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Observers for Discrete-Time Current Control of Converters Equipped With an LCL Filter

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Abstract—This paper studies the current-type observer, the prediction-type observer, and the reduced-order observer for discrete-time state-space current control of grid converters with an LCL filter. The robustness against parameter errors and the dynamic performance obtained with these three observers are compared by means of analysis and experiments. A unified control algorithm framework is developed to compare the structures of these observers. It is found that the reduced-order observer is merely a special case of the current-type observer. With the presented design examples, where the resonant poles are damped by means of radial projection, the use of either the current-type or the reduced-order observer leads to better disturbance rejection against the grid-voltage dip and harmonics as compared to the prediction-type observer. All the observers provide equal reference-tracking performance under nominal conditions. For a large parameter error in the grid-side inductance, the prediction-type observer leads to unstable operation of converter.

Index Terms—Full-order observer, grid converter, LCL filter, reduced-order observer, state-space current control, weak grid.

I. INTRODUCTION

Grid converters equipped with an LCL filter are increasingly used in modern power systems [1]. State-space current control of the converters inherently enables active damping of the LCL filter resonance [2]–[20]. In state-space current control, all the states in the LCL filter can be measured [5]–[9] or estimated using an observer [10]–[20]. The use of an observer increases reliability, reduces the number of sensors, and decreases the costs in comparison with the current control where all the states are measured.

As shown in Fig. 1, the control can be based on the converter current measurement. The other states can be estimated using either a full-order observer [10]–[16] or a reduced-order observer [17]–[20]. The full-order observer reconstructs all the states of the system, i.e., the measured state as well as the unmeasured states. In the discrete-time domain, there are two alternative structures for the full-order observer: the current-type observer and the prediction-type observer [21]. The current-type observer computes a state estimate corrected with the most recent current measurement. On the other hand, the prediction-type observer provides a state estimate based on the current measurement from the previous sampling instant. The properties of the two full-order observers approach each other as the sampling frequency approaches infinity. In contrast to the full-order observers, the reduced-order observer

Fig. 1. Current control system operating in grid-voltage oriented synchronous coordinates. The PLL is used to obtain the grid-voltage angle ϑ_g . The PWM is modeled as the zero-order hold (ZOH) in stationary coordinates. Space-vector circuit model of an LCL filter is also shown (superscript s indicates stationary coordinates).

reconstructs only the unmeasured states. The reduced-order observer computes a state estimate corrected with the most recent current measurement in a similar manner as in the case of the current-type full-order observer.

The current-type observer is identical in structure to the constant-gain Kalman filter [21]. The noise sensitivity and computational complexity of the Kalman filter have been compared with both the current-type observer and the prediction-type observer for linear quadratic Gaussian (LQG) control in [13]. However, a comparison of the observers in terms of the dynamic performance and robustness against parameter errors is not available in the context of current control.

In this paper, we compare three different observers, the current-type observer, the prediction-type observer, and the reduced-order observer, for discrete-time state-space current control. A unified control algorithm framework is first developed to compare the structures of these observers. Then, the comparison is made in terms of the robustness against parameter errors and the dynamic performance of the closed-loop system. Furthermore, the performance obtained with all three observers is compared with the performance resulting from the control without an observer, i.e., the current control where all the states are measured. The controllers are designed using the direct pole-placement method with the radial projection

for the damping of the LCL filter resonance [17]–[19]. The observers are evaluated by means of experiments using a 12.5-kVA three-phase grid converter.

II. SYSTEM MODEL

Fig. 1 shows an equivalent space-vector circuit model of an LCL filter. The LCL filter parameters are L_{fc} , C_f , and L_{fg} . The converter voltage is $\mathbf{u}_c$ and the grid voltage is $\mathbf{u}_g$. A continuous-time model of the system in synchronous coordinates rotating at the grid angular frequency ω_g is

$$\frac{d\mathbf{x}}{dt} = \underbrace{\begin{bmatrix} -j\omega_g & \frac{1}{L_{fg}} & 0 \\ -\frac{1}{C_f} & -j\omega_g & \frac{1}{C_f} \\ 0 & -\frac{1}{L_{fc}} & -j\omega_g \end{bmatrix}}_{\mathbf{A}} \mathbf{x} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \frac{1}{L_{fc}} \end{bmatrix}}_{\mathbf{B}_c} \mathbf{u}_c + \underbrace{\begin{bmatrix} -\frac{1}{L_{fg}} \\ 0 \\ 0 \end{bmatrix}}_{\mathbf{B}_g} \mathbf{u}_g \quad (1)$$

where $\mathbf{x} = [\mathbf{i}_g, \mathbf{u}_f, \mathbf{i}_c]^T$ is the state vector consisting of the grid current $\mathbf{i}_g$, the voltage across the filter capacitor $\mathbf{u}_f$, and the converter current $\mathbf{i}_c$. The model (1) is converted to a hold-equivalent discrete-time model [12]. With the discrete-time index k , the dynamics of the converter current are given by

$$\begin{aligned} \mathbf{x}(k+1) &= \Phi \mathbf{x}(k) + \Gamma_c \mathbf{u}_c(k) + \Gamma_g \mathbf{u}_g(k) \\ \mathbf{i}_c(k) &= \mathbf{C}_c \mathbf{x}(k) \end{aligned} \quad (2)$$

where $\mathbf{C}_c = [0, 0, 1]$ is the output vector and

$$\begin{aligned} \Phi &= e^{\mathbf{A}T_s} & \Gamma_c &= \int_0^{T_s} e^{\mathbf{A}\tau} e^{-j\omega_g(T_s-\tau)} d\tau \cdot \mathbf{B}_c \\ \Gamma_g &= \int_0^{T_s} e^{\mathbf{A}\tau} d\tau \cdot \mathbf{B}_g \end{aligned} \quad (3)$$

are the system matrices. The sampling period is denoted by T_s . The closed-form expressions of (3) can be found in [12]. A computational delay of one sampling period exists in standard implementations [12], [18]. The effect of the computational delay is modeled in synchronous coordinates as

$$\mathbf{u}_c(k+1) = e^{-j\omega_g T_s} \mathbf{u}_{c,\text{ref}}(k) \quad (4)$$

where $\mathbf{u}_{c,\text{ref}}$ is the converter voltage reference.

III. CURRENT CONTROL

Fig. 1 shows the overall block diagram of the current control system. The current controller operates in grid-voltage coordinates, where $\mathbf{u}_g = u_g + j0$. The DC-link voltage u_{dc} is measured for the pulse-width modulator (PWM) and the grid voltage is measured for the phase-locked loop (PLL). Only the converter current is measured for the current controller. The unknown states are estimated using an observer.

Fig. 2 shows observer-based state-space current control in more detail. The reference feedforward, integral action, and full-state feedback together with the estimated states are used. In accordance with Fig. 2, the control law is

$$\mathbf{x}_i(k+1) = \mathbf{x}_i(k) + \mathbf{i}_{c,\text{ref}}(k) - \mathbf{i}_c(k) \quad (5)$$

$$\mathbf{u}_{c,\text{ref}}(k) = \mathbf{k}_t \mathbf{i}_{c,\text{ref}}(k) + \mathbf{k}_i \mathbf{x}_i(k) - \mathbf{k}_u \mathbf{u}_c(k) - \mathbf{K} \bar{\mathbf{x}}(k) \quad (6)$$

Fig. 2. Observer-based state-space current controller. A realizable-reference anti-windup is used in the current control [22], but it is not shown in the figure for simplicity. In the case of current control without the observer, $\bar{\mathbf{x}}(k) = \mathbf{x}(k)$.

where $\mathbf{x}_i$ is the integral state, $\bar{\mathbf{x}}$ is the estimated state vector, $\mathbf{k}_t$ is the feedforward gain, $\mathbf{k}_i$ is the integral gain, $\mathbf{k}_u$ is the computational-delay gain, and $\mathbf{K}$ is the state-feedback gain. If all the states are measured, i.e., an observer is not used, the state estimate in the control law (6) is replaced with the true state vector, i.e., $\bar{\mathbf{x}}(k) = \mathbf{x}(k)$. The system model in (2) and (4) together with the control law in (5) and (6) relates the converter current $\mathbf{i}_c$ to the current reference $\mathbf{i}_{c,\text{ref}}$ and the grid voltage $\mathbf{u}_g$. Since integral action is used to eliminate the steady-state control error, the grid voltage is considered as an unknown disturbance.

In the case of an observer-based design, the observer is needed to estimate the unknown states for the current controller. Three alternative observers are considered: the current-type observer, the prediction-type observer, and the reduced-order observer. In the following subsections, the control algorithms resulting from the three observers are described in a unified framework.

A. Current-Type Observer

The current-type observer used in [13]–[16] provides an estimate of the entire state vector $\bar{\mathbf{x}}$ to be used in the control law. The control algorithm consists of four steps:

- 1) The observer updates the predicted state estimate $\hat{\mathbf{x}}$ using the measured current $\mathbf{i}_c$ as

$$\bar{\mathbf{x}}(k) = \hat{\mathbf{x}}(k) + \mathbf{K}_o [\mathbf{i}_c(k) - \hat{\mathbf{i}}_c(k)] \quad (7)$$

where $\bar{\mathbf{x}}$ is the updated state estimate and $\mathbf{K}_o = [\mathbf{k}_{o1}, \mathbf{k}_{o2}, \mathbf{k}_{o3}]^T$ is the observer gain. The predicted current is defined as

$$\hat{\mathbf{i}}_c(k) = \mathbf{C}_c \hat{\mathbf{x}}(k). \quad (8)$$

- 2) The converter voltage reference $\mathbf{u}_{c,\text{ref}}(k)$ is computed using (6).
- 3) The converter voltage $\mathbf{u}_c(k+1)$ at the next sampling instant is calculated using (4). Furthermore, the integral

state $\mathbf{x}_i(k+1)$ is updated for the next sampling instant according to (5).

- 4) The observer predicts the state estimate $\hat{\mathbf{x}}$ for the next sampling instant using the updated state $\bar{\mathbf{x}}$ as

$$\hat{\mathbf{x}}(k+1) = \Phi \bar{\mathbf{x}}(k) + \Gamma_c \mathbf{u}_c(k). \quad (9)$$

B. Prediction-Type Observer

The prediction-type observer used in [10]–[12] also estimates the entire state vector. The steps of the control algorithm are:

- 1) The state estimate $\bar{\mathbf{x}}$ used in the control law equals the predicted state estimate $\hat{\mathbf{x}}$, i.e., $\bar{\mathbf{x}}(k) = \hat{\mathbf{x}}(k)$.
- 2) The converter voltage reference is computed using (6).
- 3) The computational-delay state (4) and the integral state (5) are updated for the next sampling instant.
- 4) The observer predicts the state estimate $\hat{\mathbf{x}}$ for the next sampling instant based on the measured current $\hat{\mathbf{i}}_c$ as

$$\hat{\mathbf{x}}(k+1) = \Phi \hat{\mathbf{x}}(k) + \Gamma_c \mathbf{u}_c(k) + \mathbf{K}'_o [\hat{\mathbf{i}}_c(k) - \hat{\mathbf{i}}_c(k)] \quad (10)$$

where $\mathbf{K}'_o = [\mathbf{k}'_{o1}, \mathbf{k}'_{o2}, \mathbf{k}'_{o3}]^T$ is the observer gain and $\hat{\mathbf{i}}_c$ is the predicted current, cf. (8).

C. Reduced-Order Observer

The reduced-order observer used in [17]–[20] estimates only the unmeasured states $\mathbf{x}_r = [\hat{\mathbf{i}}_g, \hat{\mathbf{u}}_f]^T$. The steps are:

- 1) The observer updates the predicted state estimate $\hat{\mathbf{x}}_r$ using the measured current $\hat{\mathbf{i}}_c$ as

$$\bar{\mathbf{x}}_r(k) = \hat{\mathbf{x}}_r(k) + \mathbf{K}_r [\hat{\mathbf{i}}_c(k) - \hat{\mathbf{i}}_c(k)] \quad (11)$$

where $\bar{\mathbf{x}}_r$ is the updated state, $\mathbf{K}_r = [\mathbf{k}_{r1}, \mathbf{k}_{r2}]^T$ is the observer gain, and $\hat{\mathbf{i}}_c$ is the predicted current.

- 2) The converter voltage reference is computed using (6). In this case, the state estimate $\bar{\mathbf{x}}$ consists of the estimated states $\bar{\mathbf{x}}_r$ and measured state $\hat{\mathbf{i}}_c$, i.e., $\bar{\mathbf{x}} = [\bar{\mathbf{x}}_r^T, \hat{\mathbf{i}}_c^T]^T$.
- 3) The computational-delay state (4) and the integral state (5) are updated for the next sampling instant.
- 4) The observer predicts the state estimate $\hat{\mathbf{x}}_r$ and current $\hat{\mathbf{i}}_c$ for the next sampling instant using the updated state $\bar{\mathbf{x}}_r$ and measured current $\hat{\mathbf{i}}_c$ as

$$\begin{bmatrix} \hat{\mathbf{x}}_r(k+1) \\ \hat{\mathbf{i}}_c(k+1) \end{bmatrix} = \Phi \begin{bmatrix} \bar{\mathbf{x}}_r(k) \\ \hat{\mathbf{i}}_c(k) \end{bmatrix} + \Gamma_c \mathbf{u}_c(k). \quad (12)$$

The predicted current $\hat{\mathbf{i}}_c$ is used in the update step in (11).

D. Comparison of the Observer Structures

A comparison of the three above-described observer structures reveals that the converter voltage reference $\mathbf{u}_{c,\text{ref}}(k)$ depends on the most recent current measurement $\hat{\mathbf{i}}_c(k)$ in the case of the current-type and reduced-order observers, whereas $\mathbf{u}_{c,\text{ref}}(k)$ depends on the old measurement $\hat{\mathbf{i}}_c(k-1)$ in the case of the prediction-type observer. The current-type and prediction-type observers are of the third order and they have three observer gain elements. Due to structural differences, the observer gains ($\mathbf{K}_o$ and $\mathbf{K}'_o$) are different for identical

observer poles. On the other hand, the reduced-order observer needs only two gain elements.

The state estimates corresponding to (7) and (9) are the different outputs of the current-type observer, as shown in [21]. Using (7) and (9), the predicted state estimate $\hat{\mathbf{x}}$ can be formulated as

$$\hat{\mathbf{x}}(k+1) = \Phi \hat{\mathbf{x}}(k) + \Gamma_c \mathbf{u}_c(k) + \Phi \mathbf{K}_o [\hat{\mathbf{i}}_c(k) - \mathbf{C}_c \hat{\mathbf{x}}(k)]. \quad (13)$$

By comparing (10) and (13), it can be concluded that the state estimate $\hat{\mathbf{x}}$ in the current-type observer equals the state estimate $\hat{\mathbf{x}}$ in the prediction-type observer if the observer gains are related by

$$\mathbf{K}'_o = \Phi \mathbf{K}_o. \quad (14)$$

With a suitable selection of the observer gain, the current-type observer equals the reduced-order observer [23]. To show the mathematical equivalence between them, the state vector $\bar{\mathbf{x}}$ in (7) is split into $\bar{\mathbf{x}}_r = [\hat{\mathbf{i}}_g, \hat{\mathbf{u}}_f]^T$ and $\hat{\mathbf{i}}_c$. The state estimate (7) can be rewritten as

$$\bar{\mathbf{x}}_r(k) = \hat{\mathbf{x}}_r(k) + \mathbf{K}'_r [\hat{\mathbf{i}}_c(k) - \hat{\mathbf{i}}_c(k)] \quad (15)$$

$$\hat{\mathbf{i}}_c(k) = \hat{\mathbf{i}}_c(k) + \mathbf{k}_{o3} [\hat{\mathbf{i}}_c(k) - \hat{\mathbf{i}}_c(k)] \quad (16)$$

where $\mathbf{K}'_r = [\mathbf{k}_{o1}, \mathbf{k}_{o2}]^T$. According to (11), (15), and (16), the current-type and reduced-order observers become mathematically equal for the same system if the following conditions

$$\begin{aligned} \mathbf{K}'_r &= \mathbf{K}_r \\ \mathbf{k}_{o3} &= 1 \end{aligned} \quad (17)$$

are met. Then, the estimated current equals the measured current in the current-type observer, i.e., $\hat{\mathbf{i}}_c = \hat{\mathbf{i}}_c$. Furthermore, the prediction steps in (9) and (12) are equal. It is worth noticing that the conditions given in (17) are valuable in practice. If the current-type full-order observer structure has been implemented, the user can easily parametrize it to equal the standard reduced-order observer, without any changes in the implemented observer structure.

IV. PERFORMANCE ANALYSIS

In this section, we compare the control performance obtained with the three different observers. First, we compare the reduced-order and prediction-type observers. As a benchmark case, the performance is also shown for the control system without an observer. Then, the reduced-order and current-type observers are compared.

A. Comparison of the Reduced-Order and Prediction-Type Observers

1) *Design Example:* Based on the separation principle [21], the current controller can be designed in two steps. Firstly, a control law is designed assuming all the states are available for feedback. Then, an observer is designed separately.

The same control gains $\mathbf{k}_t$, $\mathbf{k}_i$, $\mathbf{k}_u$, and $\mathbf{K}$ are used for all the cases under comparison. The gains, which lead to the desired control characteristic polynomial, are calculated numerically using the direct pole-placement method, as in

TABLE I
NOMINAL PARAMETERS OF A 12.5-kVA CONVERTER SYSTEM

Parameter	Value	Value (p.u.)
<i>LCL filter</i>		
Converter-side inductance L_{fc}	3.3 mH	0.081
Grid-side inductance L_{fg}	3.0 mH	0.074
Capacitance C_f	8.8 μ F	0.036
<i>Grid</i>		
Inductance L_g	0	0
Angular frequency ω_g	$2\pi \cdot 50$ rad/s	1
Voltage (phase-neutral, peak)	$\sqrt{2/3} \cdot 400$ V	1
<i>Converter</i>		
Rated current (peak)	$\sqrt{2} \cdot 18$ A	1
DC-bus voltage u_{dc}	650 V	2
Sampling period T_s	100 μ s	$2\pi \cdot 0.005$

[18], [19]. The control poles¹ can be parametrized by means of radial projection, as [17], [19]

$$\begin{aligned} p_{1,2} &= \exp \left[\left(-\zeta_r \pm j\sqrt{1-\zeta_r^2} \right) \omega_r T_s \right] \\ p_{3,4} &= \exp(-\alpha_c T_s) \quad p_5 = 0 \end{aligned} \quad (18)$$

where $\omega_r = \sqrt{(L_{fc} + L_{fg})/(L_{fc}C_f L_{fg})}$ is the resonance angular frequency, and ζ_r and α_c are the design parameters. The resonant poles of the LCL filter are placed in a more damped location by choosing the damping ratio ζ_r , but without altering their natural resonance angular frequency ω_r . The poles $p_{3,4}$ are placed to determine the dominant dynamics. Therefore, the approximate closed-loop bandwidth α_c is selected significantly below ω_r . The pole p_5 originating from the computational delay is not moved, since it is already in an optimal location. A zero, originating from the reference feedforward in the control law (6), is placed to cancel one of the control poles at $p_{3,4}$. As an example, the control poles $p_{1...5}$ are shown in Fig. 3 for $\zeta_r = 0.7$ and $\alpha_c = 2\pi \cdot 400$ rad/s. The parameters of a 12.5-kVA converter system, given in Table I, are used.

The poles of the reduced-order observer are parametrized using the radial projection technique [17]–[19]

$$p_{o1,o2} = \exp \left[\left(-\zeta_o \pm j\sqrt{1-\zeta_o^2} \right) \omega_r T_s \right] \quad (19)$$

where ζ_o is the damping ratio. The gain K_r is obtained from (19) using the direct pole-placement method. As mentioned earlier, the reduced-order observer also follows from the more general current-type observer structure with the condition given in (17). For calculating the gain for the prediction-type observer, the third pole is placed at the origin of the z plane,

$$p_{o3} = 0. \quad (20)$$

The gain K'_o results from (19) and (20) with the direct pole-placement method. As an example, the observer poles are shown in Fig. 3 for $\zeta_o = 0.7$. Since the observer poles are placed at a higher frequency as compared to the closed-loop

¹The system model (2) has three open-loop poles. The computational delay (4) and the integral action (5) add two more poles to the closed-loop system. The number of the control poles is the same for all the cases.

Fig. 3. Nominal locations of the control (green) and observer (black) poles. The poles $p_{1...5}$ correspond to the current control without an observer. The reduced-order observer poles are given in (19) and the prediction-type observer poles in (19) and (20).

bandwidth α_c , the control poles $p_{3,4}$ dominate the closed-loop dynamics. It is worth mentioning that the poles of the closed-loop system consist of the combination of the control poles and the observer poles [21].

2) *Robustness Against Parameter Errors:* In this section, the robustness of the controllers against parameter errors in the LCL filter is examined. The robustness is analyzed by computing the eigenvalues of the closed-loop system. The nominal closed-loop poles are selected according to Fig. 3. The nominal parameters of the LCL filter, given in Table I, are used in the control design.

The effect of variations in the grid-side inductance on the stability of the control system is studied first. In this analysis, a nonzero grid inductance is taken into account, as presented in [12]. The control system sees the grid inductance as a parameter error. Fig. 4 shows the loci of the closed-loop poles as the real grid-side inductance increases in the range $L_{fg}^r = L_{fg} \dots 1$ p.u. The green crosses show the nominal pole locations, i.e., $L_{fg}^r = L_{fg}$. When the grid-side inductance increases, the closed-loop poles move from their nominal locations. The blue crosses show the pole locations under very weak grid conditions, i.e., $L_{fg}^r = 1$ p.u. All the closed-loop poles remain inside the unit circle for the control system without an observer as well as with the reduced-order observer, i.e., these systems are stable. The prediction-type observer leads to unstable operation of the converter if $L_{fg}^r \geq 0.36$ p.u. The complex conjugate poles associated with the resonance of the LCL filter move outside the unit circle in the case of the prediction-type observer [see Fig. 4(c)].

Fig. 5 shows the robustness of the controllers against the parameter errors in the converter-side inductance L_{fc} and capacitance C_f . The real converter-side inductance is varied in the range $L_{fc}^r = 0.5L_{fc} \dots 2L_{fc}$ and the real filter capacitance is varied in the range $C_f^r = 0.5C_f \dots 1.5C_f$. The grid-side inductance is nominal, i.e., $L_{fg}^r = L_{fg}$. This example indicates that all the controllers provide stable operation within a wide

Fig. 4. Loci of the closed-loop poles when the real grid-side inductance increases in the range $L_{fg}^r = L_{fg} \dots 1$ p.u.: (a) without an observer; (b) reduced-order observer; and (c) prediction-type observer. The nominal poles at $L_{fg}^r = L_{fg}$ are shown with the green crosses and the poles at $L_{fg}^r = 1$ p.u. with the blue crosses.

Fig. 5. Stability maps for the variations in the converter-side inductance L_{fc} and capacitance C_f : (a) without an observer; (b) reduced-order observer; and (c) prediction-type observer. In the shaded area, all the eigenvalues of the closed-loop system are inside the unit circle, i.e., the system is stable. The black lines show the contours of the lowest damping ratios (0, 0.05, 0.1, and 0.2) of the eigenvalues.

range of the parameter variations in L_{fc} and C_f . However, the controller without an observer provides the largest stability region. In the case of the observer-based designs, the reduced-order observer is more robust against the parameter errors under the examined range and provides better damped closed-loop system as compared to the prediction-type observer.

3) *Frequency Responses:* Reference tracking and disturbance rejection of the closed-loop systems are analyzed by means of frequency responses. The closed-loop system can be expressed as [17]

$$\mathbf{i}_c(z) = \mathbf{G}(z)\mathbf{i}_{c,\text{ref}}(z) + \mathbf{Y}(z)\mathbf{u}_g(z) \quad (21)$$

where $\mathbf{G}(z)$ is the reference-tracking transfer function and $\mathbf{Y}(z)$ is the disturbance-rejection admittance.

Fig. 6(a) shows the frequency responses of the reference-tracking transfer function $\mathbf{G}(z)$ under nominal conditions for the reduced-order and prediction-type observers as well as for the control system without an observer. As can be seen, all the cases lead to equal reference-tracking frequency responses. The reason for identical responses is that the reference-tracking dynamics are independent of an observer under nominal conditions [21]. Thus, the control poles (18) completely determine the reference-tracking dynamics. Due to the identical control poles, the frequency responses become

equal in all the cases. They are critically damped because of the real dominant control poles $p_{3,4}$.

Fig. 6(b) shows the frequency responses of the closed-loop admittance $\mathbf{Y}(z)$ under nominal conditions. It can be seen that the control system without an observer has the best disturbance-rejection capability at low frequencies. However, it has the lowest rejection capability against high-frequency disturbances. In the case of the observer-based designs, the reduced-order observer has better rejection capability against disturbances at low frequencies as well as against high-frequency disturbances, excluding frequencies around 3 kHz, as compared to the prediction-type observer.

Fig. 7 shows an example of the frequency responses for the reference-tracking transfer function $\mathbf{G}(z)$ and closed-loop admittance $\mathbf{Y}(z)$ when the real grid-side inductance is $L_{fg}^r = 3L_{fg}$. As can be seen, the control system without an observer provides the best closed-loop frequency responses, with an exception of high-frequency disturbance-rejection capability. In the case of the observer-based designs, the use of the reduced-order observer leads to larger reference-tracking bandwidth, better resonance damping, and better disturbance rejection, excluding frequencies around 3 kHz, in comparison with the prediction-type observer.

Fig. 6. Frequency responses in synchronous coordinates under nominal conditions, corresponding to the pole locations shown in Fig. 3: (a) reference-tracking transfer function $G(z)$; and (b) closed-loop admittance $Y(z)$.

B. Comparison of the Reduced-Order and Current-Type Observers

The control performance, such as the robustness against parameter errors and the dynamic performance, depends on the selected control and observer pole locations. As an example, in this section, the effect of observer pole locations on the dynamic performance is studied for the current-type observer. Furthermore, the dynamic performance is compared with the reduced-order observer.

In addition to the complex-conjugate poles, cf. (19), the designer has the freedom to choose a nominal location for the pole corresponding to the measured state in the current-type observer, i.e., the gain for the measured state is set to $k_{o3} \neq 1$. Then, the current-type observer is not equal to the reduced-order observer, cf. (17). As an example design, the pole

$$p_{o3} = \exp(-\omega_r T_s) \quad (22)$$

is placed at the resonance frequency. The complex-conjugate observer poles $p_{o1,o2}$ and the control poles $p_{1...5}$ are placed at the same locations as the previous design example shown in Fig. 3. The response of the closed-loop system is dominated by the control poles in this design also, since all the observer poles are faster than the dominant control poles $p_{3,4}$. The same design parameters ($\zeta_r = \zeta_o = 0.7$ and $\alpha_c = 2\pi \cdot 400$ rad/s), corresponding to Fig. 3, are used.

Fig. 8 shows the frequency responses of $G(z)$ and $Y(z)$ for the reduced-order and current-type observers when the real grid-side inductance is $L_{fg}^r = 3L_{fg}$. The observer poles are selected according to (19) and (22). As can be seen in Fig. 8(a), lower bandwidth and lower resonance damping are realized with the current-type observer as compared to the reduced-order observer. Furthermore, worse rejection against low-frequency disturbances is obtained [see Fig. 8(b)]. How-

Fig. 7. Frequency responses when the real grid-side inductance is $L_{fg}^r = 3L_{fg}$: (a) $G(z)$; and (b) $Y(z)$.

Fig. 8. Frequency responses for $L_{fg}^r = 3L_{fg}$, when the observer poles are placed according to (19) and (22): (a) $G(z)$; and (b) $Y(z)$.

ever, with the current-type observer pole locations (19) and (22), better rejection against high-frequency disturbances is possible. The frequency responses obtained with the reduced-order observer are the same as in Fig. 7.

Generally, with identical complex-conjugate observer poles $p_{o1,o2}$, a slower pole p_{o3} leads to better rejection against high-frequency disturbances in the current-type observer. It is worth mentioning that better robustness against the parameter errors in the converter-side inductance L_{fc} can be achieved with the current-type observer as compared to the reduced-order observer.

V. EXPERIMENTAL RESULTS

The observer-based current controllers are compared by means of experiments using a three-phase 12.5-kVA 50-Hz

Fig. 9. Measured responses of the converter current components i_{cd} and i_{cq} under nominal conditions ($L_{fg}^r = L_{fg}$): (a) reference-tracking performance; (b) grid-voltage dip response; (c) harmonic-disturbance rejection. The measured grid phase voltages (u_{ga} , u_{gb} , and u_{gc}) are shown with black lines.

Fig. 10. Measured responses of the converter current components i_{cd} and i_{cq} , when the real grid-side inductance is $L_{fg}^r = 3L_{fg}$: (a) reference-tracking performance; (b) grid-voltage dip response; (c) harmonic-disturbance rejection.

grid converter. The control methods are implemented on a dSPACE DS1006 processor board. The grid is emulated with a 50-kVA three-phase four-quadrant power supply (Regatron TopCon TC.ACS). The converter under test controls the converter current and another back-to-back connected converter provides constant DC-link voltage. The switching frequency of the converter is 5 kHz and synchronous sampling (twice per carrier) is used. The pole locations of the controllers correspond to the design example shown in Fig. 3. The system parameters are given in Table I. The design parameters of the controllers are $\zeta_r = \zeta_o = 0.7$ and $\alpha_c = 2\pi \cdot 400$ rad/s. The PLL operates in synchronous coordinates [24] and its bandwidth is $2\pi \cdot 2$ rad/s.

Fig. 9(a) shows the measured responses of the converter current components i_{cd} and i_{cq} , when a step of 0.2 p.u. is applied to the current reference $i_{cd,ref}$ at 5 ms. The compared observers, reduced-order and prediction-type, provide similar dynamics, since they have equal reference-tracking performance, cf. Fig. 6(a). The realized dynamics are very close

to the desired dynamics and as expected, the responses are critically damped. Fig. 9(b) shows the measured responses of i_{cd} and i_{cq} , when a balanced grid-voltage dip of 0.5 p.u. is applied. The converter supplies the active power of 0.4 p.u. to the grid. The reduced-order observer provides faster grid-voltage dip rejection with a smaller overshoot as compared to the prediction-type observer. It is worth mentioning that faster disturbance rejection against the grid-voltage swell can also be achieved with the reduced-order observer. Fig. 9(c) shows the measured responses when the fifth and seventh harmonic components (sixth harmonic component in synchronous coordinates) of 0.03 p.u. are superimposed on the grid voltage. As expected from Fig. 6(b), the reduced-order observer leads to better harmonic-disturbance rejection in comparison with the prediction-type observer.

Fig. 10 shows the measured responses of the converter current components i_{cd} and i_{cq} , when the real grid-side inductance is $L_{fg}^r = 3L_{fg}$. The reference-tracking responses are underdamped and become slower than the specified nominal

responses, cf. Fig. 7(a). As expected, the response obtained with the reduced-order observer is faster and better damped as compared to the prediction-type observer. Furthermore, the reduced-order observer provides faster grid-voltage dip rejection with a smaller overshoot and less settling time [see Fig. 10(b)]. As can be seen from Fig. 10(c), the difference in the disturbance rejection against the sixth harmonic component becomes minimal, cf. Fig. 7(b). Overall, the experimental results agree very well with the analysis.

VI. CONCLUSION

This paper compared the current-type observer, the prediction-type observer, and the reduced-order observer for discrete-time state-space current control of grid converters. The robustness against parameter errors and the dynamic performance obtained with these three different observers are evaluated by means of analysis and experiments. All the observers provide equal reference-tracking performance under nominal conditions. If the filter resonance is damped by means of radial projection, the use of either the current-type or the reduced-order observer leads to better disturbance rejection against the grid-voltage dip and harmonics as compared to the prediction-type observer. The reduced-order observer provides the best reference-tracking performance for a nonzero grid inductance. With a suitable selection of the observer gain, the current-type observer equals the reduced-order observer.

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