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Laakso, Sampsa V.A.; Johansson, Daniel

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There is logic in logit – including wear rate in Colding’s tool wear model

Sampsa V.A. Laakso ^{a*}, Daniel Johansson ^b

^a*Aalto University, Otakaari 1 B, Espoo, 02150, Finland*

^b*Lund University, Ole Römers Väg 1, 223 63, Lund, Sweden*

Abstract

Modelling of tool wear in metal cutting processes has been the foundation of the field since the famous Taylor’s tool life model. Tool wear models have been developed for planning machining operations and calculating production costs. Most wear models do not include wear rate in the model but rather just the final wear at end of tool life, i.e. the limiting wear. This is a major disadvantage of Taylor’s model, and practically all the other experimental models of tool wear. The wear progression over time is lost with the selection the limiting wear. This paper proposes a new logit-function based model for wear rate that can be included in the existing models. In this work, in addition to Taylor’s model, Colding’s model is used with the proposed wear rate model. The outcome is a model that can predict the tool wear at a given cutting speed, feed and at any given time within the tool life range, without selection the limiting tool wear. The model can be also used inversely so that the output is the tool life at a given cutting parameters and wear. The results show good fit to the experiment data (~10 %) and the model captures the typical wear rate shape well.

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Keywords: Tool Wear; Metal Cutting; Wear Rate; Colding’s Model; Taylor’s Model

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1. Introduction

Tool life modeling was introduced by Taylor in 1907 and it has been a topic of research over the last 100 years [1]. A more precise tool life model was proposed by Colding in 1980 [2] based on five model constants. This model has been proven to work well on modelling cutting data and tool life for several materials and cutting methods [3–5]. A model describing tool life as a function of cutting parameters and cutting time, can be used to optimize not only the selected cutting parameters, wear and its corresponding tool life, but also for optimizing tool selection, cutting methods, balancing cycle times, part cost calculations etc. By using empirical models based on model constants from

experiment data, the complex physical phenomena of tool deterioration can be modelled without fully mapping and in depth describing the physical interaction between the work material and the tool material.

One major limitation with both of the models is the need for a fixed wear criterion. A wear criterion such as $VB=0.3\text{ mm}$ is selected when establishing the tool life model. The output of the model will then be the cutting time until the tool will reach its wear criterion for a specific set of cutting parameters. This can be troublesome because the tests required to establish the tool life model must reach the specific wear criterion or they cannot be included. Also, when a tool life model is established, the wear criterion cannot be changed at a later stage if needed. The wear rate is important factor in process planning and modelling, and especially the initial wear has a great impact on cutting process, as demonstrated in Laakso et al. 2017 [6,7].

Ståhl et al. 2014 [8] proposed a solution to this issue by combining the Archard wear model [9] and the Colding model. One limitation with using the Archard-Colding model is that measuring the cutting forces acting on the tool are required when establishing the model. In this work, a more robust solution is proposed using a scaled logit function to include the tool wear rate in the model. This approach also captures the slope of the wear rate curve better than Archard's model. To illustrate the need for such a model, an analogy can be made of a vehicle traveling with a velocity over a distance in a given time; traditional wear models estimate the tool life based on time at destination and the distance to destination by calculating the average speed in this route, but the new proposed model includes acceleration (change of wear rate in time), and calculates the real velocity at given time. The benefit of such model is that it gives much more accurate estimation of the tool wear at any given time than a model that uses average wear rate. This wear rate model is based on logit-function that is fitted to the experiment data with two constant parameters using iterative optimization.

Nomenclature

v_c	Cutting speed
T	Tool life
n	Taylor's exponent
C	Taylor's tool life coefficient
K	Colding's constant 1
H	Colding's constant 2
M	Colding's constant 3
$N0$	Colding's constant 4
L	Colding's constant 5
h_e	Woxén's equivalent chip thickness
A	Chip area
l_c	Tool-chip contact length
r_n	Tool nose radius
κ	Cutting edge angle, or setup angle
f	Cutting feed
t	time
t_{max}	Maximum cutting time or maximum tool life
VB	Flank wear
a	Logit model initial wear coefficient
b	Logit model shape exponent
m	Logit model limiter

2. Materials and methods

2.1. Tool wear data

Two sets of tool wear data are used in this paper. The first set is for cast iron EN-GJS-500-14 with three different cutting speeds 400, 300 and 250 m/min with 0.2 mm/rev feed and 2.5 mm cutting depth and tool wear was measured

following the ISO-3685 standard. The tool wear was therefore measured at given time increments using digital optical microscope. The measurements are illustrated in Fig. 1. Sandvik Coromant CNMG120408-KR 3210 turning insert was used in the measurements. The experiments were continued until tool's average flank wear VB_B reached 0.3 mm. Each cutting speed was tested three times. The data points are plotted in Fig. 2. The second dataset was collected by Johansson et al. (2014), where AISI 4340 steel was machined with 5 different cutting parameter sets [10]. The cutting speed was varied between 150 and 300 m/min, feed was between 0.2 and 0.45 mm/rev and the depth of cut was 2.5 mm for each case. The tool was CNMG120408. The data for AISI 4340 is presented in Fig. 3.

Fig. 1. Tool wear measurement at the beginning and at the end of tool life with optical microscope and standard guidelines.

Fig. 2. Tool wear data for EN-GJS-500-14.

Fig. 3. Tool wear data for AISI 4340.

2.2. Tool wear models

Bertil Colding developed a more detailed tool wear model similar to the widely known Taylor's model (equation 1) in his doctoral thesis in 1959 [1,11]. Johansson et al. 2017 have used and evaluated a modified Colding model, that is also used in this paper. This Colding model is a function of cutting speed v_c , cutting feed f , tool nose radius r_n , cutting edge angle κ and the depth of cut a_p with 5-constants K , H , M , $N0$ and L . It can be written in parabolic form to describe the relationship between tool life and cutting speed equations (2) and (3). The equivalent chip thickness (4) h_e in the equations is the mean theoretical chip thickness along the tool nose, proposed by Ragnar Woxén in 1932 [12]. The models are presented in Johansson et al. 2016 [10] and the Colding model parameters used for the AISI 4340 steel, $K=5.942$, $H=-1.832$, $M=0.683$, $N0=0.296$, $L=-0.095$ are from the same source. The Taylor's equation parameters for EN-GJS-500-14 are $n=0.38$ and $C=4633.25$.

$$v_c \cdot T^n = C \quad (1)$$

$$v_c = e^{\left[K - \frac{(\ln(h_e) - H)^2}{4M} - (N0 - L \cdot \ln(h_e) \cdot \ln(T)) \right]} \quad (2)$$

$$T = e^{\left[\frac{H^2 - 2 \cdot H \cdot \ln(h_e) + \ln(h_e)^2 - 4 \cdot K \cdot M + 4 \cdot M \cdot \ln(v_c)}{4 \cdot M \cdot (N0 - L \cdot \ln(h_e))} \right]} \quad (3)$$

$$h_e = \frac{A}{l_c} = \frac{a_p \cdot f}{\frac{a_p - r_n(1 - \cos \kappa)}{\sin \kappa} + \kappa \cdot r_n + \frac{f}{2}} \quad (4)$$

2.3. Logit model

The logit wear model is developed based on the logit function, presented in equation (5), that was widely used by Joseph Berkson in statistics [13]. The function is then transferred and scaled, equations (6) and (7), so that the function is approximately zero with $x=m$ and one with $x=1-m$, where the m is a very small positive number $\ll 1$. The actual limits are negative and positive infinities that are avoided by using the limiter m . The value of m in this paper is 0.01 for all data sets. The equations 5-7 are plotted and presented in Fig. 4. The actual wear function (equation 8) is now derived from the scaled function by transferring it with initial wear a and multiplying with maximum wear VB_{max} . The x-axis is replaced with a time axis and the maximum time is limited to the tool life at maximum wear t_{max} . The shape of the wear curve is adjusted to the wear data using the exponent b and the initial wear a , using Levenberg-Marquardt method.[14,15]

Original logit	$f(x) = \log \frac{x}{1-x}$	(5)
Transferred logit	$g(x) = f(x) - f(m)$	(6)
Scaled logit	$h(x) = \frac{g(x)}{g(1-m)}$	(7)
Wear function	$VB(t) = a + h\left(\frac{t}{t_{max}}\right)^b (VB_{max} - a)$	(8)

Fig. 4. Different stages of the logit wear function development.

Final step to combine the developed logit model and existing tool wear model is to replace the maximum time t_{max} in the wear function (8) with a tool life function. This can come from either Colding's model equation (3) or from Taylor's model, T solved from equation (1), that leads to: $T = (C/v_c)^{1/n}$. Both of the models are demonstrated in this paper, Colding's model with the larger dataset of AISI 4340 and Taylor's model with the dataset with cutting speed as only variable for EN-GJS-500-14.

3. Results

The results of the model fit to the data sets using Levenberg-Marquardt method produced an excellent fit. The average errors were calculated as percent error between the experiment data points and the value given by the model at that point divided with the number of data points. The model with Taylor's tool life equation fitted to the EN-GJS-500-14 cast iron cutting data produced 10.7 % average error with model parameters $a = -0.004542$ and $b = 0.839$. The model is plotted with the tool wear data in Fig. 5. The model with Colding's tool life equation was fitted individually to each data set with different cutting parameters and another fit was done to the whole data set. The individual fit leads to slightly better overall fit, with an average error of 6.5 %. The model parameters for the individual fit are shown in Table 1. The model fit to the whole data set produced nearly as good fit with an average error of 10.9 %. The models for AISI 4340 are plotted in Fig. 6.

Table 1. Logit model parameters for AISI 4340.

	a	b
Full Data Set	0.094	1.22
$v_c=300, f=0.25$	0.067	1.263
$v_c=300, f=0.2$	0.089	1.168
$v_c=250, f=0.3$	0.081	1.124
$v_c=200, f=0.3$	0.126	1.071
$v_c=150, f=0.45$	0.098	0.985

Fig. 5. Logit tool wear model with Taylor's tool life equation for cast iron EN-GJS-500-14.

Fig. 6. Logit tool wear model with Colding's tool life equation for AISI 4340, individual fit up, general fit down.

4. Discussion and conclusions

This paper presents a new tool wear rate model that can be used together with practically any tool life equation. The model can be used with both Colding's and Taylor's wear models so that the wear rate, i.e. the flank wear with given cutting time can be included in the wear models. The model has a good performance regardless if the fit is done to the whole data set or individual data sets with different cutting parameters, and the model has a low average error $\sim 10\%$ in all tested data sets. The model accuracy was improved with few percent when a third parameter was introduced to the model to take into account the change in the parameters a and b (see Table 1), but it can be argued if the benefit of improving the accuracy with a few percent is worth the increase of the number of model parameters. There are some known issues using the Colding's model, since it has asymptotic behavior with small cutting velocities and small feed values and the model accuracy is questionable outside the experimental range, but these are more cautionary notions to remember while using the Colding's model. The model can be used with Taylor's model, but then the effect of feed, cutting depth or tool geometry is neglected. The logit model brings the benefit of including the full representation of the experimental data in the model. Instead of only the selected limiting wear, the model includes the whole wear progression history, i.e. wear rate, and this was achieved by including only two more constants.

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