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State-Space Control for LCL Filters: Converter Versus Grid Current Measurement

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Abstract—This paper deals with discrete-time state-space current control of three-phase converters equipped with an LCL filter. Either the converter or grid current is measured and the unknown states are estimated using a reduced-order observer. The stability and dynamic performance of the control designs based on these two current measurement options are compared by means of analysis and experiments at different sampling frequencies and under varying grid conditions, ranging from strong to very weak. Equal reference-tracking performance under nominal conditions is used as a basis for comparison between these two options. If a strong grid is assumed in the control tuning, the controller based on the grid current measurement (GCM) is found to be more robust against varying grid conditions in a wide range of sampling frequencies than the controller based on the converter current measurement (CCM). The CCM leads to better dynamic performance as compared to the GCM if the resonance frequency of the system falls below the critical resonance frequency.

Index Terms—Feedback, grid converter, LCL filter, state-space current control, weak grid.

NOMENCLATURE

Space Vectors

Space vectors in synchronous dq -coordinates are marked as follows.

e_g	Grid voltage.
i_c	Converter current.
i_g	Grid current.
u_c	Converter voltage.
u_g	PCC voltage.

Space vectors in stationary coordinates are marked with the superscript s .

Transfer Functions

F_c, F_g	Reference prefilters.
G	Reference-tracking transfer function.
G_c, G_g	Feedback controllers.
Y	Closed-loop admittance.

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Matrices

C_g	Output matrix for the GCM.
C_c	Output matrix for the CCM.
I	Identity matrix.
K	State-feedback gain matrix.
K_o	Observer gain matrix.
Φ	State-transition matrix.
Γ	Input matrix for the converter voltage reference.
Γ_e	Input matrix for the grid voltage.

Other Symbols

C_f	LCL-filter capacitance.
f_c	Critical frequency.
f_r	Resonance frequency.
f_s	Sampling frequency.
k_i	Integral gain.
k_t	Feedforward gain.
L_{fc}	Converter-side inductance of an LCL filter.
L_{fg}	Grid-side inductance of an LCL filter.
L_g	Grid inductance.
T_s	Sampling period.
α_c	Closed-loop bandwidth of current control.
ζ_o, ζ_r	Damping ratios.
ω_g	Grid angular frequency.
ω_r	Resonance angular frequency.

Parameter estimates used in the control tuning are marked with hats.

I. INTRODUCTION

GRID converters equipped with an LCL filter are increasingly used to connect distributed and renewable energy sources to the electric grid. The LCL filter is the preferred option to attenuate the switching harmonics because of its compact size and better grid-current quality in comparison with the L filter [1], [2]. Active resonance damping of the LCL filter by means of control [3] improves the system efficiency as compared to passive damping [4]. State-space control provides a straightforward way for active resonance damping and for setting the dominant dynamics of the closed-loop system [3], [5]–[17].

As shown in Fig. 1, the current control can be based on either the CCM or the GCM. In the CCM-based control, the current sensors can be integrated into the converter module, which reduces the amount of external cabling and simplifies signal conditioning hardware. Furthermore, the same sensors can be used for protecting the converter against overcurrent. On the other hand, with the GCM, the power factor and the current injected into the grid can be directly controlled at the

Fig. 1. Space-vector model of an LCL filter and an inductive grid in stationary coordinates (vectors marked with the superscript s). The current controller, operating in PCC-voltage coordinates is also shown. Either the converter or grid current is measured. The PLL determines the PCC-voltage angle ϑ_g .

point of common coupling (PCC). State-space control based on the CCM is presented in [3], [5]–[7] and based on the GCM in [9]–[13]. The unknown states can be estimated using a full-order observer [5], [6] or a reduced-order observer [7], [10]–[12]. The reduced-order observer provides better disturbance rejection than the full-order observer [18].

In state-space control, the closed-loop poles can be placed at desired locations, e.g., using direct pole placement [5], [8], [10]–[12], linear quadratic (LQ) control [9], [13]–[16], or $\mathcal{H}_\infty$ control [17]. The direct pole-placement method provides a convenient way to solve the controller and observer gains. The gains can be solved using numerical tools [10]–[12] or analytically [5], [8] using the system parameters and the desired pole locations. However, the actual poles of the system depend on the grid impedance, which is typically unknown. The grid impedance may degrade the dynamic performance of the closed-loop system or lead to unstable operation of the converter [19]–[22]. Current control becomes challenging in weak grids, where the grid impedance is high [21], [22].

The time delays due to sampling, pulse-width modulation (PWM), and computation play an important role in the design of the current-control loop. The ratio of the filter resonance frequency f_r to the sampling frequency f_s affects the stability of single-loop proportional-integral (PI) or proportional-resonant (PR) current control [23]–[28]. Stability problems arise for the CCM case if the filter resonance frequency f_r is above the critical resonance frequency $f_c = f_s/6$ and for the GCM case if f_r is below f_c [26]–[28]. On the contrary, a discrete-time state-space controller can stabilize the system, independently of the resonance frequency of the filter [11]. However, the robustness against grid strength variations still depends on the ratio between the resonance frequency and the sampling frequency. Even though limitations of the PI controllers with both current measurement options have been analyzed in [23]–[25], there is no comparison available for state-space control, particularly in the case of weak grids.

The main contribution of this paper is to compare the stability of discrete-time state-space controllers, based on the CCM and GCM options shown in Fig. 1. The comparison

is carried out at different sampling frequencies and under varying grid conditions, ranging from strong to very weak.¹ The grid current is chosen as the controlled variable in both options. Equal reference-tracking performance under nominal conditions is used as a basis for comparison. The design of the controllers is based on the direct pole-placement method with the radial projection for the resonance damping of the LCL filter [11], [18]. Both measurement options are experimentally compared in terms of reference tracking and disturbance rejection using a 12.5-kVA three-phase grid converter under strong and very weak grid conditions.

II. SYSTEM MODEL

A. Open-Loop System

Fig. 1 shows a space-vector circuit model of an LCL filter and a grid in stationary coordinates (vectors marked with the superscript s). The converter voltage is u_c , the voltage across the capacitor is u_f , the PCC voltage is u_g , and the grid voltage is e_g . The converter current is denoted by i_c and the grid current by i_g . The LCL filter parameters are denoted by L_{fc} , C_f , and L_{fg} . The grid inductance is L_g and the grid-side inductance seen from the capacitor terminals is $L_{gt} = L_{fg} + L_g$. The undamped resonance angular frequency of the system

$$\omega_r = 2\pi f_r = \sqrt{\frac{L_{fc} + L_{gt}}{L_{fc} C_f L_{gt}}} \quad (1)$$

depends on the grid inductance L_g , i.e., increasing the grid inductance decreases the resonance frequency.

The model of the system shown in Fig. 1, after transformation to synchronous dq -coordinates rotating at the grid angular frequency ω_g , is considered in this paper. The derivation of the model is given in Appendix A. The computational delay of one sampling period exists in standard implementations, i.e., [5], [11]

$$u_c(k+1) = e^{-j\omega_g T_s} u_{c,\text{ref}}(k) \quad (2)$$

where $u_{c,\text{ref}}$ is the converter voltage reference and $T_s = 1/f_s$ is the sampling period.

Including the delay (2), a discrete-time model for the plant shown in Fig. 1 can be expressed as

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{\Phi} \mathbf{x}(k) + \mathbf{\Gamma} u_{c,\text{ref}}(k) + \mathbf{\Gamma}_e e_g(k) \\ i_g(k) &= \mathbf{C}_g \mathbf{x}(k) \quad i_c(k) = \mathbf{C}_c \mathbf{x}(k) \end{aligned} \quad (3)$$

where $\mathbf{x} = [i_c, u_f, i_g, u_c]^T$ is the state vector and the system matrices are defined in Appendix A.

Fig. 2 shows the block diagrams of the open-loop systems (inside the gray blocks). The open-loop transfer functions can be derived from (3) for both measurement options. For example, the transfer functions from the converter voltage

¹As compared to the preliminary study [29], two significant additions have been made. Firstly, two different grid strengths (strong and very weak) are assumed for control tuning to examine the effect of the grid inductance estimate on the stability of current control. Secondly, the robustness against grid strength is analyzed at different sampling frequencies.

Fig. 2. Current control system rearranged as a 2DOF control structure: (a) GCM; (b) CCM. The open-loop systems are shown inside the gray blocks.

Fig. 3. Frequency responses of $Y_g(z)$ (blue), $Y_c(z)$ (red), and $H_c(z)$ (black dashed) in synchronous coordinates. The nominal LCL-filter parameters given in Table I are used. The grid inductance is $L_g = 0$ and the resonance frequency is $f_r = 1.34$ kHz. The sampling period is $T_s = 100$ μ s.

reference to the grid current and to the converter current, respectively, are

$$\frac{i_g(z)}{u_{c,ref}(z)} = Y_g(z) = C_g(zI - \Phi)^{-1}\Gamma \quad (4)$$

$$\frac{i_c(z)}{u_{c,ref}(z)} = Y_c(z) = C_c(zI - \Phi)^{-1}\Gamma. \quad (5)$$

Similarly, the transfer functions from the grid voltage to the grid current and to the converter current, respectively, are given by

$$\frac{i_g(z)}{e_g(z)} = Y_{eg}(z) = C_g(zI - \Phi)^{-1}\Gamma_e \quad (6)$$

$$\frac{i_c(z)}{e_g(z)} = Y_{ec}(z) = C_c(zI - \Phi)^{-1}\Gamma_e. \quad (7)$$

The transfer function from the converter current to the grid current is obtained as

$$\frac{i_g(z)}{i_c(z)} = H_c(z) = \frac{Y_g(z)}{Y_c(z)} \quad (8)$$

where the relation between the transfer functions $Y_g(z)$ and $Y_c(z)$ can also be seen. The frequency responses of the transfer functions (4), (5), and (8) are shown in Fig. 3. It can be seen that both $Y_g(z)$ and $Y_c(z)$ have resonance at f_r . Furthermore, $H_c(z)$ exhibits resonance at the same frequency as $Y_c(z)$ has its antiresonance.

B. Closed-Loop System

Fig. 2 also shows the block diagrams of the current controllers for both measurement options, rearranged as a two-

degrees-of-freedom (2DOF) control structure. The structure consists of a reference prefilter and a feedback controller. According to Fig. 2(a), the GCM loop gain is $L_g(z) = Y_g(z)G_g(z)$ and the closed-loop response is

$$i_g(z) = \underbrace{\frac{F_g(z)L_g(z)}{1 + L_g(z)}}_{G(z)} i_{g,ref}(z) + \underbrace{\frac{Y_{eg}(z)}{1 + L_g(z)}}_{Y(z)} e_g(z) \quad (9)$$

where $G(z)$ is the reference-tracking transfer function and $Y(z)$ is the closed-loop output admittance. The output admittance can be shaped only via the controller $G_g(z)$, while reference tracking can also be affected via the prefilter $F_g(z)$.

According to Fig. 2(b), the CCM loop gain is $L_c(z) = Y_c(z)G_c(z)$ and the closed-loop response is

$$i_g(z) = \underbrace{\frac{F_c(z)L_c(z)H_c(z)}{1 + L_c(z)}}_{G(z)} i_{g,ref}(z) + \underbrace{\left[\frac{H_c(z)Y_{ec}(z)}{1 + L_c(z)} + H_e(z)Y_{ec}(z) \right]}_{Y(z)} e_g(z). \quad (10)$$

By examining (9) and (10), it can be observed that the open-loop transfer functions, cf. (4)–(7), play a significant role in the characterization of the closed-loop system. It is evident from Figs. 2 and 3 that the CCM loop gain becomes unavoidably different from the GCM loop gain, leading to different closed-loop output admittances. Due to the additional degree of freedom provided by the reference prefilter, reference tracking of the CCM case can be designed to equal that of the GCM case under nominal conditions.

III. CURRENT CONTROL

Fig. 1 shows the overall block diagram of the current control system. The current controller operates in PCC-voltage coordinates, where $u_g = u_g + j0$. Fig. 4 shows the structures of the two state-space controllers to be compared. The state and parameter estimates are marked with hats. To provide the basis for the comparison, reference tracking of the CCM case is designed to equal that of the GCM case under nominal conditions. The reference-tracking transfer function $G(z)$ becomes equal in (9) and (10) if the following three conditions are met: 1) DC gains are equal; 2) closed-loop poles are equal; and 3) closed-loop zeros are equal.

Fig. 4. State-space current control. A realizable-reference anti-windup is used [30], but not shown in the figure for simplicity.

A. Control Laws

In accordance with Fig. 4, the control law for the GCM is

$$\begin{aligned} x_i(k+1) &= x_i(k) + i_{g,\text{ref}}(k) - i_g(k) \\ u_{c,\text{ref}}(k) &= k_t i_{g,\text{ref}}(k) - \mathbf{K} \hat{\mathbf{x}}(k) + k_i x_i(k) \end{aligned} \quad (11)$$

where k_i is the integral gain, k_t is the feedforward gain, $\mathbf{K}$ is the state-feedback gain, x_i is the integral state, and $\hat{\mathbf{x}}$ is the vector consisting of the measured and estimated states. The states are estimated using a reduced-order observer, as explained in Appendix B. The integral action eliminates the steady-state error between the grid current reference and measured grid current. The current controller and the reduced-order observer is presented in detail in [12].

Similarly, the control law for the CCM is

$$\begin{aligned} x_i(k+1) &= x_i(k) + i_{c,\text{ref}}(k) - i_c(k) \\ u_{c,\text{ref}}(k) &= k_t i_{c,\text{ref}}(k) - \mathbf{K} \hat{\mathbf{x}}(k) + k_i x_i(k). \end{aligned} \quad (12)$$

In this case, the integral action eliminates the steady-state error between the converter current reference and measured converter current. As already mentioned, the grid current is chosen as the controlled variable also in the CCM case. Therefore, the converter current reference $i_{c,\text{ref}}$ has to be expressed as a function of the grid current reference $i_{g,\text{ref}}$. Based on the plant model shown in Fig. 1, the control error becomes zero in the steady state with accurate parameter estimates, if the converter current reference is

$$i_{c,\text{ref}} = (1 - \omega_g^2 \hat{C}_f \hat{L}_{fg}) i_{g,\text{ref}} + j \omega_g \hat{C}_f u_{g,\text{ref}}. \quad (13)$$

The PCC voltage reference is

$$u_{g,\text{ref}} = \sqrt{e_{gN}^2 - (\omega_g \hat{L}_g i_{gd,\text{ref}})^2} - \omega_g \hat{L}_g i_{gq,\text{ref}} \quad (14)$$

where e_{gN} is the rated grid voltage magnitude. In this way, the DC gain of the transfer function $G(z)$ in the CCM case can be made equal to that of the GCM case.

B. Gain Calculation

1) *Design Process*: Fig. 5 shows a flowchart for computing the controller gains k_t , k_i , $\mathbf{K}$, and $\mathbf{K}_o$ according to the direct pole-placement method [5], [12]. The nominal parameters of the LCL filter are typically known at the design stage, while the grid inductance L_g is unknown. Typically, the controller gains are calculated assuming a strong grid under nominal

Fig. 5. Flowchart of control tuning.

conditions [5], [11], [12]. However, the grid inductance can be taken into account in control tuning, if weak grid conditions are to be expected. In this paper, the controller gains are calculated with the assumption of a strong grid ($\hat{L}_g = 0$) and a very weak grid ($\hat{L}_g = 1$ p.u.) to examine the effect of the grid inductance estimate on the stability of current control.

The selection of the sampling frequency f_s also affects the stability of current control. Two distinct stability regions of the LCL filter resonance, below and above the critical resonance frequency $f_c = f_s/6$, are identified in [26]. The high resonance frequency region occurs if $f_r > f_c$, and the low resonance frequency region occurs if $f_r < f_c$. In this paper, the robustness of the current measurement options is examined at two different sampling frequencies for which f_c becomes lower and higher than f_r . After selecting $\hat{L}_g$ and f_s , the matrices $\hat{\Phi}$ and $\hat{\Gamma}$ are computed. These matrices are used in the observer and in control gain calculation.

2) *Desired Closed-Loop Poles*: The system has three open-loop poles, located at

$$p_{ol,1,2} = \exp[-j(\omega_g \pm \omega_r)T_s] \quad p_{ol,3} = \exp(-j\omega_g T_s). \quad (15)$$

The open-loop poles depend on the grid inductance L_g and on the sampling frequency $f_s = 1/T_s$. As an example, Fig. 6 shows the open-loop pole locations for the grid inductance of $L_g = 0$ and the sampling frequency of $f_s = 10$ kHz. The parameters of the LCL filter given in Table I are used.

In addition to the three open-loop poles, the computational delay and the integral action add two more poles to the

Fig. 6. Pole locations with the nominal grid inductance of $\hat{L}_g = 0$ and sampling frequency of $f_s = 10$ kHz: open-loop poles (cyan); control poles (black); observer poles (green). The design parameters are $\alpha_c = 2\pi \cdot 400$ rad/s and $\zeta_r = \zeta_o = 0.7$.

closed-loop system. In order to design equal poles for $G(z)$ in (9) and (10), the nominal closed-loop poles are placed at the same locations for both measurement cases. The resonant poles of the LCL filter are damped using the radial projection technique, i.e., the undamped natural frequency of the resonant pole pair is not altered, as shown with the dashed arrows in Fig. 6. Thus, the closed-loop control poles are parametrized as [5]

$$\begin{aligned} p_{1,2} &= \exp \left[\left(-\zeta_r \pm j\sqrt{1-\zeta_r^2} \right) \hat{\omega}_r T_s \right] \\ p_{3,4} &= \exp(-\alpha_c T_s) \quad p_5 = 0 \end{aligned} \quad (16)$$

where ζ_r is the damping ratio of the resonant poles and α_c is the approximate closed-loop bandwidth of current control. The resonance angular frequency estimate $\hat{\omega}_r$ depends on the grid inductance estimate $\hat{L}_g$ according to (1). The observer poles are parametrized as [5]

$$p_{o1,o2} = \exp \left[\left(-\zeta_o \pm j\sqrt{1-\zeta_o^2} \right) \hat{\omega}_r T_s \right] \quad (17)$$

where ζ_o is the damping ratio.

The dominant dynamics are determined by the pair $p_{3,4}$ of double poles [5], [12]. The pole p_5 originating from the computational delay is not moved, since it is already in the optimal location. In both measurement cases, the reference-feedforward zero is placed to cancel one of the control poles at $p_{3,4}$. The plant has two resonant open-loop zeros, as explained in [5]. These zeros become inherently equal in both cases through the transfer function $H_c(z)$, cf. (8). Thus, the transfer function $G(z)$ yields equal zeros in both cases. It is worth mentioning that the transfer function $G(z)$ does not depend on the observer under nominal conditions.

The grid inductance estimate $\hat{L}_g$ in the control tuning affects the choice of the current control bandwidth α_c . The larger the grid inductance estimate $\hat{L}_g$ is, the larger the controller gains are for a given bandwidth α_c . Too large gains amplify the measurement noise and cause the converter output voltage to saturate during a change in the operating point. Therefore, the grid inductance limits the achievable bandwidth. This limitation is of a fundamental nature, and it affects other controller

TABLE I
NOMINAL PARAMETERS OF A 12.5-kVA CONVERTER SYSTEM

Parameter	Value	Value (p.u.)
<i>LCL filter</i>		
Converter-side inductance L_{fc}	3.3 mH	0.081
Grid-side inductance L_{fg}	3.0 mH	0.074
Capacitance C_f	8.8 μ F	0.036
<i>Grid</i>		
Inductance L_g		
Strong grid	0	0
Very weak grid	40.2 mH	1
Angular frequency ω_g	$2\pi \cdot 50$ rad/s	1
Voltage e_{gN} (phase-neutral, peak)	$\sqrt{2/3} \cdot 400$ V	1
<i>Converter</i>		
Rated current (peak)	$\sqrt{2} \cdot 18.3$ A	1
DC-bus voltage u_{dc}	650 V	2
Switching frequency f_{sw}	5 kHz	100

types as well. In this paper, the current control bandwidth is $\alpha_c = 2\pi \cdot 400$ rad/s for $\hat{L}_g = 0$ and $\alpha_c = 2\pi \cdot 100$ rad/s for $\hat{L}_g = 1$ p.u. The damping ratios are $\zeta_r = \zeta_o = 0.7$. After defining the closed-loop poles (16) and (17), the controller and observer gains are calculated numerically, as in [11], [12]. The nominal system parameters, given in Table I, are used in the gain calculation. As an example, the resulting closed-loop poles with the nominal grid inductance of $\hat{L}_g = 0$, the nominal control bandwidth of $\alpha_c = 2\pi \cdot 400$ rad/s, and the sampling frequency of $f_s = 10$ kHz are shown in Fig. 6.

IV. ROBUSTNESS ANALYSIS

In this section, first, the robustness of both current measurement options (GCM and CCM) against grid strength variations is examined. Further, the dynamic performance obtained with the GCM and CCM is compared by means of frequency responses under strong ($L_g = 0$) and very weak ($L_g = 1$ p.u.) grid conditions.

A. Closed-Loop Poles

The robustness against grid strength variations is analyzed at the sampling frequencies of 5 kHz and 10 kHz by computing the eigenvalues (poles) of the closed-loop systems (9) and (10). The sampling frequencies are achieved using single-update PWM and double-update PWM at the switching frequency of 5 kHz. The resonance frequency is $f_r = 1.34$ kHz in the strong grid case. Two different grids, strong ($\hat{L}_g = 0$) and very weak ($\hat{L}_g = 1$ p.u.), are assumed in the control tuning for the gain calculation, as explained in Section III-B.

1) *Strong Nominal Grid:* Fig. 7(a) shows the loci of the closed-loop poles for both measurement cases as the actual grid inductance is increased in the range $L_g = 0 \dots 1$ p.u. The sampling frequency is $f_s = 5$ kHz, i.e., $f_r > f_c$. The inductance estimate $\hat{L}_g = 0$ is used in the controller and the nominal control bandwidth is $\alpha_c = 2\pi \cdot 400$ rad/s. The corresponding nominal poles are shown with the green crosses. When the actual grid inductance increases, the poles move toward the unit circle. At $L_g = 1$ p.u., the poles remain inside the unit circle only for the GCM case, i.e., it is stable. The

Fig. 7. Loci of the closed-loop poles when the actual grid inductance increases in the range $L_g = 0 \dots 1$ p.u. The estimate $\hat{L}_g = 0$ is used in the controller and the nominal control bandwidth is $\alpha_c = 2\pi \cdot 400$ rad/s. The sampling frequency is: (a) $f_s = 5$ kHz; (b) $f_s = 10$ kHz.

CCM case is more sensitive to the grid inductance variations. The CCM case becomes unstable due to the complex conjugate poles associated with the resonance of the LCL filter. These are also the least damped poles in the GCM case.

Fig. 7(b) shows the loci of the closed-loop poles with the sampling frequency of $f_s = 10$ kHz, i.e., $f_r < f_c$. The corresponding nominal pole locations are shown with the green crosses. At $L_g = 1$ p.u., all the closed-loop poles remain inside the unit circle in both measurement cases, i.e., they are stable in the whole range of the grid inductance variation. However, the CCM provides better damping for the resonance-frequency poles.

2) *Very Weak Nominal Grid*: Fig. 8 shows the loci of the closed-loop poles as the actual grid inductance is decreased in the range $L_g = 1 \dots 0$ p.u. The inductance estimate $\hat{L}_g = 1$ p.u. is used in the controller and the nominal control bandwidth is $\alpha_c = 2\pi \cdot 100$ rad/s. The corresponding nominal poles are shown with the green crosses. Fig. 8(a) shows the loci for the sampling frequency of $f_s = 5$ kHz and Fig. 8(b) for the sampling frequency of $f_s = 10$ kHz. As expected, the closed-loop poles are well damped under weak grid conditions. Both measurement cases become unstable if the actual grid is strong ($L_g = 0$).

Fig. 8. Loci of the closed-loop poles when the actual grid inductance decreases in the range $L_g = 1 \dots 0$ p.u. The estimate $\hat{L}_g = 1$ p.u. is used in the controller and the nominal control bandwidth is $\alpha_c = 2\pi \cdot 100$ rad/s. The sampling frequency is: (a) $f_s = 5$ kHz; (b) $f_s = 10$ kHz.

3) *Effect of the Sampling Frequency*: Fig. 9 shows the stability map of the closed-loop system for both measurement cases when the sampling frequency is varied in the range $f_s = 2.5 \dots 10$ kHz and the actual grid inductance is varied in the range $L_g = 0 \dots 1$ p.u. The stability map is obtained by numerical evaluation of the eigenvalues of the closed-loop system. In the stable region, all the eigenvalues of the closed-loop system are inside the unit circle.

In Fig. 9(a), the strong grid ($\hat{L}_g = 0$) is assumed in the control tuning and the nominal control bandwidth is $\alpha_c = 2\pi \cdot 400$ rad/s. In this case, a higher sampling frequency is required for the CCM case to provide stable operation for the whole range of the grid inductance variation. The GCM case is robust against the grid inductance in a wider range of sampling frequencies.

In Fig. 9(b), the very weak grid ($\hat{L}_g = 1$ p.u.) is assumed in the control tuning and the nominal control bandwidth is $\alpha_c = 2\pi \cdot 100$ rad/s. Neither of the measurement options provides stable operation for the whole range of the grid inductance variation. However, these tuning assumptions guarantee stable operation in very weak grids for a wide range of sampling frequencies. As shown in Fig. 9(b), the CCM case provides a larger stability range for the grid inductance variation as

Fig. 9. Stability maps as a function of the actual grid inductance L_g and the sampling frequency f_s : (a) $\hat{L}_g = 0$ and $\alpha_c = 2\pi \cdot 400$ rad/s; (b) $\hat{L}_g = 1$ p.u. and $\alpha_c = 2\pi \cdot 100$ rad/s. The black crosses correspond to the nominal conditions in Figs. 7 and 8. The cyan circles refer to the conditions used in the frequency response analysis in Figs. 10 and 11 and in the experiments in Figs. 13 and 14. The green circles refer to the conditions used for the experimental validation shown in Fig. 15.

compared to the GCM case.

It is worth noticing that stable operation is ensured independently of the sampling frequency and of the grid assumption ($\hat{L}_g = 0$ or $\hat{L}_g = 1$ p.u.) under nominal conditions, marked with the black crosses in Fig. 9. This shows that the discrete-time state-space controller can stabilize the system irrespective of the ratio of the filter resonance frequency to the sampling frequency, which is not possible with the PI or PR controllers [25]–[28]. Furthermore, both measurement cases provide stable operation for actual grid inductances of $0 \dots 1$ p.u. when the resonance frequency of the system falls below the critical resonance frequency.

B. Frequency Responses

Reference tracking and disturbance rejection are analyzed by means of frequency responses under strong and very weak grid conditions. The sampling frequency of $f_s = 10$ kHz and the grid inductance of $\hat{L}_g = 0$ are used in the control tuning, since only this combination of parameters (out of the four examined combinations shown with the black crosses in Fig. 9) provides stable operation for both measurement cases for the whole range of grid strength variations.

1) *Reference Tracking*: The reference-tracking transfer function in (9) and (10) can be expressed as

$$\frac{i_g(z)}{i_{g,\text{ref}}(z)} = G(z) = G_{dd}(z) + jG_{qd}(z). \quad (18)$$

When the grid-voltage disturbances are omitted, the d -component of the grid current is

$$i_{gd}(z) = G_{dd}(z)i_{gd,\text{ref}}(z) - G_{qd}(z)i_{gq,\text{ref}}(z). \quad (19)$$

Fig. 10(a) shows the frequency responses of $G_{dd}(z)$ for both measurement cases under strong grid (nominal) conditions.

Fig. 10. Frequency response of $G_{dd}(z)$ with $\hat{L}_g = 0$, and $\alpha_c = 2\pi \cdot 400$ rad/s, and $f_s = 10$ kHz: (a) strong grid $L_g = 0$; (b) very weak grid $L_g = 1$ p.u.

Fig. 11. Frequency response of $Y_{dd}(z)$ with $\hat{L}_g = 0$, and $\alpha_c = 2\pi \cdot 400$ rad/s, and $f_s = 10$ kHz: (a) strong grid $L_g = 0$; (b) very weak grid $L_g = 1$ p.u.

The frequency responses are identical and critically damped. Fig. 10(b) shows the frequency responses of $G_{dd}(z)$ under very weak grid conditions. It can be seen that the closed-loop systems become poorly damped, as expected based on Fig. 7(b). Fig. 10(b) also shows that the CCM provides a slightly larger bandwidth and better resonance damping.

2) *Disturbance Rejection*: The closed-loop admittance in (9) and (10) can be expressed as

$$\frac{i_g(z)}{e_g(z)} = Y(z) = Y_{dd}(z) + jY_{qd}(z). \quad (20)$$

When only the grid-voltage disturbance is considered, the d -component of the grid current can be formulated as

$$i_{gd}(z) = Y_{dd}(z)e_{gd}(z) - Y_{qd}(z)e_{gq}(z). \quad (21)$$

Fig. 12. (a) Photograph of the experimental setup; and (b) external inductors used for emulating the different grid conditions.

Fig. 13. Measured step responses of the grid current components i_{gd} and i_{gq} with $f_s = 10$ kHz, $\alpha_c = 2\pi \cdot 400$ rad/s, and $\hat{L}_g = 0$: (a) strong grid $L_g = 0$; (b) very weak grid $L_g = 1$ p.u.

Fig. 11(a) shows the frequency responses of $Y_{dd}(z)$ for both measurement cases under strong grid conditions and Fig. 11(b) under very weak grid conditions. It can be seen that the CCM provides better disturbance rejection at low frequencies as compared to the GCM.

V. EXPERIMENTAL RESULTS

Both current measurement cases are compared by means of experiments using a three-phase 12.5-kVA 50-Hz grid

Fig. 14. Measured responses of i_{gd} and i_{gq} when a grid-voltage dip of 0.5 p.u. is applied at 10 ms: (a) strong grid $L_g = 0$; (b) very weak grid $L_g = 1$ p.u.

converter system (Table I). Fig. 12(a) shows a photograph of the experimental setup. The switching frequency of the test converter is 5 kHz and synchronous sampling (twice per carrier, $f_s = 10$ kHz) is used. The design parameters are the same as in the previous analyses ($\zeta_r = \zeta_o = 0.7$, $\alpha_c = 2\pi \cdot 400$ rad/s for $\hat{L}_g = 0$, and $\alpha_c = 2\pi \cdot 100$ rad/s for $\hat{L}_g = 1$ p.u.). A slow PLL having the bandwidth of $2\pi \cdot 2$ rad/s is used in order to avoid the coupling between the current control and PLL dynamics [21]. Different grid conditions, such as $L_g = 0$ (strong grid), $L_g = 0.45$ p.u., $L_g = 0.85$ p.u., and $L_g = 1$ p.u. (very weak grid), are emulated by adding external inductors shown in Fig. 12(b) between the PCC and the grid.

The experiments were carried out first assuming the strong grid ($\hat{L}_g = 0$) in the control tuning [cf. Fig. 9(a)]. Figs. 13 and 14 show the corresponding measured responses of the grid current components i_{gd} and i_{gq} . Fig. 13 shows the measured responses of i_{gd} and i_{gq} , when a step of 0.2 p.u. is applied to the reference $i_{gd,ref}$. Fig. 13(a) shows the responses under strong grid (nominal) conditions. Both measurement cases provide similar dynamics, since they have equal closed-loop poles and equal reference-tracking performance, cf. Figs. 7(b) and 10(a). The responses are critically damped and very close to the desired dynamics. Fig. 13(b) shows the measured responses under very weak grid conditions ($L_g = 1$ p.u.). As expected, the responses are underdamped and slower than the specified nominal responses, cf. Figs. 7(b) and 10(b). Furthermore, the response for the CCM is better damped.

Fig. 15. Measured responses of the grid current components and the converter voltage reference, when a stepwise change is applied in the grid inductance L_g at 0.05 s: (a) 1 p.u. $\rightarrow$ 0.85 p.u.; (b) 1 p.u. $\rightarrow$ 0.45 p.u.; and (c) 0.45 p.u. $\rightarrow$ 0. The nominal grid inductance is $\hat{L}_g = 1$ p.u., the nominal control bandwidth is $\alpha_c = 2\pi \cdot 100$ rad/s, and the sampling frequency is $f_s = 10$ kHz. The black line in the fourth subplots shows the maximum available voltage $u_{dc}/\sqrt{3}$.

Fig. 14 shows the measured responses of i_{gd} and i_{gq} obtained with the assumption of strong grid ($\hat{L}_g = 0$) in the control tuning, when a balanced grid-voltage dip of 0.5 p.u. is applied. The dip was generated using a 50-kVA three-phase four-quadrant power supply (Regatron TopCon TC.ACS). The converter supplies an active power of 0.3 p.u. to the grid. Fig. 14(a) shows the measured responses under strong grid conditions and Fig. 14(b) under very weak grid conditions. As can be seen, both measurement cases reject the grid-voltage dip well under both grid conditions. The CCM provides faster disturbance rejection in comparison with the GCM, as expected based on Fig. 11. Under very weak grid conditions, slower dynamics are encountered as compared to the strong grid conditions.

Theoretically, the CCM case leads to a steady-state error under very weak grid conditions and under grid-voltage dip operation. The steady-state error originates from the inaccurate reference estimate in (13), since the grid inductance is assumed to be zero and the grid voltage magnitude is assumed to be 1 p.u. under nominal conditions. This error can hardly be seen in Figs. 13(b) and 14.

The stability of the two controllers is validated by means of experiments with the sampling frequency of $f_s = 10$ kHz. Fig. 15 shows a set of responses to stepwise changes in the grid inductance L_g . The converter is supplying an active power of 0.6 p.u. to the grid. As in the analysis in Fig. 9(b), a very weak grid ($\hat{L}_g = 1$ p.u.) is assumed in the control tuning and the nominal control bandwidth of $\alpha_c = 2\pi \cdot 100$ rad/s is used. Fig. 15(a) shows a stepwise change in L_g from 1 p.u. to 0.85 p.u. As can be seen, both measurement cases provide

stable operation. A stepwise change in L_g from 1 p.u. to 0.45 p.u. occurs in Fig. 15(b). As expected from Fig. 9(b), the GCM case becomes unstable when $L_g = 0.45$ p.u., whereas the CCM case provides stable operation. Further, Fig. 15(c) shows a stepwise change in L_g from 0.45 p.u. to 0 in the CCM case. When L_g drops to 0, the CCM case becomes unstable also, as expected from Fig. 9(b). During unstable operation, the converter voltage reaches the maximum available voltage. As can be seen in Fig. 15(a), the CCM leads to better rejection of the sixth harmonic component as compared to the GCM.

VI. CONCLUSION

This paper presented the stability analysis of the converter and grid current measurement options for discrete-time state-space control at different sampling frequencies and under varying grid conditions, ranging from strong to very weak. Equal reference-tracking performance is designed for both cases under nominal conditions based on the direct pole-placement method. Both measurement cases stabilize the system independently of the ratio of the filter resonance frequency to the sampling frequency under nominal conditions, i.e., the state-space controller eliminates the fundamental limitation of PI and PR controllers. Furthermore, with the assumption of strong grid in the control tuning, both measurement cases can provide stable operation for actual grid inductances of 0...1 p.u. when the resonance frequency of the system falls below the critical resonance frequency. With high sampling frequencies, the CCM case leads to better disturbance rejection in comparison with the GCM case under both strong and weak grid conditions. Furthermore, the CCM case provides faster

reference tracking and better resonance damping in weak grids. However, the GCM case is found out to be robust against grid strength variations for a wider range of sampling frequencies as compared to the CCM case.

APPENDIX A DISCRETE-TIME PLANT MODEL

In synchronous dq -coordinates rotating at the grid angular frequency ω_g , a continuous-time model of the plant is

$$\frac{d\mathbf{x}_p}{dt} = \underbrace{\begin{bmatrix} -j\omega_g & -\frac{1}{L_{fc}} & 0 \\ \frac{1}{C_f} & -j\omega_g & -\frac{1}{C_f} \\ 0 & \frac{1}{L_{gt}} & -j\omega_g \end{bmatrix}}_{\mathbf{A}_p} \mathbf{x}_p + \underbrace{\begin{bmatrix} \frac{1}{L_{fc}} \\ 0 \\ 0 \end{bmatrix}}_{\mathbf{B}_p} \mathbf{u}_c + \underbrace{\begin{bmatrix} 0 \\ 0 \\ -\frac{1}{L_{gt}} \end{bmatrix}}_{\mathbf{B}_{ep}} \mathbf{e}_g \quad (22)$$

where $\mathbf{x}_p = [\mathbf{i}_c, \mathbf{u}_f, \mathbf{i}_g]^T$ is the state vector. The PWM is modeled as the zero-order hold (ZOH) in stationary coordinates, and the currents are sampled synchronously with the ZOH. Under these assumptions, the model (22) can be converted to an exact hold-equivalent discrete-time model, whose system matrices are [5]

$$\begin{aligned} \Phi_p &= e^{\mathbf{A}_p T_s} & \Gamma_p &= \int_0^{T_s} e^{\mathbf{A}_p \tau} e^{-j\omega_g(T_s - \tau)} d\tau \cdot \mathbf{B}_p \\ \Gamma_{ep} &= \int_0^{T_s} e^{\mathbf{A}_p \tau} d\tau \cdot \mathbf{B}_{ep}. \end{aligned} \quad (23)$$

The closed-form expressions for these system matrices are available in [5]. With the computational delay (2), the discrete-time model for the plant shown in Fig. 1 is

$$\begin{aligned} \mathbf{x}(k+1) &= \underbrace{\begin{bmatrix} \Phi_p & \Gamma_p \\ \mathbf{0} & \mathbf{0} \end{bmatrix}}_{\Phi} \mathbf{x}(k) + \underbrace{\begin{bmatrix} \mathbf{0} \\ \gamma \end{bmatrix}}_{\Gamma} \mathbf{u}_{c,\text{ref}}(k) + \underbrace{\begin{bmatrix} \Gamma_{ep} \\ \mathbf{0} \end{bmatrix}}_{\Gamma_e} \mathbf{e}_g(k) \\ \mathbf{i}_g(k) &= \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}}_{\mathbf{C}_g} \mathbf{x}(k) \\ \mathbf{i}_c(k) &= \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}}_{\mathbf{C}_c} \mathbf{x}(k) \end{aligned} \quad (24)$$

where $\mathbf{x} = [\mathbf{i}_c, \mathbf{u}_f, \mathbf{i}_g, \mathbf{u}_c]^T$ is the state vector and $\gamma = e^{-j\omega_g T_s}$ is the nonzero element in the input matrix Γ .

APPENDIX B REDUCED-ORDER OBSERVER

The unknown states are estimated using a reduced-order observer. Due to the integral action of the controller, the grid voltage $\mathbf{e}_g$ is considered as an unknown disturbance. Following the standard approach [18], the reduced-order observer for the GCM case becomes [12]

$$\begin{aligned} \begin{bmatrix} \hat{\mathbf{i}}_c(k) \\ \hat{\mathbf{u}}_f(k) \end{bmatrix} &= \begin{bmatrix} \hat{\phi}_{11} & \hat{\phi}_{12} \\ \hat{\phi}_{21} & \hat{\phi}_{22} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{i}}_c(k-1) \\ \hat{\mathbf{u}}_f(k-1) \end{bmatrix} + \begin{bmatrix} \hat{\phi}_{13} \\ \hat{\phi}_{23} \end{bmatrix} \mathbf{i}_g(k-1) \\ &+ \begin{bmatrix} \hat{\phi}_{14} \\ \hat{\phi}_{24} \end{bmatrix} \mathbf{u}_c(k-1) + \mathbf{K}_o \mathbf{e}_o(k) \\ \mathbf{e}_o(k) &= \mathbf{i}_g(k) - \hat{\phi}_{31} \hat{\mathbf{i}}_c(k-1) - \hat{\phi}_{32} \hat{\mathbf{u}}_f(k-1) \\ &- \hat{\phi}_{33} \mathbf{i}_g(k-1) - \hat{\phi}_{34} \mathbf{u}_c(k-1) \end{aligned} \quad (25)$$

where $\mathbf{K}_o$ is the observer gain, $\mathbf{e}_o$ is the estimation error of the current, the elements $\hat{\phi}_{ij}$ refer to those of $\hat{\Phi}$, and the converter voltage is obtained from $\mathbf{u}_c(k) = e^{-j\omega_g T_s} \mathbf{u}_{c,\text{ref}}(k-1)$. The state estimate for the state feedback is $\hat{\mathbf{x}} = [\hat{\mathbf{i}}_c, \hat{\mathbf{u}}_f, \mathbf{i}_g, \mathbf{u}_c]^T$.

The design of the observer in the CCM case is analogous to that in the GCM case,

$$\begin{aligned} \begin{bmatrix} \hat{\mathbf{u}}_f(k) \\ \hat{\mathbf{i}}_g(k) \end{bmatrix} &= \begin{bmatrix} \hat{\phi}_{22} & \hat{\phi}_{23} \\ \hat{\phi}_{32} & \hat{\phi}_{33} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{u}}_f(k-1) \\ \hat{\mathbf{i}}_g(k-1) \end{bmatrix} + \begin{bmatrix} \hat{\phi}_{21} \\ \hat{\phi}_{31} \end{bmatrix} \mathbf{i}_c(k-1) \\ &+ \begin{bmatrix} \hat{\phi}_{24} \\ \hat{\phi}_{34} \end{bmatrix} \mathbf{u}_c(k-1) + \mathbf{K}_o \mathbf{e}_o(k) \\ \mathbf{e}_o(k) &= \mathbf{i}_c(k) - \hat{\phi}_{11} \mathbf{i}_c(k-1) - \hat{\phi}_{12} \hat{\mathbf{u}}_f(k-1) \\ &- \hat{\phi}_{13} \hat{\mathbf{i}}_g(k-1) - \hat{\phi}_{14} \mathbf{u}_c(k-1). \end{aligned} \quad (26)$$

The corresponding state estimate is $\hat{\mathbf{x}} = [\mathbf{i}_c, \hat{\mathbf{u}}_f, \hat{\mathbf{i}}_g, \mathbf{u}_c]^T$.

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