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GRAPHIC EQUALIZER DESIGN WITH SYMMETRIC BIQUAD FILTERS

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ABSTRACT

A novel graphic equalizer design comprised of a single second-order section per band is proposed, where the band filters have a symmetric shape about their center frequency in the entire audio range. The asymmetry of the band filters at high frequencies close to the Nyquist limit has been one source of inaccuracy in previous designs. The interaction between the different band filters is accounted for using the weighted least-squares design, which employs an interaction matrix. In contrast to prior works, the interaction matrix is designed with a different prototype gain for each band filter, helping to keep the maximum approximation error below 1 dB at the center frequencies and between them when the neighboring command gains are the same. An iteration step can further diminish the approximation error. Comparisons of the proposed design with previous methods show that it is the most accurate graphic equalizer design to date.

Index Terms—Audio systems, equalizers, IIR filters

1. INTRODUCTION

Recent research shows an increasing number of attempts to improve graphic equalizers (GEQ), which used to be quite inaccurate [1, 2], with efforts to enhance both cascade [3–8] and parallel GEQ designs [1, 9–14]. The current state-of-the-art cascade GEQ design [7, 15] utilizes an iterative least-squares (LS) design to optimize the gains of second-order (“biquad”) equalizer (EQ) filters and then computes the filter coefficients using closed-form formulas. Its accuracy is achieved by employing an interaction matrix that accounts for the leakage of the band filter gain to neighboring bands [7]. The idea of controlling the interaction using the concept of self-similarity of the band-filter responses was first proposed by Abel and Berners [3] and Oliver and Jot [6]. Since the shape of the dB amplitude response of a biquad filter changes little with different gains, the normalized response can be used as a basis function in the LS design [3, 6, 7].

This paper proposes considerable improvements to the cascade GEQ design, including a different biquad filter design with adjustable gain at the Nyquist limit, resulting in symmetric EQ filters, an optimized prototype interaction matrix with frequency-dependent values, and a more accurate weighted LS optimization. These improvements lead to shorter design times without loss in accuracy, since the iteration step is no longer needed.

The rest of this paper is organized as follows. Sec. 2 briefly recapitulates the design of the current state-of-the-art method. Sec. 3 introduces the symmetric parametric EQ filters and the improved GEQ design, called the symmetric graphic equalizer (SGE). Sec. 4

presents the results of the proposed design and compares it with two previous methods. Sec. 5 concludes this paper.

2. ITERATIVE LEAST-SQUARES EQ DESIGN

An accurate cascade GEQ (ACGE) design was proposed by Välimäki and Liski [7, 15] that uses a single biquad section for each band m with the following transfer function [16]:

$$H_m(z) = \frac{1 + G_m\beta - 2\cos(\omega_{c,m})z^{-1} + (1 - G_m\beta)z^{-2}}{1 + \beta - 2\cos(\omega_{c,m})z^{-1} + (1 - \beta)z^{-2}}, \quad (1)$$

where $m = 1, 2, \dots, M$ is the filter index, G_m is the linear peak gain, $\omega_{c,m} = 2\pi f_{c,m}/f_s$ is the normalized center frequency, f_s is the sampling rate (44.1 kHz is used in this work), β is defined as

$$\beta = \begin{cases} \tan(B_m/2), & \text{when } G_m = 1, \\ \sqrt{\frac{|G_{B,m}^2 - 1|}{|G_m^2 - G_{B,m}^2|}} \tan\left(\frac{B_m}{2}\right) & \text{otherwise,} \end{cases} \quad (2)$$

and $G_{B,m}$ is the linear gain at the edges of the bandwidth $B_m = 2\pi f_{B,m}/f_s$. The reference gain of the filters at dc is set to 1.

For the octave GEQ design, the center frequencies $f_{c,m}$, for $m = 1, 2, \dots, 10$, are 31.25, 62.5, 125, 250, 500, 1000, 2000, 4000, 8000, and 16000 Hz [2]. The filters in (1) allow for exact control over the bandwidth and the gain at the edges of the bandwidth [7]. The ACGE method is based on an unusual definition of the EQ filter bandwidth: unlike the traditional -3 -dB point specification, the bandwidth is defined between the points at which the dB gain is $g_{B,m} = cg_m$, where c is a design parameter and g_m is the filter dB peak gain. Thus, when the bandwidths B_m are set equal to the difference between the neighboring center frequencies, resulting in $B_m = 1.5\omega_{c,m}$, the behavior of the filters is controlled exactly at three points [7]. Finally, due to the asymmetry of the filters near the Nyquist limit, the bandwidths of the three filters with the highest center frequencies are manually adjusted, resulting in the ACGE bandwidths of 46.88, 93.75, 187.5, 375.0, 750.0, 1500, 3000, 5580, 9360, and 12160 Hz. A suitable value for the design parameter in the octave ACGE design was found to be $c = 0.3$ [7, 15].

The interaction matrix $\mathbf{B}$ is formed by designing a filter for all M bands using (1) with the prototype gain $g_p = 17$ dB and by dividing the magnitude responses by the prototype gain in dB. $\mathbf{B}$ is then obtained by sampling the normalized responses at the band center frequencies and at the geometric means of the corresponding bands [7]. In the octave case, this is a 19×10 matrix. The optimal dB gains used in the filter design are obtained with a LS solution from the user-specified command gains. However, since the interaction matrix is non-square, a pseudoinverse of the interaction matrix must be used [17], resulting in

$$\mathbf{g} = \mathbf{B}^+ \mathbf{t}_1 = (\mathbf{B}^T \mathbf{B})^{-1} \mathbf{B}^T \mathbf{t}_1, \quad (3)$$

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where $\mathbf{B}^+$ is the pseudoinverse of $\mathbf{B}$ and $\mathbf{t}_1$ is a $2M - 1$ -vector containing the command gain values in odd rows and their linearly interpolated intermediate values in even rows. Finally, to achieve the desired hi-fi accuracy of ± 1 dB [4, 7], an iterative step is needed [7]: a new interaction matrix is formed using $\mathbf{g}$, and the final dB gains are obtained as in (3) except that the $\mathbf{B}$ matrix has changed.

3. EQ DESIGN WITH SYMMETRIC FILTERS

This section presents an improved GEQ design method called the symmetric graphic equalizer (SGE). Usually digital parametric EQ filters are assumed to have unity gain both at dc (0 Hz) and at the Nyquist limit (f_{Nq}). Orfanidis [18] proposed a design method of a parametric EQ (PEQ) filter allowing these gains to be different by setting the gain G_{Nq} at the Nyquist limit to that of a corresponding analog equalizing filter. Although this works well for most octave band filters, there are problems at high frequencies, where the wide filters become asymmetrical due to the warping effect close to the Nyquist frequency. We propose a modification to the PEQ design in order to allow a symmetric band-filter design instead of matching an analog design. The new SGE design method based on the symmetric band filters is then introduced.

3.1. Modified Orfanidis parametric equalizer filter

The proposed modification concerns the Nyquist gain G_{Nq} : instead of calculating the gain value during the filter design (see (21) in [18]), we determine it beforehand. The transfer function of the modified second-order PEQ filter for the m^{th} EQ band, when the dc gain is set to one and $G_{Nq,m}$ is now adjustable, is written as

$$H_m(z) = \frac{b_{0,m} + b_{1,m}z^{-1} + b_{2,m}z^{-2}}{1 + a_{1,m}z^{-1} + a_{2,m}z^{-2}}, \quad (4)$$

where the filter coefficients are

$$\begin{aligned} b_{0,m} &= V(G_{Nq,m} + W^2 + A_2), \\ b_{1,m} &= -2V(G_{Nq,m} - W^2), & a_{1,m} &= -2V(1 - W^2), \\ b_{2,m} &= V(G_{Nq,m} + W^2 - A_2), & a_{2,m} &= V(1 + W^2 - A_1), \end{aligned}$$

and

$$\begin{aligned} A_1 &= \sqrt{\frac{C + D}{|G_m^2 - G_{B,m}^2|}}, & A_2 &= \sqrt{\frac{G_m^2 C + G_{B,m}^2 D}{|G_m^2 - G_{B,m}^2|}}, \\ W^2 &= \sqrt{\frac{|G_m^2 - G_{Nq,m}^2|}{|G_m^2 - 1|}} \tan\left(\frac{\omega_{c,m}}{2}\right)^2, & V &= \frac{1}{1 + W^2 + A_1}, \\ C &= |G_{B,m}^2 - G_{Nq,m}^2| \Delta\Omega^2 - 2W^2 \left(|G_{B,m}^2 - G_{Nq,m}^2| \right. \\ &\quad \left. - \sqrt{|G_{B,m}^2 - 1| |G_m^2 - G_{Nq,m}^2|} \right), \\ D &= 2W^2 \left(|G_m^2 - G_{Nq,m}^2| - \sqrt{|G_m^2 - 1| |G_m^2 - G_{Nq,m}^2|} \right), \\ \Delta\Omega &= \left(1 + \sqrt{\frac{|G_{B,m}^2 - 1|}{|G_{B,m}^2 - G_{Nq,m}^2|}} W^2 \right) \tan\left(\frac{B_m}{2}\right). \end{aligned} \quad (5)$$

The correct Nyquist gain G_{Nq} to be used in the above equations can be estimated, e.g., from highly oversampled EQ filters by evaluating the response at 22050 Hz. We use the sample rate of 10 MHz, which corresponds to an oversampling factor of about

Figure 1: Magnitude responses of the last octave EQ band filter: (a) the original PEQ [18] with two different bandwidths and (b) the modified PEQ with optimized bandwidth. The dashed black curve in both figures is the ideal shape approximated with oversampling.

227 w.r.t. 44.1 kHz. The magnitude response of the first six band filters always reaches approximately 0 dB at f_{Nq} , even when the maximum gain in our design is applied. However, the response of the last four band filters have a non-zero value at the Nyquist frequency that must be taken into account. Designing oversampled filters before the actual filters to evaluate their responses is impractical. So, to avoid this, we use second-order polynomials to approximate the Nyquist dB gain $g_{Nq,m}$ based on the peak dB gain g_m : $g_{Nq,m} = \text{sgn}(g_m)(q_{0,m} + q_{1,m}|g_m| + q_{2,m}|g_m|^2)$, where the absolute value and the sign function $\text{sgn}(\cdot)$ are used since the function is antisymmetric. The polynomial coefficients are obtained for each of the four filters using curve fitting in MATLAB and are shown in Table 1. This way, the peak gain is the only parameter that must be provided when designing the filters for the SGE design.

The asymmetry problem of the original PEQ is shown in Fig. 1(a), where the continuous blue curve is the last EQ band filter with the default bandwidth 1.5 times the center frequency and the dashed black line is the shape of the ideal analog filter approximated by oversampling. Since the normalized bandwidth is now close to the value of π , the tangent function in (5) causes the filter response to become too wide. However, when the bandwidth parameter is decreased [the orange dash-dot curve in Fig. 1(a)] to match the ideal filter shape, the Nyquist gain is affected and the response becomes asymmetric.

The result of the proposed modification is shown in Fig. 1(b), which shows the proposed filter response of the highest band. When the bandwidth of the filter is adjusted to match the shape of the ideal equalizing filter, the filter response shown in purple is obtained. The dashed black curve is, again, the oversampled ideal shape. Now the symmetric shape is approximated nearly perfectly, and the Nyquist gain equals that of the analog design.

3.2. Proposed design method

The proposed SGE design utilizes the biquad filter described in Sec. 3.1. Similarly to the original ACGE design, these filters offer control over the bandwidth and the gain at the bandwidth edges. In order to know the effect of a filter at its two adjacent center frequencies, the bandwidth is set to the difference between these two points, i.e., $B_m = 1.5\omega_{c,m}$. However, the bandwidths of the

Table 1: Polynomial coefficients for the Nyquist gain.

m	7	8	9	10
$q_{0,m}$	0.00544	0.0169	0.0212	-0.0672
$q_{1,m}$	0.00474	0.0221	0.125	0.730
$q_{2,m}$	0.000321	0.00108	0.00184	-0.00751

Figure 2: Comparison of band filter response shapes in (a) the ACGE and (b) in the proposed SGE design. The dashed curves are the approximated ideal shapes obtained with excessive oversampling and the color curves are the filter responses with optimized bandwidths.

last four band filters must be adjusted in order to achieve symmetry near the Nyquist frequency. The resulting bandwidths are 46.88, 93.75 Hz, 187.5 Hz, 375.0 Hz, 750.0 Hz, 1500 Hz, 2991 Hz, 5910 Hz, 11150 Hz, and 10390 Hz.

In addition to the filter bandwidths, the parameter c must also be readjusted. Our tests indicate that $c = 0.29$ gives the best accuracy. These parameter values result in the band filters shown in Fig. 2(b), which can be compared to the filters used in the ACGE design in Fig. 2(a). The dashed curves correspond to the ideal target shapes obtained with oversampling. Both designs match the targets well for the first six band filters, but the ACGE filters are substantially asymmetrical, especially for the last two bands. The proposed filters, on the other hand, retain symmetry better for the last filters and are more suitable for accurate design.

Since the proposed SGE design uses the LS optimization, just as the previous ACGE method, an interaction matrix $\mathbf{B}$ is necessary. The biquad filters described in Sec. 3.1 are now used, and instead of a constant prototype gain for all filters, different prototype gains are used for each band filter, as this has been found to improve the accuracy. These gains are based on the mean values for each filter gain when 16384 gain settings were used containing both random settings and the 1024 extreme gain setting permutations (i.e., each slider set to ± 12 dB). When the relative gains were kept constant and the overall level of the prototype gain vector was adjusted, the smallest maximum error for the the 1024 extreme gain settings was obtained with the following prototype gain vector: $\mathbf{g}_p = [13.8 \ 14.5 \ 14.5 \ 14.6 \ 14.5 \ 14.5 \ 14.6 \ 14.6 \ 14.5 \ 13.6]^T$. Once the magnitude response samples are evaluated for each biquad filter, they are divided by the corresponding prototype dB-gain so that the diagonal of the interaction matrix contains ones [2].

The interaction matrix is used in the LS solution to find the optimal dB gains based on the user-selected command gains. Instead of (3), the accuracy of the proposed method is increased by using the weighted LS method [19]:

$$\mathbf{g} = (\mathbf{B}^T \mathbf{W} \mathbf{B})^{-1} \mathbf{B}^T \mathbf{W} \mathbf{t}_1, \quad (6)$$

where $\mathbf{t}_1$ is, again, the vector with $2M - 1$ elements containing the command gain values in odd rows and their linearly interpolated intermediate values in even rows and $\mathbf{W}$ is the 19×19 weighting matrix with the frequency-specific non-negative weights on its diag-

onal and zeros elsewhere. The weights for the filter center frequencies were set to unity, and for the midpoint frequencies different small values w_2 were tested:

$$\mathbf{W} = \text{diag}(1 \ w_2 \ 1 \ \dots \ w_2 \ 1). \quad (7)$$

For the unweighted case ($w_2 = 1$), the maximum error for the 1024 extreme gain configurations was 1.10 dB. The error was 1.03 dB for $w_2 = 0.707$. Finally, when the weight for the midpoint frequencies was set to $w_2 = 0.5$, the maximum error was 0.96 dB, which fits the tolerance scheme, and thus, this value was chosen. Since we noted that the maximum error occurs with these extreme gain settings, the proposed method can be stated to have the desired ± 1 -dB accuracy without an iterative step (see the error definition in Sec. 4.1). Thus the interaction matrix can be computed a priori and stored, so reducing the computational load when calculating $\mathbf{g}$ in (6).

4. COMPARISON WITH EXISTING METHODS

In this section, the proposed SGE design is compared against the ACGE method [7] described in Sec. 2 and a higher-order cascade GEQ utilizing a fourth-order filter section for each band, which we here call EQ4 [4]. The filter bandwidths in EQ4, which are used to optimize the filter center frequencies, were manually adjusted to match the center frequencies of the two other designs, thus making the comparison fair.

GEQs generally allow the gains to be adjusted in the range of ± 12 dB [2]. The largest errors typically occur when using only these extreme values in some combination [2, 6]. This is caused by the steep slopes with plateaus between them, which demand great precision from the EQ. For an octave EQ, testing all of the $2^{10} = 1024$ command gain configurations consisting of the extreme settings is straightforward, so they are used here to compare the accuracy of the selected designs.

4.1. Maximum error

The different designs are compared based on the maximum approximation error. The approximation error is defined here as the absolute difference between the user-set command gain and the GEQ magnitude response at the center frequencies and in the frequency

Figure 3: Magnitude responses obtained with the (a) EQ4, (b) ACGE, and (c) proposed SGE methods in a challenging test case.

interval between adjacent two center frequencies when their sliders are set to the same value. There are multiple reasons for this type of error definition: We observe whether or not the GEQ design is able to match the user-specified target, which is only defined explicitly at the command gain frequencies. When neighboring command gains are set to identical values, one can expect minimal undulation between these points [15]. Finally, the target response is not well defined between the center frequencies whose command gains are set to different values, and thus, evaluating the error at those is not fair. Additionally, since the EQ designs tested here employ low-order filters, the magnitude responses can be assumed to change monotonically between these points without causing any major deviation from a hypothetical target.

Figure 3 shows an example extreme command-gain setting realized with the three EQ designs. The gain configuration contains both steep slopes and plateaus, which presents a demanding shape for the EQs to approximate. In order to enable a fair comparison, none of the designs use any iterations in the design. Figure 3(a) shows that EQ4 cannot reach all the command gain markers and overshoots the plateaus, producing a maximum error of 2.17 dB at 9270 Hz. The ACGE design of Fig. 3(b) matches the target shape better. However, it still exceeds the ± 1 -dB tolerance on the plateau at 8635 Hz with an error of 1.24 dB. The largest error of the ACGE method often occurs at frequencies above 8 kHz. Finally, the response of the proposed design, shown in Fig. 3(c), looks similar to that of ACGE, but overall the command gains are better matched. The proposed design is the only one that fulfills the target accuracy (without iterations) having a maximum error of 0.79 dB at 58 Hz.

When testing the designs with all the 1024 extreme cases, we can conclude that the proposed design is the most accurate. The maximum errors are 2.54, 1.24, and 0.96 dB for EQ4, ACGE, and the proposed design, respectively. Depending on the design used, these maximum errors may occur with different command gain configurations. Thus, the proposed SGE is the most accurate design, and since the iterative step is no longer necessary, the design time of the SGE is shorter than that of the ACGE. Thus, it is suitable for demanding applications where the design time is important, such as unmasking of music in heavy ambient noise [20].

Since ACGE and the proposed design support iterative steps, their accuracy can be increased. Figure 4 shows the effect of the number of iterations on the maximum error when using the extreme gain configurations. Only the proposed design achieves the target accuracy without iteration, and with an arbitrary number of iterative steps, the proposed design is always more accurate than the ACGE. It is also worth noting that the iterative steps after the first one increase the accuracy very little. Finally, Fig. 4 also shows that with one or more iterative steps, the proposed method approaches

Figure 4: Effect of iteration steps on the maximum error. Blue curve (triangles) is ACGE and orange curve (crosses) is the proposed SGE design. The black line indicates the desired accuracy of 1 dB, and the dashed line is the error baseline obtained with oversampling.

but cannot quite reach the maximum accuracy obtainable with the ideally-shaped symmetric filters. The smallest possible maximum error of 0.73 dB, indicated in Fig. 4 with the dashed line, has been obtained with the SGE design using excessive oversampling.

Using a neural network to replace the design, as proposed in [21], a large number of iterations and, thus, a long design time, will not be a problem. A neural network can be trained with the user-specified command gains as the input and the optimized filter gains as the desired output. This leads practically to the same accuracy as the LS design, but with a highly reduced design time.

5. CONCLUSION

This paper introduced an improved cascade GEQ design comprising a single second-order IIR section per band. The proposed SGE design uses symmetric biquad filters, i.e., filters whose magnitude responses are symmetric about their center frequency on the log-frequency axis, to minimize the error caused by the warping close to the Nyquist frequency. The filters feature an adjustable gain at the bandwidth edges, which allows strict control of their response at the neighboring band centers. The weighted LS design is applied to solve for the optimal filter gains in order to account for the interaction between the band filters. The LS procedure requires an interaction matrix, which here is designed using a prototype gain vector with varying values instead of a constant value as done previously. In contrast to the previous state-of-the-art method, an iterative step is no longer needed to achieve the desired ± 1 -dB accuracy, and thus, the interaction matrix can be calculated beforehand and stored. The SGE design is the simplest GEQ fulfilling the hi-fi accuracy and thus suitable for many audio and music applications. The relevant MATLAB code is available online [22].

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