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Reducing time headway in platoons under the MPF topology when using sensors and wireless communications

Elham Abolfazli, Bart Besselink, and Themistoklis Charalambous

Abstract—In platoons under the multiple-predecessor following (MPF) topology, the exchange of information is usually assumed to be via Vehicle-to-Vehicle (V2V) communication links. Communication delays, however, deteriorate internal and string stability and, as a consequence, to guarantee stability of the platoon the time headway between vehicles should be increased. Autonomous vehicles are nowadays equipped with a multitude of sensors, the three primary being radar, lidar and camera. The combination of such sensors allows a vehicle, among others, to detect the distance and speed of nearby objects. In this paper, we incorporate the available sensors of vehicles to anticipate the communication delays vehicles experience with their preceding vehicles, thus improving the stability conditions and subsequently reducing the time headway. More specifically, by incorporating sensors we alleviate the communication delays between neighboring vehicles. We demonstrate that (i) the system is internally stable irrespective of the size of the communication delays, and (ii) the time headway can be reduced, by deriving a sufficient condition which provides a lower bound on the time headway that guarantees string stability. With this new lower bound on the time headway, platoons do not need to massively increase the time headway in order to compensate for the effects of communications delays. Simulations demonstrate the effectiveness of the proposed lower bound.

Index Terms—Platooning, multiple-predecessor following, communication delays, sensors, string stability, time headway.

I. INTRODUCTION

In intelligent transportation systems, the idea of vehicle platooning, especially for heavy-duty vehicles, has attracted a lot of attention from both academia and industry due to the potential in increasing the capacity of roads, reducing the fuel consumption and CO₂ emission, while reducing the manual labor required [1]. Note that heavy-duty vehicles are responsible for about a quarter of CO₂ emissions from road transport in the EU and for some 6% of total EU emissions, which is a considerable amount. Towards reducing these CO₂ emissions, for the first time standards for heavy-duty vehicles were adopted in 2019 [2]. Platooning serves as another way of reducing CO₂ emission and a promising commercially viable application of heavy-duty vehicle automation. Technological advancements have made the deployment of such platooning systems technically feasible and major vehicle manufacturers are actively developing such systems that are expected to be widely adopted in the near future [3].

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In order to construct a platooning, a cruise control system (controller) should be designed for each vehicle ensuring that i) all the vehicles move at a desired speed, while maintaining the desired inter-vehicle distance (internal stability [4]) and ii) the error, coming from any disturbances or changes in input or reference, should not amplify upstream, guaranteeing that no vehicles will collide (string stability [5], [6]).

One of the parameters affecting string stability is the spacing policy. It determines the desired distance between vehicles in a platoon. Many works in the literature, such as [7] and [8], provided definitions about the desired inter-vehicle distance. The most prevalent policies are Constant Spacing Policy (CSP) and Constant Time Headway Spacing policy (CTHP) [9], [10]. In CSP, the desired distance between vehicles is constant, while for CTHP, the desired inter-vehicle distance is a linear function of the speed of the vehicles. The proportional gain in CTHP is called *time headway*. Although the road capacity increases by choosing a smaller value for the time headway, it compromises string stability. A challenge faced by safely-driving automated vehicles is that the anticipated headway is often quite large, resulting in lower or no reduction of fuel consumption. Additionally, they would not harmonize well with traffic flow, since large headways would allow other vehicles – not participating in the platoon – to cut in, thus intercepting the platoon and even putting passengers of following automated vehicles at risk. Therefore, finding the minimum (or at list a good lower bound) time headway plays an important role in practically implementing a vehicle platoon.

Technological advances have provided Vehicle-to-Vehicle (V2V) communication and hence, each vehicle can be connected to multiple vehicles. In [11], a vehicle platoon with V2V communication in the presence of parasitic lags is considered and the minimum time headway is proposed for two scenarios, connecting to ‘ r ’ predecessor vehicles and connecting to immediate predecessor and also the r -th predecessor. In [12], a new definition for the desired inter-vehicle distance in platoons under the MPF topology and in the presence of time lag is introduced and also a lower bound for acceptable time headway is found, which shows that by increasing the number of predecessors, this minimum time headway can be reduced. In [13], a minimum time headway is proposed for a platoon, in which there are parasitic delays and lags and each vehicle is only connected to its predecessor. The lower bound on time headway proposed in [13] results in a higher throughput; however, it does not include the effects of multiple connections between vehicles. In [14],

internal stability and string stability of platoons under the MPF topology, in presence of lags and also communication delay, which is a challenge of V2V communications, are analyzed and a lower bound on the time headway is provided. Based on [14], larger values of communication delay result in a larger time headway, which is undesirable and in contrast with the concept of platooning.

To alleviate the impact of communication delays, V2V communications can be used together with on-board sensors, such as radar, lidar and camera. The use of these different sensors can provide instantaneous updates of vehicles status [15], [16] and lead to more robust cruise control systems with increased driver comfort in vehicle platooning systems. Then, supposing that all vehicles have access to their own position, velocity and acceleration without any delays, by using onboard sensors, the position and velocity of their immediate predecessor can be measured easily and do not have to be communicated through the wireless network. As a result, and hence, the position and velocity of their immediate predecessor are not subject to a delay. Of course, the acceleration of the immediate predecessor [17], as well as the position, velocity and acceleration of further vehicles are obtained from V2V communications, which inevitably suffer from time delays.

In this paper, for such a system (which combines sensors and V2V communications) and under the MPF topology, internal and string stability are analyzed. More specifically, the contributions of the paper are as follows.

- It is shown that this type of vehicle platoon can be internally stable, independently of the size of the delay.
- Moreover, string stability is investigated and a lower bound on the time headway is proposed.
- Numerical simulations corroborate the proposed theorems and shows that the minimum time headway is smaller than the case in which all information comes from V2V communications, as it is in [14].

The rest of the paper is structured as follows. In Section II the notations and some mathematical preliminaries necessary for the development of our results are presented. In Section III the vehicle model and control structure are given. In Section IV, the control law for a platoon with homogeneous time delays is proposed and the objectives of the paper are formulated. In Section V the string stability of the system is analyzed, providing a lower bound for the time headway. In Section VI simulation results show that if the proposed lower bound in headway is satisfied, string stability is guaranteed. Finally, in Section VII, we draw conclusions and discuss future directions.

II. NOTATION AND MATHEMATICAL PRELIMINARIES

A. Notation

Vectors and matrices are denoted by lowercase and uppercase letters, respectively. Integer and natural numbers sets are denoted by $\mathbb{Z}$ and $\mathbb{N}$, respectively. $\mathbb{Z}_0 \triangleq \{0, 1, 2, \dots\}$, $\mathbb{Z}_0^n \triangleq \{0, 1, 2, \dots, n\}$, and $\mathbb{N}^n \triangleq \{1, 2, \dots, n\}$. Real and nonnegative real numbers sets are denoted by $\mathbb{R}$ and $\mathbb{R}_+$,

respectively. $m \times n$ real matrices are denoted by $\mathbb{R}^{m \times n}$. For any matrix $A \in \mathbb{R}^{m \times n}$, $(m, n) \in \mathbb{N} \times \mathbb{N}$, we denote its transpose by A^T and its entries by a_{ij} , $i \in \mathbb{N}^m$, $j \in \mathbb{N}^n$ (i.e., $A = [a_{ij}]$). The $n \times n$ identity matrix is denoted by I_n .

B. Mathematical Preliminaries

Lemma 1: [18] Suppose A , B , C and D are matrices of dimension $n \times n$, $n \times m$, $m \times n$ and $m \times m$, respectively. Then, if A is invertible, for the block matrix we have

$$\det \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \det(A) \det(D - CA^{-1}B). \quad (1)$$

In this paper, we consider the vehicle platoon as a directed graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{v_1, v_2, \dots, v_N\}$ is a set of nodes representing all the following vehicles and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is a set of edges representing the connections between each pair of following vehicles. The following matrices characterize some properties of $\mathcal{G}$. First, the Laplacian matrix associated with $\mathcal{G}$ is defined as $\mathcal{L} = [l_{ij}]$, $i, j \in \mathbb{N}^N$, with

$$l_{ij} = \begin{cases} -a_{ij}, & i \neq j, \\ \sum_{k=1}^N a_{ik}, & i = j, \end{cases} \quad (2)$$

where $a_{ij} = 1$ if $(v_i, v_j) \in \mathcal{E}$ and $a_{ij} = 0$, otherwise. Also, the connections between the vehicles and the leader can be modeled by

$$\mathcal{P} = \text{diag}\{p_{11}, p_{22}, \dots, p_{NN}\}, \quad (3)$$

where $p_{ii} = 1$ when vehicle i is connected to the leader and $p_{ii} = 0$, otherwise. Then, a new information topology matrix can be defined as

$$\mathcal{L}_p := \mathcal{L} + \mathcal{P}. \quad (4)$$

III. VEHICLE MODEL AND CONTROL STRUCTURE

A. Vehicle Model

Consider a platoon of N vehicles with the following model for their longitudinal dynamics, as in, e.g., [19]

$$\begin{cases} \dot{p}_i(t) = v_i(t), \\ \dot{v}_i(t) = a_i(t), \\ \tau_i \dot{a}_i(t) + a_i(t) = u_i(t), \end{cases} \quad (5)$$

where $p_i(t)$, $v_i(t)$, $a_i(t)$ and $u_i(t)$ are the position, velocity, acceleration and control input of the i th vehicle, respectively and $\tau_i > 0$ is the time lag in the powertrain. Also, $p_0(t)$, $v_0(t)$, $a_0(t)$ and $u_0(t)$ are the position, velocity, acceleration and control input of the leader vehicle, respectively.

It is assumed that vehicle i can use information from multiple predecessor vehicles, as shown in Fig. 1, where vehicle i , $i \in \mathbb{Z}_3^N$, is connected to three predecessor vehicles. For this topology, the desired distance between vehicle i and the l -th vehicle ahead of it is considered as [12]

$$d_{i,i-l}(t) = \sum_{k=i-l+1}^i (h_k v_k(t) + d_k), \quad (6)$$

where $h_k \geq 0$ is the time headway of vehicle k and $d_k > 0$ is the desired standstill gap between vehicle k and $k-1$.

Fig. 1: A platoon under MPF topology, using V2V communications and onboard sensors.

B. Control Structure

The following linear feedback controller is used in [12], for vehicle i when there are no time-delays:

$$u_i(t) = - \sum_{l=1}^{r_i} \left(k_{pi} \left(p_i - p_{i-l} + \sum_{k=i-l+1}^i (h_k v_k + d_k) \right) + k_{vi} (v_i - v_{i-l}) + k_{ai} (a_i - a_{i-l}) \right), \quad (7)$$

where $r_i \leq i$ is the number of the vehicles directly ahead of vehicle i that send their information to it. Control parameters k_{pi} , k_{vi} and k_{ai} are positive tunable gains for feeding back distance, velocity and acceleration errors between vehicle i and its l -th vehicle ahead.

IV. PROBLEM FORMULATION

It is assumed that each vehicle has access to all its own states and also position and velocity of the preceding vehicle without any delay, i.e., $\{p_i(t), v_i(t), a_i(t), p_{i-1}(t), v_{i-1}(t)\}$ are available for the controller on vehicle i . But, the acceleration of the previous vehicle as well as state values of all other connected vehicles are communicated through the wireless network and thus, are subject to time delay, i.e., $\{a_{i-1}(t-\Delta), p_{i-l}(t-\Delta), v_{i-l}(t), a_{i-l}(t-\Delta)\}, \forall 2 \leq l \leq r_i$, where Δ denotes the homogeneous time-delay. Using the controller proposed in [12], the actual form of the controller which utilizes the available data for vehicle i will be

$$\begin{aligned} u_i(t) = & -k_{pi} \left(p_i(t) - p_{i-1}(t) + h_i v_i(t) + d_i \right) \\ & -k_{pi} \sum_{l=2}^{r_i} \left(p_i(t) - p_{i-l}(t-\Delta) - \Delta v_0 \right. \\ & \quad \left. + \sum_{k=i-l+1}^i (h_k v_k(t - \beta_{ki} \Delta) + d_k) \right) \\ & -k_{vi} (v_i(t) - v_{i-1}(t)) \\ & -k_{vi} \sum_{l=2}^{r_i} (v_i(t) - v_{i-l}(t-\Delta)) \\ & -k_{ai} \sum_{l=1}^{r_i} (a_i(t) - a_{i-l}(t-\Delta)), \end{aligned} \quad (8)$$

where

$$\beta_{ki} = \begin{cases} 0, & k = i \quad \text{or} \quad k = i-1 \\ 1, & \text{otherwise.} \end{cases} \quad (9)$$

The communication time delay can be computed in automated vehicles, and thus Δv_0 as a supplement is proposed in the literature, such as [20], [21], in order to mitigate the negative impact of communication delay and reduce the difference between $p_{i-l}(t-\Delta)$ and $p_{i-l}(t)$. The speed of the lead vehicle v_0 is assumed to be constant and also available for all vehicles.

Control parameters should be carefully chosen so that the vehicles could track the desired inter-vehicle distance and also keep the desired speed. We investigate this objective in Section V-A. In addition, the controllers should have the ability to prevent the amplification of disturbances throughout the vehicle string. We examine this goal in Section V-B.

V. STABILITY ANALYSIS

As the first step for finding the closed loop form of the system, we define the following errors, with respect to the lead vehicle's states

$$\begin{cases} \bar{p}_i(t) = p_i(t) - p_0(t) + \sum_{k=1}^i (h_k v_k(t) + d_k), \\ \bar{v}_i(t) = v_i(t) - v_0, \\ \bar{a}_i(t) = a_i(t) - a_0. \end{cases} \quad (10)$$

Due to the constant speed of the leader, we have $u_0(t) = 0$ and $a_0(t) = 0$. Using (5), the error dynamics is obtained as

$$\begin{cases} \dot{\bar{p}}_i(t) = \bar{v}_i(t) + \sum_{k=1}^i h_k \bar{a}_k(t), \\ \dot{\bar{v}}_i(t) = \bar{a}_i(t), \\ \dot{\bar{a}}_i(t) = -\frac{1}{\tau_i} \bar{a}_i(t) + \frac{1}{\tau_i} u_i(t). \end{cases} \quad (11)$$

Also, after algebraic manipulations, the control input (8) can be rewritten as

$$u_i(t) = -k_{pi} (\bar{p}_i(t) - \bar{p}_{i-1}(t)) \quad (12a)$$

$$-k_{pi} \sum_{l=2}^{r_i} (\bar{p}_i(t) - \bar{p}_{i-l}(t-\Delta)) \quad (12b)$$

$$+k_{pi} \sum_{l=2}^{r_i} \left(\sum_{k=1}^{i-2} h_k (\bar{v}_k(t) - \bar{v}_k(t-\Delta)) \right) \quad (12c)$$

$$-k_{vi} (\bar{v}_i(t) - \bar{v}_{i-1}(t)) \quad (12d)$$

$$-k_{vi} \sum_{l=2}^{r_i} (\bar{v}_i(t) - \bar{v}_{i-l}(t-\Delta)) \quad (12e)$$

$$-k_{ai} \sum_{l=1}^{r_i} (\bar{a}_i(t) - \bar{a}_{i-l}(t-\Delta)). \quad (12f)$$

Term (12c) can be rewritten as

$$k_{pi} (r_i - 1) \left(\sum_{k=1}^{i-2} h_k (\bar{v}_k(t) - \bar{v}_k(t-\Delta)) \right). \quad (13)$$

Then, let $\bar{p} = [\bar{p}_1, \dots, \bar{p}_N]^\top$, $\bar{v} = [\bar{v}_1, \dots, \bar{v}_N]^\top$, $\bar{a} = [\bar{a}_1, \dots, \bar{a}_N]^\top$ and $\xi = [\bar{p}^\top, \bar{v}^\top, \bar{a}^\top]^\top$. Using (12), the closed loop platoon dynamics can be obtained as

$$\begin{aligned} \dot{\xi}(t) &= A\xi(t) + A_\Delta \xi(t-\Delta), \\ \xi(t) &= \Phi(t), \quad t \in [-\Delta, 0], \end{aligned} \quad (14)$$

where $\Phi(\cdot) \in \mathcal{C}([-\Delta, 0], \mathbb{R}^\nu)$ represents the initial state of the system, and A and $A_\Delta \in \mathbb{R}^{\nu \times \nu}$, $\nu = 3N$, are given as

$$A = \begin{bmatrix} 0 & I_N & H \\ 0 & 0 & I_N \\ -TK_p\mathcal{L}_{p_1} & -T(K_v\mathcal{L}_{v_1} + K_pFH) & -T-TK_a\mathcal{L}_{a_1} \end{bmatrix},$$

$$A_\Delta = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -TK_p\mathcal{L}_{p_2} & -T(K_v\mathcal{L}_{v_2} - K_pFH) & -TK_a\mathcal{L}_{a_2} \end{bmatrix},$$

with

$$K_m = \text{diag}\{k_{m1}, \dots, k_{mN}\}, \quad m \in \{p, v, a\},$$

$$T = \text{diag}\{1/\tau_1, \dots, 1/\tau_N\},$$

and

$$H = \begin{bmatrix} h_1 & 0 & \dots & 0 \\ h_1 & h_2 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ h_1 & h_2 & \dots & h_N \end{bmatrix}.$$

Moreover, we have

$$\mathcal{L}_{p_1} = \mathcal{L}_{v_1} = \begin{bmatrix} r_1 & 0 & 0 & \dots & 0 \\ -1 & r_2 & 0 & \dots & 0 \\ 0 & -1 & r_3 & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & -1 & r_N \end{bmatrix},$$

$$\mathcal{L}_{a_1} = \begin{bmatrix} r_1 & 0 & 0 & \dots & 0 \\ 0 & r_2 & 0 & \dots & 0 \\ 0 & 0 & r_3 & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & 0 & r_N \end{bmatrix},$$

and then, $\mathcal{L}_{p_2}$, $\mathcal{L}_{v_2}$ and $\mathcal{L}_{a_2}$ can be computed using

$$\mathcal{L}_{p_1} + \mathcal{L}_{p_2} = \mathcal{L}_{v_1} + \mathcal{L}_{v_2} = \mathcal{L}_{a_1} + \mathcal{L}_{a_2} = \mathcal{L}_p, \quad (15)$$

where $\mathcal{L}_p$ is defined in (4). Also, we have

$$F = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 \\ f_3 & 0 & 0 & \dots & 0 & 0 \\ 0 & f_4 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & f_N & 0 & 0 \end{bmatrix},$$

where $f_i = r_i - 1$.

A. Internal Stability Analysis

We provide necessary and sufficient conditions, which guarantee the asymptotic stability of the vehicle platoon (14). More specifically, we show that the internal stability of the system does not depend on the magnitude of the delays.

Theorem 1: Closed loop system (14) with $\Delta \geq 0$, is internally stable if and only if

$$\frac{1}{\tau_i}(1 + k_{ai}r_i)(k_{vi} + k_{pi}h_i) > k_{pi}, \quad \forall i \in \mathbb{N}^N. \quad (16)$$

Proof. See Appendix I.

B. Homogeneous String Stability Analysis

Assuming a homogeneous platoon, in which $\tau_i = \tau > 0$, $r_i = r$, $h_i = h$, $k_{pi} = k_p$, $k_{vi} = k_v$ and $k_{ai} = k_a$, $\forall i \in \mathbb{Z}_0^n$, we have

$$\tau \ddot{p}_i(t) + \ddot{p}_i(t) = u_i(t) \quad (17)$$

and

$$\tau \ddot{p}_{i-1}(t) + \ddot{p}_{i-1}(t) = u_{i-1}(t). \quad (18)$$

The time derivative of (17) is

$$\tau \ddot{v}_i(t) + \ddot{v}_i(t) = \dot{u}_i(t). \quad (19)$$

Then, after calculating (17) – (18) + $h \times$ (19), substituting from (8) and also defining the spacing error as $e_i(t) = p_i(t) - p_{i-1}(t) + hv_i(t) + d_i$, we obtain

$$\begin{aligned} \tau \ddot{e}_i(t) + (1 + rk_a)\ddot{e}_i(t) + r(k_v + k_ph)\dot{e}_i(t) + rk_pe_i(t) = \\ k_a\ddot{e}_{i-1}(t - \Delta) + (k_v - k_ph(r - 1))\dot{e}_{i-1}(t) + k_pe_{i-1}(t) \\ + \sum_{l=2}^r \left(k_a\ddot{e}_{i-l}(t - \Delta) + (k_v - k_ph(r - l))\dot{e}_{i-l}(t - \Delta) \right. \\ \left. + k_pe_{i-l}(t - \Delta) \right). \end{aligned} \quad (20)$$

After taking the Laplace transform, we have

$$E_i(s) = H_1(s)E_{i-1}(s) + \sum_{l=2}^r H_l(s)E_{i-l}(s), \quad (21)$$

where $E_i(s)$ is the Laplace transformation of $e_i(t)$ and

$$H_1(s) = \frac{k_as^2e^{-\Delta s} + (k_v - k_ph(r - 1))s + k_p}{\tau s^3 + (1 + rk_a)s^2 + r(k_v + k_ph)s + rk_p} \quad (22)$$

and

$$H_l(s) = \frac{k_as^2e^{-\Delta s} + (k_v - k_ph(r - l))se^{-\Delta s} + k_pe^{-\Delta s}}{\tau s^3 + (1 + rk_a)s^2 + r(k_v + k_ph)s + rk_p}. \quad (23)$$

Using the string stability definition in [12], platoon (5) controlled by (8) is string stable if both of the following conditions hold

$$\|H_1(j\omega)\|_\infty \leq \frac{1}{r}, \quad (24a)$$

$$\|H_l(j\omega)\|_\infty \leq \frac{1}{r}, \quad \forall 2 \leq l \leq r, \quad (24b)$$

where $H_1(j\omega)$ and $H_l(j\omega)$ are derived by substituting $s = j\omega$ in (22) and (23).

Theorem 2: Vehicle platoon (5) with the control input (8) and the control parameters that satisfy internal stability condition (16), is string stable if all the following conditions hold

$$r(1 - (l - r)^2)h^2k_p + 2r(1 + r - l)hk_v - 2 \geq 0, \quad (25a)$$

$$k_v - k_ph(r - 1) \geq 0, \quad (25b)$$

$$rk_a\Delta \leq \tau, \quad (25c)$$

$$2r^2hk_v \geq 2(1 + 2rk_a) + r^3k_ph^2 - 2r^2k_ph^2, \quad (25d)$$

$$1 + 2r(k_a - \tau(k_v + k_ph)) \geq 2r^2k_a(k_v - k_ph(r - 1))\Delta. \quad (25e)$$

For the region defined by (25), string stability conditions (24) holds if

$$h \geq h_{\min} = \max \{h_{\min,1}, h_{\min,l}\}, \quad (26)$$

where

$$h_{\min,1} = \frac{2(\tau + rk_a\Delta)}{r}, \quad h_{\min,l} = \frac{2\tau}{2rk_a + 1}. \quad (27)$$

$h_{\min,1}$ and $h_{\min,l}$ are the minimum time headways that guarantee (24a) and (24b) respectively.

Proof. See Appendix II.

Remark 1: The minimum time headway proposed in [14], when all the information only comes from V2V communications and is subject to time delay, is

$$h_{\min} = \frac{2(\tau + \Delta)}{2rk_a + 1}. \quad (28)$$

It is clear that when in (26), $h_{\min} = \frac{2\tau}{2rk_a + 1}$, this minimum time headway is smaller than (28) and hence, the use of onboard sensors has resulted in a higher throughput. If $h_{\min} = \frac{2(\tau + rk_a\Delta)}{r}$, the values of time delay Δ and also τ , k_a and r will define whether (28) is smaller or (26).

VI. NUMERICAL RESULTS

To illustrate the proposed theorems, numerical presentations are proposed in this section. We consider two scenarios, where in both, vehicles have the linear model as (5), controller (8) is used to make the platoon internally and string stable and the number of connected predecessors is $r = 3$. Vehicles start from rest and move to reach the desired velocity, which is the leader's velocity, and also the desired inter-vehicle distance, based on (6). At $t = 60$, when the platoon has reached its stable equilibrium, an external disturbance in the form of a sinusoidal perturbation $u_0(t) = A_0 \sin(\omega_0 t)$ with the duration of one cycle, acts on the leader. The numerical values for simulation parameters are given in Table I.

TABLE I:
Model Parameters

N	r	d	v_0	A_0	ω_0
5	3	5 m	20 m/s	10 m/s ²	1 rad/s

Also, the numerical values for the two different scenarios are considered as Table II and the control parameters are chosen in such a way that the internal stability condition (16) holds for the both scenarios.

TABLE II:
Model Parameters for Different Scenarios

	τ	Δ	k_a	$h_{\min,1}$	$h_{\min,l}$
Scenario 1	0.5 s	0.1 s	0.18	0.37 s	0.48 s
Scenario 2	0.4 s	0.3 s	0.3	0.45 s	0.29 s

In the first scenario, where a small value for the time delay is considered, we obtain $h_{\min,1} < h_{\min,l}$ and, based on Theorem 2, the final minimum time headway will be

$h_{\min} = h_{\min,l} = 0.48s$. To verify this result, three different values for the time headway are considered in Fig. 2. It can be seen in Fig. 2(a) and Fig. 2(d) that when $h < h_{\min,1}$, the magnitudes of all string stability functions surpass $1/r$ and there are some collisions in the platoon. In Fig. 2(b) and Fig. 2(e), when $h_{\min,1} < h < h_{\min,l}$, although the magnitudes of $H_1(j\omega)$ and $H_2(j\omega)$ are smaller than $1/r$, the magnitude of $H_3(j\omega)$ is not acceptable for string stability and the external disturbance results in a collision between the leader and the first vehicle. Finally, when $h > h_{\min,l}$, the magnitudes of all string stability functions are smaller than $1/r$ and the platoon is string stable, as it is shown in Fig. 2(c) and Fig. 2(f).

In the second scenario, there is a larger communication delay between the vehicles and based on the values in Table II, $h_{\min,1} > h_{\min,l}$ and according to Theorem 2, $h_{\min} = h_{\min,1} = 0.45s$. Therefore, unlike the previous scenario, in which the magnitude of $H_l(j\omega)$ was defining the minimum time headway, in this scenario the magnitude of $H_1(j\omega)$ defines the acceptable time headway. To corroborate these results, simulations have been done for three different values of the time headway, as it is shown in Fig. 3. In the first case, when $h < h_{\min,l}$, it is demonstrated in Fig. 3(d) that all the string stability functions surpass $1/r$. By increasing the time headway and having $h_{\min,l} < h < h_{\min,1}$, Fig. 3(e) shows that although the behavior of $|H_2(j\omega)|$ and $|H_3(j\omega)|$ is acceptable, there are some frequencies at which $|H_1(j\omega)|$ is larger than $1/r$ and hence, the vehicles cannot move with a safe inter-vehicle distance and the platoon becomes string unstable after acting of the external disturbance. In the last case, when $h > h_{\min,1}$, Fig. 3(c) and Fig. 3(f) depicts that the platoon is string stable.

Remark 2: If we calculate the minimum time headway introduced in [14], we will have $h_{\min} = 0.58s$ and $h_{\min} = 0.5s$ for the first and the second scenario respectively, which are larger than the values obtained from Theorem 2. Therefore, for these two scenarios, the proposed lower bound in (26) is more beneficial.

VII. CONCLUSION AND FUTURE DIRECTIONS

A. Conclusions

In this paper, we made use of the available sensors of vehicles to anticipate the communication delays that vehicles experience with their preceding vehicles. As a result, we improved the stability conditions and reduced the time headway. We showed that the system is internally stable irrespective of the size of the communication delays, and the time headway can be reduced, by deriving a sufficient condition which provides a lower bound on the time headway that guarantees string stability. With this new lower bound on the time headway, platoons do not need to massively increase the time headway in order to compensate for the effects of communication delays. We demonstrated the effectiveness of the proposed lower bound via simulations.

B. Future directions

Next, we plan to derive sufficient conditions that guarantee internal and string stability in platoons with time-varying

Fig. 2: Simulation results for a vehicle platoon, with $h_{\min,l} > h_{\min,1}$.

Fig. 3: Simulation results for a vehicle platoon, with $h_{\min,1} > h_{\min,l}$.

heterogeneous delays, which constitutes a more realistic scenario, since packet drops and number of retransmissions (which determine the delay) might differ from vehicle to vehicle. Also, since it is possible that a channel might be in

deep fading for long, we intent to study the effect of packet drops in the (internal and string) stability as well.

Finally, we are interested in the scenario in which platoons under the MPF topology have bidirectional connectivity.

APPENDIX I
PROOF OF THEOREM 1

To find the stability conditions of the platoon, we take the Laplace transform of (14) and we obtain

$$\Xi(s) = (sI_{3N} - A - A_\Delta e^{-\Delta s})^{-1} \xi(0). \quad (29)$$

The characteristic equation of the platoon can be written as

$$\begin{aligned} G(s) &= \det(sI_{3N} - A - A_\Delta e^{-\Delta s}) \\ &= \det \begin{bmatrix} sI_N & -I_N & -H \\ 0 & sI_N & -I_N \\ \tilde{G}_1 & \tilde{G}_2 & sI_N + \tilde{G}_3 \end{bmatrix} \end{aligned} \quad (30)$$

where

$$\tilde{G}_1 = \text{TK}_p(\mathcal{L}_{p_1} + \mathcal{L}_{p_2} e^{-\Delta s}), \quad (31a)$$

$$\tilde{G}_2 = \text{TK}_v(\mathcal{L}_{v_1} + \mathcal{L}_{v_2} e^{-\Delta s}) + \text{TK}_p FH(1 - e^{-\Delta s}), \quad (31b)$$

$$\tilde{G}_3 = \text{T} + \text{TK}_a(\mathcal{L}_{a_1} + \mathcal{L}_{a_2} e^{-\Delta s}). \quad (31c)$$

By using Lemma 1, the determinant in (30) can be obtained as

$$\begin{aligned} G(s) &= \det(sI_N) \det \left(\begin{bmatrix} sI_N & -I_N \\ \tilde{G}_2 & sI_N + \tilde{G}_3 \end{bmatrix} \right. \\ &\quad \left. - \begin{bmatrix} 0 \\ \tilde{G}_1 \end{bmatrix} (sI_N)^{-1} \begin{bmatrix} -I_N & -H \end{bmatrix} \right) \\ &= \det(sI_N) \det \begin{bmatrix} sI_N & -I_N \\ \tilde{G}_2 + \frac{1}{s}\tilde{G}_1 & sI_N + \tilde{G}_3 + \frac{1}{s}\tilde{G}_1 H \end{bmatrix}. \end{aligned} \quad (32)$$

Again, by using Lemma 1, we have

$$\begin{aligned} G(s) &= \det(sI_N) \det(sI_N) \det \left[sI_N + \tilde{G}_3 + \frac{1}{s}\tilde{G}_1 H \right. \\ &\quad \left. + (\tilde{G}_2 + \frac{1}{s}\tilde{G}_1)(sI_N)^{-1} \right] \\ &= \det \left[s^3 I_N + s^2 \tilde{G}_3 + s(\tilde{G}_1 H + \tilde{G}_2) + \tilde{G}_1 \right]. \end{aligned} \quad (33)$$

Substituting from (31), one can see that $s^3 I_N + s^2 \tilde{G}_3 + s(\tilde{G}_1 H + \tilde{G}_2) + \tilde{G}_1$ is a lower triangular matrix and hence, its determinant equals the product of diagonal entries. Also, since the diagonal elements of $\mathcal{L}_{p_1}$, $\mathcal{L}_{v_1}$, $\mathcal{L}_{a_1}$ are same as the diagonal elements in $\mathcal{L}_p$, based on (15), the diagonal elements of $\mathcal{L}_{p_2}$, $\mathcal{L}_{v_2}$, $\mathcal{L}_{a_2}$ are zero. Moreover, matrix F has zero diagonal and hence, it is clear that $\tilde{G}_1$, $\tilde{G}_2$ and $\tilde{G}_3$ do not have any delay terms on their diagonals and as a result, the diagonal elements of (33) do not include the delay term. Eventually, (33) can be decoupled to N subsystems in the form of

$$\begin{aligned} G(s) &= \prod_{i=1}^N \left(s^3 + s^2 \frac{1}{\tau_i} (1 + k_{ai} l_{ii}) + s \frac{1}{\tau_i} l_{ii} (k_{vi} + k_{pi} h_i) \right. \\ &\quad \left. + \frac{1}{\tau_i} l_{ii} k_{pi} \right). \end{aligned} \quad (34)$$

For $i \geq r$, the final polynomial is defined as

$$\begin{aligned} g_i(s) &:= s^3 + s^2 \frac{1}{\tau_i} (1 + k_{ai} r_i) + s \frac{1}{\tau_i} r_i (k_{vi} + k_{pi} h_i) \\ &\quad + \frac{1}{\tau_i} r_i k_{pi}. \end{aligned} \quad (35)$$

Then, the closed loop system (14) is stable if and only if the roots of (35) lie in the left half plane. As proposed in [12], using the Routh Hurwitz stability criterion and also knowing that all the control parameters are positive, $g_i(s)$ is stable if and only if

$$\left(\frac{1}{\tau_i} (1 + k_{ai} r_i) \right) \left(\frac{1}{\tau_i} r_i (k_{vi} + k_{pi} h_i) \right) > \frac{1}{\tau_i} r_i k_{pi}, \quad (36)$$

which is equivalent to (16). $\square$

APPENDIX II
PROOF OF THEOREM 2

Inequalities (24a) and (24b) can be re-written as

$$\sup_{\omega > 0} |H_1(j\omega)|^2 \leq \frac{1}{r^2}, \quad (37a)$$

$$\max_{2 \leq l \leq r} \|H_l(j\omega)\|_\infty^2 = \max_{2 \leq l \leq r} \sup_{\omega > 0} |H_l(j\omega)|^2 \leq \frac{1}{r^2}. \quad (37b)$$

From [12], it can be easily seen that to satisfy condition (37b) and accordingly (24b), the following conditions should hold, for all $2 \leq l \leq r$:

$$1 + 2r(k_a - \tau(k_v + h k_p)) \geq 0, \quad (38a)$$

$$r(1 - (l - r)^2) h^2 k_p + 2r(1 + r - l) h k_v - 2 \geq 0, \quad (38b)$$

and also, the corresponding minimum time headway is

$$h \geq h_{\min, l} = \frac{2\tau}{2rk_a + 1}. \quad (39)$$

The next step is to find control parameters and their corresponding minimum time headway, which guarantee condition (37a). We define

$$|H_1(j\omega)|^2 \triangleq \frac{N_1}{D_1}, \quad (40)$$

where

$$\begin{aligned} N_1 &= (k_p - k_a \omega^2 \cos(\Delta\omega))^2 \\ &\quad + \left((k_v - k_p h(r - 1))\omega + k_a \omega^2 \sin(\Delta\omega) \right)^2, \end{aligned} \quad (41)$$

and

$$D_1 = (rk_p - (1 + rk_a)\omega^2)^2 + (r(k_v + k_p h)\omega - \tau\omega^3)^2. \quad (42)$$

After some calculations, condition (37a) can be written in the form of

$$D_1 - r^2 N_1 = M_6 \omega^6 + M_4 \omega^4 + M_3 \omega^3 + M_2 \omega^2 \geq 0, \quad (43)$$

where

$$M_6 = \tau^2, \quad (44a)$$

$$M_4 = 1 + 2rk_a - 2r\tau(k_v + k_ph), \quad (44b)$$

$$M_3 = -2r^2k_a(k_v - k_ph(r-1))\sin(\Delta\omega), \quad (44c)$$

$$M_2 = -2rk_p(1 + rk_a) + r^2(k_v + k_ph)^2 + 2r^2k_pk_a\cos(\Delta\omega) - r^2(k_v - k_ph(r-1))^2. \quad (44d)$$

Considering the fact that $\sin(\Delta\omega) \leq \Delta\omega$ for $\omega \geq 0$ and $\cos(\Delta\omega) \geq -1$, if

$$k_v - k_ph(r-1) \geq 0, \quad (45)$$

then we can write

$$D_1 - r^2N_1 \geq \omega^2(\overline{M}_4\omega^4 + \overline{M}_2\omega^2 + \overline{M}_0), \quad (46)$$

where

$$\overline{M}_4 = \tau^2, \quad (47a)$$

$$\overline{M}_2 = 1 + 2rk_a - 2r\tau(k_v + k_ph) - 2r^2k_a(k_v - k_ph(r-1))\Delta, \quad (47b)$$

$$\overline{M}_0 = -2rk_p(1 + rk_a) + r^2(k_v + k_ph)^2 - 2r^2k_pk_a - r^2(k_v - k_ph(r-1))^2. \quad (47c)$$

Then, if $\overline{M}_2, \overline{M}_0 \geq 0$, condition (46) holds and the platoon will be string stable. Therefore, we need

$$\overline{M}_2 \geq 0 \iff k_v \leq \frac{1 + 2rk_a - 2rk_ph(\tau - r\Delta k_a(r-1))}{2r(\tau + rk_a\Delta)}, \quad (48a)$$

$$\overline{M}_0 \geq 0 \iff k_v \geq \frac{2(1 + 2rk_a) + r^3k_ph^2 - 2r^2k_ph^2}{2r^2h}. \quad (48b)$$

The upper bound of k_v in (48a) should be larger than its lower bound in (48b). Then, after some simplifications, we obtain

$$(1 + 2rk_a)(rh - 2(\tau + rk_a\Delta)) + r^3k_ph^2(rk_a\Delta - \tau) \geq 0. \quad (49)$$

Considering

$$rk_a\Delta \leq \tau, \quad (50)$$

then, the following condition satisfies (49)

$$rh - 2(\tau + rk_a\Delta) \geq 0, \quad (51)$$

and the minimum acceptable time headway, which guarantees (37a) and accordingly (24a), can be written as

$$h \geq h_{\min,1} = \frac{2(\tau + rk_a\Delta)}{r}. \quad (52)$$

Finally, the minimum time headway which guarantees the string stability of platoon (5) controlled by (8) is

$$h \geq h_{\min} = \max\{h_{\min,1}, h_{\min,l}\}. \quad (53)$$

By considering (45), conditions (38a) and (47b) can be merged into (25e). $\square$

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