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Graph Sets Method (GSM) for Multi-Coil Wireless Power Transfer Systems, Part II: Simulated and Experimental Results

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Abstract—This paper is the continuation of Graph Sets Method (GSM) proposed in the companion paper Part I for deriving equations and analyzing Multi Coil (MC) Wireless Power Transfer (WPT) systems. After explaining GSM, categorizing WPT coils into three different types, and proposing some approximation techniques in Part I, this article, as the second part of this work, presents numerical simulations and experimental results for a three-coil MCWPT system similar to what is explained in Part I. The simulations are conducted in MATLAB/Simulink and ANSYS environments, and for the experimental model, three ferrite-less cylindrical coils are built and compensated to behave as Voltage Driven (VD), Current Driven (CD), and Passive (PS) coils respectively at the specific frequency of 200 kHz. After analyzing and comparing the simulation and experimental results, it is shown that GSM can be an effective and illustrative technique that facilitates comprehensive analysis of interactions amongst the coils and their characteristics, especially when WPT systems contain multiple coils.

Index Terms—Graph Sets, Signal Flow Graphs, Wireless Power Transfer.

I. INTRODUCTION

IN Multi-Coil Wireless Power Transfer (MCWPT) systems, knowing how different coils interact with each other through their couplings is an important aspect in analyzing and designing WPT systems. Such an understanding is particularly useful when the effect of other coils on a specific coil, in terms of induced voltage and current, is to be studied. Couplings amongst the coils can form different power flow paths, and they desirably or undesirably transfer the power to different coils. One undesirable example of this phenomenon is the effect of cross couplings between the closely spaced transmitters on the system soft switching. In closely spaced multi-transmitter WPT dynamic chargers, cross coupled induced voltages in the transmitters can influence soft-switching in their driving power converters, and they can lead to an increased thermal losses, reduction of efficiency, and failure of switches [1]. Nagnedra *et al.* [2] have addressed this issue by decreasing the space between the transmitters and making cross-coupled links to be negligible. However, this approach

will influence other useful power flow paths amongst the system, namely trans couplings, which are formed between transmitters and receiver(s), and reduce their effectiveness in power transfer. Thus, in [1], through a more detailed study on the cross coupling phenomenon and with the use of coupled inductor compensators, the undesirable cross-coupled power flow paths are compensated. This has been done in such a way that the power can still be transferred without affecting the desirable trans-coupled power flow paths.

On the other hand, GSM can be used to identify the beneficial effects of induced voltages and currents on MCWPT systems to increase the desirable power flow paths of the system. Towards this end, Zhong *et al.* [3], inspired by [4], have introduced an idea of adding fully compensated coils (equipped with series capacitor) as intermediate repeaters to increase the effective power flow paths and consequently enhance the level of transferred power. Another example of enhancing the desirable power flow path is the introduction of DD and DDQ as new coil arrangements to the MCWPT systems [5]. The proposed DD method can extend the effective range of power flow between DD transmitters and receiver. Moreover, by adding the quadrature Q coil in the DD arrangement, known as DDQ, it can remove the power null seen by the receiver at certain positions. Similar approach has been followed in [6] to extend and improve the wirelessly transferred power profile with the use of unequal DD coils.

However, in none of the aforementioned approaches, the power flow paths are explicitly used to analyze and design the system. Hence, in the first volume of this work [7], all possible patterns that can form power flow paths are derived with the use of GSM. Then, by proposing different methods of approximation, the system is simplified by its dominant graph sets (GSs). GSs have some meaningful patterns amongst the sourcing and sinking coils in an MCWPT system, which are characterized in GSM laws. Using this concept and categorizing WPT coils into Voltage Driven (VD), Current Driven (CD), and Passive Coils (PS), help better understanding of the system. It also can simplify modelling MCWPT systems in a more conceptual way, which is explained as AP1 and AP2 in Part I. The simplifications can be further expanded by omitting the effect of negligible couplings (AP3). In Part I, main principles of GSM and different steps of simplification are explained. In this part, the same three-coil MCWPT system explained in Part I is simulated and experimentally tested. Then the obtained results are compared and studied to show

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the validity of the concept. Therefore, in Section II, both state-space and GSM based models of the proposed three-coil MCWPT system are derived. Finally, Section III compares the numerical and experimental results for different gains of the MCWPT system, and it proves that GSM is a valid tool to derive the governing equations of MCWPT systems.

II. STANDARD STATE-SPACE MODEL AND GSM

In this section, how the three-coil MCWPT system shown in Fig. 1 is modelled with the use of standard state-space matrix equation and GSM is explained.

Fig. 1: Three-coil MCWPT system, consisting of VD, CD, and PS coils at its driving frequency.

Fig. 2: Different steps for GSM.

Similar to what has been carried out in Part I, to form the state-space equation of the system, the state variables, which are voltages across the capacitors and currents in the inductors are to be found, as highlighted in Fig. 1. Then the first order state-space matrix equation for all of these variables is formed as in (1).

$$\begin{aligned}
 \underbrace{\begin{bmatrix} \mathbf{I}_L \\ \mathbf{I}_{LS} \\ \mathbf{V}_{CS} \\ \mathbf{V}_{CP} \end{bmatrix}}_{\mathbf{X}} &= \underbrace{\begin{bmatrix} -\mathbf{L}^{-1}\mathbf{ESR}_2 & \mathbf{L}^{-1}\mathbf{ESR}_{CP} & -\mathbf{L}^{-1} & \mathbf{L}^{-1} \\ \mathbf{L}_S^{-1}\mathbf{ESR}_{CP} & -\mathbf{L}_S^{-1}\mathbf{ESR}_1 & \mathbf{O}_{3 \times 3} & -\mathbf{L}_S^{-1} \\ \mathbf{C}_S^{-1} & \mathbf{O}_{3 \times 3} & \mathbf{O}_{3 \times 3} & \mathbf{O}_{3 \times 3} \\ -\mathbf{C}_P^{-1} & -\mathbf{C}_P^{-1} & \mathbf{O}_{3 \times 3} & \mathbf{O}_{3 \times 3} \end{bmatrix}}_{\mathbf{A}} \times \underbrace{\begin{bmatrix} \mathbf{I}_L \\ \mathbf{I}_{LS} \\ \mathbf{V}_{CS} \\ \mathbf{V}_{CP} \end{bmatrix}}_{\mathbf{X}} + \\
 &\underbrace{\begin{bmatrix} \mathbf{O}_{3 \times 1} & \mathbf{L}^{-1} \\ \mathbf{L}_S^{-1} & \mathbf{O}_{3 \times 1} \\ \mathbf{O}_{3 \times 1} & \mathbf{O}_{3 \times 1} \\ \mathbf{O}_{3 \times 1} & \mathbf{O}_{3 \times 1} \end{bmatrix}}_{\mathbf{B}} \times \underbrace{\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}}_{\mathbf{U}}
 \end{aligned} \quad (1)$$

where $\mathbf{L}$ is the inductance matrix of the MCWPT system, $\mathbf{L}_S$ is the diagonal matrix of the primary loop series inductor.

$\mathbf{C}_S$ is the diagonal matrix of the series capacitor of $\mathbf{C}_{Si}$, $\mathbf{C}_P$ is the diagonal matrix of the parallel capacitors of $\mathbf{C}_{Pi}$, $\mathbf{ESR}_{LS}$ is the diagonal matrix of the equivalent series resistors (ESRs) of the primary loop series inductors, $\mathbf{ESR}_{CP}$ is the diagonal matrix of the parallel capacitor ESRs, $\mathbf{ESR}_L$ is the diagonal matrix of the coil ESRs, $\mathbf{ESR}_{CS}$ is the diagonal matrix of the series capacitors, $\mathbf{ESR}_1 = \mathbf{ESR}_{LS} + \mathbf{ESR}_{CP}$ and $\mathbf{ESR}_2 = \mathbf{ESR}_L + \mathbf{ESR}_{CP} + \mathbf{ESR}_{CS}$ are the primary loop and secondary loop ESRs respectively, and V_1 and V_2 in the input vector of $\mathbf{U}$ are the input variables related to the first and second coil voltage sources respectively.

This equation is the typical way of modeling all linear systems, including WPT systems with a low speed pickup. The reason is that this system can elaborately provide many useful information, such as the dynamic behavior of the system by taking the eigenvalues and eigenvectors of the state matrix ($\mathbf{A}$), and finding the transfer function between the input and output variables. However, as explained before, most of the works have been limited to numerically solving this equation, and obtaining the possible patterns for power flow paths is somehow overlooked. Therefore, by parametrically solving the state-space matrix equation representing the behavior of the MCWPT system and following the analytical steps shown in Fig. 2, inspired by [8] and [9], Part I of this work has established GSM to explicitly analyze MCWPT systems by considering the effect of power flow paths to transfer power.

As shown in Fig. 2, GSM includes four main steps of identification, categorizing, graph sets, and analysis. In identification step, system parameters, such as MCWPT self-inductance and mutual inductance profiles, coil resistances and other equivalent series resistances (ESRs) in the system, and values of capacitors and inductors in the system are to be determined. This step is necessary to categorize and simplify the graph sets. In the next step of categorizing, the type of each coil, in terms of VD, CD, or PS coil, according to its driving compensator is classified. Then in the third step, all possible graph sets for the characteristic function and the gains between independent variables (driving voltages and currents of the coils) and dependent variables (induced voltages and currents amongst the coils) can be formed to obtain the detailed model (DM) of the system. This has to be done according to the principles that are established in Part I. Finally, the system will be analyzed and simplified in the forth step. The simplifications can be done by approximating the behavior of an active coil to an ideal VD or CD coil and by omitting the effect of negligible mutual inductances in the MCWPT system inductance profile. According to the nature of the MCWPT system, any mixture of these approximations can be used to simplify modelling of the system.

In this work, in the first step of approximation (AP1), CD coils are considered to be ideal, then in the next step of approximation (AP2), both CD and VD coils are assumed to be ideal, and in the third step (AP3), the negligible mutual inductances are removed. The reason of current sequence of approximation is that achieving an ideal CD coil is more feasible than VD coil when typical passive compensators are used. Therefore, adding the ideal VD coils to the assumptions (AP2) will lead to a lower accuracy in the obtained results

Fig. 3: Experimental results for mutual inductances of the three-coil MCWPT prototype for different displacements of coil 2.

compared to DM and AP1. However, the assumption of ideal behavior for all CD and VD coils can significantly simplify the analysis of the system by removing more graphs from the GSM. Although AP2 is not as accurate as AP1, still it can represent the behavior of the dominant power flow paths of the system. Omission of negligible mutual inductances is also another way to reduce the number of graphs in each graph set, and it can be used along with AP1 or AP2. However, in this work, negligible inductances are removed from the DM to simplify the characteristic function for dynamic analysis of the system. It is worthy to mention that this approach is valid for arbitrary number of coils, and the reason of choosing the three-coil MCWPT case study is that showing the graph sets and explaining the concept for a higher number of coils will take too much of space.

Therefore, according to the GSM flowchart shown in Fig. 2, the three-coil MCWPT system is identified by the specifications given in Table I, in Appendix, and mutual inductance profile shown in Fig. 3. As shown in Fig. 1, the first coil is compensated with an LCC network and the second and third coils are compensated with series capacitors. Moreover, coil 2 is excited with a voltage source, and a passive impedance R_L is connected to the terminals of coil 3. If these compensators are tuned at ω_0 , coil 1 is excited by an independent current source, and by fully compensation of the self-inductance in the second coil, it can be made to be driven by an independent voltage source. Therefore, to categorize the types of coils based on the GSM flow chart, coils 1, 2, and 3 can be considered as CD, VD, and PS coils. In the third phase of the flowchart and with the use of GSM rules explained in Part I, three main graph sets of the system can be obtained as shown in Appendix.

The final phase of GSM is to analyze and simplify the system. As previously mentioned, the simplifications have been done by taking CD coil (coil 1) as an ideal coil, in AP1, and then both CD coil (coil 1) and VD coil (coil 2) as ideal coils, in AP2. To omit the effect of negligible mutual inductances, referring to Fig. 3, L_{13} is removed from DM in AP3. The removed graphs in each graph set is highlighted in grey for each step of approximation, as shown in Tables II to VI, in Appendix.

Fig. 4: The three-coil MCWPT experimental setup.

III. SIMULATION AND EXPERIMENTAL RESULTS

To verify the effectiveness of GSM, a three-coil MCWPT system same as what has been explained in Part 1 is modelled in ANSYS finite element simulator and Matlab/Simulink, as shown in Fig. 1. Simulations are experimentally verified using a laboratory prototype shown in Fig. 4. The numerical simulations and experiments are done according to the specifications given in Table I. This is the simplest possible example for an MCWPT system to show how GSM can be applied in MCWPT systems, and the same method can be used for MCWPT systems with a higher number of coils.

The scenario of simulations and experiments have been organized in such a way that all the important aspects of GSM are dealt with. According to the flowchart of this technique shown in Fig. 2, these important aspects are characteristic function, GS gains (GVP, GCP, and Z_{ii}), and approximations.

All three different types of coils are also included in the model to evaluate the approximation steps. Referring to Fig. 1, symbols that are used in Tables II to VI show the type of each coil as explained in Part I. In this setup, the middle VD coil (coil 2) moves from $g_{12}=7$ cm to $g_{12}=29$ cm with reference to coil 1. The distance between the CD coil (coil 1) and the PS coil (coil 3) is fixed and $g_{13}=36$ cm. The displacement (g_{12}) of coil 2 can change L_{12} and L_{23} , as shown in Fig. 3 for experimentally obtained inductance profiles. Therefore, the effect of coil 2 positioning on transmission and reflection of power can be studied. Coils 1 and 3 are positioned far enough to make a negligible L_{13} compared to other mutual inductances. In this way negligible inductances can be omitted in AP3. The comparisons with DM of GSM will show that AP3 can be a valid approximation for simplifying MCWPT systems.

The following statements describe how this section of the simulated and experimental results is organized. Subsection (A) studies the validity of GSM to derive the natural frequencies of the MCWPT system (eigenvalues). In subsection (B), gains of current paths (GCPs) related to sourcing coils VD and CD and the sinking coil PS are investigated. In continuation of studying the gains, subsection (C) studies the reflected impedances in each of these coils.

A. Characteristic Function

According to the linear system analysis, eigenvalues and eigenvectors completely describe the behavior of a linear system. This concept is especially useful when the dynamic behavior of a system is to be investigated. Therefore, in this section, the efficacy of the proposed GSM to extract eigenvalues is studied. In order to do so, s-plane poles of the characteristic function (Δ) of the three-coil system are compared with the eigenvalues obtained from the standard state-space model.

To achieve the s-plane poles of an MCWPT system from its standard state space model, its state space matrix needs to be derived as given in (1). On the other hand, to extract s-plane poles (eigenvalues) of an MCWPT system with the use of GSM, its characteristic function graph set can be formed according to what has been elaborated in Part I, and briefly shown in Table II in the appendix. Moreover, as shown in Fig. 3, for different positions, L_{13} is significantly smaller than other mutual inductances. Therefore, as an approximation in dynamic modeling of the system, the negligible effect of L_{13} is omitted in AP3. Finally, the resultant eigenvalues obtained from the standard state-space model, detailed model (DM), and AP3 are compared in Fig. 5.

In the given three-coil MCWPT system, overall number of oscillating poles at each position is always eight as there are eight energy storage elements in the system shown in Fig. 1, which are C_{S1} and L_{11} in the series compensated VD coil (coil 1), L_{S2} , C_{P2} , C_{S2} , and L_{22} in the LCC compensated CD coil (coil 2), and C_{S3} and L_{33} in the series compensated passive coil. Therefore, these poles can be categorized into three sets of VD, CD, and PS poles, as shown in Fig. 5.

From Fig. 5, it is apparent that the poles obtained from GSM well match those of the standard model. Moreover, removing the effect of L_{13} does not make a significant difference between DM and AP3 of the GSM. Obviously, DM of GSM is obtained from the KVL and KCL equations of the MCWPT system, and the same laws are used to form the state-space matrix equation. Therefore, with the use of GSM, eigenvalues that partly describes dynamic behavior of the system (eigenvalues) can be analyzed by forming the characteristic equation (Δ) of the MCWPT system.

B. Transferred Gain of Current Paths for PS coil

To evaluate the transferred gains between different coils, which can be obtained from the graph sets shown in Table IV and Table VI, P_{13} and P_{23} graph sets for gain of current path (GCP) are studied in this section. If transferred GCPs yield the same results that are expected from the experimental results, then it can be concluded that the same is valid for transferred gain of voltage paths (GVPs). As explained in Part I, the only difference between the transferred GCPs and GVPs is the absence of overall impedance of starting coil or ending coil in their NCP or NVP graph sets respectively.

For this purpose, the three-coil MCWPT is considered as a three port system and impedance matrix (Z) of each coil input terminals, $a_1 - b_1$, of the VD coil, $a_2 - b_2$ of the CD coil, and $a_3 - b_3$ of the CD coil, shown in Fig. 1, are measured.

The simulated (obtained from DM of GSM) and experimental results of the amplitude and phase angle of GCP_{13} at different frequencies and displacements are shown in Figs. 6 and 7 respectively. Then, as AP1 and AP2 are only valid for the tuning frequency of compensators $f_0 = 200$ kHz, the experimental results and DM of GSM at $f_0 = 200$ kHz are compared with other approximations for the amplitude and angle of GCP_{13} in Fig. 8. Similarly, the simulation and experimental results for the amplitude and phase angle of GCP_{23} are shown in Fig. 9 and Fig. 10 respectively, and these results are compared with AP1, AP2, and AP3 at $f_0 = 200$ kHz in Fig. 11. Comparing the experimental and numerical results of GCP_{13} , in Figs. 6 and 7, and GCP_{23} , in Figs. 9 and 10, it can be seen that DM can completely model the behavior of the system. These figures show that DM of GSM can clearly represent the phenomena, such as bifurcation of the dominant poles and change of phase angle at the place of dominant poles at different positions and frequencies, and these results are consistent with the experimental results. The same consistency is valid for other gains and for clarity, they are not presented in this paper. As previously mentioned, this consistency of behavior in the frequency domain reflects that GSM can produce the gains of the system, which means eigenvectors are appropriately mapped into the model. Therefore, GSM can be used to determine transferred gains.

Figs. 8 and 11 show that the approximation steps of GSM can effectively model the behavior of the system. Moreover, as previously mentioned, due to the ESR of the coils, mimicking the ideal behavior of a VD coil in the real world is not an easy task. That is why AP2 results are significantly different from the other types of approximation at small displacements. On the other hand, for AP1 and AP3, the assumption of the ideal behavior of CD coils (AP1) and the omission of the negligible couplings (AP3) are more realistic, and their obtained results are so close to the DM and experimental results.

To further elaborate the effect of sub-terms on gains, Fig. 12(a) shows the contribution of each sub-term (associated with the corresponding graph) in making the overall GCP_{13} and GCP_{23} . Referring to Fig. 3 and Table VI, it is obvious that the sub-terms that have a negligible couplings produce weaker contributions and as a result they can be removed from the gain calculations. For example, in Fig. 12(a), $GCP_{13}(1)$ (which is related to $P_{13}(1)$ in Table VI) and $GCP_{23}(2)$ (which is related to $P_{23}(2)$ in Table VI), include the negligible coupling L_{13} in their terms, and consequently, they yield insignificant sub-terms for GCPs. Similarly, in Fig. 13(a), $\Delta(2)$ and $\Delta(5)$, which have L_{13} in their graphs, as shown in Table II in the appendix, are significantly small compared to other Δ sub-terms. This explains why negligible couplings can be removed in AP3. The following are some insight analysis that have been carried out using the proposed GSM for the transferred gains in this study.

Therefore, the MCWPT system can be modeled only considering its dominant graph sets. In this particular case, $GCP_{13}(2)$ and $GCP_{23}(1)$ are the dominant sub-terms of GCP_{13} and GCP_{23} respectively. Between these gains, $GCP_{23}(1)$ does not have a consistent behavior with displacement. This sub-term increases from $g_{12} = 7$ cm, and it reaches its peak at $g_{12} = 12$

Fig. 5: Depiction of s-plane poles for the MCWPT system obtained from the standard model and Δ roots for (a) all poles on the s-plane, (b) VD and PS poles, (c) first set of CD poles, and (d) second set of CD poles.

cm. After that, it starts to decrease with the increase in g_{12} . This phenomenon occurs due to the bifurcation of poles in VD (coil 2) and PS (coil 3) coils, as shown in Fig. 5(b). Therefore, it can be concluded that the coils that have a direct contribution to the bifurcated poles are more sensitive compared to those of the other coils. This can also be seen in Figs. 7 and 9, which shows that bifurcation in GCP_{13} is not as distinctive as in GCP_{23} .

Moreover, Fig. 12(b) shows $GCP_{13}(2)$ links I_{NR1} to I_3 with -180° of phase angle, and $GCP_{23}(1)$ links V_{TH2} to I_3 with -90° of phase angle. Therefore, if there is a power electronic converter at the PS coil synchronized with one of the CD or VD coils, the effect of these two GCPs on the soft-switching of the converter switches can be prominent.

In Fig. 13(a), the influence of GSM loops on Δ sub-terms is also observable. It is clear that when g_{12} increases, L_{23} increases and L_{12} decreases. Therefore, the loops that contribute to these couplings follow the same pattern, as shown in Table II in the appendix, for $\Delta(3)$ sub-terms (which contributes to L_{23}) and $\Delta(4)$ (which contributes to L_{12}). The phase angle of these dominant sub-terms of Δ is always 0° and they make a dominant real part for Δ , as shown in Fig. 13(b).

C. Reflected Impedance

In addition to the derivation of transferred gains between different coils of an MCWPT system, GSM can also be used

to determine the reflected impedances. This can be especially useful when the distribution of power amongst the coils is to be analyzed. To achieve the reflected impedance seen by coil i , the voltage drop due to the flow of current in that coil, when other coils are deactivated (off), is calculated. As previously mentioned in Part I, for PS coils, a virtual active source (V_{TH}^* or I_{NR}^*) can be considered for supplying the PS coil while other coils are deactivated. This is to reveal the reflected impedance seen from the PS coil. As explained in Part I, using GSM, this impedance can be obtained from $Z_{rfl,ii} = \frac{GVP_{ii}}{GCP_{ii}} = Z_{ii} \frac{NVP_{ii}}{NCP_{ii}}$. Therefore, evaluation of these impedances covers all reflective gains, which are formed by loops, such as NVP_{ii} , NCP_{ii} , and Δ .

For this evaluation, the input impedance of each coil seen from the coil terminals, as shown in Fig. 1 for $Z_{in,ii}$, is calculated from the reflected impedance and compared with the values measured by the network analyzer. The obtained results for different frequencies and displacements are plotted for $Z_{in,11}$ in Figs. 14 and 15, and different steps of approximations for this coil at $f_0 = 200$ kHz are compared with each other in Fig. 16. Similar scenario is repeated for coils 2 and 3, and the results are shown in Figs. 17 to 22.

Comparing the simulated and experimental results for $Z_{in,ii}$, it is obvious that they are well matched, and GSM is also capable of modeling the reflective gains. Therefore, GSM can be considered as an effective method to model the behavior of

Fig. 6: Simulation results showing the variation of GCP_{13} obtained from DM at different positions and frequencies in terms of (a) amplitude and (b) angle. In both figures, the red lines represent the values at $f_0 = 200$ kHz.

Fig. 7: Experimental results showing the variation of GCP_{13} at different positions and frequencies in terms of (a) amplitude and (b) angle. In both figures, the red lines represent the values at $f_0 = 200$ kHz.

Fig. 8: Amplitude (a) and phase angle (b) of GCP_{13} for DM (red line), AP1 (dashed blue line), AP2 (dashed violate line), and experimental results (starred markers) at $f_0 = 200$ kHz.

Fig. 9: Simulation results showing the variation of GCP_{23} obtained from DM at different positions and frequencies in terms of (a) amplitude and (b) angle. In both figures, the red lines represent the values at $f_0 = 200$ kHz.

Fig. 10: Experimental results showing the variation of GCP_{23} at different positions and frequencies in terms of (a) amplitude and (b) angle. In both figures, the red lines represent the values at $f_0 = 200$ kHz.

Fig. 11: Amplitude (a) and phase angle (b) of GCP_{23} for DM (red line), AP1 (dashed blue line), AP2 (dashed violate line), and experimental results (starred markers) at $f_0 = 200$ kHz.

Fig. 12: The gain components of GCP_{13} and GCP_{23} for (a) amplitude and (b) angle at $f_0 = 200$ kHz.

Fig. 13: Amplitude (a) and angle (b) of Δ sub-terms at $f_0 = 200$ kHz.

MCWPT systems. In the following paragraphs, further analysis of the three-coil MCWPT system using GSM is conducted.

From $Z_{rfl,11}(DM)$ in Fig. 16, it can be seen that VD coil can increase the reflected impedance seen from the CD coil when it gets closer to the CD coil. Considering the phase angle, this impedance behaves as almost a resistive load when VD coil is close to CD coil, and its behavior changes to a capacitive load when g_{12} gets larger. This impedance is calculated in Part I for DM, AP1, AP2, and AP3, and the reason of its capacitive behavior is the presence of $P_{11}(3)$ in its graph sets, which appears when the amplitude of Z_{33} becomes insignificant. Due to the presence of L_{13} , $P_{11}(2)$ and $P_{11}(3)$ of NVP do not make a dominant contribution, as shown in Fig. 16 for (AP3). Moreover, when coil 2 in AP2 is assumed to be an ideal VD coil, $Z_{rfl,11}$, can be obtained from $\left(\frac{L_{12}}{L_{23}}\right)^2 Z_{33}$. The resultant reflected impedance can magnify the overall impedance of the PS coil Z_{33} when $L_{12} > L_{23}$. As a result, when $\frac{L_{12}}{L_{23}}$ increases, transferred power from the CD coil to the PS coil will increase. This is aligned with P_{13} graph forming NVP_{13} and NCP_{13} , and can be considered as one of the interesting features of the proposed arrangement of the coils to increase the transferred power.

The other phenomenon that can be described with the use of GSM is the ineffectiveness of the ideal CD coil overall impedance on the reflected impedances of the other coils. As explained in Part I, the overall impedance of a CD coil tends to infinity. Therefore, other coils cannot induce an effective current in the CD coil, and consequently, this coil behaves as an open circuit when it is viewed from the other coils.

This behavior is obvious by comparing the reflected impedance results of the VD and PS coils, which have the same overall impedances according to Table I, as shown in Figs. 17, 18, and 19 (for VD coil) in comparison to Figs. 20, 21, and 22 (for PS coil). Comparing these figures, it can be seen that $Z_{rfl,22}$ and $Z_{rfl,33}$ are almost similar, and different values of L_{12} and L_{13} while g_{12} changes do not make a difference between coils 2 and 3 reflected and self-inductances.

Compared to the traditional ways of analysis of WPT systems, GSM can offer certain advantages. One such advantage is its ability to break down the voltage and current gains into sub-term graphs in an MCWPT system, so that GSM can yield all the possible combinations of mutual inductances and overall impedances (coils) responsible for the power transfer. This is especially useful when the influence of couplings and coils are to be analyzed on different variables of the system. For instance, Fig. 12 shows the sub-terms of GCP_{13} in the three-coil system under study, in which $GCP_{13}(2)$ has the dominant influence. Therefore, the system tends to follow $GCP_{13}(2)$ pattern. This characteristic is apparent when the amplitude and phase angle of $GCP_{13}(2)$ in Fig. 12 and GCP_{13} in Fig. 8 are compared. Similar characteristic is true for $GCP_{23}(1)$ in Fig. 12, when it is compared with GCP_{23} in Fig. 11.

Having the access to the dominant sub-terms which are reasonably linked to a graph based pattern provides an effective observability to analyze and design MCWPT systems. Take a segmented dynamic MCWPT system as an example, to have a proper transferred power profile, transmitter pads should be

Fig. 14: Numerical results obtained from the DM of GSM for (a) amplitude and (b) phase angle of the input impedance seen from the first coil.

Fig. 15: Experimental results for (a) amplitude and (b) phase angle of the input impedance seen from the first coil.

Fig. 16: Reflected impedance seen from the first coil obtained from AP1, AP2, and AP3 at $f_0 = 200$ kHz.

Fig. 17: Numerical results obtained from the DM of GSM for (a) amplitude and (b) phase angle of the input impedance seen from the second coil.

Fig. 18: Experimental results for (a) amplitude and (b) phase angle of the input impedance seen from the second coil.

Fig. 19: Reflected impedance seen from the second coil obtained from AP1, AP2, and AP3 at $f_0 = 200$ kHz.

Fig. 20: Numerical results obtained from the DM of GSM for (a) amplitude and (b) phase angle of the input impedance seen from the third coil.

Fig. 21: Experimental results for (a) amplitude and (b) phase angle of the input impedance seen from the third coil.

Fig. 22: Reflected impedance seen from the third coil obtained from AP1, AP2, and AP3 at $f_0 = 200$ kHz.

properly positioned next to each other. The adjacency of the transmitter pads, however, can create some undesirable influence on the operation of the system, and consequently, some undesirable power can be transferred amongst the tra pads. This power can result in hard-switching in the tra side power electronic converters and finally can lead failure. Therefore, by finding the corresponding graph negative influence can be properly addressed [1].

As another example, similar to what has been explained the three-coil system under study, by driving the graph in a generic MCWPT system, and finding the desirable undesirable power flow graphs, the transmission of power can be further improved. To this end, on one hand, the undesirable links or nodes can be nullified by choosing proper CD coils, and on the other hand, the desirable links can be boosted by using high quality VD repeaters, and different arrangement of compensators [3].

IV. CONCLUSION

As a continuation of the proposed idea of GSM presented in Part I, in this Part, further studies have been conducted on different steps of this approach. For this purpose, a three-coil MCWPT system consisting of a CD coil, a VD coil, and a PS coil, is considered as the case study. Then, by dividing the GSM graph sets into transferred and reflective GVPs, and GCPs, each set is experimentally evaluated. For this purpose, two GCP sets and all reflected impedances are evaluated at different frequencies and in different positions of the VD coil, and it is shown that this technique can derive governing equations of MCWPT systems, and it can effectively simplify these equations. To show the validity of approximation steps, each step is compared with the detailed model, which perfectly matches the experimental results. Then the effectiveness of these approximation steps on modeling an MCWPT system is investigated. Finally, the results have been studied and the effect of displacement and variation of frequency on the system behavior is analyzed. During the analysis of the results, some useful applications of this technique, such as studying the influence of graph sets on soft-switching, formation of power flow paths, and influence of different coils on each other (which is useful for estimation of receiver parameters from the transmitter coils) are revealed. Therefore, along with direct derivation of the system equation based on known and unknown variables, by following the analytical steps of GSM, some informative and useful patterns in WPT systems are revealed that helps designing and analyzing process of these systems in a more insightful way.

APPENDIX

TABLE I: Simulation and Experimental Specifications.

Parameter	Value
Coil diameter (1, 2, 3) (cm)	10
Coil height (1, 2, 3) (cm)	2.5
Airgap between coils 1 and 3 (cm)	36
Number of turns for each coil (1, 2, 3)	10
Self-inductance plus series compensating inductance of each coil (μH)	$[L_{11}, L_{22} = L_{33}]$ $= [28.5, 19.2]$
Equivalent series resistor of the coil plus its series compensating inductor (mH)	$[R_{C1}, R_{C2} = R_{C3} = R_L]$ $= [49, 33]$
Series compensating capacitance (nF)	$[C_{S1} = C_{S2} = C_{S3}]$ $= 33$
LCC parallel compensating capacitance (nF)	$C_{P1} = 68$
LCC series compensating inductance (μH)	$L_{S1} = 9.31$
L_{S1} and source series equivalent	$D \rightarrow D - AC$

LP(1) for $\Delta(1)$	LP(2) for $\Delta(2)$	LP(3) for $\Delta(3)$	LP(4) for $\Delta(4)$	LP(5) for $\Delta(5)$
1	$-\frac{L_{13}^2 s^2}{ Z_{11} Z_{33}}$	$-\frac{L_{23}^2 s^2}{Z_{22}Z_{33}}$	$-\frac{L_{12}^2 s^2}{ Z_{11} Z_{22}}$	$\frac{2L_{12}L_{13}L_{23}s^3}{ Z_{11} Z_{22}Z_{33}}$

TABLE III: Graph sets of the three-coil MCWPT for reflected NVP.

$P_{11}(1)$ for NVP ₁₁ (1)	$P_{11}(2)$ for NVP ₁₁ (2)	$P_{11}(3)$ for NVP ₁₁ (3)	$P_{22}(1)$ for NVP ₂₂ (1)	$P_{22}(2)$ for NVP ₂₂ (2)
$-\frac{L_{12}^2 s^2}{ Z_{11} Z_{22}}$	$-\frac{L_{13}^2 s^2}{ Z_{11} Z_{33}}$	$\frac{2L_{12}L_{23}L_{13}s^3}{ Z_{11} Z_{22}Z_{33}}$	$-\frac{L_{12}^2 s^2}{ Z_{11} Z_{22}}$	$-\frac{L_{23}^2 s^2}{Z_{22}Z_{33}}$
$P_{22}(3)$ for NVP ₂₂ (3)	$P_{33}(1)$ for NVP ₃₃ (1)	$P_{33}(2)$ for NVP ₃₃ (2)	$P_{33}(3)$ for NVP ₃₃ (3)	
$\frac{2L_{12}L_{23}L_{13}s^3}{ Z_{11} Z_{22}Z_{33}}$	$-\frac{L_{13}^2 s^2}{ Z_{11} Z_{13}}$	$-\frac{L_{23}^2 s^2}{Z_{22}Z_{33}}$	$\frac{2L_{12}L_{23}L_{13}s^3}{ Z_{11} Z_{22}Z_{33}}$	

TABLE IV: Graph sets of the three-coil MCWPT for transferred NVP.

$P_{12}(1)$ for NVP ₁₂ (1)	$P_{21}(1)$ for NVP ₂₁ (1)	$P_{23}(1)$ for NVP ₂₃ (1)	$P_{13}(1)$ for NVP ₁₃ (1)
$\frac{L_{12}s}{ Z_{11} }$	$\frac{L_{12}s}{Z_{22}}$	$\frac{L_{23}s}{Z_{22}}$	$\frac{L_{13}s}{ Z_{11} }$
$P_{12}(2)$ for NVP ₁₂ (2)	$P_{21}(2)$ for NVP ₂₁ (2)	$P_{23}(2)$ for NVP ₂₃ (2)	$P_{13}(2)$ for NVP ₁₃ (2)
$-\frac{L_{13}L_{23}s^2}{ Z_{11} Z_{33}}$	$-\frac{L_{13}L_{23}s^2}{Z_{22}Z_{33}}$	$-\frac{L_{12}L_{13}s^2}{ Z_{11} Z_{22}}$	$-\frac{L_{12}L_{23}s^2}{ Z_{11} Z_{22}}$

TABLE V: Graph sets of the three-coil MCWPT for reflected NCP.

1	$-\frac{L_{23}^2 s^2}{Z_{22} Z_{33}}$	1	$-\frac{L_{13}^2 s^2}{ Z_{11} Z_{33}}$
1	$-\frac{L_{12}^2 s^2}{ Z_{11} Z_{22}}$		

TABLE VI: Graph sets of the three-coil MCWPT for transferred NCP.

$-\frac{L_{12} s}{Z_{22}}$	$-\frac{L_{12} s}{ Z_{11} }$	$-\frac{L_{23} s}{Z_{33}}$	$-\frac{L_{13} s}{Z_{33}}$
$\frac{L_{13} L_{23} s^2}{Z_{22} Z_{33}}$	$\frac{L_{13} L_{23} s^2}{ Z_{11} Z_{33}}$	$\frac{L_{12} L_{13} s^2}{ Z_{11} Z_{33}}$	$\frac{L_{12} L_{23} s^2}{Z_{22} Z_{33}}$

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