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Winding Function Based Analytical Model of Squirrel Cage Induction Motor for Fault Diagnostics

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Abstract— In this paper, a detailed analytical model of a three-phase squirrel cage induction motor is derived. The geometrical parameters of the motor are considered and couple magnetic circuit theory is used for the calculation of inductances, resistances and other performance parameters. Although analytical models are not as accurate as numerical models, their lower calculation time makes them important in the field of performance analysis and fault diagnostics, particularly in hardware-in-the-loop environment and inverse problem theory implementation. The model is made general for any number of rotor bars even if they are not integral number per pole and stator geometry considering the end ring parameters, leakage inductances, slotting and end winding effects.

Keywords—Analytical model, induction motors, fault diagnosis

I. INTRODUCTION

Electrical machines, particularly induction motors, are playing a key role in modern day society. This argument can be supported by the fact that they are consuming more than 60% of total generated energy worldwide [1]. The induction machines are dominating all other types of electrical machines, working as motors such as squirrel cage induction motors and as generators such as doubly fed induction generator [2]. This is because of their simple structure, high efficiency, easy maintenance, efficient controllability, high torque and ability to work in rough industrial environment. This increasing importance of the induction motors make their fast and accurate mathematical models equally important. These models are very important for performance analysis [3], fault simulations under transient and steady state conditions [4], parameters estimations [5], hardware-in-the-loop environment [6], design of electrical machine drives[7], etc.

Since these motors are associated with mechanically moving parts, they are always subject to failures [8]. The faults in electrical machines are degenerative in nature, inviting fast and accurate diagnostic algorithms, to avoid any catastrophic situation. Although finite element analysis based numerical models are more accurate, they are not suitable for diagnostic algorithms, where the mathematical model of the system is necessary. This is so because numerical models

require more computational time and memory as compared to analytical models. Moreover, the conventional d-q models are not always suitable to be used in diagnostic algorithms, because they neglect the effect of spatial harmonics. The time and space harmonics have impact on speed, torque, currents and other performance parameters of electrical machines. The faults at the incipient stage are so small that any approximation can lead to failure in their detection.

In this paper, a detailed analytical model of squirrel cage induction motor is derived. Unlike the conventional mathematical models, as presented in [9][10][11][12], the stator slots opening effect on the air gap is considered, the stator and rotor electrical parameters are computed using their geometrical dimensions, the stator end winding and rotor end ring parameters are calculated using analytical functions. The model is presented in a systematic manner for ease of understanding and implementation.

II. FLOW CHART DIAGRAM

Induction motor is a complex system where a number of parameters are interrelated with each other. It is recommended to follow a systematic way while making its mathematical model. To avoid complexity the entire parameters can be divided into rotor, stator and mutual parameters as shown in fig. 1.

On the stator side the electrical parameters such as number of phases, voltage, connection scheme, frequency and the mechanical parameters such as number of slots, dimensions of slot and winding configuration are taken as input and stator per phase resistances and leakage inductances are calculated. The stator self and mutual inductances are calculated using winding function approach as discussed in subsequent section.

Similarly, on the rotor side, the bar and end ring resistances and the leakage inductances are calculated based on the geometry of rotor slots. The self and mutual inductance among various rotor loops are calculated using winding function approach.

In order to consider the slotting effect the air gap is taken as a function of rotor and stator angles and their mutual inductances are calculated using winding function approach.

All inductances and resistances are calculated in the form of matrices for ease of implementation. At the end, the performance parameters like torque, currents, speed and rotor position is calculated. The calculated rotor position is used in feedback manner to calculate air gap and mutual inductances.

Fig. 1. Flow chart diagram of mathematical model

III. MATHEMATICAL MODELING

A. Air gap

Because of the fact that stator and rotor slot openings generate frequencies in current spectrum called spatial harmonics, the careful calculation and implementation of air gap is very important. If these slot frequencies are not considered effectively, they can become a potential candidate for errors in diagnostic algorithms. The total effective flux crosses the air gap to generate air gap flux linkage and connects different parts of machine. It means that the air gap and slot openings of stator and rotor have an impact on total flux linkage, torque, radial and tangential forces, speed and current. The total air gap of motor can be divided into two parts, the constant air gap g_c and slot air gaps variable with stator and rotor geometry (φ) and relative position (θ) as shown in equation 1.

$$g = g_c + g(\varphi, \theta) \quad (1)$$

The calculation procedure is shown in fig. 2, where distance between rotor outer and stator inner surface is taken as constant g_c . The stator slots based air gap, shown by solid line, is calculated from geometry of stator and slots. Since the motor is squirrel cage, its rotor surface is smooth having no effect on physical air gap.

Fig. 2. Physical air gap

B. Leakage Inductance

In electrical machines, total flux can be divided into two parts, the magnetization and leakage flux. The magnetization flux (φ_m) is responsible to generate air gap flux linkage (Ψ_m) between rotor and stator and participates in energy conversion. The leakage flux (φ_l) creates leakage flux

linkage (Ψ_l) and is associated with both stator and rotor as stator leakage flux (ϕ_{sl}) and rotor leakage flux (ϕ_{rl}). This is a common perception about leakage flux that it has a negative role in electrical machines due to increase in losses which is not always true. This argument can be justified by the fact that the transient inductance of induction motors depends mainly on leakage inductances as shown by following equation.

$$L'_s \cong L_{sl} + L_{rl} \quad (2)$$

These leakage inductances should be calculated carefully because any error in their calculation can lead to wrong transient analysis of motor.

The leakage induction of a machine can be divided into air gap leakage inductance L_g , slot leakage inductance L_q , tooth tip leakage inductance L_t , end winding leakage inductance L_{ew} and skew leakage inductance L_{sq} as shown by the following equation.

$$L_l = L_g + L_q + L_t + L_{ew} + L_{sq} \quad (3)$$

In case of asynchronous machines

$$L_{ew} > L_g > L_q > L_t > L_{sq} \quad (4)$$

The detailed analytical formulas of these inductances can be found in [13].

C. Resistance Calculation

The per phase stator resistance is calculated by counting the number of turns per phase, the resistance and length of a single turn along with the number of parallel paths. The effective slot area A_s , can be calculated by multiplying the actual area of slot A_A with filling factor K_f as shown in equation below.

$$A_s = A_A \times K_f \quad (5)$$

The cross sectional area of conductor can be calculated by dividing effective slot area with number of conductors (N_c) in it as given by following equation.

$$A_c = A_s / N_c \quad (6)$$

The average length of a single turn can be calculated using analytical expression as discussed in [13].

$$l_{av} \approx (2l + 2.4W + 0.1)m \quad (7)$$

Where l is the effective length of the machine and W is the average coil span. The per phase stator resistance can be calculated as,

$$R_s = N_s(\rho l_{av}/aAc) \quad (8)$$

Where, a is the number of parallel paths and N_s is effective number of series turns per phase, which is the function of total number of series turns per phase (N_t) and winding factor K_w as described by following equations.

$$N_s = K_p K_d K_s N_t \quad (9)$$

Where K_p , K_d and K_s are pitch, distribution and skewing factors respectively.

The rotor bar resistance is also calculated in a conventional way by calculating the slot area whereas, the end ring resistance is calculated and divided by number of rotor bars (n_b) to calculate the resistance of a sector between two consecutive rotor bars.

$$r_s = R_e/n_b \quad (10)$$

D. Inductance Calculation

The self and mutual inductances associated with various coils are calculated using conventional winding function approach [14].

$$L_{ij}(\theta) = \mu_o r l \int_0^{2\pi} g^{-1}(\varphi, \theta) N_i(\varphi, \theta) N_j(\varphi, \theta) d\theta \quad (11)$$

Where θ is rotor's angular position with respect to some reference point, φ is some point along air gap, $g^{-1}(\varphi, \theta)$ is the inverse air gap, $N_i(\varphi, \theta)$ is the winding function of i th coil and can be calculated by using following formula,

$$N_i(\varphi, \theta) = n_i(\varphi, \theta) - \langle n_i(\varphi, \theta) \rangle \quad (12)$$

Where $n_i(\varphi, \theta)$ is turn function of coil, which is spatially distributed along stator or rotor surface and $\langle n_i(\varphi, \theta) \rangle$ is the average of this turn function. Since the shape of the turn function depends upon the reference point, hence it is advisable to make it in such a way that it fulfills following conditions,

$$N_i(\theta^*) = N_i(-\theta^*) \quad (13)$$

$$\int_0^{2\pi} N_i(\theta^*) d\theta^* = 0 \quad (14)$$

Where $\theta^* = 0$ is the unique angle for which $N_i(0)$ has a maximum value and condition shown by equation (13) ensures even symmetry of function.

i. Stator Inductances

For a full pitch sinusoidal distributed stator winding shown in fig. 3, the winding function can be calculated using equation (12) as given below.

$$N_i(\theta_e) = \frac{N_s}{p} \cos(\theta_e) \quad (15)$$

For constant air gap the magnetization inductance can be calculated using equation (11) as,

$$L_m = \frac{\mu_o r l}{g} \int_0^{2\pi} \left(\frac{N_s}{p} \cos(\theta_e) \right)^2 d\theta_e \quad (16)$$

Upon integrating.

$$L_m = \frac{\mu_o r l}{g} \left(\frac{N_s}{p} \right)^2 \pi \quad (17)$$

Fig. 3. Winding function of single phase sinusoidally distributed winding in stator of p poles machine

In addition, the self-inductances can be calculated by adding magnetization inductance (16) and leakage inductance (3).

$$L_{AA} = L_{BB} = L_{CC} = (L_m + L_{ls}) \quad (18)$$

Fig. 4. Winding function of two phases for sinusoidally distributed winding in stator of p poles machine

Since phase windings are α ($2\pi/3$) electrical radians apart from each other as shown in fig. 4, the mutual inductance can be calculated as follows.

$$L_{AB} = \frac{\mu_o r l}{g} \int_0^{2\pi} \left(\frac{N_A}{p} \cos(\theta_e) \frac{N_B}{p} \cos(\theta_e + \alpha) \right) d\theta_e \quad (19)$$

After integration and simplification

$$L_{AB} = \frac{\mu_o r l}{g} \frac{N_A N_B}{p^2} \pi \cos(\theta_e) \quad (20)$$

$$L_{AB} = -\frac{\mu_o r l}{g} \frac{N_A N_B}{p^2} \frac{\pi}{2} = -\frac{L_m}{2} \quad (21)$$

So far a symmetrical three phase system

$$L_{AB} = L_{BC} = L_{CA} = -\frac{L_m}{2} \quad (22)$$

ii. Rotor Inductances

The turn function of one rotor loop can be represented by an analytical function given below.

$$n(\theta) = \begin{cases} 1, & \theta_i \leq \theta_e \leq \theta_i + \alpha_r \\ 0, & \theta_i > \theta_e > \theta_i + \alpha_r \end{cases} \quad (23)$$

$$\begin{aligned} \langle n(\theta) \rangle &= \frac{1}{2\pi} \int_{\theta_i}^{\theta_i + \alpha_r} 1 \cdot d\theta_e \\ &= \frac{\alpha_r}{2\pi} \end{aligned} \quad (24)$$

Where $\alpha_r = \frac{2\pi}{nb}$, is the angle span between two consecutive rotor bars. The following equation shows the winding function of single loop analytically shown in fig. 5.

$$N_r(\theta_e) = \begin{cases} -\frac{\alpha_r}{2\pi}, & 0 \leq \theta_e < \theta_i \\ 1 - \frac{\alpha_r}{2\pi}, & \theta_i \leq \theta_e \leq \theta_i + \alpha_r \\ -\frac{\alpha_r}{2\pi}, & \theta_i + \alpha_r < \theta_e \leq 2\pi \end{cases} \quad (25)$$

Fig. 5. Winding function of a single rotor loop

The following equation shows how to calculate self-inductance of rotor's kth loop.

$$L_{rkk} = \frac{\mu_o r l}{g} \int_0^{2\pi} (N_r(\theta_e))^2 d\theta_e \quad (26)$$

$$\begin{aligned} &= \frac{\mu_o r l}{g} \left[\int_0^{\theta_i} \left(-\frac{\alpha_r}{2\pi} \right)^2 d\theta_e + \int_{\theta_i}^{\theta_i + \alpha_r} \left(1 - \frac{\alpha_r}{2\pi} \right)^2 d\theta_e + \right. \\ &\quad \left. \int_{\theta_i + \alpha_r}^{2\pi} \left(-\frac{\alpha_r}{2\pi} \right)^2 d\theta_e \right] \end{aligned} \quad (27)$$

$$L_{rkk} = \frac{\mu_o r l}{g} \alpha_r \left[1 - \frac{\alpha_r}{2\pi} \right] \quad (28)$$

The equation (28) can be modified as a function of magnetization inductance (17) as follows.

$$L_{rkk} = \frac{\mu_o r l}{g} N_s^2 N_r \frac{\pi}{4} \alpha_r \left[1 - \frac{\alpha_r}{2\pi} \right] \frac{1}{N_s^2} \frac{4}{\pi} \quad (29)$$

$$L_{rkk} = \frac{4}{\pi} \left(\frac{N_r}{N_s} \right)^2 L_m \alpha_r \left[1 - \frac{\alpha_r}{2\pi} \right] \quad (30)$$

Fig. 6, shows the winding function of any two rotor-loops i and j . While the mutual inductance can be calculated using following equations.

Fig. 6. Winding function of two rotor loops placed at arbitrary locations.

$$L_{rni} = \frac{\mu_0 r l}{g} \int_0^{2\pi} (N_{rn}(\theta_e))(N_{ri}(\theta_e))d\theta_e \quad (31)$$

$$= \frac{\mu_0 r l}{g} \left[\int_0^{\alpha_r} \left(1 - \frac{\alpha_r}{2\pi}\right) \left(-\frac{\alpha_r}{2\pi}\right) d\theta_e + \int_{\alpha_r}^{\theta_i} \left(-\frac{\alpha_r}{2\pi}\right)^2 d\theta_e + \int_{\theta_i}^{\theta_i + \alpha_r} \left(1 - \frac{\alpha_r}{2\pi}\right) \left(-\frac{\alpha_r}{2\pi}\right) d\theta_e + \int_{\theta_i + \alpha_r}^{2\pi} \left(-\frac{\alpha_r}{2\pi}\right)^2 d\theta_e \right] \quad (32)$$

$$L_{rni} = \frac{\mu_0 r l}{g} \left[-\frac{\alpha_r^2}{2\pi} \right] \quad (33)$$

After modification as a function of magnetization inductance (17), equation (33) can be represented as.

$$L_{rni} = \frac{4}{\pi} \left(\frac{N_r}{N_s} \right)^2 L_m \left[-\frac{\alpha_r^2}{2\pi} \right] \quad (34)$$

This equation shows that the mutual inductance between any two rotor loops does not depend on the angle between them.

iii. Rotor and Stator Mutual Inductances

The mutual inductance between rotor and stator is the function of rotor's angle and can be calculated as follows. The graphical representation of mutual inductance using winding functions is shown in fig. 7.

Fig. 7. Winding function of a stator winding and ith rotor loop

$$L_{asri} = \frac{\mu_0 r l}{g} \int_0^{2\pi} (N_{as}(\theta_e))(N_{ri}(\theta_e))d\theta_e \quad (35)$$

$$= \frac{\mu_0 r l N_s}{g p} \left[\int_0^{\theta_i} \left(-\frac{\alpha_r}{2\pi}\right) \cos(\theta_e) d\theta_e + \int_{\theta_i}^{\theta_i + \alpha_r} \left(1 - \frac{\alpha_r}{2\pi}\right) \cos(\theta_e) d\theta_e + \int_{\theta_i + \alpha_r}^{\frac{\pi}{2}} \left(-\frac{\alpha_r}{2\pi}\right) \cos(\theta_e) d\theta_e + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \left(-\frac{\alpha_r}{2\pi}\right) \cos(\theta_e) d\theta_e + \int_{\frac{3\pi}{2}}^{2\pi} \left(-\frac{\alpha_r}{2\pi}\right) \cos(\theta_e) d\theta_e \right] \quad (36)$$

$$= \frac{\mu_0 r l N_s}{g p} [-\sin \theta_i + \sin(\theta_i + \alpha_r)] \quad (37)$$

$$= \frac{\mu_0 r l N_s}{g p} [\sin \alpha_r \cos \theta_i - \sin \theta_i (1 - \cos \alpha_r)] \quad (38)$$

For a two-pole machine, this equation can be simplified using trigonometric identities as in [15].

$$L_{sri} = \frac{\sin(\frac{\alpha_r}{2})}{N_s} \frac{4}{\pi} L_m \cos(\theta_r + (i-1)\alpha_r + \frac{\alpha_r}{2}) \quad (39)$$

The mutual induction between *i*th rotor loop and phases *b* and *c* can be calculated in same way by shifting phase *a* by ± 120 degrees.

E. Speed and Torque

The final equations in matrices form can be shown as;

$$\begin{bmatrix} V_s \\ 0 \end{bmatrix} = \begin{bmatrix} R_s & 0 \\ 0 & R_r \end{bmatrix} \begin{bmatrix} I_s \\ I_r \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} L_{ss} & L_{sr} \\ L_{rs} & L_{rr} \end{bmatrix} \quad (40)$$

From where currents can be calculated as

$$\begin{bmatrix} I_s \\ I_r \end{bmatrix} = \frac{1}{\begin{bmatrix} L_{ss} & L_{sr} \\ L_{rs} & L_{rr} \end{bmatrix}} \int \left[\begin{bmatrix} V_s \\ 0 \end{bmatrix} - \begin{bmatrix} R_s & 0 \\ 0 & R_r \end{bmatrix} \begin{bmatrix} I_s \\ I_r \end{bmatrix} \right] dt \quad (41)$$

$$T_e = I_s^T \left(\frac{d}{d\theta} L_{rs} \right) I_r \quad (42)$$

In matrices form it is very easy to implement the model in Matlab environment.

$$T_{e1} = \begin{bmatrix} I_s \\ I_r \end{bmatrix}^T \frac{d}{d\theta} \begin{bmatrix} L_{ss} & L_{sr} \\ L_{rs} & L_{rr} \end{bmatrix} \begin{bmatrix} I_s \\ I_r \end{bmatrix} \quad (43)$$

$$\begin{aligned} &= \begin{bmatrix} I_s^T & I_r^T \end{bmatrix} \begin{bmatrix} 0 & \frac{d}{d\theta} L_{sr} \\ \frac{d}{d\theta} L_{rs} & 0 \end{bmatrix} \begin{bmatrix} I_s \\ I_r \end{bmatrix} \\ &= I_r^T \frac{d}{d\theta} L_{rs} I_s + I_s^T \frac{d}{d\theta} L_{sr} I_r \\ &= 2T_e \end{aligned} \quad (44)$$

The generated torque becomes double using the matrices approach hence it should be divided by two to get actual result.

$$T_e = \frac{T_{e1}}{2} \quad (45)$$

Finally the speed and angular displacement of motor can be calculated using following equation where T_e is generated torque, T_L is load torque, B is friction coefficient J is moment of inertia of rotor and ω_m is motor's electrical speed.

$$J \frac{d}{dt} \omega_m = T_e - T_L - B \omega_m \quad (46)$$

CONCLUSIONS

A detailed systematic approach to make the mathematical model of squirrel cage induction motor is presented in this paper. Unlike pre-calculated values of resistances, leakage and magnetization inductances; the motor's geometry and winding configurations are used to calculate all performance parameters analytically.

The air gap of motor is proposed as a function of rotor position and stator angle. The inclusion of slots opening effect is very important for mathematical models designed for fault diagnostics. This is so because slot openings produce high frequency components in current spectrum, which should be handled carefully. Although for the sake of simplicity, it is taken as constant for various inductances calculations however, it can be easily handled using Taylor and Fourier series. This is a general model for any number of rotor bars regardless to the integral number of bars per pole. Moreover, this model can be used for any number of poles, parallel paths and number of conductors per slot.

It is good to study transient and steady state response of system and simulate common types of faults such as broken rotor bar, stator short circuit and eccentricity. The equations are compiled in the form of matrices, which reduces the complexity while implementing them in computer program.

The non-sinusoidal distribution of rotor circuits improves the accuracy of model. The equations work good for two pole machine while they can be updated for general p pole machine by multiplying L_m with $(p/2)$ and changing L_{sr} and L_{rr} correspondingly.

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