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A STRESS MAGNIFICATION FACTOR FOR PLATES WITH WELDING-INDUCED CURVATURES

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ABSTRACT

The fatigue strength of thin-walled structures can be reduced significantly by non-linear secondary bending effects resulting from geometrical imperfections such as axial and angular misalignments. The welding-induced distortions can cause a critical increase of the structural hot-spot stress in the vicinity of the weld. Traditionally, the classification society rules for the fatigue strength assessment of welded ship structures suggest an analytical formula for a stress magnification factor k_m for axial and angular misalignment under axial loading condition. Recently, the well-known analytical solution for the angular misalignment has been extended to account for the curvature effect. The present paper analyses the effect of non-ideal, intermediate boundary conditions between fixed and pinned ends. In this regard, the fixity factors ρ (with $0 \leq \rho \leq 1$ from ideally pinned to clamped conditions) are introduced in order to model the actual constraint on the rotation close to the ends. Under tension, a non-negligible decrease of the k_m factor is observed in relation to the reduction of the fixity factor at the welded end, while the fixity factor related to the loaded end has a minor effect on the k_m factor. Under compression, the reduction of the beam end fixity factors results into lower buckling resistance.

Keywords: Stress magnification factor – Welding distortions – Thin plate – Non-linear geometry – Non-ideal boundary conditions.

1. INTRODUCTION

Over the last decades, in the scientific research there has been a push towards sustainable metal structures. Aiming at reducing the economic and environmental impact, whilst maintaining highly performing mechanical properties, the

manufacturing of thin-walled steel structures currently stands as one of the most targeted engineering lightweight solution, especially for large structures such as cruise ships.

Thin-walled structures are particularly prone to structural stability and durability problems, as they show considerable susceptibility to manufacturing-induced imperfections. Depending on the engineering field, there are different thresholds to quantify what is a thin component. In shipbuilding industry, given that the code specifications (i.e. [1]) state a minimum allowed thickness $t_0 = 5$ mm for ship deck components, a wall is defined *thin* when its thickness is less than 5 mm [2],[3]. The fatigue stress assessment of these structures is a major challenge for current research. This mainly relates to the detrimental effect on the global fatigue strength caused by welding processes, which are inevitable in the assembly of the block-based, modular superstructure [3]. Part of the challenge also comes from the interaction between the local mechanical behavior of the welds and the global response of the ship [3],[4]. For this reason, recognizing the influencing factors in the ship performance is of fundamental importance. In this regard, even though low-heat-input welding methods such as laser-hybrid welding can guarantee a less severe impact (see [2][5][6]), geometric and material imperfections generate a non-negligible stress concentration in the weld region. Since this may eventually cause fatigue failures, attention has been devoted to the presence of such imperfections.

The currently preferred methodology for the fatigue strength assessment of large structures such as ships is the structural stress approach [7]. The analysis identifies a stress magnification factor as the ratio between the hot-spot structural stress and the nominal stress, i.e. $k_m = \sigma_{HS}/\sigma_N$. This factor is referred to as k_m , and its analytical definition is provided by shipbuilding code specifications and recommendations (see [1][7][8]).

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The computation of the k_m factor for plates with transverse butt joints currently accounts for the axial, e , and the angular, α , misalignment (see Figure 1). The assessment is limited to the presence of a global angular misalignment, i.e. to flat distortions.

Figure 1. Geometrical parameters describing the welding-induced imperfections. [7]

The analytical models provided by the design rules (see Eqs (1) and (2)) consider a one-dimensional, linearly distorted Euler-Bernoulli beam under axial tension. The k_m factors derived from the analytical models for fixed and pinned boundary conditions (BCs) at the plate supports were introduced in [9] as:

$$k_m = 1 + \frac{3al}{2t} \frac{\tanh \frac{\beta}{2}}{\frac{\beta}{2}}, \quad (1)$$

$$k_m = 1 + \frac{3al}{t} \frac{\tanh \beta}{\beta}, \quad (2)$$

respectively, where $\beta = (2l/t)\sqrt{(3\sigma_m/E)}$. Parameters l , t and α are the length, the thickness and the angular misalignment, respectively; σ_m is the tensile membrane stress (i.e. the nominal stress) applied to the structure, and E is the modulus of elasticity. These solutions reveal a non-linear relationship between the k_m factor and the remotely applied load (see Figure 2). This non-linearity is analytically derived by including the developing deflection into the equilibrium equation of the structure as a secondary bending moment.

Figure 2. Non-linear relationship between the stress magnification factor and the applied nominal stress. [2]

Recent research has clearly demonstrated that the simple models (in Eqs (1) and (2)) do not accurately predict the stress concentration for welded thin plates used in ship deck stiffened panels. In fact, the plates may take very irregular shapes (see e.g. Figure 3), including local angular misalignment at the weld location [2][3][5]. Therefore, design codes nowadays, do not recommend the utilization of plates thinner than 5 mm.

Figure 3. Measured welding-induced distortion over the length of butt-welded plates in a ship deck stiffened panel. [5]

Non-linear Finite Element Analyses (FEA) is successfully used to calculate structural stress for welded thin plates (see e.g. [5]). Compared to a time-consuming FEA, an analytical model for the stress magnification factor prediction considerably eases the concept design phase, which is limited in time. On this matter, Shen *et al.* [10] have introduced a modified k_m factor to include the effect of a curvature defined as a quadratic function, while Zhou *et al.* [11] defined equivalent dummy loads applied to a straight beam in order to account for a non-linear distortion. These studies have shown valid geometrically non-linear analysis approaches for the structural stress problem, but their engineering use is challenged by the complexity of the analytical formulations. Thus, Mancini *et al.* [12] have recently developed a more compact solution. The study considers a non-linear beam theory, which utilizes the von Kármán kinematic assumption. This assumption allows the analysis to consider moderate rotations of the beam model, thus including the secondary bending moments caused by the welding-induced curvature. This analytical solution applies to a pre-deformed Euler-Bernoulli beam, modelled with ideally pinned or clamped boundary conditions (BCs) and a linear lateral deflection superimposed to a half-sine curvature.

This paper extends the analytical approach introduced in [12] to non-ideal slope BCs at the beam ends. This novel study utilizes the definition of fixity factors to include the rotational stiffness of the joints, see also [13] and [14]. The analytical study considers a half-sine and a quarter-sine curvature and aims at understanding the impact of non-ideal BCs on the k_m factor computation.

2. METHODOLOGY

2.1 The assumptions of the analytical model

The three-dimensional continuum problem of the distorted plate in a stiffened panel is reduced to a one-dimensional beam problem, given that thin plates with a geometric slenderness $l/t \geq 10$ (i.e. the ratio between the beam length and thickness) are considered; see [15], [16] and [17]. Small elastic

displacements and moderate rotations are assumed, meaning that the Euler-Bernoulli Theory is considered along with the von Kármán assumption in the definition of the axial strain ε_x . As shown below, the axial strain becomes:

$$\varepsilon_x = \frac{du}{dx} + \frac{1}{2} \left(\frac{dw}{dx} \right)^2 - z \frac{d^2w}{dx^2}, \quad (3)$$

where x and z are the axial and vertical coordinates, while u and w are the axial displacement and the deflection of the beam. Based on the Principle of Virtual Displacement (see e.g. [15][16]), the equilibrium of the beam of length l and virtual displacements δu and δw , considering the external (dW_E) and internal (dW_I) works, is:

$$\begin{aligned} dW_{TOT} &= dW_I + dW_E = \\ &= \int_0^l \left[N_x \frac{d\delta u}{dx} + \frac{dw}{dx} \frac{d\delta w}{dx} N_x - \frac{d^2\delta w}{dx^2} M_x \right] dx \\ &\quad \pm P\delta u(l) = 0, \end{aligned} \quad (4)$$

where N_x and M_x are the internal normal force and moment. The parameter P is the external axial load. Including an initial distortion w_0 , a double integration of the w -dependent terms in Equation (4) gives:

$$M_x + N_x(w_0 + w) = Ax + B, \quad (5)$$

where A and B are two integration constants, which depend on specific beam configurations, as shown afterwards (see Sec.3). Figure 4 shows the used sign convention for the internal normal and shear (V_x) forces and moments.

Figure 4. Sign convention

The analytical derivation assumes the constitutive relation described by the Hooke's law ($\sigma_x = E\varepsilon_x$) for linear-elastic materials. Furthermore, the curvature of the deflected beam axis, ρ , is linearized as (see, e.g. [17]):

$$\frac{1}{\rho} = \frac{w''}{(1 + w'^2)^{\frac{3}{2}}} \approx w''. \quad (6)$$

According to [17], considering the equation $\varepsilon_x = -\rho/z$, the bending moment distribution can be approximated as:

$$M_x = \int_A z\sigma_x dA = -E \int_A \frac{z^2}{\rho} dA = -\frac{EI_{xx}}{\rho} = -EI_{xx}w''. \quad (7)$$

In the equation, I_{xx} is the principal moment of inertia of the cross-section of area A , while w'' indicates the second derivative of the deflection w with respect to the x -coordinate. The substitution of Equation (7) in Equation (5) leads to the derivation of an Ordinary Differential Equation (ODE), which is solved by imposing the deflection and the slope BCs of the beam.

The ideal slope BCs of the end a and b (indicated as $\theta_{a,b}$) are extended to non-ideal conditions as a function of the beam ends rotational stiffnesses $k_{a,b}$. That is:

$$w'(x_{a,b}) = \theta_{a,b} - \frac{M_{a,b}}{k_{a,b}}. \quad (8)$$

The parameters $k_{a,b}$ ranges between 0, i.e. a pinned end configuration, to ∞ , i.e. a fully fixed end configuration. According to the relation

$$k_{a,b} = \frac{3}{\frac{1}{\rho_{a,b}} - 1} \frac{EI_{xx}}{l}, \quad (9)$$

the fixity factors $\rho_{a,b}$ will substitute the rotational stiffnesses, as they limit their variability range between 0 (for a pinned end) and 1 (for a fixed end). A fixity factor represents the intermediate BC as the amount of allowed rotation. For instance, $\rho_{a,b} = 0.5$ means that 50% of the rotation hindered by an ideally clamped condition is free to occur.

This factor can be estimated from the numerical analysis of a stiffened panel as an alteration of the curvature, i.e. of the bending moment, along the centerline. By using the Central Difference method formulation [18],

$$w'' \approx \frac{w_{i+1} - 2w_i + w_{i-1}}{(\Delta x^2)} = -\frac{(M_i + M_{\rho,i})}{EI_{xx,i}}, \quad (10)$$

the effect of the fixity factor ($M_{\rho,i}$) is related to the structure deflection. Thereby, the fixity factor relates to the departure from the final deflection expected for an ideally fixed condition.

2.2 The stress magnification factor definition

The k_m factor is defined as:

$$k_m = \frac{\sigma_{HS}}{\sigma_N}, \quad (11)$$

i.e. the ratio between the hot-spot structural stress and the nominal applied stress. For a beam element subjected to an axial load P , the nominal applied stress is the membrane stress:

$$\sigma_N(x) = \sigma_m(x) = \frac{P}{bt} \quad \forall x. \quad (12)$$

The sum of the membrane and the bending stress, i.e.

$$\sigma_b(x, z) = \frac{M(x, z)}{I_{xx}} z = \frac{12M(x, z)}{bt^3} z, \quad (13)$$

defines the structural stress. Thus, the hot-spot structural stress is determined as the maximum structural stress (see Eq. (14)).

$$\sigma_{HS} = \max(\sigma_m(0) + \sigma_b(0, z)) = \max\left(\frac{P}{bt} + \frac{12M_a}{bt^3} z\right) \quad (14)$$

Considering the weld location (i.e. $x = 0$), whether the hot-spot is on the top or bottom surface depends on the through-the-thickness, linear distribution of the structural stress.

3. STUDY OF BEAMS WITH NON-LINEAR DISTORTION

3.1 Modelling of the initial curvature

As in [12], the welding-induced distortion geometry is modelled by the superimposition of a half-sine curvature (HSC) and a linear lateral sway, thus including both a local and a global angular misalignment. This geometry, shown in Figure 5, is indicated as HSC model and expressed as:

$$w_0(x) = w_{0,y_0}(x) + w_{0,a_0}(x) = \frac{y_0 x}{l} + a_0 \sin\left(\frac{\pi x}{l}\right). \quad (15)$$

An additional geometry includes a linear lateral sway and a quarter-sine curvature (QSC). It is shown in Figure 6 and described as:

$$w_0(x) = w_{0,y}(x) + w_{0,a_0}(x) = \frac{y_0 x}{l^*} + a_0 \sin\left(\frac{\pi x}{2l^*}\right), \quad (16)$$

Considering $l = 2l^*$, the QSC model can reveal the role of the structure rotational stiffness at the plate mid-length.

Figure 5. 1-D beam distorted by a linear lateral sway and a half-sine curvature (HSC model).

Figure 6. 1-D beam distorted by a linear lateral sway and a quarter-sine curvature (QSC model).

3.2 Analysis of HSC model under axial compression

In a step-by-step procedure, the deflection BCs are firstly applied. Given that the half-sine curvature shape fulfils the zero-deflection BC at the beam ends (required for both pinned and fixed ends), it is possible to decouple the analysis of the non-linear and linear geometric distortions; see *Step 1* and *2*, respectively. Once the Principle of Superimposition is applied (see *Step 3*), the problem is solved through the Slope Deflection Method development [13] in *Step 4*. The study is completed with the computation of the related stress magnification factor for specific cases explained in Section 3.5.

Step 1 – Equation (5) is adapted to a half-sine curved beam under axial compression, i.e. to the internal actions and total deflection BCs written as:

$$\begin{cases} N_x(0) = -P; N_x(l) = -P \\ M_x(0) = M_a; M_x(l) = M_b \\ V_x(0) = 0; V_x(l) = 0 \end{cases} \quad \begin{cases} w_1(0) + w_{0,a_0}(0) = 0 \\ w_1(l) + w_{0,a_0}(l) = 0 \end{cases} \quad (17)$$

with w_1 indicating the beam deflection in *Step 1*, and w_{0,a_0} the imposed initial curvature. By applying the approximation in Equation (7), the resulting ODE is

$$w_1''(x) + k^2 w_1(x) = -k^2 a_0 \sin\left(\frac{\pi x}{l}\right), \quad (18)$$

where $k = \sqrt{P/EI_{xx}}$. The homogeneous solution to this ODE can be assumed as $w_{h1}(x) = C_1 \cos(kx) + C_2 \sin(kx)$, while the particular solution takes the form $w_{p1}(x) = b \sin(\pi x/l)$. The constants C_1 , C_2 and b are found by imposing

$$\begin{cases} w_1(0) = 0 \\ w_1(l) = 0 \end{cases} \quad (19)$$

as deflection BCs. Thus, the deflection and slope-deflection equations are derived.

Step 2 – Equations (5) and (7) are now adapted to the BCs:

$$\begin{cases} N_x(0) = -P; N_x(l) = -P \\ M_x(0) = M_a; M_x(l) = M_b \\ V_x(0) = 0; V_x(l) = 0 \end{cases} \quad \begin{cases} w_2(0) + w_{0,y_0}(0) = 0 \\ w_2(l) + w_{0,y_0}(l) = Y \end{cases} \quad (20)$$

where Y is the final position reached by the right end of the beam. Thereby, the ODE for the laterally deflected beam under compression becomes:

$$w_2''(x) + k^2 w_2(x) = -\frac{M_a}{EI} + \frac{M_a + M_b + P(Y - y_0)x}{EI} \frac{1}{l}. \quad (21)$$

The equations for the beam deflection and slope-deflection are computed according to the particular solution $w_{2,p}(x) = C_3 + C_4 x$ and to the deflection BCs:

$$\begin{cases} w_2(0) = 0 \\ w_2(l) = Y - y_0 \end{cases} \quad (22)$$

Step 3 – The total deflection and slope deflection equations related to the HSC beam model are derived by applying the Principle of Linear Superposition, that is:

$$w(x) = w_1(x) + w_2(x), \quad (23)$$

$$w'(x) = w_1'(x) + w_2'(x). \quad (24)$$

Based on Equation (7), an x -dependent equation for the bending moment follows. The solution is undetermined, as the two moments M_a and M_b are unknown.

Step 4 – A system of two equations is obtained by imposing the slope BCs in Equation (8). The fixity factors act as an alteration of the fully fixed ends, which implies $\theta_{a,b} = 0$. By isolating the two unknown bending moments M_a and M_b , the system can be written in matrix form as:

$$\begin{Bmatrix} M_a \\ M_b \end{Bmatrix} = \begin{bmatrix} S_{aa} & S_{ab} \\ S_{ba} & S_{bb} \end{bmatrix} \frac{EI}{l} \begin{Bmatrix} -\frac{Y - y_0}{l} - \left(\frac{\beta^2}{\pi^2 - \beta^2} \right) \frac{\pi a_0}{l} \\ -\frac{Y - y_0}{l} + \left(\frac{\Phi^2}{\pi^2 - \beta^2} \right) \frac{\pi a_0}{l} \end{Bmatrix}, \quad (25)$$

where β is the quantity also defined in the IIW recommendations (see Sec. 1). Notice that, for the beam equilibrium:

$$Y = -\frac{M_a + M_b}{P}. \quad (26)$$

Moreover, the terms S_{aa} , S_{ba} , S_{ab} and S_{bb} are not shown in his paper, as their explicit form can be found in the analytical study conducted by Aristizabal-Ochoa in [13].

3.3 Analysis of QSC model under axial compression

In the procedure for the QSC model, the linear and non-linear distortions are studied together in *Step 1**, as the quarter of sine does not satisfy the zero-deflection boundary conditions of the HSC model.

Step 1* – The resulting ODE in Equation (28) is obtained by applying the force and total deflection BCs in Equations (27). Equation (29) reports the deflection BCs of the model.

$$\begin{cases} N_x(0) = -P; N_x(l) = -P \\ M_x(0) = M_a; M_x(l) = M_b \\ V_x(0) = 0; V_x(l) = 0 \end{cases} \quad \begin{cases} w_1(0) + w_0(0) = 0 \\ w_1(l) + w_0(l) = Y \end{cases} \quad (27)$$

$$w''(x) + k^2 w(x) = \quad (28)$$

$$= -\frac{M_a}{EI} + \frac{M_a + M_b + P(Y - y_0)}{EI} \frac{x}{l} - k^2 a_0 \sin\left(\frac{\pi x}{2l}\right)$$

$$\begin{cases} w(0) = 0 \\ w(l) = Y - y_0 - a_0 \end{cases} \quad (29)$$

Step 2* – The same slope BCs applied to the HSC model are utilized to reach the system of two equations in the two unknowns M_a and M_b , which in matrix form becomes:

$$\begin{Bmatrix} M_a \\ M_b \end{Bmatrix} = \begin{bmatrix} S_{aa} & S_{ab} \\ S_{ba} & S_{bb} \end{bmatrix} \frac{EI}{l}. \quad (30)$$

$$\begin{Bmatrix} -\frac{Y - y_0}{l} - \left(\frac{1}{\pi^2 - 4\beta^2} \right) \left(\frac{4\beta^2 \pi a_0}{2l} + \frac{a_0 \beta \pi^2}{l \sin \beta} \right) \\ -\frac{Y - y_0}{l} + \left(\frac{\pi^2}{\pi^2 - 4\beta^2} \right) \frac{a_0 \beta}{l \tan \beta} \end{Bmatrix}.$$

3.4 Adaptation of the solution to tensile axial load

The procedures shown in Sections 3.2 and 3.3 equally apply to tensile axial load (i.e. $P > 0$). The final solutions for tensile loading condition are obtained by changing the sign of the load P . This implies that: $\beta \rightarrow i\beta$. Thus, for the properties of the trigonometric functions: ($\sin i\beta = i \sinh \beta$) and ($\cos i\beta = \cosh \beta$).

3.5 Stress magnification factor evaluation by varying one fixity factor at a time

The impact of intermediate slope BCs at the beam ends is studied by varying one fixity factor at a time. So, two case studies are defined for both the HSC and QSC models:

- *Case 1*. The fixity factor related to the loaded end, ρ_b , of the beam is varied between 0 and 1, while the weld is modelled as an ideal clamp (i.e. $\rho_a = 1$).
- *Case 2*. The weld is modelled as a non-ideal BC, meaning that ρ_a varies between 0.5 and 1, while the loaded end is ideally clamped (i.e. $\rho_b = 1$).

In *Case 2*, the weld is never treated as an ideally pinned BC (i.e. $\rho_a > 0$), as such a case would not be physically meaningful. The analytical formulations for the stress magnification factor k_m derived in *Case 1* ($\rho_a = 1, \rho_b = \rho$) and *Case 2* ($\rho_b = 1, \rho_a = \rho$) for the two models are shown in Table 1. These formulations have been verified with less than 2% error by comparison with the FEA for ideally clamped and pinned beam configurations.

Notice that, for the given beam distortions (see Sec. 3.1), the hot-spot structural stress arises at the top surface. So that the formulations in the table are determined as:

$$k_m = \frac{\sigma_{hs}}{\sigma_n} = \frac{\sigma_m}{\sigma_m} + \frac{\sigma_{b,top}}{\sigma_m} = 1 + \frac{\frac{12M_a t}{bt^3} \frac{t}{2}}{\frac{P}{bt}} = 1 + \frac{6M_a}{tP}. \quad (31)$$

3.6 Parameter selection for the case studies

Two beam configurations are considered: one is based on common small-scale specimen dimensions (see Table 2), while

Case 1 ($\rho_a = 1, \rho_b = \rho$)	
HSC	$k_m = 1 + \left(\frac{6y_0}{t}\right)\left(\frac{1}{\beta}\right)\left(\frac{3\rho(1 - \cos\beta) + (1 - \rho)\beta\sin\beta}{\gamma}\right) + \left(\frac{6\pi a_0}{t}\right)\left(\frac{\beta}{\pi^2 - \beta^2}\right)\left(\frac{3\rho(1 + \cos\beta) - (1 - \rho)\beta\sin\beta}{\gamma}\right)$
QSC	$k_m = 1 + \left(\frac{6y_0}{t}\right)\left(\frac{1}{\beta}\right)\left(\frac{3\rho(1 - \cos\beta) + (1 - \rho)\beta\sin\beta}{\gamma}\right) + \left(\frac{6\pi a_0}{t}\right)\left(\frac{2\beta}{\pi^2 - 4\beta^2}\right)\left(\frac{3\rho\cos\beta - (1 - \rho)\left(\beta\sin\beta - \frac{\pi}{2}\right)}{\gamma}\right)$
Case 2 ($\rho_b = 1, \rho_a = \rho$)	
HSC	$k_m = 1 + \left(\frac{6y_0}{t}\right)\left(\frac{1}{\beta}\right)\left(\frac{3\rho(1 - \cos\beta)}{\gamma}\right) + \left(\frac{6\pi a_0}{t}\right)\left(\frac{\beta}{\pi^2 - \beta^2}\right)\left(\frac{3\rho(1 + \cos\beta)}{\gamma}\right)$
QSC	$k_m = 1 + \left(\frac{6y_0}{t}\right)\left(\frac{1}{\beta}\right)\left(\frac{3\rho(1 - \cos\beta)}{\gamma}\right) + \left(\frac{6\pi a_0}{t}\right)\left(\frac{2\beta}{\pi^2 - 4\beta^2}\right)\left(\frac{3\rho\cos\beta}{\gamma}\right)$
With $\gamma = 3\rho\sin\beta + (1 - \rho)\beta\cos\beta$. If $P > 0$, $\beta \rightarrow i\beta$.	

Table 1. Analytical formulation for the stress magnification factor including one non-ideal slope BC

the other presents a high geometric slenderness (see Table 3) that is typical for ship deck structures. The tables refer to the HSC model. In the study of the QSC model, the geometric slenderness ratios are halved to $l/t = 21$ and $l/t = 150$, respectively. The beams are made of structural steel S355, which has a module of elasticity E of 206.8 GPa, Poisson's ratio $\nu = 0.3$ and a yielding strength of 355 MPa.

Thickness	t	3	[mm]
Width	b	20	[mm]
Global angle	α_G	0.25	[deg]
Slenderness ratio	l/t	~ 42	[mm/mm]
Angle ratio	α_L/α_G	5	[deg/deg]
Membrane stress	σ_m	-100 to 100	MPa

Table 2. Small-scale specimen configuration. HSC model.

Thickness	t	4	[mm]
Width	b	400	[mm]
Global angle	α_G	0.25	[deg]
Slenderness ratio	l/t	300	[mm/mm]
Angle ratio	α_L/α_G	20	[deg/deg]
Membrane stress	σ_m	0 to 300	MPa

Table 3. High geometric slenderness configuration. HSC model.

4. RESULTS AND DISCUSSION

In this section, the results from the case studies are presented and discussed in order to describe the influence of non-ideal BCs on the stress magnification factor for curved thin plates. Firstly, the bending stress on the top and bottom surfaces in the weld location are compared to the applied membrane stress. This

reveals the hot-spot location and whether the beam deformation is bending-dominated or not. Then, the stress magnification factor is shown as a function of the applied membrane stress to study the non-linear relationship between the k_m factor and the applied load.

The stress state of the slender HSC model under tensile load in Figure 7 is strongly dominated by the bending stress. In *Case 1* (see Figure 7(a)), the variation of the fixity factor at the loaded end does not affect the bending stress felt at the weld location. This suggests that remote BCs do not affect the stress state, and so the deformation mechanism, in the weld region. A different trend is shown for *Case 2* in Figure 7(b), where the rotation of the loaded end remains fully fixed ($\rho_b = 1$). In fact, for $\rho_a \neq 1$, the bending stress values become constant at low applied membrane stress. For instance, at about $\sigma_m = 100$ for $\rho_a = 0.5$, as well as at about 200 MPa for $\rho_a = 0.75$, the bending stress is not the dominant action anymore.

The same characteristics are observed for the stress state of the QSC model with high geometrical slenderness ($l/t = 150$). Only the *Case 2* is shown in Figure 8, where, compared to the HSC model, half of the l/t ratio results in much lower bending stress.

For lower geometric slenderness ratio ($l/t = 42$), the HSC model undergoes the state of stress described in Figure 9. The bending stress becomes comparable to the membrane stress as a result of the increased beam bending stiffness. In *Case 1* (with $\rho_a = 1$), it is always slightly higher than the tensile membrane stress. Under compression, the bending behavior remains the dominant one whenever the constraints react to the rotation to some extent. In fact, only in case of ideally pinned condition at the loaded end, the membrane stress temporarily dominates the stress field. This happens during the transition of the stress state from compression to tension on the top surface (see dot-dashed lines in Figure 9(a)). Differently, in *Case 2*, with intermediate BC at the weld location (i.e. $0.5 \leq \rho_a \leq 1$), the bending stress,

σ_b , becomes smaller than the tensile membrane stress, σ_m , while it remains the dominant action in compression (see Figure 9(b)).

7(a) Case 1, rotation is fixed at the welded end.

7(b) Case 2, rotation is fixed at the loaded end.

Figure 7. Bending stress compared to the membrane stress on the top and bottom surfaces of the beam at the weld location. HSC model, $l/t = 300$.

Case 2, rotation is fixed at the loaded end.

Figure 8. Bending stress compared to the membrane stress on the top and bottom surfaces of the beam at the weld location. QSC model, $l/t = 150$.

In Figure 10, the plots illustrate the state of stress felt by the QSC model with low slenderness ratio. In Case 2, the bending stress slightly dominates the tensile membrane stress for the ideally clamped configuration, while it becomes smaller than the other

for intermediate BCs at the weld location (i.e. smaller ρ_a , see Figure 10(b)). Under compression, the bending action leads the beam mechanical response. This also occurs in Case 1 in Figure 10(a). However, in this case, to smaller bending stiffness at the loaded end (i.e. smaller ρ_b) corresponds a higher bending stress.

9(a) Case 1, rotation is fixed at the welded end.

9(b) Case 2, rotation is fixed at the loaded end.

Figure 9. Bending stress compared to the membrane stress on the top and bottom surfaces of the beam at the weld location. HSC model, $l/t = 42$.

In general, the stress magnification factor k_m is always computed on the top surface, as it is the most excited one for the chosen beam configurations. In fact, the top bending stress is always consistent in sign with the applied membrane stress, so that their summation is higher than the one at the bottom surface. The reason behind this phenomenon is in the definition of the initial curvatures of the HSC and QSC models. Indeed, the resulting lever arm for the axial tensile load determines a bending moment that generates a tension on the top surface. While, under compression, the initial curvature sign leads the top surface to be the one under compression at the weld location (i.e. the curvature sign tends to stay negative). Furthermore, the analysis of the models with reduced geometrical slenderness ($l/t = 42$ for HSC model and $l/t = 21$ for QSC model) shows that the definition of the beam end BCs can also change the stress state of the component from bending- to membrane-dominated.

In Figures 11 and 12, the stress magnification factor as a function of the applied membrane stress is shown for those

conditions in which the variation of the fixity factors have a significant impact.

10(a) Case 1, rotation is fixed at the welded end.

10(b) Case 2, rotation is fixed at the loaded end.

Figure 10. Bending stress compared to the membrane stress on the top and bottom surfaces of the beam at the weld location. QSC model, $l/t = 21$.

The $(k_m \text{ vs. } \sigma_m)$ trend for a very slender configuration is shown in Figure 11 in relation to Case 2 and under tensile load only. The figure refers to the HSC model, which is comparable to the QSC model in the same conditions. If the loaded end is ideally clamped against rotation and the weld is modelled as a non-ideal connection, the stress magnification factor significantly changes. For instance, when $\rho_a = 0.5$, the k_m factor is more than halved if compared to the fully rigid configuration. At high tensile load, the k_m factor stabilizes around 5 for ideally clamped ends (i.e. $\sigma_b \approx 4\sigma_m$ as in Figure 7(b)) and around 2 (i.e. $\sigma_b \approx \sigma_m$) for the other cases.

Figure 12 highlights the impact of non-ideal BCs on the HSC model with low slenderness ratio. Under compression, the buckling resistance of the structure is equally altered in Case 1 and Case 2. The only difference relates to the ideally pinned configuration at the loaded end, which undergoes a change in the dynamic of deformation of the beam (see dot-dashed line in Figure 12(a)).

According to the results, having intermediate BCs at the loaded end matters mostly under compression. Under tension, anything that happens at a relatively large distance from the weld

does not affect the stress concentration at the hot-spot. On the contrary, if the joint cannot be assumed as an ideal clamp (i.e. in Case 2), the prediction of the k_m factor must consider the fixity factor ρ_a , especially in case of very slender structures. In fact, a reduced bending stiffness implies higher sensitivity to secondary bending moments and to the related influencing factors.

Case 2, rotation is fixed at the loaded end.

Figure 11. Stress magnification factor as function of the membrane stress. HSC model, $l/t = 300$.

12(a) Case 1, rotation is fixed at the welded end.

12(b) Case 2, rotation is fixed at the loaded end.

Figure 12. Stress magnification factor as function of the membrane stress. HSC model, $l/t = 42$.

5. CONCLUSION

This paper presented an analytical solution to estimate the stress magnification factor k_m for thin plates with welding-induced, non-linear distortions. To consider the impact of the actual rotation constraint at the plate short edges, a beam model with intermediate, non-ideal BCs was proposed. According to this study, the following conclusions are drawn:

- In case of an ideally clamped configuration, the stress state of a curved, thin and slender beam under tensile axial load is generally bending-dominated. While for $\rho_a < 1$ or smaller l/t ratios, the membrane stress tends to overcome the bending stress (i.e. $k_m < 2$) already at low loading conditions (i.e. $\sigma_m < 100$ MPa).
- Varying the fixity factor at the welded end has an impact on the stress concentration at the weld location. The stiffer is the constraint, the higher becomes the bending reaction of the constraint itself. Such impact becomes more evident as the geometric slenderness ratio increases.
- Under axial compression, the reduction of the fixity factors results into the reduction of the beam buckling resistance.

This study suggests that further developments are required in order to identify a reliable range of variability for the fixity factors over the plate field in a stiffened panel. This aim can be pursued by relying on FE simulations of stiffened panels.

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