This is an electronic reprint of the original article.
This reprint may differ from the original in pagination and typographic detail.

Filz, Günther H.

Making-of Pringle : Hybrid structure from equitangential, bending active, wooden frame and minimal surface robe-net

Published in:
Structures and Architecture

DOI:
[10.1201/9781315229126-140](https://doi.org/10.1201/9781315229126-140)

Published: 08/07/2019

Document Version
Peer reviewed version

Please cite the original version:

Filz, G. H. (2019). Making-of Pringle : Hybrid structure from equitangential, bending active, wooden frame and minimal surface robe-net. In P. J. S. Cruz (Ed.), *Structures and Architecture: Bridging the Gap and Crossing Borders* (Vol. 1, pp. 1171-1178). (Proceedings in Structures and Architecture). CRC Press.
<https://doi.org/10.1201/9781315229126-140>

This material is protected by copyright and other intellectual property rights, and duplication or sale of all or part of any of the repository collections is not permitted, except that material may be duplicated by you for your research use or educational purposes in electronic or print form. You must obtain permission for any other use. Electronic or print copies may not be offered, whether for sale or otherwise to anyone who is not an authorised user.

Making-of Pringle : hybrid structure from equitangential, bending active, wooden frame and minimal surface robe-net

G. H. Filz

ENG | Department of Civil Engineering ° ARTS | Department of Architecture, Aalto University, Finland

ABSTRACT: This paper presents the experimental research structure “Pringle”, a hybrid structure of minimal surface and its equitangential boundary frame. The background of this research is rooted in the competition winning entry for the permanent tensile cover for the Guatemala relief map, and the design proposal for a semi-permanent tensile cover in a historic courtyard. The successfully accomplished structure right-proofs the equitangential coexistence of a minimal surface within its bending active, annular boundary frame, and showcases the use of their geometrical match in two regards: firstly, the utilization of the strong axis of the annular stripe’s cross-section perfectly absorbing all axial forces from the minimal surface, and secondly, the use of the annular stripe’s weak axis for elastic bending into its selforganized Pringle-shape. Consequently, this paper highlights the theoretical background of the Pringle-shape, its equitangentiality with the enclosed minimal surface, the used materials, the assembly and installation and the retrospective structural analysis.

1 INTRODUCTION, CONTEXT AND OVERVIEW

The experimental research structure “Pringle”, its concept and general idea are rooted in the unrealized, competition winning entry (Filz, 2009) for the permanent tensile cover for the Guatemala relief map (Fig.1). In 2009, this competition project proposes a merger of a complex, bordering frame-like structure and its filling membrane surface, and raises the architectural demand for an integrated solution, where the boundary structure and its enclosed membrane surface form an architectural and structural unit. In 2011, the design proposal for a semi-permanent tensile cover in the environment of the historic courtyard at the University of Innsbruck again catches up the above mentioned concept, but simplifies the undulating, steel-frame by using 2dimensionally bent circular tubes only. Research and experience with other experimental structures in the field of bending active structures fostered the Pringle project and resulted in its realization as equitangential, bending active, wooden frame at the University of Innsbruck, Austria in 2017.

This paper is based on the question: Is it possible to find an equitangential solution for a minimal surface and its bending active, annular boundary-frame, which uses the strong axis of the cross-section to absorb the axial forces from the minimal surface and which uses the cross-section’s weak axis for elastic bending into its selforganized Pringle-shape?

In this context we present the “Pringle” as a result of above mentioned studies and design proposals, which highlights following aspects: firstly the theoretical background of the Pringle-shape, which has been realized as bending active, wooden frame, secondly its equitangentiality with the enclosed minimal surface, which has been represented by a robe-net, thirdly used materials, material properties and detailing, fourthly the assembly and installation process and fifthly

a structural analysis, which has been accomplished in retrospective by Filz, Shahzad, and Niiranen at Aalto University, Finland in 2018.

Figure 1. The relief map of Guatemala (Francisco Vela, 1904) in Minerva Park, Guatemala City and the competition winning entry (Filz, 2009) for its permanent tensile cover.

1.1 *A permanent tensile cover for the relief map of Guatemala in Minerva Park, Guatemala City, Guatemala*

The relief map of Guatemala in Minerva Park, Guatemala City is a scaled model in an open-air public monument that represents the geography of Guatemala (Fig.1). It was designed in 1904 by Engineer Francisco Vela and covers 1,800 square meters. This relief map shows all natural and non-natural geographical features like rivers, lakes, volcanoes, ports, railways, roads, provinces, districts and important locations. This work is unique in the world, as no other country has a relief map of such dimensions. Due to its historical and artistic value, it is considered national heritage. Having been exposed to environmental conditions like sun, wind and rain for more than 100 years it has been decided to protect the relief map by a freely spanning, tensile cover. In this context, the authorities of Guatemala City decided in 2009 to organize an international competition for architects and engineers.

Figure 2. Membrane Stresses s11 - formfind response (left) and Combined Loadcases Wind & Rain – displacements (m) (right).

The unrealized, competition winning entry's boundary-layout follows two separated, closed 3d spline-curves (Fig.1). They have been designed according to the needs of not only to roof but to enclose the relief map as well as to guarantee the functionality regarding the flow of visitors around the relief map, providing shade and allowing for two viewing platforms. Instead of being structural support for the membrane only, the boundary frame has been considered to be integral part of the architectural design too. In order to increase the overall structural and architectural performance, at first, rough iterative steps have been made with the help of Rhinocero's plugins Grasshopper and Rhino Membrane. Structural analysis and fine-tuning has been done with help of Forten 3000 (Fig.2).

The final, undulating progression of the boundary curve, as well as the frame's elliptical cross-section represent the result from feedback loops, which consider above mentioned parameters and the frame's structural inter- and coaction with the membrane surface in different load-cases. (Fig.2) In this sense, the shape of the frame's cross-section derived from and responds to the unidirectional flow of information from the membrane surface.

1.2 *A semi-permanent tensile cover for the historic courtyard at the University of Innsbruck, Austria*

Since the year 1848, the big ceremony day is an integral part of the academic year at the University of Innsbruck, founded in 1669. It finds its valuable follow-up, awarding honorary doctorates, honorary medals, honorary membership and honorary senator status as well as awarding outstanding students in the historic courtyard at the University of Innsbruck. Due to weather uncertainties, the ceremonies have been difficult to organize, and the university commissioned a semi-permanent roof, which should consider aspects like, the historical context and facades, the limited accessibility and space for big machinery for construction work, fast and frequent assembly, easy to store during winter, flexibility in use, and limited financial resources.

In 2011, the design proposal for the semi-permanent tensile cover in the historic courtyard at University of Innsbruck again catches up the above described approach and concept of the cover for the relief map of Guatemala in Minerva Park, Guatemala City. The design, proposed by Filz and students at the University of Innsbruck, suggests a curved, spatially articulated boundary line, which is materialized as a frame in stainless steel. (Fig.3) In order to minimize the storage space for the structure during winter months, this frame is proposed to permanently stay in place as a sculptural part of the structure. This frame will span a membrane surface during summer month or if temporary needed, and it has been connected to the historic facades in a short line only, since there is a stair, which was needed to be covered. The translucency of the membrane material well responded to the courtyard-related, light-from-above situation and the existing windows facing the courtyard. For economic reasons the structurally optimized shape of the undulating, steel-frame has been simplified in geometry and fabrication by using 2dimensionally bent, circular tubes only. The frame owes its surprisingly slender cross-section firstly to the temporary use, excluding winter and therefore excluding snow load, and secondly to the reduced wind loads according to its location in the courtyard. Having only the membrane as temporary part of the structure, resulted in minimized storage space and minimized assembly times, which increased the cost efficiency.

Figure 3. Design proposal for the semi-permanent tensile cover in the historic courtyard at University of Innsbruck, winter- and summer scenario, as proposed by Filz and students.

2 PRINGLE : EQUITANGENTIAL, BENDING ACTIVE, WOODEN FRAME AND MINIMAL SURFACE MEMBRANE

The experimental structure “Pringle” has been composed from a minimal surface within an actively bent, annular stripe by Filz and students at the University of Innsbruck in 2017. The project seeks to right-proof the hypothesis of the coexistence of an equitangential, bending active, annular frame and minimal surface, and showcasing the use of their geometrical match in 3D in mainly two regards: firstly, the utilization of the strong axis of the annular stripe’s cross-section, perfectly absorbing all axial forces from the minimal surface, and secondly, the use of the annular stripe’s weak axis for elastic bending into its selforganized Pringle-shape.

2.1 *Why applying the Principles of Active Bending on a thin, annular Stripe?*

Lienhard et al. describe that active-bending is applied intentional for the shaping process to achieve a predefined geometry. It is one advantage of actively-bent elements that the same straight/flat elements can be used for different curvatures. Actively-bent elements are defined as elements that are transformed from the stress-free start-geometry to their end-geometry by elastic bending. Possible elements are beams and plates with a usually straight respectively flat start-geometry (Lienhard 2012). The authors of this paper agree with above mentioned definition in the sense, that the shape/geometry of actively-bent elements, which basically follow elastica curves, can be predicted. At the same time, the geometrical result of active bending is the result of a self-organizing process, where firstly, its shape cannot be directly designed but it can only be influenced by predefining the setup of boundary conditions, and secondly, the result represents a physically informed state of equilibrium. Accordingly, the self-organized, bending active shape would cover both, a homogeneous, aesthetic shape as well as a structurally efficient form, thus following the homogeneous flow of forces.

As a physical model, we can achieve the Pringle-shape by the following steps. An annular, circular and elastic stripe of any width is divided at one or a couple of locations. At these divisions further segments with the same geometry of the annular circular stripe are added. When assembling the elastic stripe-segments to a self-overlapping, planar “ring”. By linking the two ends of the ring to closed configuration, the elastic material is frustrated by geometry, forcing it to buckle out of plane and basically forms the Pringle-shape. Its non-planarity would increase both the curvature and torsion depending on the magnitude of its over-length but the resulting shapes are always determined by minimizing the total elastic energy.

Dias et al. “quantify this shape in terms of the radius of the circle, the dihedral angle of the fold, and the mechanical properties of the sheet of paper and the fold itself.” (Dias 2012) When this problem was introduced to Williams and Naicu by the first author at the FORM-RULE/RULE-FORM Workshop and Symposium 2013 (Filz 2013), they describe their more specific investigation in “The bending of very thin, narrow circular strips”. The “very thin” means that it has been assumed that the bending stiffness of the strip is small in comparison to the membrane stiffness, which in turn means that the surface of the strip has zero Gaussian curvature and remains developable. The theory applies to both wide and narrow strips, but the solution of the equations is easier if the strip is “narrow” in comparison to the distance between the centerline of the strip and the edge of regression of the tangent developable surface.” The resulting shape is maintained and stabilized by the distribution of internal forces, which tend to be harmonized. If this ring is cut at any point it releases its evenly distributed stresses, develops into a planar geometry and forms an overlapping annular circular stripe.

Naturally, the spatially undulating ring would form two “low-points” and two “high-points”. For self-standing purposes (tripod) the experimental structure of the “Pringle” was realized as a three-three version – three times 120° –, which we assume to behave symmetrically.

2.2 *Equitangentiality of Minimal Surface and its bending active, annular Boundary*

The physical minimal surface is also the result from selforganization. Its surface geometry, surface tension and curvature can be indirectly designed only, by choosing and adapting the constellation of boundary conditions.

By means of physical models and digital simulation, we were analyzing the bidirectional interaction of minimal surface and its boundary-defining Pringle-shape, and looking for the setup, in which the tangential direction of the minimal surface in perpendicular to its boundary curve matches the inclination of the boundary-stripe as shown in Figure 4b.

With increasing over-length of the stripe, the high points of the Pringle rise, and the inclination-angle at the high- and low-points increases, but they have equal value. (Fig.4a). If the inserted segments are too long, the stripe flips over and touches itself. Depending on the applied over-length, the total rise of the stripe and the inversely inclining angles at high- and low-points have been terminated, and iteratively approximated to the tangential direction of the minimal surface.

The shape matching with the minimal surface results from a stripe with an over-length of about 1/6 (Fig.4c). Simultaneously, we can recognize increased stability of the 3-dimmmensional form as an equilibrium of curvature and elastic material properties.

Figure 4. a) rise of high points and increase of inclination-angle at the high- and low-points at increasing over-length, b) equitangential match of the undulating, annular circular stripe and its filling by soapfilm, c) equitangential matching stripe and minimal surface.

2.3 Feedback from Physical Modelling

As mentioned above, the natural shape of the undulating stripe is forming a boundary of two high- and two low-points. The according minimal surface is resulting in a classical saddle-shape. In order to generate a self-supporting tripod, a configuration of three high-points and three low-points has been chosen for realization, which was extensively tested by scaled, physical models. There, we observed a strong “rolling-effect”, which we also expected to happen in the full size structure. This means that the boundary stripe tends to relax into its initial two-two configuration. On the other hand, the Pringle-shaped boundary stripe has been fully locked by geometrical constraints, when attaching the membrane or robe-net to it. Still, a spherical triangle was introduced, which connected the low-points and predetermined their distance. This way, a foundation for the Pringle-shape could be provided, the global shape of the stripe was supposed to be stabilized, and the clearance of the structure could be increased at the same time. For the later installation process this triangular connection has been helpful, although and as expected, the “rolling-effect” has been totally blocked by the prestressed, anticlastically curved robe-net.

Figure 5. Vertical displacement field of the strip without (top) and with (bottom) the rope net.

Figure 6. Top surface bending von Mises stress field without (top) and with (bottom) the rope net.

2.4 Feedback from the Structural Analysis

The structural analysis for the structure has been accomplished by Filz, Shahzad and Niiranen in retrospective at Aalto University, Finland in two parts. First, the realized final shell configuration with the minimal surface rope net is analyzed for the self-weight of the plywood strip. In this analysis, the initial stresses of the structure are not taken into account, i.e., the geometry of the structure is considered as a given configuration not as a results of a bending process for an originally planar structure. Second, a part of the deformation process for forming the “pringle”-like annular geometry of the strip is analyzed as a quasi-static geometrically nonlinear deformation. This process gives a view on the initial stresses of the bending-active annular strip structure. With the geometrically nonlinear quasi-static structural analysis of the bending process, we targeted at demonstrating the roles of geometry and initial stresses of the actively bent frame structure, but in the context of this paper the vertical displacement fields of the annular strip without and with the rope net (top and bottom, respectively) seem to be more relevant. The results are compared in Figure 5. As expected, the fields are qualitatively identical but the maximum displacement with the net (41.5 mm) is about one fourth of the maximum displacement of the net-augmented one (155 mm). Bending von Mises stress distribution of the shell strip is presented in Figure 6. As can be seen, the stress level with the net is very low (maximum level about 10^{-2} MPa) and the distribution is fairly uniform across the width and perimeter of the strip. Without the net, the stresses increase near the ground points (maximum level about 10^1 MPa).

3 MATERIAL AND MATERIAL PROPERTIES

We tested and compared different kinds of wood products for the Pringle-shaped frame of our experimental structure. Accordingly, the dimensions and choice of material for the full scale experimental structure has been the following:

3.1 All-Beech Plywood

The annular stripe was made from three layers of 600mm x 6mm plywood and its pieces were screwed together in three double rows, as illustrated in Figure 8c. No glue was added between the layers and the holes for the screws did not allow for longitudinal sliding. The selection and final decision on the used all-beech plywood was taken with an eye on smoothness, respectively homogeneity of bending curvature and on its bending-strength. The selected, orthotropic material is made of five plies of rotary-cut beech veneers with three layers in longitudinal and with two layers in cross direction. The plies are glued together with an urea-formaldehyde adhesive. According to the provided data sheet, the class of gluing is class 1 (IF 20) with a quality of gluing of $f_v \geq 1.0$ N/mm². It is supposed to be a health harmless plywood complying with the A (E1) Class of formaldehyde escape according to ČSN EN 1084 norm - by the method of gas analysis. The grains of adjoining layers are at right angles to each other, and its volumetric mass density (including glue) is around 740 kg/m³. For its characteristics such as good compactness, persistence, flexibility and size stableness, beech plywood is an often used material in furniture-making production and joinery, for the production of casings, wooden toys and even for musical instruments. Usually it is used for interior applications, but due to the short-term exposure of our structure to exterior weather conditions possible negative effects were neglected. Table 1 is showing the material properties of used all-beech plywood, according to the data sheet, which has been provided by the manufacturer.

Thickness	Number of Plies	Approximate volume weight	Thickness tolerance	Bending-strength surface veneer grain orientation	
6 mm	3/2	740 kg/m ³	+0.2/-0.5 mm	lengthwise 85.43 N/mm ²	crosswise 53.73 N/mm ²

Table 1. Material properties of used all-beech plywood. Provided by the manufacturer.

3.2 Robe material

For the net-structure we compared products like steel cables and robe-material. A closed membrane surface would have been even more beneficial for the stability because it would provide a sort of shear stiffness compared to the robe net, which allows for an angular change of the mesh. The decision to use robes instead of steel cables has been taken for mainly four reasons, firstly the needed breaking strength compared to applied prestress of 1 kN/m for warp and weft, secondly the ease of installation and prefabrication of the net, thirdly the selfweight of the material and its connection details and fourthly for economic reasons.

The material that most precisely met our requirements has been a Liros-Prestretch high performance yacht robe, which is usually in use as halyards and control lines for yachts of all sizes. Its breaking strength is 420kg at a diameter of 4mm only. Due to its prestretch-characteristics, the working stretch is $< 5\%$, which is in the average range of this sort of heat set Polyester robes. There is a variety of 8-plaited and 3-strand twisted robes available, but the twisted robes offered a very simple knotting technique, which has been inspired from the Ashley Book of Knots. By twisting the robe against its fabrication-twist, the strands separate and allow for example the warp-robe to intersect with the weft-robe, which is as well self-intersecting in the knotting process. Therefore, the position of each individual knot, which represents the intersection-node of warp and weft, is fully fixed in position, and it is not relying on the friction between a straight robe and a surrounding knot.

4 ASSEMBLY AND INSTALLATION

An inverted spherical triangle was assembled and served as a kind of foundation, connecting the low-points of the Pringle-shape and lifting the whole structure from the ground for functional reasons. The spherical triangle consisted of three times two vertical boards of beech plywood. Horizontal plywood boards were snapped into the spherical triangle, connecting them at different angles. This way, the theoretical intersections of vertical and horizontal boards, were forming curved intersection lines, which were not aligning, and therefore not forming a mechanism, but blocking each other by geometrical constraints. So, the horizontal plates were preventing the vertical boards from torsional buckling, and providing seating at the same time.

Figure 7. a) planned structure incl. spherical triangle foundation b) leaf-spring-like supports c) annular stripe assembled from three layers of beech plywood c) annular self-overlapping planar stripe

As shown in Figure 7d, in planar state, the self-overlapping wooden annular stripe had a diameter of 9m. When assembling the wooden stripe to a closed “ring”, the resulting shape had a total height of 2.8m, and it was cantilevering 2.5m from the three low-points. Compared to earlier test installations, the wooden ring has been supported by leaf-spring-like elements (Fig. 7b), in order to avoid buckling and potential snap-through due to the rings total self-weight of about 400kg, although the later inserted robe-net would have counteracted.

The whole installation process has been managed by man power only (Fig.8 left). The annular stripe’s weak axis allowed for connecting the ends of the overlapping as well as for elastic bending of the stripe into its final shape. The strong axis of the annular stripe perfectly absorbed all axial forces from the equitangential minimal surface robe-net (Fig.8 right) and even allowed for climbing on it (Fig.8 center). An act of vandalism caused a complete snap-through of the whole structure, but was reversible without any visual damage of the structure.

Figure 8. installation process (left), assembled structure (center) and anticlastically curved prefabricated robe-net equitangentially matching the boundary (right)

5 CONCLUSIONS

In conclusion, this research right-proofed the hypothesis of the coexistence of an equitangential, bending active, annular frame and minimal surface. At the same time, the experimental structure showcased the potential of an integrated thinking of formgenerating boundary conditions as architectural and structural elements.

The realization of the Pringle took advantage of the strong axis of the annular stripe, which perfectly absorbs all axial forces from the minimal surface and simultaneously used the annular stripe's weak axis for elastic bending into its selforganized shape as well as for the benefit of the installation process.

Altogether, the structural analysis supports the following conclusion. The rope net serves as a tensional membrane-like structural system supporting the shell frame against its self-weight and other prospective vertical loadings. In this sense, the Pringle is a good showcase for a bending active hybrid structure.

The role of the initial bending stresses stemming from the geometrically nonlinear bending process for forming the frame geometry, now simulated only partially, remains as an open question. In addition, the structural performance could significantly be increased by optimizing the materials grain direction and by rethinking the layered assembly of the annular stripe.

6 REFERENCES

- Dias, M. A.; Dudte, L. H.; Mahadevan, L.; Santangelo, Ch. D.; (2012): "Geometric Mechanics of Curved Crease Origami". In: *Phys. Rev. Lett.* 109, 114301 – Published 13 September 2012
- Filz, G. H.; et.al (eds.); (2013): "*FORM – RULE | RULE – FORM 2013*". ISBN: 978-3-902936-23-3, Publisher: Innsbruck University Press, iup, 2014.
- Filz, GH & D'Anza, G 2018, 'Soft Spaces : Hybrid systems from structural membranes/cable-nets and conventional building technologies' TENSINET Symposium, Milano, Italy, 03/05/2019 - 05/05/2019.
- Lienhard, J.; Alpermann, H.; Gengnagel, Ch.; Knippers, J.; (2012): "Active Bending, a review on Structures where Bending is used as a Selfformation Process". In: Kim, Seung Deong: *From Spatial Structures to Space Structures*, IASS-APCS 2012. Abstract Book. Seoul: Korean Association for Spatial Structures, ISBN 978-89-968907-1-3, S. 213.
- Soriano, E., Tornabell, P., Naicu, D. I. & Filz, G. H., 2015, Topologically-based curvature in thin elastic shell networks. in Onate, E., Kroplin, B. & Bletzinger, K-U. (eds.), *Proceedings of the 7th International Conference on Textile Composites and Inflatable Structures, STRUCTURAL MEMBRANES 2015*. Barcelona, p. 167-176 10 p. International Center for Numerical Methods in Engineering (CIMNE), 7th International Conference on Textile Composites and Inflatable Structures, STRUCTURAL MEMBRANES 2015. Barcelona, Spain, 2015.
- Ylirisku, S & Filz, GH 2018, Resolving 7 Tensions in-between Design and Engineering Education : Cases for Reflective Studio Practice. in I Payá Zaforteza & A Bernabeu Larena (eds), *IV International Conference on Structural Engineering Education Without Borders : ACHE 2018* . Madrid, pp. 618-627, International Conference on Structural Engineering, Madrid, Spain, 20/06/2018.