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Instantaneous Control of Scattering From a Time-Modulated Meta-Atom

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Abstract – Time-modulation of material parameters is a powerful tool that enables ultimate control over scattered light. Proper description of scattering from time-modulated materials should be written rigorously in the time domain. This becomes possible if the properties of a single time-varying meta-atom are also considered in the time domain. In this talk we will present a theoretical model which describes a time-variant meta-atom and its interaction with an incident electromagnetic wave. Based on the developed theory, we will present several peculiar applications of time-varying meta-atoms, such as cancellation of scattering and shifting the frequency of the scattered wave.

I. INTRODUCTION

Spatial modulation of materials' properties enables control of electromagnetic radiation by changing meta-atom's parameters such as size, shape and mutual orientation. Modulation of materials' properties in time domain represents a relatively new direction of research. Time-varying external modulation is another degree of freedom that can open novel possibilities in various fields such as microwave techniques [1], optoelectronics and photonics. Currently existing studies consider time-modulation of permittivity [2] or conductivity [3], which correspond to the properties of bulk materials. However, modulation of a single meta-atom in time has not yet been properly studied. Furthermore, external modulation function in the majority of studies is limited to time-harmonic [4]. In this talk we present a theoretical model that correctly describes instantaneous power balance in a single radiating meta-atom. In the proposed model, modulation function is not limited to time harmonic, and modulation rate is not necessarily slow in comparison with the carrier frequency. In order to verify the validity of the developed model we present instantaneous power balance for conventional (not modulated) dipoles in the case of time-harmonic excitation. Finally, we present several examples of applications that can be realized by using a single time-varying reactive element as a load of a dipole.

II. INSTANTANEOUS POWER BALANCE

If the properties of a radiating dipole particles are changing is time, one needs to find field solutions in time domain. A straightforward approach is to use the well-known equivalent circuit in frequency domain and Fourier transform to the time domain. However, this simple method does not give physically correct results due to the fact that the radiation resistance (proportional to ω^2 in the frequency domain) measures the cycle-averaged radiated power. However, to find a consistent time-domain solution, we need an expression for the instantaneous power. To develop such model, we consider instantaneous power balance of a radiating electrical dipole.

Instantaneous power balance in the circuit theory reads that the summation of all instantaneous powers, related to resistances, capacitances, inductances, and sources in the circuit, must be zero ($\sum_k P_k(t) = 0$). In other words, this law expresses the conservation of energy at each moment of time. Existing analogy between the small electric dipole and the *RLC* circuit allows to use the power balance principle. The external instantaneous power $P_{in}(t)$ supplied to the dipole splits into three parts: the radiated power $P_{rad}(t)$, the reactive power $P_{reac}(t)$, and the dissipated power $P_{diss}(t)$. Therefore, we can write the following equation which represents the power balance for the dipole:

$$P_{in}(t) + P_{reac}(t) + P_{rad}(t) + P_{diss}(t) = 0. \quad (1)$$

Dissipated power in the resistive load of the dipole measures absorbed power due to the inherent losses in the metal or dielectric. Conventionally, in the case of time-harmonic excitation with time-invariant elements, we use values

of power that are averaged over a period. Average of the reactive power is always zero and it does not contribute to the power balance in the frequency domain. However, for time-modulated meta-atoms the instantaneous reactive power even in the case of a time-harmonic source is not zero and, since we treat the problem in the time domain, the reactive power cannot be dropped out. This issue is important because the reactive energy represents the stored energy near the dipole in the form of electric and magnetic field energies. The reactive power can be expressed using effective parameters such as L (inductance) and C (capacitance) and current in the center of the antenna, $i(t)$. The instantaneous reactive power then reads:

$$P_{\text{reac}}(t) = \frac{Q(t)}{C}i(t) + L\frac{di(t)}{dt}i(t), \quad (2)$$

where $Q(t)$ is charge stored in one half of the dipole.

Regarding the radiated power, it is defined as the surface integral of the Poynting vector over a closed surface including the dipole [5]. According to the antenna theory terminology, the radiation of the dipole is introduced through the time-averaged radiated power and the time-average of the square of the instantaneous electric current carried by the dipole: In other words,

$$R_{\text{rad}} = \langle P_{\text{rad}} \rangle / \langle i(t)^2 \rangle, \quad (3)$$

where the brackets denote time averaging over one period. Obviously, this model is applicable only for time-harmonic currents or for modulations which are very slow as compared with the carrier frequency. For arbitrary time modulations, this description of radiation is not suitable and we must find a general expression for the radiation resistance or the instantaneous radiated power such that the expression for the radiated power is valid for any moment of time. To this end, it is not enough to calculate the fields created by the dipole and subsequently compute the instantaneous Poynting vector. The instantaneous radiated power has to be calculated considering energy and momentum transfer from moving charges to the electromagnetic fields at any moment of time. Following this approach, the expression for the instantaneous radiated power reads [6]:

$$P_{\text{rad}}(t) = -\frac{\mu_0}{6\pi c} \dot{\mathbf{p}}(t) \cdot \ddot{\mathbf{p}}(t). \quad (4)$$

In the presentation we will show that computation of instantaneous power through only integration of Poynting vector can give rise to wrong interpretations even in the case of time-harmonic excitation and in the absence of time modulation (the conventional case).

The input power can be calculated again multiplying current in the antenna gap by the electromotive force over the gap. The electromotive force $\mathcal{E}(t)$ can be created by either external electric field of waves illuminating the meta-atom or by an external source connected to the dipole arms. In the case of a lossless dipole, the power balance equation can be obtained by substituting all the terms in Eq. (1) by the expressions in Eqs. (2) and (4). Therefore the expression for power balance reads:

$$-\frac{\mu_0}{6\pi c} \dot{\mathbf{p}}(t) \cdot \ddot{\mathbf{p}}(t) + \frac{Q(t)}{C}i(t) + L\frac{di(t)}{dt}i(t) - \mathcal{E}(t)i(t) = 0. \quad (5)$$

The established power balance equation is valid for transmitting and receiving regimes of the dipole antenna. As a check, it is possible to simplify this equation considering a resonant dipole antenna. In this case the sum of reactive powers exchanges by the inductance and capacitance is zero at all moments of time. Figure 1 graphically represents the power balance for a resonant dipole antenna in receiving and transmitting regimes.

The established time-domain power balance equation can be used as the governing time-domain equation for radiating and scattering time-modulated meta-atoms. In the reception regime, from the dipole-model point of view, we can write the complete version of the instantaneous power balance equation

$$-\frac{\mu_0}{6\pi c} \dot{\mathbf{p}}(t) \cdot \ddot{\mathbf{p}}(t) + \frac{Q(t)}{C}i(t) + L\frac{di(t)}{dt}i(t) + \frac{Q(t)}{C_{\text{load}}(t)}i(t) + \frac{d}{dt} \left(L_{\text{load}}(t)i(t) \right) i(t) - \mathcal{E}(t)i(t) = 0 \quad (6)$$

in terms of the effective capacitance C and inductance L of the meta-atom. Here, $C_{\text{load}}(t)$ and $L_{\text{load}}(t)$ are the time-varying load capacitance and inductance, respectively, operating as the load of the dipole in the receiving regime. The load is in series connection with the reactive elements of the antenna. Solutions of this equation give accurate relations for the time dependence of induced electric dipole moment of a radiating meta-atom which is arbitrarily modulated in time. During the talk we will present examples of intriguing effects of time-modulation of meta-atoms.

Fig. 1: Instantaneous power balance for transmitting and receiving regime regarding a resonant dipole. The length of the cylindrical wire, made of perfect conductor, is $l = 93.9$ mm and the radius of the wire is $r = 0.3$ mm. (a) - Power balance in transmitting regime. (b) - Power balance in receiving regime. In both cases (a) and (b) the orange curve shows the instantaneous radiated power, while the blue curve corresponds to supplied power.

III. CONCLUSION

We have introduced a theoretical model on the basis of the instantaneous power balance for studying time-varying meta-atoms. In particular, we have considered a cylindrical wire antenna and confirmed the validity of the proposed model in two different regimes: Transmitting regime where the wire is excited by an external lumped source, and receiving regime where the wire is excited by an incident electromagnetic plane wave. Initial studies of modulated electric dipoles reveal rather interesting effects. We hope that the developed consistent time-domain model of time-varying meta-atoms will allow full understanding of time-modulation control over the properties of metasurfaces and metamaterials made of time-modulated meta-atoms.

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