Wave into TM Leaky Wave by Metasurface. In *15th European Conference on Antennas and Propagation, EuCAP 2021* [9411277] IEEE. <https://doi.org/10.23919/EuCAP51087.2021.9411277>

Efficient Conversion of TM Surface Wave into TM Leaky Wave by Metasurface

Svetlana Tsvetkova¹, Enrica Martini², Sergei Tretyakov¹, Stefano Maci²

¹ Department of Electronics and Nanoengineering, Aalto University, P.O. 15500, FI-00076 Aalto, Finland

² Department of Information Engineering and Mathematics, University of Siena, 53100 Siena, Italy (e-mail: macis@dii.unisi.it)

Abstract—In this paper we discuss possible practical implementations of exact solutions for a lossless and reciprocal penetrable metasurface which ensures full conversion of a single-mode surface wave to an arbitrarily directed single plane leaky wave of the same polarization. In contrast to known leaky-wave antennas, we have found the optimal surface reactance modulation which ensures the absence of higher-order modes of the Floquet wave expansion. Therefore, there is no accumulation of reactive energy in higher order modes and all the energy carried by the surface wave is used for launching the single plane wave into space.

Index Terms—metasurfaces, surface waves, leaky waves, antennas

I. INTRODUCTION

One of the important problems in antenna theory is the problem of launching the energy carried by a surface wave along a boundary into space in a form of a leaky wave. Conventionally, this is done by leaky-wave antennas, which are usually realized as periodically modulated open and partially open waveguides or other structures with periodic spatial perturbations, e.g. metasurfaces. In all these periodic solutions the vicinity of the reactive boundary contains the fields of higher-order Floquet harmonics which store reactive energy. This produces dispersive effects which limit the bandwidth of the antenna. Thus, it is of importance to find a particular metasurface design which limits the storage of the reactive energy near the surface, in other words, the periodic metasurface which supports only 0 and -1 modes of the Floquet expansion.

Recently, metasurfaces that allow for an efficient conversion of the surface wave into a homogeneous plane wave [1] and leaky wave [2] (or vice versa) were proposed. However, to achieve a perfect conversion (or in case of [1], the conversion with the maximum possible efficiency) the polarization of the plane wave should be different from that of the surface wave. In practice, it means that the metasurface is anisotropic, which adds another degree of difficulty to the structure realization. In [3], the routing of space waves via surface waves with phase-gradient metasurfaces based on a momentum transfer approach was introduced and tested experimentally. In [4], a solution for launching one propagating plane wave from a partially transparent wall of a waveguide was described. However, the

practical realization of such structure is not feasible due to its complexity and bulkiness.

Here, we introduce a theoretically ideal TM surface wave to TM leaky wave converter based on isotropic periodic metasurface, which can be practically realized as an array of metal patches or slots printed on a grounded dielectric slab.

II. PERIODIC BOUNDARY VALUE PROBLEM

Let us consider a metasurface modeled by a periodically modulated reactance X with period d over a lossless grounded dielectric slab with relative permittivity ϵ_r and thickness h . A surface wave (0-mode), launched along the surface, will leak energy, transforming into a leaky wave (-1-mode). The goal is to synthesize a periodic reactance X in such a way that only these two modes of the Floquet waves expansion can exist.

The current distributions of 0- and -1 modes $J^{(0)}$ and $J^{(-1)}$ are given by

$$J^{(0,-1)} = J_{0,-1} e^{-jk_x^{(0,-1)} x}, \quad (1)$$

where J_0 , J_{-1} are constant amplitudes, $k_x^{(0)} = \beta_x^{(0)} - j\alpha_x$, $k_x^{(-1)} = k_x^{(0)} - 2\pi/d$ are complex wavenumbers of 0- and -1-modes, respectively. Attenuation constant α_x is related with the energy transfer from a guided mode into a radiating mode along the propagating path. Figure 1 depicts the equivalent circuit for coupling of two transmission lines associated with 0- and -1-Floquet wave TM modes.

Fig. 1 Equivalent circuit for coupling two transmission lines (along the normal to the metasurface) with the (0) and (-1) TM modes.

The spectral Green's function of a grounded slab locally defines the Floquet wave electric field, i.e.,

$$E_x = E^{(0)} + E^{(-1)}, \quad (2)$$

$$E^{(0)} = jX_{GF}^{(0)} \cdot J^{(0)}, \quad E^{(-1)} = jX_{GF}^{(-1)} \cdot J^{(-1)}, \quad (3)$$

$$X_{GF}^{(0)} = \frac{X^+(k_x^{(0)})X^-(k_x^{(0)})}{X^+(k_x^{(0)}) + X^-(k_x^{(0)})}, \quad (4)$$

$$X_{GF}^{(-1)} = \frac{X^+(k_x^{(-1)})X^-(k_x^{(-1)})}{X^+(k_x^{(-1)}) + X^-(k_x^{(-1)})}, \quad (5)$$

$$X^+(k_x^{(0,-1)}) = -\zeta \frac{\sqrt{(k_x^{(0,-1)})^2 - k_0^2}}{k_0}, \quad (6)$$

$$X^-(k_x^{(0,-1)}) = \zeta \frac{\sqrt{\epsilon_r k_0^2 - (k_x^{(0,-1)})^2}}{k_0 \epsilon_r} \tan \left(h \sqrt{\epsilon_r k_0^2 - (k_x^{(0,-1)})^2} \right) \quad (7)$$

where $X^\pm(k_x^{(0,-1)})$ are TM reactances of transmission lines (in the direction normal to metasurface) towards free space and towards the ground, k_0 is the free-space wavenumber, and ζ is the free-space impedance.

The exact solution of the boundary value problem with two waves only can be found as

$$X(J^{(0)} + J^{(-1)}) = X_{GF}^{(0)} \cdot J^{(0)} + X_{GF}^{(-1)} \cdot J^{(-1)} \quad (8)$$

which is the transverse resonance equation for the problem. The interpretation of (8) can be seen in Fig. 1. Due to the absence of forced excitation and the assumption on existence of only two modes, the real solution for the real-valued surface reactance reads

$$X = \text{Re } X_{GF}^{(0)} + \text{Im } X_{GF}^{(0)} \tan(K(x - x_0)/2), \quad (9)$$

where x_0 is an arbitrary shift of the reference system (further we assume it to be 0 for simplicity). Surface reactance X is a real-valued function only if $J_{-1} = J_0 e^{-j\gamma}$ and $X_{GF}^{(-1)} = X_{GF}^{(0)*}$, where γ is a constant phase. This condition implies that there is only exchange of power between two waves, and no active power is produced or lost. Therefore, for a given slab, the solution of a perfect TM-to-TM conversion exists if there is such value of $k_x^{(0)}$ that satisfies all the above conditions. Some of the solutions $k_x^{(0)}/k_0$ are shown in Fig. 2. An example of normalized surface reactance for different slab thicknesses is illustrated in Fig. 3.

Fig. 2. Real and imaginary parts of $k_x^{(0)}/k_0$ for dielectric permittivity of the slab $\epsilon_r = 15$ and frequency 10 GHz. Continuous line: broadside beam; dashed lines: backward beam at 30° counterclockwise from the normal. Insets illustrate the directions of the wavevectors of the two waves. Also, the regions for which the average impedance $X_0 = \text{Re } X_{GF}^{(0)}$ is capacitive and inductive are shown.

Fig. 3. Normalized surface reactance at the frequency $f = 10$ GHz for the dielectric permittivity of the slab $\epsilon_r = 15$, for the following thicknesses of the slab $h/\lambda = 0.1$ (green dotted line) and $h/\lambda = 0.02$ (blue dotted line) for the broadside beam case.

III. CONSIDERATIONS ON IMPLEMENTATION

The tangent function in Eq. (9) exhibits periodic singularities. This behaviour can be approximated by printed elements described by an equivalent LC circuit. In case of the reactance solution in Fig. 3 ($h/\lambda = 0.02$), printed slots close to the resonance (parallel type) appear as a promising option. The slots (see Fig. 4) are placed orthogonally to the propagation direction. In case of the

reactance curve for $h/\lambda = 0.1$, printed dipoles that exhibit a series type resonant behavior are required. However, their placement should be along the propagation direction for waves of TM polarization, meaning that reproducing of the impedance profile is critical. Convolved dipoles are investigated as a possible alternative.

Fig. 4 Possible layout of a slot-type metasurface to reproduce the reactance shown in Fig. 3 (blue dotted line). The layout is only indicative of a possible solution.

IV. FULL-WAVE ANALYSIS

A full-wave code has been developed for an arbitrary homogenized penetrable reactance X with period d to test the solution and study the sensitivity performance. A periodic method of moments is used in the algorithm, where the currents and fields are represented by the Floquet wave basis functions. The analysis is a generalization of the Oliner's method presented in [5] for penetrable impedance and arbitrary periodic profile.

The sensitivity analysis has been performed for the slab of relative permittivity $\epsilon_r = 15$ and $h/\lambda = 0.08$ for the broadside leaky wave direction. Figure 5 shows the amplitudes of the Fourier coefficients of the current assuming a reactance given by eq. (9). For the sensitivity analysis, the ideal function is approximated using the Taylor expansion around $x=0$ of the tangent function with the result truncation over a period and periodical repetition of the resulting function. The amplitudes of the higher-order Floquet wave coefficients increase with the decreasing of the approximation accuracy and the amplitude of the -1-mode coefficient decreases with respect to 0-mode.

V. CONCLUSIONS

We have introduced a theoretically ideal TM surface wave to TM leaky wave converter based on isotropic periodic metasurface that can be practically realized as an array of metal patches or slots printed on a grounded dielectric slab. This solution has been found by imposing the power balance with only two waves in the absence of losses.

Fig. 5 Amplitude of the FW spectrum of the currents flowing in the metasurface when the reactance deviates from the form in (9). The Taylor expansion of the tangent function is used around zero, truncated in one period, and periodically replicated. The results are obtained by the full-wave analysis. (a), (b), (c) correspond to $N=2$, $N=4$, $N=6$ terms in the Taylor expansion, respectively. The resulting approximation of the reactance is given in the inset in red; (d) presents the results for the exact representation of the tangent function.

REFERENCES

- [1] S. N. Tsvetkova, D. H. Kwon, A. Diaz-Rubio, and S. A. Tretyakov, "Near-perfect conversion of a propagating plane wave into a surface wave using metasurfaces," *Phys. Rev. B*, vol. 97, p. 115447, Mar 2018.
- [2] S. N. Tsvetkova, S. Maci, and S. A. Tretyakov, "Exact solution for surface wave to space wave conversion by periodical impenetrable metasurface" *IEEE Transactions on Antennas and Propagation*, Vol 67, issue 5, May 2019.
- [3] G. Minatti, F. Caminita, M. Casaletti, S. Maci, "Spiral leaky-wave antennas based on modulated surface impedance," *IEEE Trans on Antennas Propagat.*, vol. 59, no. 12, pp. 4436-4444, 2011.
- [4] E. Abdo-Sánchez, M. Chen, A. Epstein and G. V. Eleftheriades, "A Leaky-Wave Antenna With Controlled Radiation Using a Bianisotropic Huygens' Metasurface," in *IEEE Transactions on Antennas and Propagation*, vol. 67, no. 1, pp. 108-120, Jan. 2019.
- [5] A. Oliner, A. Hessel, "Guided waves on sinusoidally-modulated reactance surfaces," *IRE Trans. Antennas Propag.*, vol. 7, no. 5, pp. 201-208, Dec. 1959.