Eskelinen, Aleksi; Zakharov, Alexey; Jämsä-Jounela, Sirkka-Liisa; Hearle, Jonathan

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Dynamic modelling of a multiple hearth furnace for kaolin calcination

Aleksi Eskelinen\textsuperscript{a}, Alexey Zakharov\textsuperscript{a,*}, Jonathan Hearle\textsuperscript{b}, Sirkka-Liisa Jämsä-Jounela\textsuperscript{a}  
\textsuperscript{a} Aalto University, School of Chemical Technology, Research group of Process Control and Automation, P.O. Box 16100, FI-00076 Espoo, Finland (* corresponding author; email: alexey.zakharov@aalto.fi)  
\textsuperscript{b} Imerys Minerals Limited

ABSTRACT

A dynamic model of a multiple hearth kaolin calciner has been developed and is presented in this paper. This model describes the physical-chemical phenomena taking place in the six furnace parts: the solid phase, gas phase, walls, cooling air, rabbit arms and the central shaft. The solid phase movement, in particular, is described by a novel mixing model. The mixing model divides the solid bed in a hearth into volumes and the distribution of their contents, after one full central shaft rotation, is identified by the pilot experiments. First, the model is validated by the industrial data, and then the dynamics of the multiple hearth furnace (MHF) is studied by introducing step changes to the three manipulated variables: the feed rate, and the gas and air flows supplied. The responses of the gas phase temperature and solid bed component profiles are analysed and the results are discussed.

Keywords: Multiple hearth furnace (MHF), dynamic modelling, kaolin calcination, pilot experiments, industrial application, parameter estimation

\begin{footnotesize}
\begin{tabular}{ll}
\textbf{Roman letters} & \textbf{Greek letters} \\
A & \(\varepsilon\) Emissivity, radiative exchange ratio \\
A_i & \(\sigma\) Stefan-Boltzmann-constant (\(=5.67\cdot10^{-8}\) W/(m\(^2\)K\(^4\))) \\
C_i & Concentration of solid component i (kg) \\
c_i & Concentration of gas component (mol/m\(^3\)) \\
D_i & Solid bed movement matrix of hearth i \\
E_{A,i} & Activation energy of reaction i (kJ/kg) \\
F & Total gas flow (m\(^3\)) \\
h & Heat transfer coefficient (W/m\(^2\)K) \\
g & Gas \\
s & Solid \\
w & Wall \\
gs & Gas-solid
\end{tabular}
\end{footnotesize}
1. Introduction

Multiple hearth furnaces (MHF) are widely used in industry for the calcination of clay minerals, such as kaolin (Thomas et al., 2009). Calcination enhances the properties of kaolin ensuring its applicability for a wide variety of products, such as paper, rubber, paint and refractory items (Murray, 2005). However, mineral processing continues to provide challenges particularly in the area of maintaining efficient process operations (Jämsä-Jounela, 2001). Specifically, growing global competition in the mineral processing industry has increased the need for higher grade products. To improve the quality of calcined kaolin, more knowledge is needed on the solid temperature profile inside the furnace (Thomas, 2010) and on the physico-chemical phenomena occurring during calcination.

Mechanistic models have proven to be an excellent tool for gaining deeper understanding of processes and their behaviour (Luyben, 1990; Ogunnaike, 1994; Järvensivu et al., 2001). The development of mechanistic models is supported by the numerous studies of the reactions related to kaolin calcination. For example, the fundamentals of kaolin calcination and classification of different calcined kaolin grades have been previously introduced in Murray (2005) and Murray & Kogel (2007). Furthermore, the kinetics of kaolin calcination was examined by Ptocek et al. (2010; 2011), whilst both Langer (1967) and Castelein et al. (2001) have analysed the influence of the heating rate on the properties of calcined kaolin.

Despite the abundance of research examining the calcination reactions, significant research is still required to quantitatively describe the phenomena occurring in the MHF calciner. In the literature, several mathematical models with necessary elements have been reported. Meisingset and Balchen (1995) developed a steady-state mechanistic model for a single hearth rotary coke calciner, which outlined mass and energy balance equations for the coke bed, gas phase and the lining. Martins et al. (2001) described a one-dimensional steady-state model of the petroleum coke calcination in a rotary kiln that predicts the temperature profiles for the bed, gas phase and the kiln wall in the axial direction. Additionally, it also predicted the composition profiles for the gas and solid phase.
The model included rheological characteristics of the system of particles for modelling the axial flow of the bed. Voglauer and Jörgl (2004) presented a dynamic model of a multiple hearth furnace used for the roast process to recover vanadium consisting of a mass transfer model, thermodynamic model and a chemical model for the characterization of decisive states like mass, temperatures and the concentrations of sodium, and soluble and insoluble vanadium, respectively. Liu and Jiang (2004) have also developed a mathematical model involving the mass and heat transfer in a continuous plate dryer. The authors presented equations to calculate the key parameters of the solid mass transfer model, such as the height and the volume of each granular heap, and retention time in the dryer. The study showed that the optimization of a plate dryer should concentrate on maximizing the effective covering ratio of the plates. More recently, Ginsberg and Modigell (2011) developed a dynamic model of a rotary kiln used for the calcination of titanium dioxide. The overall heat transfer phenomena and the reactions occurring in the process were described. The model was validated by a dynamic test case representing a 15-day period of plant operation and the work succeeded in proving that the dynamic behaviour of a furnace can be modelled with quantitative accuracy.

The aim of this paper is to present the development of the dynamic model describing kaolin calcination in a MHF operated by a UK plant. In the model, the solid bed in a hearth is divided into volumes, as presented earlier by Meisingset and Balchen (1995), as the temperature difference through a solid bed in a hearth is over 150 °C. The solid phase movement, particularly, is described by a novel mixing model. In addition to dividing the solid bed in a hearth into volumes, the mixing model provides the distribution of their contents, after one full central shaft rotation, identified by the pilot experiments.

This paper comprises of the following. First, the kaolin calcination process in a MHF is described in Section 2, whilst Section 3 presents the physical and chemical relationships for the calcination reactions, and the equations for the heat and mass transfer phenomena within the furnace. Section 4 introduces the experimental work used to determine the parameters of the calcination reactions and the mixing model. The next Section 5 introduces the estimation of the model parameters that cannot be calculated from the physical-chemical laws and also reports the steady-state temperature and solid bed composition profiles and presents the model validation results. In Section 6, the dynamic behaviour of the MHF model is analysed and finally, Section 7 concludes the paper.

2. Description of kaolin calcination in a multiple hearth furnace

The MHF studied in this paper consists of eight hearths and it has counter-current solid and gas flows. The heat required for calcination is supplied to the furnace through four tangentially aligned methane burners located on Hearths 4 and 6. The temperature in the 'fried hearths’ is controlled by varying the fuel gas flow, which determines the amount of combustion air. The furnace walls are constructed of bricks and enclosed by a cylindrical steel shell with refractory lining. Figure 1 presents the cross-sectional view of the furnace.
In re-crystallization, the material flow through the furnace is stirred spirally and moved across the hearths by a centrally located vertical rotating shaft carrying arms with rabble blades. Four arms are used on each hearth, and each arm carries three to five rabble blades, whilst material is fed into the top hearth through a single inlet from the weigh feed hopper to the periphery of the hearth. On the odd numbered hearths, the material is stirred by the rabble blades towards the centre of the hearth, and the material drops down to the next hearth from the centre through a single annulus around the shaft. In contrast, the material on the even numbered hearths is moved outwards to be dropped through the drop holes at the periphery of the hearth to the following hearth. The stirring pattern is repeated until the lowest hearth is reached, from which the calcined product is extracted through the two exit holes.

During calcination, kaolin undergoes four physical-chemical processes (Ptacek et al., 2010). First the evaporation of the free moisture occurs (T≤100 °C).

\[ H_2O(l) \rightarrow H_2O(g) \] (1)

Next, kaolin undergoes a dehydroxylation reaction, in which the chemically bound water is removed and amorphous metakaolin is formed at 450 – 700 °C.

\[ Al_2O_3 \cdot 2SiO_2 \cdot 2H_2O \rightarrow Al_2O_3 \cdot 2SiO_2 + 2H_2O(g) \] (2)

The third physical-chemical process involves a reaction leading to the transformation of metakaolin to the ‘spinel phase’ by exothermic re-crystallization at 925-1050 °C.

\[ 2(Al_2O_3 \cdot 2SiO_2) \rightarrow 2Al_2O_3 \cdot 3SiO_2 + SiO_2(\text{amorphous}) \] (3)

In the fourth and final process, the nucleation of the spinel phase occurs and the material transforms into mullite at temperatures above 1050 °C.

\[ 3(2Al_2O_3 \cdot 3SiO_2) \rightarrow 2(3Al_2O_3 \cdot 2SiO_2) + 5SiO_2 \] (4)
Mullite is hard and abrasive, and as a result it can cause damage to process equipment (Thomas, 2010). The desired final consistent product which is within the specification limits has both a low mullite and metakaolin content. The differential scanning calorimetry (DSC) and thermo gravimetric (TGA) curves presenting the kaolin calcination are given in Figure 2.

![Figure 2. DSC and TG curves of kaolin](image)

### 3. Dynamic model of a multiple hearth furnace

In this modelling work, the MHF is divided to six parts: the solid bed, gas phase, walls, central shaft, rabble arms, and the cooling air. The overall mathematical model of the furnace includes the reaction kinetics for the four reactions outlined earlier, as well as models for the mass transfer and heat transfer mechanisms for each of six parts. In addition, equations for calculating the temperature dependent parameters, such as the heat capacities of the gas components, the gas emissivities and the solid bed emissivities, are incorporated in the model. The solid bed and the gas phase both have five components: the solid bed includes kaolin, water, metakaolin, spinel phase (product), and mullite (offspec), whereas the gas phase includes methane, oxygen, nitrogen, water and carbon dioxide. The first three subsections present the modelling equations and then Subsection 3.4 introduces a summary of the solving procedure.

#### 3.1. Modelling of the calcination reactions and methane combustion

The four reactions presented in Equations 1-4 occur in the solid bed and the reaction rate \( R_i \) of each reaction is described by the Arrhenius equation \( R_i = k_i C_i = A_i e^{-E_{ai} / RT_i} C_i \). The frequency factors and activation energies of each reaction are selected by fitting the reaction rate curves to the DSC and TGA curves of kaolin, as further explained in Section 4.1.

The burning of methane is assumed to be incomplete proximal to the burners and the heat provided is distributed along the gas phase in the hearths. Equation 5 presents the combustion reaction of methane:

\[
\text{CH}_4(g) + 2\text{O}_2(g) \rightarrow \text{CO}_2(g) + 2\text{H}_2\text{O}(g) \tag{5}
\]

The amount of combustion energy (\( \Delta H_{\text{comb}} \)) that the reaction provides is given by the lower heating value of 802 kJ/mol (CRC Handbook, 2005).
3.2. Modelling of solids and gas phase mass transfer

A few simplifying assumptions have to be made in order to model the mass transfer inside the furnace. Firstly, the solid bed on each hearth is split into 4 (Hearth 3 to 8) or 5 (Hearths 1 and 2) homogenous annular volumes according to the furnace rabble arm configuration. Secondly, the volumes are assumed to be identical in width in the radial direction and the mass content. Thirdly, the mixing model assumes that one full shaft rotation distributes the contents of a volume between the original volume and its neighbour volumes (one of which is the next and the other is the previous). The distribution of solid among the three volumes is determined by the experimental tests which are described in Section 4.2 and in addition, the density of the solids is assumed to remain constant.

Under these assumptions, the solid mass distribution after one time step (representing a full central shaft rotation) is calculated with Equation 6:

\[ m_{i+1}^{f} = D_{j} \cdot (m_{i}^{f} - R_{i+1}^{f}) + m_{\text{feed}}^{f} \]  

(6)

where the mass movement matrix \( D \) describes the distribution of the contents of each compartment after one central shaft rotation in the hearth \( j \). Specifically, the column \( i \) represents the distribution of volume \( i \) between the volumes of the hearth. Thus, the columns of the matrix sum up to unity, except the column representing the exit volume of the hearth. Moreover, because of the assumption that solids in a certain volume are distributed between the current volume and its neighbour volumes after one rotation, the matrix \( D \) has three nonzero central diagonals. Thus, the central diagonal represents the contents staying in the same volume, whereas the upper and lower diagonals indicate the proportion of solid moving inwards and outwards respectively. In the mixing model (Equation 7), the feed to a hearth can be found as the exit from the previous one:

\[ m_{\text{feed}}^{f} = (1 - \sum D_{j-1}^{K}) (m_{i}^{f-1,K} - R_{i+1}^{f-1,K}) \]  

(7)

where \( K \) is the exit volume of hearth \( j - 1 \) and \( D_{j-1}^{K} \) is the column \( K \) of matrix \( D_{j-1} \).

The solid bed movement matrix is generated for each hearth individually. As an example, Equation 8 presents the solid bed movement matrix of the upmost hearth. The parameter \( \alpha \) is introduced to describe the net forward flow through a hearth, which is the feed rate minus the mass loss caused by evaporation and dehydroxylation reactions. The upper diagonal matrix elements \( a_{1} \) present the full forward flow from the current volume to the next volume during one full central shaft rotation. Thus, the lower diagonal matrix elements \((a_{1} - \alpha)\) present the backward flow equal to the difference of the full forward and the net forward flows. In Equation 8, the parameter \( \alpha \) is only present in the second and third volume since the feed is introduced to volume 4 and the direction of flow is in the descending order of volumes on the first hearth.

\[
D_{1} = \begin{bmatrix}
1 - a_{1} & a_{1} & 0 & 0 & 0 \\
\alpha_{1} - \alpha & 1 - 2 a_{1} + \alpha & a_{1} & 0 & 0 \\
0 & \alpha_{1} - \alpha & 1 - 2 a_{1} + \alpha & a_{1} & 0 \\
0 & 0 & \alpha_{1} - \alpha & 1 - 2 a_{1} + \alpha & a_{1} \\
0 & 0 & 0 & a_{1} & 1 - a_{1}
\end{bmatrix}
\]  

(8)
The gas phase in the model is assumed to be ideal and the pressure inside the furnace atmospheric. As was the case with the solid bed, also the gas phase on a specific hearth is divided into 4 or 5 homogenous annular volumes, with the number of gas phase volumes in the hearth the same as the corresponding number of solid volumes in the hearth. Additionally, the gas phase volumes are assumed to be uniform regarding composition and temperature. The gas phase has a very short residence time in the furnace, and its temperature follows the steady-state values defined by the temperature profiles of the other model parts, allowing, the gas phase to be described with a steady-state model.

The gas phase mass balance equation (Equation 9) is derived for each primary gas component:

$$\dot{n}_{i,in} - \dot{n}_{i,out} - R_i = 0 .$$  \hspace{1cm} (9)

where $n_{i,in}$, denoting the incoming moles of component $i$, is obtained from Equation 10:

$$n_{i,in}^j = c_i^{j+1} F_j$$  \hspace{1cm} (10)

where $j$ is the number of the hearth.

The total gas flow in Hearths 7 and 8 is equal to the draft through the drop holes at the bottom hearth. The total gas flow in Hearths 5 and 6 comprises of the combustion gas and air fed to Hearth 6 as well as the gases from evaporation and dehydroxylation, and the air flow from the previous hearth. The total gas flow at Hearths 1 to 4 consists of the gas flow from Hearth 5, the combustion gas and air fed to Hearth 4 and the gases from evaporation and dehydroxylation. The real volumetric gas flow, $F_{real}$, on each hearth having different temperature can be calculated as:

$$F_{real} = F_{NTP} \frac{T_{real}}{T_{NTP}}$$  \hspace{1cm} (11)

3.3. Modelling of the energy balances

Heat exchange is assumed to occur in the model between solid and gas, solid and wall, gas and wall, wall and environment, and gas and arms. In addition, heat is conducted inside the walls in the radial horizontal direction and heat exchange exists between the cooling air, central shaft and the rabble arms. In the model, heat exchange between the hearth bottom surface and the solid bed, and heat conduction inside the solid bed are neglected. Furthermore, the bed volumes are assumed to be homogenous in temperature. The heat transfer paths and mechanisms in the multiple hearth furnace are shown in Figure 3.
The model considers energy balances for the gas phase, walls, central shaft, rabble arms, cooling air, and for the solid bed. The energy balance equations, containing the heat flows, are first resolved after which the temperature values can be acquired. The energy balance equation for the gas phase is:

\[ \dot{Q}_{gas} - \dot{Q}_{gas, out} + \dot{Q}_{combustion} + \dot{Q}_{gs} + \dot{Q}_{gw} + \dot{Q}_{shaft} + \dot{Q}_{arms} = 0 \] (12)

The molar heat capacities of the incoming and exiting gas components are calculated as a function of temperature as previously outlined in the DIPPR project 801 (Design Institute for Physical Properties, Sponsored by AIChE, 2012). The combustion energy is calculated using Equation 13:

\[ \dot{Q}_{combustion} = h_i \Delta H_{combust} \] (13)

where \( h_i \) is an estimated combustion ratio, which is described in more detail in Section 5.

The direct heat transfer between the solid bed and the gas phase occurs by both radiation and convection:

\[ Q_{gs} = \sigma X_e A_{gs} \varepsilon_s \varepsilon_g (T_g^4 - T_s^4) + h_{cgw} X_e A_{gs} (T_g - T_s) \] (14)

The calculation of \( \varepsilon_s \) and \( \varepsilon_g \) is described in Section 3.3.1 and the determination of the heat transfer coefficient \( h_{cgw} \) is presented in Appendix A, which also describes the calculation of all the other heat exchange coefficients. \( X_e \) is an estimated surface view factor, which is outlined further in Section 5.

Heat transfer between the inner walls and the gas phase occurs by convection and radiation and the heat flux can be written as:

\[ Q_{gw} = \sigma A_{gw} \left( \frac{\varepsilon_w + 1}{2} \right) \varepsilon_g (T_g^4 - T_w^4) + h_{cgw} A_{gw} (T_g - T_w) \] (15)

The radiation term is written as described in (Meisingset & Balchen, 1995) and the effective emissivity as presented in (Perry & Green, 1997) that can be used if the emissivity of the material is above 0.7.
To calculate the heat transfer between the gas, the central shaft and the rabble arms, the central shaft is divided into eight sections according to the hearths, so that each section has a fixed temperature and there is no vertical heat exchange in the shaft. The heat exchange flux between the gas phase and the central shaft, and between the gas phase and rabble arms consist of the radiative and the convective heat transfer terms respectively:

\[
Q_{\text{gas} shaft} = \sigma A_{gas shaft} \varepsilon_{gas} (T_{gas}^4 - T_{shaft}^4) + h_{c gas shaft} A_{gas shaft} (T_{gas} - T_{shaft})
\] (16)

\[
Q_{\text{arms}} = \sigma A_{arms} \varepsilon_{arms} (T_{gas}^4 - T_{arms}^4) + h_{c arms} A_{arms} (T_{gas} - T_{arms})
\] (17)

where \(Z\) is an estimated constant describing the insulation of the central shaft, which is further explained in Section 5.

The energy balance equation for the walls is given by:

\[
\frac{\partial Q_w}{\partial t} = \dot{Q}_{wg} - \dot{Q}_{ws} - \dot{Q}_{wa}
\] (18)

where the heat transfer between the solid bed and the walls occurs only by radiation since the two surfaces are not connected to each other. The radiative heat flux is:

\[
Q_{ws} = \sigma X_s A_{sw} \varepsilon_{sw} (T_s^4 - T_w^4)
\] (19)

where the emissivity between the solid bed and the walls is affected by the gas phase emissivity and its calculation is described in Appendix A. The heat transfer between the outer wall and the ambient air consists only of a convective term as the radiative term is neglected due to the relatively low outer wall temperature:

\[
Q_{wa} = h_{c wa} A_{wa} (T_{ambient} - T_{outer wall})
\] (20)

In order to describe the conductive heat transfer inside the wall and to calculate its temperature profile, the furnace wall is divided into eight sections according to the hearths, so that each section has a fixed temperature and there is no vertical heat exchange in the wall. In addition the furnace wall consists of four layers as illustrated in Figure 4.
The heat flux through the different layers of the wall is to be assumed constant:

\[ Q = Q_{12} = Q_{23} = Q_{34} \]  \hspace{1cm} (21)

The calculation of the wall layer temperatures is further discussed in Appendix A.

The energy balance equation for the central shaft and the rabble arms are provided by:

\[ \frac{\partial Q_{\text{shaft}}}{\partial t} = Q_{g\text{shaft}} - \dot{Q}_{\text{shaft cool}} \]  \hspace{1cm} (22)

where the heat transfer between cooling air and the central shaft and the rabble arms consist of the radiative and the convective heat transfer terms respectively:

\[ Q_{\text{shaft cool}} = \sigma A_{\text{shaft cool}} e_{\text{shaft}} (T_{\text{shaft}}^4 - T_{\text{cool}}^4) + h_{\text{cshaft cool}} A_{\text{shaft cool}} (T_{\text{shaft}} - T_{\text{cool}}) \]  \hspace{1cm} (24)

\[ Q_{\text{arms cool}} = \sigma A_{\text{arms cool}} e_{\text{arms}} (T_{\text{arms}}^4 - T_{\text{cool}}^4) + h_{\text{carsm cool}} A_{\text{carsm cool}} (T_{\text{arms}} - T_{\text{cool}}) \]  \hspace{1cm} (25)

The energy balance of the cooling air is described by Equation 26:

\[ \frac{\partial Q_{\text{cooling air}}}{\partial t} = \dot{Q}_{\text{cooling air in}} + \dot{Q}_{\text{arms cool}} + \dot{Q}_{\text{shaft cool}} - \dot{Q}_{\text{cooling air out}} \]  \hspace{1cm} (26)

Finally, the solid phase energy balance equation is written as:

\[ \frac{\partial Q}{\partial t} = \dot{Q}_{\text{mass in}} - \dot{Q}_{\text{mass out}} - \dot{Q}_{\text{reactions}} - \dot{Q}_{\text{evaporation}} + \dot{Q}_{\text{sw}} + \dot{Q}_{\text{sg}} \]  \hspace{1cm} (27)
When considering the solid bed movement during one time step (one central shaft rotation), the energy in each solid bed compartment can be calculated using Equation 28:

$$Q_{t+1}^{j,k} = D_j \cdot Q_t^{j,k} + Q_{\text{feed},t}$$

(28)

For each component in the solid phase, the heat capacity is assumed to be constant, see Table 1.

**Table 1. The heat capacities of the solid components.**

<table>
<thead>
<tr>
<th>Component</th>
<th>Heat capacity (kJ/kg K)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>4.18</td>
<td>(Perry &amp; Green, 1997)</td>
</tr>
<tr>
<td>Kaolin</td>
<td>1.16</td>
<td>(Schietz &amp; Soliman, 1966)</td>
</tr>
<tr>
<td>Metakaolin</td>
<td>1.19</td>
<td>(Schietz &amp; Soliman, 1966)</td>
</tr>
<tr>
<td>Spinel phase</td>
<td>0.93</td>
<td>[The plant operator]</td>
</tr>
<tr>
<td>Mullite</td>
<td>1.11</td>
<td>(Schietz &amp; Soliman, 1966)</td>
</tr>
</tbody>
</table>

### 3.3.1. Solid bed and gas phase emissivities

The solid bed consists of kaolin which consists mainly of a mixture of aluminium oxide and silicon oxide. The literature values for the emissivity of aluminium oxide and silicon oxide at various temperatures are given in (Sheyndlin, 1974) (Engineering.com, 2006) (MIKRON Vertretung Schweiz, n.d.). From them a correlation between the emissivity of the kaolin and the temperature is determined (see Figure 5) and the derived result is given in Equation 29:

$$\varepsilon_{\text{solidbed}} = 2.10 \times 10^{-7} \cdot T_{\text{solidbed}}^2 - 0.00064 \cdot T_{\text{solidbed}} + 0.88$$

(29)

**Figure 5. Graph shows the literature values of emissivities of aluminium oxide and silicon oxide and the curve of Equation 29.**
The gas emissivity in Hearth 1 to 6 is affected by the water and carbon dioxide content and the temperature of the gas phase. Other gas phase components are not considered as they all are small two atomic components and as such, do not affect the radiation properties of gas (Perry & Green, 1997). The gas phase in Hearths 7 and 8 has only oxygen and nitrogen, thus the emissivity is not calculated for those hearths, but it was assumed to be 0.2. The gas phase emissivities, calculated according to Perry and Green (1997, Table 5.7), are summarized in Table 2.

<table>
<thead>
<tr>
<th>Hearth</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>ε</td>
<td>0.34</td>
<td>0.32</td>
<td>0.3</td>
<td>0.27</td>
<td>0.27</td>
<td>0.23</td>
<td>0.2</td>
<td>0.2</td>
</tr>
</tbody>
</table>

3.4. Solution procedure for the MHF model

The solution procedure for the MHF model consists of five main steps. Firstly, the reaction rates are calculated using the reaction parameters obtained from the experimental data. Secondly, the solid mass balance is computed, which requires the mixing model. Thirdly, the mass and energy balances of the gas phase are solved using the estimated parameters, such as the burning ratios, solid bed surface view factors and the extraneous air flow to Hearth 8. The next step consists of energy balance calculations for the walls, cooling air, central shaft and the rabble arms. The fifth and final step solves the energy balance of the solid bed by combining the heat fluxes determined in the previous steps by utilizing the mixing model. The model solution cycle is presented in Figure 6 summarizing the mass and energy flows between the model parts and the usage of the current model states. In addition, Figure 6 shows in which step the experimentally determined and the estimated parameters are used in the calculations.

During one solution cycle, representing one full rotation of the central shaft, the solving procedure calculates the current states of the six model parts (the solid phase, gas phase, walls, cooling air, rabble arms and the central shaft). The first step, the computation of the reaction rates in the solid phase, is done by direct calculations using the current temperature profile distribution. In the second step, the solid bed mass balance is solved using the computed reaction rates and the experimentally determined mixing model. After the first two steps, the energy produced by the reactions in the solid bed, the energy flows caused by the convection in the solid bed and the water transferred to the gas phase are available. In the next step, the water transferred to the gas phase is first used to calculate the mass balance, and subsequently the energy balance of the gas phase is determined. Specifically, the steady-state values of the heat fluxes between the gas phase and other model parts are calculated by solving algebraic equations achieved by an interval division method. Subsequently, the fourth step involves solving the temperature profiles of the walls, central shaft, rabble arms and the cooling air using the obtained heat fluxes. Finally, the last step of the solution cycle combines the heat fluxes in the solid bed, the heat exchange between the solid bed and other model parts, and the composition of the solid bed to calculate its energy balance and temperature profile through the furnace. In these last two steps, Euler's method is employed. The solving algorithms have been implemented in MATLAB environment.
The solving procedure is developed to compute the mass and energy flows between the model parts once during a solution cycle. Thus, while resolving the balance equations of two model parts, the same value of the mass and heat flow between them is used. A conservative solution scheme is attained and the mass and energy conservation is achieved in the solution.

Figure 6. Scheme of the solution procedure for the MHF model.

4. Calculating model parameters from the experimental data

4.1. Determination of the reaction parameters

The reaction parameters, namely the frequency factor and activation energy of the Arrhenius equation, were determined for the four reactions (Equations 1 – 4) by identifying the reaction rate curves from the experimental data provided by the company operating the furnace.

The experimental reaction data obtained by using the DSC and TGA is shown in Figure 2. The reaction parameters were determined by approximating the reaction rate curves while using a heating rate of 40 °C/min. Table 3 lists the obtained values of the parameters and the values of the reaction enthalpies. These obtained reaction parameters were compared with the values in Matrins et al., 2001,
Petzold et al., 1985, Ptacek et al., 2010, Mazumdar & Mukherjee, 1983 and Ptacek et al., 2012 to confirm the parameters are consistent with the literature. The obtained reaction rate curves are presented in Figure 7.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Frequency factor (1/s)</th>
<th>Activation energy (kJ/mol)</th>
<th>Enthalpy of the reaction (kJ/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evaporation</td>
<td>$5 \times 10^7$</td>
<td>61.0</td>
<td>2258.2</td>
</tr>
<tr>
<td>Dehydroxylation</td>
<td>$1 \times 10^7$</td>
<td>145.0</td>
<td>891.0</td>
</tr>
<tr>
<td>Spinel formation</td>
<td>$5 \times 10^{13}$</td>
<td>856.0</td>
<td>-212.9</td>
</tr>
<tr>
<td>Mullite formation</td>
<td>$1 \times 10^{17}$</td>
<td>522.0</td>
<td>1301.7</td>
</tr>
</tbody>
</table>

Figure 7. Reaction rate curves for the four reactions, left to right: evaporation of free moisture, consumption of kaolin during dehydroxylation, consumption of metakaolin in the spinel formation reaction, and the mullite formation presenting the consumption of spinel phase.

### 4.2. Determination of the solid bed movement matrix D

The movement of the solid bed was studied to determine the residence time of every hearth. The mass distribution between the hearths was studied with a pilot furnace. The residence time distribution through the whole furnace was taken from industrial experiments (Thomas et al., 2009) utilizing tracer material to determine the residence time distribution. Subsequently, these results were combined, the residence time for each hearth was obtained, and the solid movement matrices were defined.

The main equipment used for the testing includes a pilot scale furnace and a continuous weight proportioner. The pilot scale multiple hearth furnace is a 1:12 ratio replica of the industrial furnace (without the heating equipment) having the same rabble arm configuration as the industrial size MHF. The mass distribution experiments were executed using a feed rate of 70 g/min corresponding to the real feed rate of 120 kg/min. The rotation speed of the central shaft was 3 RPM. After the mass profile inside the pilot had achieved its steady state, the mass of the kaolin on each hearth was weighted.
The mass distributions obtained in the four experiments performed are shown in Figure 8. Clearly, Hearths 3 and 7 have the largest portions of kaolin, whereas Hearths 1 and 2 have the smallest portions indicating that the material spends a relatively short time on the first two hearths and a much longer time on Hearths 3 and 7. The results of the four tests showed a good correlation and are shown in Figure 8.

![Figure 8. Mass distribution between the hearths from the tests 1, 2, 3, and 4.](image)

The net forward flow coefficients $\alpha_f$ used in the solid bed movement matrix, were chosen to achieve consistency between the experimental results and the mixing model prediction of the mass distribution between the hearths and the residence time distribution. The backward flow parameters $\alpha_r$ were selected to be 5 percent of the total net flow for Hearths 1 and 2, and 10 percent for the rest of the other hearths, since higher values of the backward flow in the first two hearths are inconsistent with the short residence time in these particular hearths.

Figure 9 presents the resulting residence time distribution according to the identified solid movement model and Figure 10 shows the residence time distribution curve of the industrial experiment. Comparison between the model and the industrial setting shows that both of the curves are consistent. Additionally, in both cases, the tracer starts reaching the outlet approximately 20 minutes after introduction into the system and the last particles leave the furnace within 80 minutes.
Figure 9. Graph showing the residence time distribution curve in the MIF model.

Figure 10. Graph showing the change in talc content (inferred by MgO content) of the calcined product during the talc tracer dosing experiment (Thomas et al., 2009).

5. Estimation of parameters and model validation

Four different types of model parameters introduced in Equations 13, 14, 16 and 19 need to be determined from the experimental data. These four parameters are:

1) The burning ratios $b_i$ (at Hearths 4, 5 and 6 as shown in Table 4) distributing the combustion energy obtained between the different volumes, are identified from the process data. The burning ratios are used for simplification of the complex fluid dynamics calculation in the presence of combustion flame. In fact, if all of the methane would be burned in the volume next to the walls its temperature would increase too high.
2) Form and view factors $X_{x_i}$ for the solid bed surface areas, which describe the surface shape of the solid bed and it is used to follow the energy transfer between the gas and solid phases in each hearth $i$.

3) The central shaft has insulation, but its characteristics are unknown. If the insulation is ignored in the model, the cooling air temperature increases to an unrealistically high value. Therefore, a heat exchange area multiplier $Z$ is introduced to lower the heat exchange between the central shaft and the gas phase and between the rabble arms and the gas phase. The multiplier $Z$ can be estimated from the energy the cooling air absorbs from the furnace since the amount of the cooling air and its inlet and exit temperatures are known.

4) Extraneous air flow to Hearth 8 through the bottom drop holes is unknown. This extraneous air flow is introduced for cooling down the solid bed in Hearths 7 and 8, since no other mechanisms for cooling are available. Specifically, it is known that the solid bed reaches temperature up to 1050 °C at the end of Hearth 6 and the solids exit the furnace at a temperature around 750 °C. This information can be used to estimate the amount of the extraneous air flow. In addition, it was confirmed from the operative personnel at the plant that there a draft through the drop holes exists.

For determination of these model parameters, the steady-state process data from May 2013, at which the feed rate was 120 kg/min, was used. The steady values of the input variables are summarized in Table 5.

Table 4. The burning ratios.

<table>
<thead>
<tr>
<th>Hearth</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>$b_1^*(1 - b_1)^3$</td>
</tr>
<tr>
<td>5</td>
<td>$b_2^*(1 - b_2)^4$</td>
</tr>
<tr>
<td>6</td>
<td>$b_2^*(1 - b_2)^3$</td>
</tr>
</tbody>
</table>

Table 5. The steady-state inputs and outputs used for model adaptation.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feed rate</td>
<td>120 kg/min</td>
</tr>
<tr>
<td>Gas flow H4</td>
<td>221.03 kg/h</td>
</tr>
<tr>
<td>Air flow to H4</td>
<td>4191.4 kg/h</td>
</tr>
<tr>
<td>Gas flow to H6</td>
<td>133.17 kg/h</td>
</tr>
<tr>
<td>Air flow to H6</td>
<td>2381.4 kg/h</td>
</tr>
</tbody>
</table>
Next, the model parameters were estimated by minimizing the sum of the mass and energy balance errors and the errors between the model outputs and the actual measurement data. Specifically, the modelling error minimization procedure was conducted utilizing nonlinear least-squares as shown in Equation 30:

$$\min_{x,p} (f_1(x)^2 + f_2(x)^2)$$  \hspace{1cm} (30)$$

where \( p \) contains the burning ratios, form factors, insulation multiplier and the extraneous air flow to Hearth 8, \( x \) represents a model steady state, \( f_1 \) denotes the error of the mass and energy balances and \( f_2 \) is the error between the output variables of the state \( x \) and the actual measured variables. The estimated model parameters are given in Table 6.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( b_1 )</th>
<th>( b_2 )</th>
<th>( X_{a,1} )</th>
<th>( X_{a,2} )</th>
<th>( X_{a,3} )</th>
<th>( X_{a,4} )</th>
<th>( X_{a,5} )</th>
<th>( X_{a,6} )</th>
<th>( X_{a,7} )</th>
<th>( X_{a,8} )</th>
<th>( Z )</th>
<th>Extraneous air to H8 (kg/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0.77</td>
<td>0.5</td>
<td>0.52</td>
<td>0.46</td>
<td>1.9</td>
<td>1.34</td>
<td>1.39</td>
<td>0.5</td>
<td>1.28</td>
<td>0.56</td>
<td>0.72</td>
<td>2086.8</td>
</tr>
</tbody>
</table>

The estimated parameter values were applied and a simulation at the steady-state was performed. The resulting temperature profiles of the solid bed and of the gas phase predicted by the model are presented in Figure 11 with the measured values of the gas phase temperatures next to the walls. In addition, Figure 12 illustrates in which hearth each of the calcination reactions occurs in the furnace and how the composition changes in the solid bed throughout the MHF.

Figures 12 and 13 show that in Hearths 1 and 2, the solids temperature rises to 170 °C and only the evaporation of free moisture occurs. This is because of the short residence time on these hearths observed from the mass distribution tests. When the solids enter the third hearth, their temperature rises rapidly and the dehydroxylation reaction begins. The metakaolin formation reaction lasts until the second half of the fifth hearth, by which time the solids temperature becomes increasingly get higher. The ‘spinel phase’ or the product starts to be produced in the middle of the sixth hearth, since the solids temperature is high enough. Now, the exothermic reaction increases the temperature even faster and also the endothermic mullite formation reaction begins and leads to a cooling of the solids. From the beginning of the seventh hearth, the solids temperature decreases until the material exits the furnace at 750 °C having a mullite content of approximately 5 mass-percent.
The steady-state temperature profile of the MHF at the feed rate of 120 kg/h. The x-axis presents the volumes of the solid and the gas phases organised in the order of the solid flow from top to bottom.

Next, the dynamic model was validated using the industrial data. Figure 13 presents the comparison of the gas phase temperatures next to the walls in different hearths between the model estimate and the industrial measurement data. As the graph shows, the model is able to produce good estimates of the current gas phase temperature and the model performance can be evaluated as satisfactory.
Figure 13. The model simulation compared with industrial measurement data.

6. Dynamic behaviour of the MHF

The dynamic behaviour of the model was investigated to study the effect of the process inputs on the reactions occurring in the solid bed and the process outputs. Feed rate and the gas and air flows to Hearths 4 and 6 were chosen as manipulated variables while studying changes in the gas temperature profile through the furnace, and the quality variables: product and offspec mass-percentage in the lower hearths. The model response was studied by introducing a step change at 833 minutes in the input variables when the model had settled into a steady operation mode.
6.1. Furnace response under feed rate changes

The model behaviour was first studied by introducing a 2.4 kg/min (2 percent) step increase in the feed rate. At the beginning, the mullite formation in Hearth 7 slows down, which can be observed as the slight increase in the amount of the spinel phase in the final product, see Figure 16.

As the product formation reaction is not receiving enough heat, it moves from Hearth 6 to the entrance of Hearth 7 at around 1200 minutes, as can be seen in the product content dynamics displayed in Figure 15-16. The shift of product formation to the beginning of Hearth 7 also affects the gas temperature profile in Hearths 5, 6 and 7 in Figure 15. However, the final product is still composed almost entirely of the spinel phase.

After 1560 minutes of the simulation time, the spinel formation partly moves to the middle of Hearth 7, which is confirmed by Figure 16. As a result, the metakaolin conversion to the spinel phase is not complete and the gas temperature in Hearth 7 decreases rapidly.

During the whole test, the temperatures of the exhaust and the gas phases on Hearths 2 to 4 decrease steadily until settling to a new steady state, as Figure 14 shows. In contrast, the gas temperature in Hearth 8 slowly increases as a result of the rising temperature of the solid that enters to the last hearth.

Next, a step decrease of 2.4 kg/min (2 percent) was introduced to the feed rate and the model response is shown in Figure 17 to Figure 19. At the beginning, the mullite formation is activated in Hearth 7 due to the temperature rise, which can be seen as a decrease of the spinel phase contents in Figure 19.

After 1260 minute of the simulation time, the spinel formation partly moves towards the middle of Hearth 6, as demonstrated by Figure 18. Simultaneously, the mullite formation in Hearth 7 increases rapidly and the share of off-spec in the final product rises above 5%, which is demonstrated in Figure 19. As a result, the gas temperature in Hearths 6 and 7 changes its dynamics as it is seen in Figure 17.

In this case study, the temperatures of the exhaust and the gas temperatures of Hearths 2 to 5 increase steady until reaching a new steady state, since more specific energy is supplied to the furnace.
Figure 14. The response of the gas phase temperature next to the wall in the feed rate increase test.

Figure 15. The response of the product content of the solid bed in Hearth 6 during the feed rate increase test.
Figure 16. The response of the product content of the solid bed in Hearth 7 during the feed rate increase test.

Figure 17. The response of the gas phase temperature next to the wall in the feed rate decrease test.
6.2. Furnace response under gas and air flows changes on Hearth 4

Next, a 11.0 kg/h (5 percent) step increase was introduced to the gas and air flows to Hearth 4. The model response is presented in Figure 20 to Figure 22. At the beginning, the spinel phase formation in Hearth 6 and its conversion to mullite in Hearth 7 slowly increase their rates, as seen in Figure 21 and Figure 22.

As the gas temperature on Hearth 6 rises, the product formation rapidly moves towards the earlier volumes at 1180 minutes, as it is demonstrated in Figure 21. It causes a small change in the dynamics of the gas temperature in Hearth 6. Simultaneously, the drop of the temperature in Hearth 7 is caused by the increased reaction rate of the endothermic mullite formation, which can be seen in Figure 22 as a decrease in the product content. The rise of the gas temperature in Hearth 8 before the 1180 minute of the simulation and its afterwards decrease also follows the phenomena occurring in Hearth 7.
The exhaust gas temperature and the temperatures of Hearth 2, 3, and 4 increase instantly after the step change and afterwards they settle to a higher steady state than the initial temperature was. As the extra energy is supplied to Hearth 4, the gas temperatures of Hearth 5 grow smoothly, which is caused by the slow increase of the temperature of the solid phase entering this hearth.

Subsequently, the model response was studied using a 11.0 kg/h (5 percent) step decrease in the gas and air flows to Hearth 4 and the results are shown in Figure 23 to Figure 25. Due to the lack of energy supplied to the furnace, the product formation reaction first moves to the beginning of Hearth 7 at about 1130 minutes of simulation. At 1310 minutes, the reaction moves towards the middle of the hearth, as shown in Figure 24 and Figure 25. Additionally, the gas phase temperature in Hearths 6 and 7 follow these reaction shifts, see Figure 23.

Next, the temperature in the solid phase drops, and does not achieve in any part of the furnace the temperature of 925 °C required for the spinel phase reaction to occur. Thus, the reaction collapses around 1420 minutes, as observed in Figure 25. Now, the solid bed on Hearth 7 and the final product consist only of metakaolin. Since the reaction is not releasing heat anymore, it greatly affects the gas and solid temperature in the last three hearths of the furnace.

As the gas and air flow to Hearth 4 drops, the temperatures of the exhaust gas and Hearths 2, 3 and 4 decrease first with a steep drop and then more steadily until a new steady state is reached. In contrast, the temperature of Hearth 5 decreases steadily for the whole recorded time period.
Figure 20. The response of the gas phase temperature next to the wall while increasing the gas and air flows to Hearth 4.

Figure 21. The response of Hearth 6 product content while increasing the gas and air flows to Hearth 4.
Figure 22. The response of Hearth 7 product content while increasing the gas and air flows to Hearth 4.

Figure 23. The response of the gas phase temperature next to the wall while decreasing the gas and air flows to Hearth 4.
Figure 24. The response of Hearth 6 product content while decreasing the gas and air flows to Hearth 4.

Figure 25. The response of Hearth 7 product content while decreasing the gas and air flows to Hearth 4.

6.3. Furnace response under gas and air flows changes on the Hearth 6

The gas and air flows to Hearth 6 were introduced a 6.6 kg/h (5 percent) step increase to investigate the response of the system presented in Figure 26 to Figure 28. The mullite formation reaction begins immediately to intensify on Hearth 7 as seen in Figure 28.

As the gas temperature on Hearth 6 rises the product formation intensifies at 870 minutes, especially in the third volume on the same hearth as shown in Figure 26. Since, the amount of gas fed to Hearth 6 is approximately half of what is fed to Hearth 4, the observed changes in the reactions are not as intensive as in the previous case.

The exhaust gas temperature and the temperatures of Hearths 2, 3, and 5 increases instantly after the step change and afterwards they settle to a higher steady state than the initial temperature was. The Hearth 4 temperature decreases slightly as a result of the increased...
gas flow from previous hearth, but afterwards increases steadily to a higher temperature. The gas temperatures on Hearths 6 to 8 increase over the time and settle to new steady state values.

The final single input step test was a 6.6 kg/h (5 percent) decrease in the gas and air flows to Hearth 6. The model response is given in Figure 29 to Figure 31. As a result of decreased heating the spinel phase formation moves to Hearth 7 at the time of 1540 minutes as seen in Figure 30 and 32. This also noticeably increases the Hearth 7 temperature and decreases the Hearth 6 temperature as seen in Figure 29.

As the gas and air flow to Hearth 6 drops, the temperatures of the exhaust gas and Hearths 2, 3 and 5 decrease first with a steep step and then more steadily before reaching a new steady state. In contrast, the temperature of Hearth 4 increases first slightly and then decreases steadily for the whole recorded time period.

Figure 26. The response of the gas phase temperature next to the wall while increasing the gas and air flows to Hearth 6.
Figure 27. The response of Hearth 6 product content while increasing the gas and air flows to Hearth 6.

Figure 28. The response of Hearth 7 product content while increasing the gas and air flows to Hearth 6.
Figure 29. The response of the gas phase temperature next to the wall while decreasing the gas and air flows to Hearth 6.

Figure 30. The response of Hearth 6 product content while decreasing the gas and air flows to Hearth 6.
6.4. Discussion of the process dynamics

The performed tests have demonstrated that the model outcomes are adequate. In particular, the spinel phase formation takes place in the last two volumes of Hearth 6 and first two volumes of Hearth 7, which agrees with the process knowledge. More specifically, the reaction moves towards the entrance of Hearth 6 when higher specific energy is supplied to the furnace, and the reaction shifts towards Hearth 7 as the amount of the supplied energy decreases. In the former case, higher temperature is reached in the solid phase which activates the mullite formation reaction in Hearth 7. On the other hand, if the specific energy is decreased too much, the temperature in the solid phase drops below the temperature required for spinel formation, and the reaction collapses. In this case, the output of the furnace consists of pure metakaolin. Thus, it can be concluded that the furnace must be controlled such a way as to direct the spinel formation at the end of Hearth 6 and the first volume of Hearth 7, which would deliver both highest content of the spinel phase in the product and stable operating conditions.

The dynamics of the temperature profile through the furnace is the result of complex interactions between the fast dynamics of the gas phase, the slower dynamics of the solid content and the even slower dynamics of the furnace walls. As an example, during the changes in the gas and air feed flows, the gas phase temperatures show first fast responses and then slow dynamics while settling to their new steady values. On the other hand, it takes a considerably longer time for the gas phase temperatures to react to steps in the feed rate. In some cases, significant shifts in the temperature dynamics occur after a much longer time, even after 700 minutes, which denotes the ability of the solid bed and the walls to act as a reservoir of huge amounts of heat.

Comparing the dynamics of different furnace hearths, it was noticed that the temperatures of the exhaust gases and Hearths 2, 3, and 4 show qualitatively simple behaviour. In contrast, the temperature of the gas phase in Hearths 5 to 8 depends on the process inputs in a more complex and nonlinear way. This is explained by the shifts in the spinel phase and mullite formation reactions through Hearths 6 and 7.
7. Conclusions

A dynamic model of kaolin calcination in a Multi Hearth Furnace (MHF) was developed in this paper. This model describes the physical-chemical phenomena taking place in the six furnace parts: the solid phase, gas phase, walls, cooling air, raffle arms and the central shaft. A simple mixing model was proposed to describe the dynamics of the solid phase, and experiments with a pilot plant have been conducted to estimate the parameters of the mixing model.

Simulation studies have confirmed that the outcomes of the developed model are qualitatively adequate and its performance can be evaluated as fairly good. Thus, the model can be used to study the physico-chemical phenomena occurring inside the furnace and it provides temperature profile through the hearths. In particular, the model behaviour was studied by introducing step changes in the three input variables: the feed rate, and the gas and air flows to Hearths 4 and 6. A discussion regarding the observed process dynamics was provided. In addition, as the simulation results have demonstrated, the conditions in different volumes in a hearth vary greatly, a conclusion can be made that introducing the division into volumes is required to properly represent the actual chemical-physical phenomena.

The developed model has proven to give valuable information about the calcination reactions which can be utilized to design an enhanced control of furnace operations. Next, the properties of the feed material will be considered in the model along with a more detailed study of the solid bed dynamics which will take into consideration the effect of the raffle arm configuration.

Acknowledgement

The research leading to these results has received funding from the European Union Seventh Framework Programme (FP7/2013-2016) under Grant Agreement No. 310645.
Appendix A – Heat transfer coefficients

Nomenclature

Roman letters
A  Area (m$^2$)
$c_p$  specific heat capacity (J/molK)
$D_H$  characteristic diameter (m)
g  gravimetric constant (=9.81 m/s$^2$)
$k_d$  thermal conductivity (W/mK)
L  characteristic dimension (m)
P  perimeter (m)
v  average velocity (m/s)

Greek letters
$\alpha$  kinetic viscosity
$\beta$  volumetric expansion coefficient (1/T(K))
$\varepsilon$  emissivity, radiative exchange ratio
$\rho$  density (kg/m$^3$)
v  thermal diffusivity
$\mu$  dynamic viscosity (Pa s)

Heat transfer between the solid bed and the gas phase

The heat transfer coefficient $h_{cg}$ is calculated using the Nusselt’s number presented in Equation A.1.

$$N \alpha = \frac{hL}{k_g}$$  \hspace{1cm} (A.1)

In this case the characteristic dimension is the length of the heat transfer surface shown in Figure A.1.
For the hearths 1 to 6 the conditions for the Nusselt number calculation are assumed to be forced convection, turbulent (Re > 1*10^5) and internal flow. For this case the Nusselt number is given by Equation A.2 (Incropera et al., 2007)

\[ N_u = 0.23 \times Re^4 \times Pr^{0.35} \]  \hspace{1cm} (A.2)

The Reynolds number in Equation A.1 is calculated as

\[ Re = \frac{\rho u D_h}{\mu} \]  \hspace{1cm} (A.3)

The Prandtl number describing the ratio between the momentum diffusivity and thermal diffusivity of the fluid is calculated as

\[ Pr = \frac{c_p \mu}{k} \]  \hspace{1cm} (A.4)

The conditions on the hearths 7 and 8 are assumed to be natural convection on the top of a hot surface. Now, the solid bed is assumed to be in a higher temperature than the gas phase, otherwise there would not be a cooling effect. According to (Incropera et al., 2007) the Nusselt number for this particular case can be evaluated as

\[ N_u = 0.54Ra^{1/4} \]  \hspace{1cm} (A.5)

The Rayleigh number is written as

\[ Ra = \frac{g \beta (T_s - T_0) L^3}{\nu \alpha} \]  \hspace{1cm} (A.6)

The characteristic length L is defined as

\[ L = \frac{A}{F} \]  \hspace{1cm} (A.7)

The gas-solid heat exchange is calculated separately for each four or five solid volumes depending on the hearth.

**Heat transfer between the solid bed and the walls**

The emissivity between the solid bed and the walls \( \varepsilon_{sw} \) is calculated as in (Ginsberg & Modigell, 2011):
\[ \varepsilon_{sw} = \frac{\varepsilon_s \varepsilon_w (1 - \varepsilon_g)}{1 - U} \] (A.8)

where \( \varepsilon_s, \varepsilon_w \) and \( \varepsilon_g \) present emissivities of the solid bed, walls and gas phase, respectively. The variable \( U \) is defined as in Equation A.9.

\[ U = (1 - \varepsilon_w)(1 - \varepsilon_g)[v(1 - \varepsilon_g)(1 - \varepsilon_s) + (1 - v)] \] (A.9)

where the variable \( v \) presents the ratio between the heat exchange area of the solid bed and the gas phase \( A_{sg} \) and the heat exchange area between the walls and the gas phase \( A_{gw} \):

\[ v = \frac{A_{sg}}{A_{gw}} \] (A.10)

The emissivity of the walls is assumed to be 0.90 as with a refractory material being an average radiator (Perry & Green, 1997)(page 5-29).

**Heat transfer between the inner walls and the gas phase**

The heat transfer coefficient \( h_{gw} \) is calculated using the same equations as for the heat exchange between the gas phase and the solid bed for the hearths 1 to 6.

For the hearths 7 and 8, the heat exchange surface is assumed to be a vertical plate with natural convection due to the stagnant gas phase. The Nusselt number for the hearths 7 and 8 is calculated as

\[ N_u = 0.68 + \frac{0.677 Ra \frac{1}{\text{Pr}^{1/3}}}{1 + \left( \frac{0.492}{\text{Pr}} \right)^{9/16}} \] (A.11)

The Raleigh number (Ra) is calculated with Equation A.6 where the characteristic length is now the height of the wall. The Prandtl number (Pr) is calculated with Equation A.4.

**Conduction inside the walls**

To solve the boundary surface temperatures of the wall layers, the heat fluxes between the wall and the gas phase and between the wall and the solid bed are calculated first. Next, the outer wall temperature is iterated so that the heat flux inside the wall and the heat flux to the ambient air are equal. The wall is considered as a multi-layered cylinder. As the heat flux through the different layers of the wall was assumed constant, the heat flux inside the wall can be calculated using Equation A.12 (Engineers Edge, LLC, 2014)

\[ Q_w = \frac{2\pi L (T_{\text{inner wall}} - T_{\text{outer wall}})}{\frac{1}{L_1} \ln \left( \frac{r_2}{r_1} \right) + \frac{1}{L_2} \ln \left( \frac{r_3}{r_2} \right) + \frac{1}{L_3} \ln \left( \frac{r_4}{r_3} \right) + \frac{1}{L_4} \ln \left( \frac{r_5}{r_4} \right)} \] (A.12)
where \( L \) is the height of the wall section, \( \lambda_i \) is the thermal conductivity of a wall layer and, \( r_j \) is the radius of the surface from the centre of the hearth. After updating the inner wall surface temperature, the temperatures of the other surface layers can be calculated using Equation A.13:

\[
\frac{\partial T_{\text{wall section}}}{\partial t} = -Q_w + \ln\left(\frac{r_i}{r_j}\right) \frac{1}{2\pi \lambda_i h_{\text{wall}}} \tag{A.13}
\]

where subscript \( i \) is the number of the wall layer (number 1 presents the most inner layer and the inner surface) and \( h \) presents the height of the wall section. The heat exchange properties of the wall materials are given in Table A.1.

<table>
<thead>
<tr>
<th>Material</th>
<th>Width (mm)</th>
<th>Thermal conductivity (W/m K)</th>
<th>Heat capacity (J/kg K)</th>
<th>Density (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refractory brick DSF 59 (DSF Refractories, 2009)</td>
<td>152</td>
<td>1.6</td>
<td>1000</td>
<td>2480</td>
</tr>
<tr>
<td>Fire brick HTJ (BNZ Materials, Inc., 2009)</td>
<td>114</td>
<td>0.32</td>
<td>1000</td>
<td>769</td>
</tr>
<tr>
<td>Backfill Caldecast XL 106 (Calderys, 2010)</td>
<td>75</td>
<td>0.16</td>
<td>1000</td>
<td>2350</td>
</tr>
<tr>
<td>Mild steel (0.5% carbon) (Engineers Edge, LLC, 2013)</td>
<td>25</td>
<td>54</td>
<td>465</td>
<td>7833</td>
</tr>
</tbody>
</table>

**Heat transfer between the outer wall and the ambient air**

The heat transfer between the outer wall surface and ambient air is assumed to be natural convection on a vertical hot plate. The heat exchange coefficient is calculated using the Rayleigh number, Equation A.6. Now, \( Ra_w > 10^9 \) and equation for the Nusselt number becomes (Incropera et al., 2007)

\[
Nu_w = 0.10 \times Ra_w^{1/3} \tag{A.14}
\]

**Heat transfer between the gas and the central shaft and the rabbles arms**

For the heat exchange between the gas phase and the central shaft at the hearths 1 to 6, the following assumptions are made: the shaft is in external gas flow, the shaft is considered as a flat plate parallel to the flow and the gas flow is turbulent. Now, the Nusselt number for the heat transfer coefficient can be calculated using Equation A.15.
The Reynold number is an average of the Reynold number in the annulus between the central shaft and the hearth and of the Reynold number in the gas phase close to the shaft.

For the hearths 7 and 8, the heat exchange coefficient is approximated to be the same as the coefficient between the walls and the gas phase since, in both cases, the heat exchange conditions are the same: natural convection on a vertical plate.

For the heat exchange between the rabble arms and the gas phase, the Nusselt number is calculated with the same equation A.14 on the hearths 1 to 6. In the hearths 7 and 8, the Nusselt number is calculated using Equation A.16, which applies to a long horizontal cylinder for a wide Rayleigh number range $Ra<10^{12}$ (Incropera et al, 2007):

\[
Nu = 0.60 + \frac{0.387Ra^{\frac{1}{9}}}{\left( 1 + \left( \frac{0.559}{Pr} \right)^{\frac{9}{16}} \right)}
\]  

(A.16)

Heat transfer between cooling air and the central shaft and the rabble arms

Nusselt number for the heat transfer between cooling air and the central shaft and the rabble arms is calculated using Equation A.17, which applies for a flow inside a tube in turbulent conditions.

\[
Nu = 0.023 * Re^{0.8} * Pr^{\frac{1}{3}} \left( \frac{\mu}{\mu_w} \right)^{0.14}
\]  

(A.17)

For the rabble arms the characteristic length $D_h$ is given by Equation A.18:

\[
D_h = 4 * \frac{\text{Cross sectional area of the arm}}{\text{Perimeter of the arm}}
\]  

(A.18)

References


