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A constraint-based optimization technique for estimating physical parameters of Jiles – Atherton hysteresis model

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Abstract

Purpose – Improperly fitted parameters for the Jiles–Atherton (JA) hysteresis model can lead to non-physical hysteresis loops when ferromagnetic materials are simulated. This can be remedied by including a proper physical constraint in the parameter-fitting optimization algorithm. This paper aims to implement the constraint in the meta-heuristic simulated annealing (SA) optimization and Nelder–Mead simplex (NMS) algorithms to find JA model parameters that yield a physical hysteresis loop. The quasi-static B(H)-characteristics of a non-oriented (NO) silicon steel sheet are simulated, using existing measurements from a single sheet tester. Hysteresis loops received from the JA model under modified logistic function and piecewise cubic spline fitted to the average M(H) curve are compared against the measured minor and major hysteresis loops.

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Design/methodology/approach – A physical constraint takes into account the anhysteretic susceptibility at the origin. This helps in the optimization decision-making, whether to accept or reject randomly generated parameters at a given iteration step. A combination of global and local heuristic optimization methods is used to determine the parameters of the JA hysteresis model. First, the SA method is applied and after that the NMS method is used in the process.

Findings – The implementation of a physical constraint improves the robustness of the parameter fitting and leads to more physical hysteresis loops. Modeling the anhysteretic magnetization by a spline fitted to the average of a measured major hysteresis loop provides a significantly better fit with the data than using analytical functions for the purpose. The results show that a modified logistic function can be considered a suitable anhysteretic (analytical) function for the NO silicon steel used in this paper. At high magnitude excitations, the average $M(H)$ curve yields the proper fitting with the measured hysteresis loop. However, the parameters valid for the major hysteresis loop do not produce proper fitting for minor hysteresis loops.

Originality/value – The physical constraint is added in the SA and NMS optimization algorithms. The optimization algorithms are taken from the GNU Scientific Library, which is available from the GNU project. The methods described in this paper can be applied to estimate the physical parameters of the JA hysteresis model, particularly for the unidirectional alternating $B(H)$ characteristics of NO silicon steel.

Keywords Jiles–Atherton (J–A), Magnetic hysteresis, Non-oriented (NO) steel, Physical constraint, Simulated annealing (SA), Nelder–Mead simplex (NMS), Soft magnetic materials, Material modeling

Paper type Research paper

1. Introduction

The Jiles–Atherton (JA) model of ferromagnetic hysteresis is a widely used hysteresis model (Jiles and Atherton, 1983; Jiles *et al.*, 1992). The model has gained popularity because of its physical origin and simplicity of computational implementation (Philips *et al.*, 1995). Compared to more complex mathematical models, such as the modified Preisach-type models (Mayergoyz, 1991), the JA model is usually computationally cheaper (Benabou *et al.*, 2003). Also, a vector extension has rendered the JA model suitable for magnetic field simulations using the finite element method (Bergqvist, 1996; Sadowski *et al.*, 2002; Gyselinck *et al.*, 2004; Padilha *et al.*, 2016). It has also influenced the development of other hysteresis models, such as the energy-based model (Henrotte *et al.*, 2006). However, an accurate estimation of the JA model parameters, e.g. for FeSi electrical steel, is still challenging.

A measured hysteresis loop of a non-oriented (NO) silicon steel sheet includes a sharp rising curve until the knee region and it flattens quickly in the saturation region (Vaseghi *et al.*, 2013a). This typical magnetic behavior presents difficulties when fitting the curve with a (combination of) transcendental function(s) (Kokornaczyk and Gutowski, 2015; Steentjes *et al.*, 2017). Another serious problem is that a certain combination of the model parameters can lead to an unphysical hysteresis loop (Zirka *et al.*, 2012). In particular, when estimating parameters for sharp rising $B(H)$ -characteristics, the unphysical loops are often received (Leite *et al.*, 2004a; Rasilo *et al.*, 2015). Thus, it is of great importance to have a constraint included in the parameter optimization algorithm, which could effectively reduce the chances of getting undesirable parameters.

The anhysteretic magnetization can be considered to be the base on which the JA model is set up (Jiles and Atherton, 1983; Jiles *et al.*, 1992). In the classical JA model, a modified Langevin function represents the anhysteretic characteristic. It is sufficient for modeling a broad class of isotropic magnetic materials, such as NO silicon steel. Besides the modified Langevin function, several other analytical functions are also used to represent the anhysteretic magnetization, such as the Brillouin, hyperbolic tangent, inverse tangent, modified logistic and double Langevin (Ivanyi, 1997; Krah and Bergqvist, 2004; Raghunathan *et al.*, 2009; Kokornaczyk and Gutowski, 2015; Steentjes *et al.*, 2017).

It should be noted that a direct measurement of the anhysteretic curve is difficult (Pearson *et al.*, 1997; Kvasnica and Kundracik, 1996). Therefore, in its absence, we consider the average curve from the major hysteresis loop as a reasonably close approximation to the actual anhysteretic magnetization (Bozorth, 2003). This choice maintains the simplicity of the JA model, simultaneously allowing a better characterization of the sharply rising magnetization curve of NO silicon steel. The average $M(H)$ curve is fitted with a modified logistic function and the piecewise cubic spline. The parameters are estimated for these anhysteretic representations in the JA model.

This article implements a physical constraint in the optimization algorithm that helps in rejecting a combination of model parameters, which result in non-physical hysteresis loops. The parameters of the JA model are estimated using the combination of a global and local heuristic optimization methods, such as simulated annealing (SA) (Kirkpatrick *et al.*, 1983) and Nelder–Mead simplex (NMS) (Nelder and Mead, 1965). The parameters of the JA model are estimated in four steps:

- (1) The initial set of parameters is obtained from the measured major hysteresis loop.
- (2) After that, the parameters of the anhysteretic (analytical) functions are re-estimated by fitting the average $M(H)$ curve.
- (3) In this step, the global optimization is performed using the SA method.
- (4) Finally, the result of the SA method is fine-tuned with the NMS method.

2. Methodology

2.1 Jiles–Atherton hysteresis model

The classical JA model gives the expression of $\frac{dM}{dH}$, which is known as differential susceptibility. The derivation of the JA model is summarized in studies by Jiles and Atherton (1983) and Jiles *et al.* (1992). The following equations (1)–(8) describe the inverse scalar JA model (Sadowski *et al.*, 2002; Gyselinck *et al.*, 2004).

$$M = M_{\text{rev}} + M_{\text{irr}}, \quad (1)$$

$$M_{\text{rev}} = c(M_{\text{an}} - M_{\text{irr}}), \quad (2)$$

$$M_{\text{irr}} = \frac{M - cM_{\text{an}}}{1 - c}, \quad (3)$$

$$M_{\text{an}} = F_{\text{an}}(H_{\text{eff}}), \quad (4)$$

$$H_{\text{eff}} = H + \alpha M, \quad (5)$$

$$\frac{dM}{dB} = \begin{cases} \frac{c\xi}{\mu_0[1 + c\xi(1 - \alpha)]}, & \text{if } (M_{\text{an}} - M_{\text{irr}})dH_{\text{eff}} \leq 0, \\ \frac{(1 - c)\chi + c\xi}{\mu_0[1 + (1 - c)\chi(1 - \alpha) + c\xi(1 - \alpha)]} & \text{otherwise,} \end{cases} \quad (6)$$

$$\chi_d = \frac{dM_{\text{irr}}}{dH_{\text{eff}}} = \frac{|M_{\text{an}} - M_{\text{irr}}|}{k}, \quad (7)$$

$$\xi_d = \frac{dM_{\text{an}}}{dH_{\text{eff}}} = \frac{dF_{\text{an}}(H_{\text{eff}})}{dH_{\text{eff}}}. \quad (8)$$

where M , M_{an} , M_{rev} and M_{irr} represent the bulk, anhysteretic, reversible and irreversible magnetizations, respectively. H and H_{eff} represent the applied and effective field strengths, respectively. χ_d and ξ_d represent the differential irreversible and anhysteretic susceptibilities, respectively. The model parameters M_s , k , α , c and μ_0 are, respectively, the technical saturation magnetization, inter-domain coupling, reversible factor, the pinning strength and the permeability of free space.

For NO silicon steel, the anhysteretic magnetization is usually described by the modified Langevin function L (Cullity and Graham, 2011):

$$M_{\text{an}} = M_s L\left(\frac{H_{\text{eff}}}{a}\right), \quad (9)$$

$$L(x) = \coth(x) - \frac{1}{x}, \quad (10)$$

$$\frac{dL(x)}{dx} = \left[1 - \coth^2(x) + \left(\frac{1}{x}\right)^2\right]. \quad (11)$$

where a is the shape parameter. Finally, the differential reluctivity $\frac{dH}{dB}$ is obtained by differentiating the material constitutive equation (12):

$$B = \mu_0(H + M), \quad (12)$$

$$\frac{dH}{dB} = \frac{1}{\mu_0} - \frac{dM}{dB}, \quad (13)$$

where B is the magnetic-flux density. Given the initial states H_b , B_b and the excitation $B_{t+\Delta t}$, the field strength $H_{t+\Delta t}$ is solved from the differential equation (12) by the explicit fourth-order Runge–Kutta (RK4) method (Abramowitz and Stegun, 1972).

2.2 Magnetic measurements

In this article, the alternating $B(H)$ -characteristics are used, which are obtained for a NO silicon steel sheet of grade M400-50A. The quasi-static magnetic measurements are performed on rotational single sheet tester. The measured data consists of 16 symmetric hysteresis loops with amplitude of B being 0.1 T, 0.2 T, ..., 1.6 T. The details of the used measurement setup are summarized in studies by Gorican *et al.* (2000) and Handgruber *et al.* (2015). The parameters of the JA model are estimated only for the major hysteresis loop. Minor hysteresis loops are simulated using the parameters identified for the major loop.

2.3 Anhyseretic magnetization

The magnetization in an ideal paramagnetic material can be derived using statistical mechanics, which results in $M_{\text{an}}(H_{\text{eff}})$ being given by the Langevin function (Cullity and Graham, 2011). However, the Langevin function cannot provide an adequate fit for certain electrical steels (Rasilo *et al.*, 2015; Kokornaczyk and Gutowski, 2015; Steentjes *et al.*, 2017). Studies performed by Krah and Bergqvist (2004), Steentjes *et al.* (2017), Rasilo *et al.* (2015) and Kokornaczyk and Gutowski (2015) suggest that the $B(H)$ -loop fitting could be improved by increasing the number of involved parameters.

The sum of two Langevin functions has been preferred over a single Langevin function (Steentjes *et al.*, 2012; Rasilo *et al.*, 2015; Steentjes *et al.*, 2017). The addition of a second Langevin function increases the degrees of freedom, which is ideal in minimizing the fitting error after the knee region of a typical anhyseretic curve. Thus, we consider other options than the conventional Langevin function to model the anhyseretic magnetization, such as modified logistic function and the cubic spline fitted to the average $M(H)$ curve extracted from the measured major hysteresis loop.

A reasonable estimate of the effective field strength H_{eff} is obtained by averaging the measured field strength in ascending and descending branches of the measured major hysteresis loop (Bozorth, 2003). In doing so, a $M(H_{\text{avg}})$ curve is obtained and it is assumed that the $M(H_{\text{avg}})$ curve acts as a reasonable estimate of $M_{\text{an}}(H_{\text{eff}})$ anhyseretic magnetization curve (Krah and Bergqvist, 2004). In the implementation, the $M(H_{\text{avg}})$ curve is fit with the modified logistic function and a piecewise cubic spline (Steffen, 1990; Galassi *et al.*, 2019). The modified logistic function presented by Krah and Bergqvist (2004) is considered in this article. For simplicity, we denote the average $M(H_{\text{avg}})$ curve as AVMH and the modified logistic function as MLGS.

The MLGS function and its derivative are expressed as (Krah and Bergqvist, 2004):

$$M_{\text{an}} = M_{\text{s1}} \left[\frac{2}{1 + e^{-H_{\text{eff}}/a}} - 1 \right] + M_{\text{s2}} H_{\text{eff}}, \quad (14)$$

$$\xi_{\text{d}} = M_{\text{s1}} \frac{2e^{-H_{\text{eff}}/a}}{a(1 + e^{-H_{\text{eff}}/a})^2} + M_{\text{s2}}, \quad (15)$$

where M_{s1} , M_{s2} and a are the model parameters, which can be determined from the measurement data. It should be noted that the parameters M_{s1} and a have the unit of A/m, whereas M_{s2} is a dimensionless parameter.

2.4 Parameter estimation

In the literature, several different optimization techniques are applied for estimating parameters of the JA model (Lederer *et al.*, 1999; Hernandez *et al.*, 2000; Wilson *et al.*, 2001; Kis and Ivanyi, 2004; Leite *et al.*, 2004b; Cao *et al.*, 2004; Fulginei and Salvini, 2005; Marion *et al.*, 2008; Thoman *et al.*, 2008; Bai *et al.*, 2011; Baghel and Kulkarni, 2012; Naghizadeh *et al.*, 2012; Vaseghi *et al.*, 2013a, Zaman and Matin, 2015). Both the local optimization methods, such nonlinear least-square fitting, NMS and pattern-search, and the global optimization techniques, such as SA, genetic algorithm (GA), differential evolution (DE), particle swarm method (PSO), shuffled frog leaping algorithm (SFLA) and Taguchi's method (TGU) are used to estimate parameters of the JA model. In particular, the heuristic methods are frequently used for optimization of JA model parameters.

We present a summary of relevant literature. [Lederer et al. \(1999\)](#), [Hernandez et al. \(2000\)](#), [Boukhtache et al. \(2009\)](#) and [Bai et al. \(2011\)](#) demonstrate the use of SA method in estimating parameters of the NO silicon steel. Another, popular metaheuristic method based on genetics is used by [Wilson et al. \(2001\)](#), [Leite et al. \(2004b\)](#), [Cao et al. \(2004\)](#) and [Fulginei and Salvini \(2005\)](#). Likewise, a method based on swarm intelligence is used by [Marion et al. \(2008\)](#). They show that the PSO method converges faster to the global minima than the GA method. [Baghel and Kulkarni \(2012\)](#) combine existing GA with the nonlinear least-square method, which is based on Lavenberg–Marquardt (LM) algorithm, to estimate parameters of the JA model. They demonstrate that a hybrid (GA + LM) optimization technique improves convergence as well as shortens the simulation time.

[Naghizadeh et al. \(2012\)](#), in their work, apply a meta-heuristic SFLA method to estimate parameters of the JA model. They compare the results of SFLA with those of SA, DE, GE and PSO methods. In their comparison, the SFLA method yielded better results than other heuristic optimization methods. [Zaman and Matin \(2015\)](#) demonstrate the use of Taguchi's technique, which is based on the concept of orthogonal arrays, to optimize JA parameters. They show that the performance of the TGU method is comparable to DE, PSO and GA. In contrast, [Kis and Ivanyi \(2004\)](#), [Vaseghi et al. \(2013a\)](#), [Vaseghi et al. \(2013b\)](#) and [Rasilo et al. \(2015\)](#) use computationally efficient methods, such as nonlinear least-square and pattern-search.

In general, local methods, such as nonlinear least-square, NMS and pattern-search, are computationally efficient compared to SA, GA, PSO, DE, SFLA and TGU optimization methods. However, the local methods require a good starting point that is close to the global optimum. If there are several local minima, then the chances of being trapped in one of those minima are higher for the local methods as compared to the global methods. For this reason, in this article, we consider a combined method that includes both global and local optimization methods. First, a metaheuristic SA method is applied and, second, the result of the SA method is used as a starting point for the NMS method. The combined technique allows locating the global minimum. However, the SA method is computationally expensive, so the primary goal of this work is to locate the best possible parameters that yield physical hysteresis loops and low-fitting errors.

The mean square error (MSE) is considered for the quantitative representation of the error between two observations:

$$\text{MSE} = \frac{1}{N} \sum_{i=1}^N (H_{s,i} - H_{m,i})^2, \quad (16)$$

where H_s and H_m represent the simulated and measured field strengths and N represents the total number of samples for a given period of the measurement. The MSE and constraint (bounds and physical) are used as the decision-making quantities in both the SA and NMS optimization algorithms.

2.4.1 Initial estimate. The initial values of the model parameters are obtained from the measured major hysteresis loop, and the AVMH curve, following the equations described by [Jiles et al. \(1992\)](#) and [Gmyrek \(2014\)](#).

$$M_s = \frac{B_{\text{sat}}}{\mu_0} - H_{\text{sat}}, \quad (17)$$

$$c = \frac{\chi_d(0)}{\xi_d(0)}, \quad (18)$$

$$k = \frac{H_c}{1 - c}, \quad (19)$$

$$\alpha = \frac{1}{\xi_d(0)}, \quad (20)$$

$$a = \frac{M_s}{3\xi_d(0)}, \quad (21)$$

where B_{sat} and H_{sat} represent the maximum (measured) magnitude of flux density and field strength, H_c is the coercive field strength, $\chi_d(0)$ and $\xi_d(0)$ are the initial normal susceptibility and initial anhysteretic susceptibility, respectively. Given the values of $B_{\text{sat}} = 1.6$ T, $H_{\text{sat}} = 4063$ A/m, $H_c = 65$ A/m, $\chi_d(0) = 1235.9$ and $\xi_d(0) = 2162.4$, the initial set of parameters are received from [equations \(17\) to \(21\)](#): $M_s = 1.269 \times 10^6$ A/m, $c = 0.57156$, $k = 151.7$ A/m, $\alpha = 46.24 \times 10^{-5}$ and $a = 195.5$ A/m. It should be noted that the shape parameter a is specifically evaluated for the Langevin function and $\xi_d(0)$ is obtained from the AVMH curve.

The initial guesses of parameters M_{s1} , M_{s2} , and a of the MLGS function are re-estimated by fitting the AVMH curve. After that, the initial estimate of the inter-domain coupling parameter α is determined using the physical criteria summarized in Section 2.4.4. It should be noted that the initial estimates are essential to set the upper and lower bounds of the search space.

2.4.2 Simulated annealing method. The SA method is a popular meta-heuristic optimization technique based on the annealing process of the solid materials ([Kirkpatrick et al., 1983](#)). The method proceeds with the random steps in the problem space, looking for points with lower energies. During these random steps, the probability of taking a step is determined by the Boltzmann distribution:

$$P = e^{-\Delta E/(K_B T)}, \quad (22)$$

where $\Delta E = E(\mathbf{x}_{p,i+1}) - E(\mathbf{x}_{p,i})$, E represents the energy, K_B is the Boltzmann constant and T is the temperature.

2.4.2.1 Metropolis criterion

$$P(\Delta E, T) = \begin{cases} e^{-\Delta E/(K_B T)} & \text{if } \Delta E > 0 \\ 1 & \text{otherwise.} \end{cases} \quad (23)$$

Furthermore, a step can only occur if the new energy function value is lower. If the new energy function value is higher, the step can still occur at the initial phase of the problem and gradually the chances of taking a step diminish at lower temperatures.

2.4.2.2 Search strategy. The search strategy in the SA algorithm is established as follows:

$$\mathbf{x}_{p,i+1} = \mathbf{x}_{p,i} + \Delta\mathbf{x}_p(2r - 1) \quad \forall r \in (0, 1), \quad (24)$$

where r is a random number and $\Delta\mathbf{x}_p = \mathbf{x}_{p,\max} - \mathbf{x}_{p,\min}$ is the search space volume. The new candidate points $\mathbf{x}_{p,i+1}$ are generated around the current points $\mathbf{x}_{p,i}$ by perturbing it randomly (Reeves, 1995; Galassi *et al.*, 2019).

2.4.3 Nelder–Mead simplex. The NMS is a heuristic optimization method that relies on simplices, i.e. polytopes of dimension $n + 1$. The method starts from an initial simplex, and at each step, it updates the current simplex based on the value of the cost function. These updates are carried out using four operations: reflection, expansion, contraction and multiple contractions (Nelder and Mead, 1965).

Using the step size vector $\mathbf{s} = (s_0, s_1, \dots, s_n)$ and the initial vector $\mathbf{x} = \mathbf{p}_0$, the algorithm constructs an additional n vectors $\mathbf{p}_i$:

$$\mathbf{p}_0 = (x_0, x_1, \dots, x_n)$$

$$\mathbf{p}_1 = (x_0 + s_0, x_1, \dots, x_n)$$

$$\dots = \dots$$

$$\mathbf{p}_n = (x_0, x_1, \dots, x_n + s_n).$$

The NMS algorithm presented by Galassi *et al.* (2019) proceeds with the following steps:

- (1) *Reflection* – In this step, the vertex that yields the maximum value of the cost function is moved as:

$$\tilde{\mathbf{p}}_i = \bar{\mathbf{p}}_c + \alpha_r(\bar{\mathbf{p}}_c - \mathbf{p}_i),$$

where $\tilde{\mathbf{p}}_i$ is the new trial vertex, $\bar{\mathbf{p}}_c$ is the middle point of all the vertices except the worst vertex $\mathbf{p}_i$ and $\alpha_r > 0$ is the reflection coefficient.

- (2) *Expansion* – If the reflected vertex from (1) yields the lowest value of the cost function, it is further expanded as:

$$\tilde{\mathbf{p}}_i = \bar{\mathbf{p}}_c + \alpha_e(\bar{\mathbf{p}}_c - \mathbf{p}_i),$$

where $\alpha_e > 1$ is the expansion coefficient. The worst vertex $\mathbf{p}_i$ is replaced by a new vertex even if the reflection or reflection followed by expansion improves the cost function.

- (3) *Contraction* – If the reflection or reflection followed by expansion does not improve the cost function, then the simplex is contracted such that the worst vertex is moved:

$$\tilde{\mathbf{p}}_i = \bar{\mathbf{p}}_c - \alpha_c(\bar{\mathbf{p}}_c - \mathbf{p}_i),$$

where $0 < \alpha_c < 1$ is the contraction coefficient.

- (4) *Multiple contraction* – If the abovementioned steps (1)–(3) do not improve the cost function, then the simplex is contracted with respect to the best vertex $\mathbf{p}_l$ as:

$$\tilde{\mathbf{p}}_i = \frac{(\mathbf{p}_l + \mathbf{p}_i)}{2}.$$

The abovementioned steps (1)–(4) are repeated until either the size of simplex is less than the user-supplied tolerance or the number of iterations reaches the set limit. The size of simplex $\bar{s}$ is calculated as the average sum of length of vectors from simplex center $\bar{\mathbf{p}}_0$ to vertices:

$$\bar{s} = \frac{1}{n+1} \sum_{i=0}^n \|\bar{\mathbf{p}}_0 - \mathbf{p}_i\|.$$

In the implementation, the NMS coefficients' values are set as follows: $\alpha_r = 1$, $\alpha_e = 2$ and $\alpha_c = 0.5$. Note that the NMS method approximates a local optimum of a problem, so it is sensitive to the initial guess $\mathbf{p}_0$.

2.4.4 Physical constraint. The parameter estimation technique described in the preceding section proceeds with the random steps. Consequently, these steps could possibly lead to a non-physical hysteresis loop (Figure 1). For this reason, equation (25) is considered to reject the combination of the parameters, which leads to the non-physical behavior (Jiles *et al.*, 1992; Iyer and Krishnaprasad, 2005):

Figure 1. Unphysical $B(H)$ -characteristics produced by the JA hysteresis model with (a) Langevin ($M_s = 1.39 \times 10^6$ A/m, $a = 84.96$ A/m, $k = 77.51$ A/m, $\alpha = 218.22 \times 10^{-6}$, $c = 192.22 \times 10^{-3}$) and (b) double Langevin ($M_{s1} = 223 \times 10^3$ A/m, $M_{s2} = 1186 \times 10^3$ A/m, $a_1 = 2130$ A/m, $a_2 = 27.8$ A/m, $k = 112$ A/m, $\alpha = 1.1 \times 10^{-4}$, $c = 0.371$) functions as the anhysteretic magnetization. Leite *et al.* (2004a) and Rasilo *et al.* (2015)

$$\alpha \xi_d(0) = \frac{1}{1 + \frac{1}{\alpha \chi_{an}}}, \quad (25)$$

where $\chi_{an} = \frac{dM_{an}}{dH}|_{H=0}$ is the anhysteretic susceptibility at the origin. According to [equation \(25\)](#), the lower and upper bounds of $\alpha \xi_d(0)$ are 0 and 1.

To avoid the undesirable parameters, the following condition is incorporated in the SA search algorithm:

$$\mathbf{x}_{p,i+1} = \begin{cases} \mathbf{x}_{p,i} + \Delta \mathbf{x}(2r_i - 1) & \text{if } 0 < \alpha \xi_d(0) < 1, \\ \mathbf{x}_{p,i} + \Delta \mathbf{x}(2r_{i+1} - 1) & \text{otherwise.} \end{cases} \quad (26)$$

For instance, $\frac{\alpha M_s}{3a} < 1$ is the constraint for the Langevin function. [Table 1](#) depicts expression of constraints for commonly used anhysteretic functions (Langevin [LGV], Brillouin [BRLN], hyperbolic tangent [TANH], inverse tangent [ATAN], double Langevin [DLGV], MLGS and Ep [[Kokornaczyk and Gutowski, 2015](#)]). [Figure 2](#) shows the SA optimization algorithm flowchart that includes physical constraint.

The parameters M_s , α and a primarily affect the slope of the anhysteretic curve that could lead to negative differential permeability. The remaining two parameters, k and c , which are related to the coercive field strength $H_c \approx k(1 - c)$, can also yield unphysical hysteresis loop. However, their effects on the differential permeability can be pronounced if [equation \(25\)](#) is violated. Therefore,

Function name	M_{an} expression	Constraint
LGV	$M_s \left[L \left(\frac{H_{eff}}{a} \right) \right]$	$0 < \frac{\alpha M_s}{3a} < 1$
BRLN	$M_s \left[\frac{2J+1}{2J} \coth \left(\frac{2J+1}{2J} \frac{H_{eff}}{a} \right) - \frac{1}{2J} \coth \left(\frac{1}{2J} \frac{H_{eff}}{a} \right) \right]$	$0 < \left(\frac{1+J}{J} \right) \frac{\alpha M_s}{3a} < 1$
TANH	$M_s \left[\tanh \left(\frac{H_{eff}}{a} \right) \right]$	$0 < \frac{\alpha M_s}{a} < 1$
ATAN	$M_s \left[\frac{2}{\pi} \arctan \left(\frac{H_{eff}}{a} \right) \right]$	$0 < \frac{2}{\pi} \frac{\alpha M_s}{a} < 1$
DLGV	$M_{s1} \left[L \left(\frac{H_{eff}}{a_1} \right) \right] + M_{s2} \left[L \left(\frac{H_{eff}}{a_2} \right) \right]$	$0 < \frac{\alpha(M_{s1}a_2 + M_{s2}a_1)}{3a_1a_2} < 1$
MLGS	$M_{s1} \left[\frac{2}{1 + e^{-H_{eff}/a}} - 1 \right] + M_{s2} H_{eff}$	$0 < \alpha \left(\frac{M_s}{2a} + M_{s2} \right) < 1$
Ep	$M_s \left[1 + \frac{e^{-(H_{eff}/a)} - p^2 e^{-(H_{eff}/pa)}}{p^2 - 1} \right]$	$0 < \frac{\alpha M_s}{(p+1)a} < 1$

Table 1.
Anhysteretic
functions and
constraint for
physical hysteresis
loop

Notes: The vector^x represents the parameters of the JA model, E represents the MSE, i and n are the iteration indices, T_0 and T_f are the initial and final temperatures, and $\tau > 1$ is the cooling schedule, r is the random number, $H_s = f(B, H, dB, x)$ is the JA model simulation. The dashed box implements bound and physical constraints. The variables x_{min} and x_{max} are the lower and upper bounds of model parameters

Figure 2.
Algorithm flowchart
for the SA
optimization (Galassi
et al., 2019)

equation (25) must be valid for a physical hysteresis loop (Iyer and Krishnaprasad, 2005).

For the NMS method, the following condition is implemented in the routine that evaluates the cost function:

$$\text{MSE} = \begin{cases} \text{FLT}_{\max} & \text{if } \alpha \xi_d(0) \geq 1, \\ \text{FLT}_{\max} & \text{if } \mathbf{x} \notin [\mathbf{x}_{\min}, \mathbf{x}_{\max}] \\ \frac{1}{N} \sum_{i=1}^N (H_{s,i} - H_{m,i})^2 & \text{otherwise,} \end{cases} \quad (27)$$

where $\text{FLT}_{\max} = 3.4 \times 10^{38}$ is the IEEE 754 32-bit base-2 maximum floating-point value.

3. Results and discussion

Global optimization is carried out by setting bounds as well as a physical constraint in the SA algorithm (Figure 2). Table 2 depicts the initial set of JA model parameters. Table 3 shows the control parameters for the SA optimization.

Table 2.
Initial estimate of JA
parameters

Parameter	MLGS	AVMH
M_{s1} (A/m)	1.169×10^6	—
k (A/m)	151.7	151.7
α	46.24×10^{-5}	46.24×10^{-5}
a (A/m)	257.2	—
c	0.57156	0.57156
M_{s2} (A/m)	23.28	—

Table 3.
SA control
parameters

Name	SA parameter	Value
Initial temperature	T_0	1,000 K
Final temperature	T_f	0.1 K
Cooling schedule	τ	1.002
Iterations per step	N_{iter}	1,000

Figure 3.
AVMH curve fitted
with MLGS function
($M_{s1} = 1.169 \times 10^6$
A/m, $a = 257.2$ A/m
and $M_{s2} = 23.28$)

Figure 3 compares the AVMH curve with the result produced by the MLGS function. The fitting produced by the MLGS function is in a close agreement with the AVMH curve. However, the small discrepancy can be observed, particularly after the knee region of the AVMH curve. The parameters of MLGS functions are received from the SA and NMS optimizations. The initial values determined from equations (17) to (21) were used in the process.

Table 4 shows the parameters of the JA model, which are received after the SA optimization process. The result shows a significant change from the initial guess. The parameters optimized by the SA method are used as the initial values for the NMS optimization method. Table 5 depicts the parameters received from the NMS method. Before starting the NMS optimization, the lower and upper bounds are determined such that $\mathbf{x}_{\min} = 0.7\mathbf{x}_0$ and $\mathbf{x}_{\max} = 1.3\mathbf{x}_0$, where $\mathbf{x}_0$ is the parameter received from the SA optimization. Next, the initial guesses and the bounds are normalized such that the parameters are searched within (0.53, 1). The set of parameters that do not meet the essential criteria is rejected. In the implementation, the maximum step size is set to be 0.2. The NMS optimization is terminated, if either the step size drops to 10^{-6} or the number of iterations reaches a set limit. The maximum number of allowed iterations was set to be 1,000. The desired tolerance is reached in 452 iterations for MLGS and 111 for AVMH (Figure 7).

Figure 4 compares the simulated and measured hysteresis loops. The JA model that implements the AVBH as the anhysteretic magnetization has produced a close fitting with the measured major hysteresis loop [Figure 4(c)]. In contrast, hysteresis loops received under MLGS function show a significant fitting error. At around 1.4 T (knee region), the major hysteresis loop has a sharp bend, which is typical for M400-50A. After that, the curve flattens.

Figure 5 compares the simulated and measured iron losses. It is interesting to note that the parameters estimated for the major hysteresis loop yield narrower minor loops as compared to the measured data [Figure 4(b) and (d)]. As a result, the simulated losses are significantly lower as compared to the measured ones. The parameters, k and c , which are related to the coercive field strength, affect the width of the hysteresis loop and should be

Parameter	MLGS	AVMH
M_{s1} (A/m)	1.137×10^6	—
k (A/m)	345.3	341.3
α	39.54×10^{-6}	49.14×10^{-6}
a (A/m)	210.9	—
c	0.73169	0.77792
M_{s2}	33.19	—

Table 4.
JA parameters
optimized by SA
method

Parameter	MLGS	AVMH
M_{s1} (A/m)	1.144×10^6	—
k (A/m)	370.3	343.2
α	27.67×10^{-6}	48.28×10^{-6}
a (A/m)	2 26.4	—
c	0.78467	0.78078
M_{s2}	31.09	—

Table 5.
JA parameters
optimized by NMS
method

Figure 4. Measured (solid) and simulated (dashed) hysteresis loops (see Table 5 for the parameters)

Notes: The measured hysteresis loops are received under alternating excitation with magnitude of B being 1.4 T and 1.6 T. It should be noted that at $B = 1.6$ T and $H = 4.063$ kA/m. (a) $B = 1.6$ T; (b) $B = 1.4$ T; (c) $B = 1.6$ T; (d) $B = 1.4$ T

Figure 5. Measured and simulated hysteresis losses received under alternating field excitations

re-estimated independently for the minor hysteresis loops (Wlodarski and Wlodarska, 2001; Gmyrek, 2014). The loop-dependent parameters, however, will depend on the magnitude of the input excitation. In particular, parameter c determines the return point-slope, which in our case is too large for the minor hysteresis loops. The value of c for the minor loops should be lower than those valid for the major hysteresis loop.

Figure 6 shows the evolution of MSE during the SA optimization process. When simulating the major hysteresis loop, the JA model supplied with AVMH yields the lowest fitting error as compared to MLGS. It is expected because the AVMH fitted with a piecewise cubic spline has a higher number of degrees of freedom than the MLGS. In contrast, during the NMS optimization, the MSE is lower for the MLGS function than the AVMH spline

(Figure 7). A small fitting error in the saturation region contributes to a larger value of MSE. Although the JA model supplied with the AVMH yield has slightly higher MSE, the fitting is proper for the part before the knee of the hysteresis loop.

For the annealing process facilitated by the control parameters shown in Table 3, the optimization took 2,434 s (Intel® Core™ i3-4170 CPU @3.7 GHz) for MLGS and 1,669 s for AVMH. In both the SA and NMS optimizations, the JA model is simulated for two periods of input excitations, which has 1,000 steps per period. The RK4 method requires four calls to the JA model, so a single iteration during the optimization process requires 8,000 JA model evaluations. Bounds, as well as physical constraints, increase the computation time. Alternatively, the SA algorithm could be run parallelly using multiple cores, which can significantly improve the computational efficiency.

Instead of using SA and NMS separately, at each step, the global best of SA could be improved by feeding the local best (of SA) to the NMS. The algorithm must be modified to accommodate such an approach. One drawback could be the increase in computation time.

Equation (25) is crucial for a sharp rising hysteresis loop. It ensures that the hysteresis loop remains physical for a given combination of the model parameters. Of course, adding additional constraints in the optimization algorithm will take longer time for the process to terminate. Alternatively, the evaluation of constraint can be completely omitted if the AVMH curve (spline fitted) is applied instead of the analytical functions.

4. Conclusion

Aiming to fit the parameters for the JA model, a physical constraint was added in the heuristic optimization algorithms. The constraint prevented non-physical hysteresis

Figure 6.
Evolution of MSE
during SA
optimization process

Figure 7.
Evolution of MSE
during NMS
optimization process

loops from being formed in the simulations. The constrained optimization was implemented using freely available GNU Scientific Library. Compared to analytical expression for the anhysteretic magnetization, such as modified logistic function, a spline fitted to the average of a measured major loop yielded the most accurate simulated major hysteresis loop. However, the parameters estimated for the major hysteresis loop do not yield better fitting for the minor loops, particularly for the NO steel of grade M400-50A.

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