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Published in:

Proceedings of the 22nd International Conference on Electrical Machines and Systems, ICEMS 2019

DOI:

[10.1109/ICEMS.2019.8921847](https://doi.org/10.1109/ICEMS.2019.8921847)

Published: 01/01/2019

Document Version

Peer reviewed version

Please cite the original version:

Ren, X., Thabuis, A., Belahcen, A., & Perriard, Y. (2019). Topology Optimization for Coils of Electric Machine with Level-set Method. In *Proceedings of the 22nd International Conference on Electrical Machines and Systems, ICEMS 2019* [8921847] (Journal of international conference on electrical machines and systems). IEEE. <https://doi.org/10.1109/ICEMS.2019.8921847>

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Topology Optimization for Coils of Electric Machine with Level-set Method

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Abstract—This work studies the level-set based topology optimization for the optimal layout of winding in electric motor. The functional to optimize is to maximize the torque in the air-gap. The coil domain is initiated by a level set function. The time evolutionary equation is used to update the function. The level set model forms clearly the dynamic boundaries between the conductor and the insulation which determine the optimal distribution of coil by means of the topological changes. This proposed optimization strategy is applied to the winding of a Permanent Magnet Synchronous Motor. An optimal layout of coil is generated from this proposed method.

Index Terms—level set function, topology optimization, torque

I. INTRODUCTION

Topology optimization initiates innovative design. It is initially developed in mechanics structural design [1], [2]. The two most developed and applied topology optimization methods are SIMP and LS. Solid Isotropic Material with Penalization (SIMP) method is a density-based approach. It is easy to implement and therefore is widely studied and applied [3]. But intermediate material is obtained from this optimal design, which needs a post-processing step to be manufactured. On the other hand, Level-set (LS) method defines the interfaces between material phases implicitly through boundaries that can evolve during the optimization process. The front propagation in this method makes sure that the boundaries keep crisp during all the optimization process. Therefore, it avoids intermediate domains that occur in SIMP optimization. But this method views the problem in a higher dimension, and therefore is not as straightforward to implement.

Topology optimization first aims at saving material by generating holes while maintaining the desired stiffness of the structure. It extends rapidly into electromagnetic engi-

neering [4]–[6]. At the beginning, it is material-void design. Gradually, the optimization extended into multi-material domain [7]. Level-set function was initiated to represent boundary movement [8]. It is especially suitable for multi-material condition, since it is convenient to define complex boundaries between different material with a range of values of level-set.

The development of topology optimization technologies open a new door for creative motor design [9]. As for winding design, there are conductor and the insulation consolidation. A crisp description of the boundary is critical in order to solve Maxwell's equation correctly. Level-set method avoids ambiguities of gray material phases. The solution determines directly the objective function and next step of the optimizer. Since the boundaries are represented by an implicit level-set function, there are variable formulations to build the function. Different choice of level-set function leads to distinctive convergence behavior. Special care is needed to parameterize the function and regularization is employed in the optimization process.

In this study, we explore the level-set based topology optimization for winding design of electric motor. The objective is to maximize the torque exerted in the air-gap from an optimal coil design in the slot. A time-evolution level-set function is applied so as to move the boundary between the insulation matrix and the conductor domain. A velocity forcing item drives the boundary and a perimeter regularization maintains the convexity of the conductor zone. A topological derivative function generates the conductor in the insulation.

II. OPTIMIZATION METHOD

A. Objective Function

The objective function of this optimization is to maximize torque generated in the air gap, denoted by Ω_a . We apply Arkkio's equation (1) for the torque T calculation [10].

$$T = \frac{l}{\mu_0(r_1 - r_2)} \int_{\Omega_a} r B_r B_\phi dS \quad (1)$$

where l is the machine length, r_1, r_2 are the radii of the air gap, as shown in Fig. 1, and B_r, B_ϕ denote respectively the radial and tangential flux density in the infinitesimal element of dS at the position $\mathbf{r}$ in the air gap. In order to obtain the flux density $\mathbf{B}$, we solve the Maxwell's equation by vector potential $\mathbf{A}$ with $\mathbf{B} = \nabla \times \mathbf{A}$.

We study a 24-slot Permanent Magnet Synchronous Machine (PMSM). The stator carries a three-phase winding in the slots in positive phase sequence: $i_u - i_v - i_w$. By tuning the value of the stator current, a sinusoidal distribution of magneto motive force could be produced. It is 8-pole machine, with magnets mounted on the surface of the rotor core. The torque produced in the air-gap is determined by many parameters such as the length of the air gap, the shape of the stator tooth, the phase angle of the current, and the position of the rotor. In this study, separation of variables is applied so as to focus on the influence of the conductor distribution, and we fix the rotor as shown in Fig. 1. At this position, we further choose the moment so that the three phase currents are $i_v = 0$, $i_w = -i_u$. Right at this moment, the output torque is the maximum if the current is the only variable parameter. Denote Γ_v the line that passes both the machine center and the center of the v -phrase coil. Let Γ_3 be the line-section Γ_v slicing the machine. Because $i_w = -i_u$, there would a anti-symmetry electromagnetic behavior with respect to Γ_3 in the machine. With spatial periodicity, it permits us to numerically calculate $\frac{1}{8}$ part of the machine. We could further simplify the numerical calculation by half. It suffices to module only $\frac{1}{16}$ portion of the machine, as illustrated in Fig. 1.

Fig. 1. One half of a sector of a PMSM motor. The design domain Ω is indicated as a rectangle in the slot in blue.

With the structure shown Fig. 1, it allows us to focus on one slot, which is the design domain, denoted by Ω . In this 2D case, assume vector potential $\mathbf{A}$ has one non-zero component perpendicular to the domain and set $\mathbf{A} = (0, 0, u)$. Let Ω_c denote the conductor domain in the slot carrying a source

current density J . The optimization problem is to find the optimal distribution of conductor in Ω :

max T subject to

$$\begin{cases} -\nabla \cdot \left(\frac{1}{\mu} \nabla u \right) = J + \nabla \times \mathbf{M} & \text{in } \Omega_{\text{motor}} \\ \partial_{\mathbf{n}} u = 0 & \text{on } \Gamma_1 \\ u = 0 & \text{on } \Gamma_2, \Gamma_3, \Gamma_4 \\ J - J_{\max} \leq 0 & \text{in } \Omega_c \\ \int_{\Omega_c} dS - S_{\max} \leq 0 & \text{in } \Omega_c \end{cases} \quad (2)$$

where $\mathbf{M}$ is the magnetization of the permanent magnet (PM). $J_{\max}$ is a given current density according to the maximum value of the conductor material considering the cooling condition. $S_{\max}$ is the allowable surface (volume) of the conductor.

B. Level-set Method

The name of the level-set method is derived from its applying the level-set function to represent the structure. For a point $\mathbf{r}$ in a design domain Ω , the level set function ϕ satisfies,

$$\begin{cases} \phi(\mathbf{r}) < 0 & \forall \mathbf{r} \in \Omega_m \\ \phi(\mathbf{r}) = 0 & \forall \mathbf{r} \in \Gamma \\ \phi(\mathbf{r}) > 0 & \forall \mathbf{r} \in \Omega_c \end{cases} \quad (3)$$

where Ω_m denotes respectively the insulation matrix, and $\Omega = \Omega_m \cup \Omega_c$. Γ is the interface between these two domains.

The level-set function is propagated by a time evolution equation [11],

$$\frac{\partial \phi}{\partial t} = v \|\nabla \phi\| + \omega g \quad (4)$$

where t is the pseudo time and $\|\cdot\|$ is the Euclidean norm operator. The level-set function value of each element is calculated in this article as the minimum distance to an element in the domain of the opposite phase. Signed distance is introduced such that an insulation element has a negative distance while the distance of a conductor element is positive, as defined in (3). g is a function reflecting the topological derivative. This function provides a mechanism to determine the generation of conductor domain in the insulation matrix. ω is a weight factor for function g . In this study, g is only non-trivial in the insulation matrix so that conductor can be generated and the generation of insulation in the domain of conductor can be avoided. A simplified evolution of the level set function is described in Fig. 2. When $\omega = 0$, (4) is the standard Hamilton-Jacobi equation.

The velocity v is calculated using the shape derivative. When the optimization converges, the optimum of the objective functional is obtained. The velocity approaches zero and the propagation of level-set function stops. A clear boundary between the two materials is determined. Since the objective functional is the torque exerted in the air-gap, the magnetic energy in the design domain is chosen as the velocity parameter to reflect the shape sensitivity, as was studied in [4].

Fig. 2. An illustration of time-evolution of level-set function. At time t_0 , the function is indicated by solid blue curve, and at $t + t_0$ it evolve to the function represented by dashed red curve.

Let N_T denote the number of conductor domains in the slot. Each conductor domain $\Omega_i \in \Omega_c$, $i = 1, \dots, N_T$ is considered a turn of the coil with a cross-section surface S_i . The turns are connected in series. Each turn of the coil carries the same current I_0 . The torque output is proportional to the total current input $N_T I_0$. Therefore, a maximum total current is required to produce the maximum torque. The current is determined as $I_0 = J_{max} \cdot \min S_i$.

The Lagrangian of the problem writes,

$$L = \int_{\Omega} \frac{1}{2\mu} \|\mathbf{B}\|^2 dS + \lambda_J \int_{\Omega} (J - J_{max}) dS + \lambda_S (S - S_{max}) \quad (5)$$

where λ_J , λ_S are Lagrangian parameters for the respective constraints of current density and surface. λ_J , λ_S update with each iteration of the optimization process.

For each element e , the velocity writes as,

$$v|_e = -\frac{1}{2\mu} \|\mathbf{B}_e\|^2 - \lambda_J (J_e - J_{max}) - \lambda_S \quad (6)$$

The front propagation depends on the magnetic flux density, current density, and the total conductor surface. A bigger magnetic energy is desired so as to obtain a higher torque in the air-gap. The current density reflect the conductor surface, as in (7). It is a factor to shrink a conductor domain. The Lagrangian parameter λ_S works as a global parameter representing the total conductor surface. It generally expands a conductor domain. For a domain with smooth enough boundary, if a uniform current density is imposed perpendicular, the induced magnetic field has the greatest value near the boundary. The elemental magnetic energy in the velocity is to reduce the expansion of conductor domain. It especially avoid the situation that there is one conductor domain that attains S_{max} . Take the example in Fig. 2, at the moment the velocity of the boundary movement would be: $v_a < 0$ and $v_b > 0$.

Now that the vector potential is calculated by using finite element method with first-order basis function, the magnetic flux density is piece-wise constant for each element. The current density, the source for Maxwell's equation, is a uniform value for a conductor domain Ω_i . The direction of the current density is perpendicular to the domain and the value is calculated by,

$$J_e = \frac{I_0}{S_i}, \quad \forall e \in \Omega_i, \quad i = 1, \dots, N_T \quad (7)$$

In order to maintain convergence of the problem and to enhance the performance of the optimization, regularization is introduced to LS method. Regularization is employed to establish uniqueness, well-posedness, and geometry control of the optimization [12]. Perimeter regularization is applied in each iteration to smooth the LS function and to make the conductor domain convex.

III. NUMERICAL IMPLEMENTATION

Apply the level-set based optimization method to the conductor distribution in the slot. Since for this stage, it is statics calculation. The losses dissipated in Iron, Copper, and PM are not considered. The critical machine parameters are listed:

- Machine depth: $l = 100$ mm, stator radius: $r_0 = 50$ mm;
- Air-gap: $r_1 = 36.5$ mm, $r_2 = 35$ mm;
- Stator and rotor material: Iron, permeability: $\mu_{Fe} = 4000 \mu_0$;
- Magnetization: $M_r = 750$ kA/m in polar coordinate with machine center as the pole.
- Coil material: Copper, permeability: $\mu_{Cu} = \mu_0$.

The constraints for the optimization iteration are given in Table I.

TABLE I
CONSTRAINTS FOR OPTIMIZATION OF CONDUCTOR MATERIAL DISTRIBUTION IN PMSM SLOT.

Physical quantity	Symbol	Value
Design domain	Ω	9 mm $\times$ 5 mm
Conductor volume	S_{max}	65 % Ω
Current density	J_{max}	5 A/mm ²

The optimization routine is written in FreeFEM [13]. The main motivation is to solve the particle differential equation in (2). Besides, this software package provides an operator `convect`, which is ready for the time evolution problem. It takes about 1.5 s to finish the finite element analysis for 471696 degrees of freedom. A magnetic flux calculated is shown in Fig. 3. In this case, there are 4322 triangles covering the design domain in the slot. Different types of conductor distribution are obtained.

For the finer mesh, a better torque can be calculated. A finer mesh also means a better surface resolution. The surface discrepancy between two conductor domains closely depends on the mesh size. With infinitesimal discrepancy, each turn of coil carries the current at full capacity. Otherwise, a larger conductor than other ones would be a waster of material. It takes more time to solve the Maxwell's equation and more optimization iteration is needed to converge.

Fig. 3. The magnetic flux in the machine. The plot is rotated $\frac{\pi}{24}$ counter-clockwise about the machine axis (now Γ_1 becomes horizontal).

As a summary, the torque is proportional to the total current flowing in the coil. Different distribution of conductor brings negligible influence to the torque, with torque variation less than 3%. Thus the optimization problem turns from maximizing torque into maximizing total current.

IV. DISCUSSION

A finer mesh is required to obtain better optimization result. It is highly related to the size of the separated conductor. Besides, the mesh size near the boundary between the two materials is critical for the optimization. In the future work, we would integrate adaptive mesh refinement into the optimization iteration. At the step when the level-set function is reinitialized, the refined mesh would be removed in order not to multiple the calculation burden. With the mesh refinement, the boundary can be smoothed, and the influence of mesh is reduced. Another option to deal with the mesh-dependent problem is the multigrid method [14]. This method is useful in problems exhibiting multiple scales of behavior. With the help of multigrid method, the optimization iterates by solving a coarse problem and fine grid at the boundary between the two materials.

In the two-dimensional situation, the torque is proportional to the sum of current the coil carries. So long as the total current remains invariable, the variance of the torque in the air-gap could be negligible. The objective is to flow the maximum current in the design domain. One conductor is one turn of the coil. Each turn carries the same current. It is desired to, on one hand, place the maximum number of turns in the slot, and, on the other hand, the variance of surface between any two conductor should be as small as possible. In theory, the optimization reduces to packing problem with the same inclusion. For the further work, the optimization would be performed in the three-dimensional scenario. It would be an integration of this work and another one [15]. The innovative conductor path will enhance the performance of the motor, as shown in [16].

V. CONCLUSION

The level-set topology optimization strategy is applied in this paper to the optimum design of coil in electric motor. Optimized designs are generated from this method according to different sets of constraints. Traditional geometry could be retrieved from this approach. In this work, magnetic linearity is assumed. A special position for the rotor is chosen and kept invariable. In this condition, the variation of torque with respect to the conductor distribution in the slot does not stand out so long as the total current flowing in the conductor is kept constant. In the future work, magnetic non-linearity and time step should be considered. It is necessary to kept the rotor blocked for the iteration of the optimization otherwise the coupling between the optimization routine and the mechanical movement would make it hard to handle.

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