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State-Space Control for LCL Filters: Comparison Between the Converter and Grid Current Measurements

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Abstract—This paper deals with discrete-time state-space current control of converters equipped with an LCL filter. Either the converter or grid current is measured and the unknown states are estimated using a reduced-order observer. These two current measurement options are compared by means of analysis and experiments under strong and weak grid conditions. To provide the basis for the comparison, equal reference-tracking performance is designed for both cases under strong grid conditions. In weak grids, the converter current measurement provides faster reference tracking, faster disturbance rejection, and better resonance damping in the case of the studied system where the LCL-filter resonance frequency is below the critical frequency.

Index Terms—Grid converter, LCL filter, reduced-order observer, state-space current control, weak grid.

I. INTRODUCTION

Grid converters equipped with an LCL filter are increasingly used to connect distributed and renewable energy sources to the electric grid. The LCL filter is the preferred option to attenuate the switching harmonics because of its compact size and better grid-current quality in comparison with the L filter [1], [2]. Active resonance damping of the LCL filter by means of control [3] makes the system more efficient than passive damping [4]. State-space control provides a straightforward way for active resonance damping and for setting the dominant dynamics of the closed-loop system [3], [5]–[10].

As shown in Fig. 1, the current control can be based on either the converter current measurement (CCM) or the grid current measurement (GCM). State-space control based on the CCM is proposed in [3], [5], [6] and based on the GCM in [8]–[11]. The unknown states can be estimated using a full-order observer [5], [6] or a reduced-order observer [8], [10]. The reduced-order observer provides better disturbance rejection than the full-order observer [12]. However, the reduced-order observer is more sensitive to noise. In state-space control, the closed-loop poles can be placed at the desired locations using the direct pole-placement method [5], [8], [10]. The actual poles of the system depend on the grid impedance, which is typically unknown. The grid impedance may degrade the dynamic performance of the closed-loop system or lead to unstable operation of the converter [13]–[16]. Current control

Fig. 1. Measurement options for controlling the grid current of a converter equipped with an LCL filter.

becomes challenging in weak grids, where the grid impedance is high [15], [16].

The time delays due to sampling, pulse-width modulation (PWM), and computation play an important role in the design of the current-control loop. The ratio of the filter resonance frequency to the sampling frequency f_s affects the stability of single-loop proportional-integral (PI) or proportional-resonant (PR) current control [17]–[20]. For example, when the GCM is used, stability problems arise if the filter resonance frequency is below the critical resonance frequency $f_c = f_s/6$. On the contrary, a direct discrete-time state-space controller can stabilize the system, independently of the resonance frequency of the filter [8]. However, the robustness against grid impedance variations still depends on the ratio between the resonance frequency and the sampling frequency. Even though limitations of the PI controllers with both current measurement options have been analyzed in [17], [20], there is no comparison available for state-space control, particularly in the case of weak grids.

This paper compares the current measurement options, shown in Fig. 1, for discrete-time state-space control. The grid current is chosen as the controlled variable in both cases, since the power factor can be directly controlled at the point of common coupling (PCC) [11], [21]. To provide the basis for the comparison, equal closed-loop poles and equal reference-tracking performance are designed for both cases under nominal (strong grid) conditions. The comparison is made in terms of reference-tracking performance and disturbance-rejection capabilities focusing on very weak grids. The sensitivity of

Fig. 2. Space-vector circuit model of an LCL filter and a grid in stationary coordinates (vectors marked with the superscript s).

the control methods to the parameter errors in the LCL filter is also compared. The designs are experimentally evaluated using a 12.5-kVA grid converter.

II. SYSTEM MODEL

A. Open-Loop System

Fig. 2 shows an equivalent circuit of the LCL filter connected to the inductive grid. The converter voltage is u_c^s , the PCC voltage is u_g^s , and the grid voltage is e_g^s . The converter current is denoted by i_c^s and the grid current by i_g^s . The LCL filter parameters are denoted by L_{fc} , C_f , and L_{fg} . The losses of the filter are neglected. The grid inductance is L_g . The total grid-side inductance seen from the capacitor terminals is $L_{gt} = L_{fg} + L_g$. The undamped resonance angular frequency of the system

$$\omega_p = \sqrt{\frac{L_{fc} + L_{gt}}{L_{fc} L_{gt} C_f}} \quad (1)$$

depends on the grid inductance L_g , i.e., increasing the grid inductance decreases the resonance frequency.

A state-space model of the plant in synchronous dq -coordinates rotating at the grid angular frequency ω_g is

$$\frac{dx}{dt} = \underbrace{\begin{bmatrix} -j\omega_g & -\frac{1}{L_{fc}} & 0 \\ \frac{1}{C_f} & -j\omega_g & -\frac{1}{C_f} \\ 0 & \frac{1}{L_{gt}} & -j\omega_g \end{bmatrix}}_{\mathbf{A}} \mathbf{x} + \underbrace{\begin{bmatrix} \frac{1}{L_{fc}} \\ 0 \\ 0 \end{bmatrix}}_{\mathbf{B}_c} u_c + \underbrace{\begin{bmatrix} 0 \\ 0 \\ -\frac{1}{L_{gt}} \end{bmatrix}}_{\mathbf{B}_g} e_g$$

$$\mathbf{i}_g = \underbrace{\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}}_{\mathbf{C}_g} \mathbf{x}, \quad \mathbf{i}_c = \underbrace{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}}_{\mathbf{C}_c} \mathbf{x} \quad (2)$$

where $\mathbf{x} = [\mathbf{i}_c, u_f, \mathbf{i}_g]^T$ is the state vector and $\mathbf{A}$, $\mathbf{B}_c$, $\mathbf{B}_g$, $\mathbf{C}_g$, and $\mathbf{C}_c$ are the system matrices. The plant model is converted to a hold-equivalent discrete-time model [5]

$$\mathbf{x}(k+1) = \Phi \mathbf{x}(k) + \Gamma_c u_c(k) + \Gamma_g e_g(k)$$

$$\mathbf{i}_g(k) = \mathbf{C}_g \mathbf{x}(k), \quad \mathbf{i}_c(k) = \mathbf{C}_c \mathbf{x}(k) \quad (3)$$

where the system matrices are

$$\Phi = e^{\mathbf{A}T_s}, \quad \Gamma_c = \left(\int_0^{T_s} e^{\mathbf{A}\tau} e^{-j\omega_g(T_s-\tau)} d\tau \right) \mathbf{B}_c$$

$$\Gamma_g = \left(\int_0^{T_s} e^{\mathbf{A}\tau} d\tau \right) \mathbf{B}_g.$$

The sampling period is $T_s = 1/f_s$. The closed-form expressions of the system matrices are given in [5].

Fig. 3. Current control system rearranged as a 2DOF control structure: (a) GCM; (b) CCM. The reference prefilters are F_g and F_c and the feedback controllers are G_g and G_c . The open-loop systems are shown inside the gray blocks.

Fig. 4. Frequency responses of the transfer functions Y_{gc} (blue), Y_{cc} (red), and H_c (green) in synchronous coordinates. The nominal LCL-filter parameters are given in Table I.

The computational delay of one sampling period exists in standard implementations, i.e., [5]

$$u_c(k+1) = u_{c,\text{ref}}(k) \quad (4)$$

where $u_{c,\text{ref}}$ is the converter voltage reference.

Fig. 3 shows the block diagrams of the open-loop systems (inside the gray blocks). The open-loop transfer functions can be derived from (3) for both measurement options. For example, the transfer function from the converter voltage u_c to the grid current i_g is

$$Y_{gc}(z) = \mathbf{C}_g(z\mathbf{I} - \Phi)^{-1} \Gamma_c. \quad (5)$$

The other transfer functions can be obtained similarly. According to Fig. 3, the relations between the open-loop transfer functions can be written as

$$Y_{gc}(z) = H_c(z) Y_{cc}(z) \quad (6)$$

$$Y_{gg}(z) = [H_c(z) + H_e(z)] Y_{cg}(z). \quad (7)$$

As an example, the frequency responses of the transfer functions Y_{gc} , Y_{cc} , and H_c are shown in Fig. 4. The frequency response of H_c exhibits the resonance behavior at the antiresonance frequency of Y_{cc} .

Fig. 5. Control system. The sampling is synchronized with the PWM. The PCC-voltage angle ϑ_g is obtained using the PLL. The effect of the computational delay on the voltage reference angle is compensated for in the coordinate transformation using $\vartheta'_g = \vartheta_g + T_s\omega_g$.

B. Closed-Loop System

Fig. 3 also shows the block diagrams of the current controllers for both measurement cases, rearranged as a two degrees-of-freedom (2DOF) control structure [23]. The structure consists of a reference prefilter and a feedback controller. By comparing Figs. 3(a) and 3(b), it can be realized that the loop gains of the closed-loop system become unavoidably different for the measurement options. The loop gain for the GCM is [cf. Fig. 3(a)]

$$L_g(z) = z^{-1}Y_{gc}(z)G_g(z) \quad (8)$$

and the closed-loop response is

$$i_g(z) = \underbrace{\frac{F_g(z)L_g(z)}{1+L_g(z)}}_{G_r(z)} i_{g,ref}(z) + \underbrace{\frac{Y_{gg}(z)}{1+L_g(z)}}_{Y_d(z)} e_g(z) \quad (9)$$

where G_r is the closed-loop reference-tracking transfer function and Y_d is the closed-loop admittance for the disturbance rejection. According to Fig. 3(b), the loop gain for the CCM is

$$L_c(z) = z^{-1}Y_{cc}(z)G_c(z) \quad (10)$$

and the closed-loop response can be written as

$$i_g(z) = \underbrace{\frac{F_c(z)L_c(z)H_c(z)}{1+L_c(z)}}_{G_r(z)} i_{g,ref}(z) + \underbrace{\left[\frac{H_c(z)Y_{cg}(z)}{1+L_c(z)} + H_e(z)Y_{cg}(z) \right]}_{Y_d(z)} e_g(z). \quad (11)$$

The reference prefilter (F_g or F_c) and the loop gain (L_g or L_c) define the reference-tracking performance. In order to provide the basis for the comparison between the current measurement cases, an equal closed-loop reference-tracking transfer function G_r is designed for them. The closed-loop

admittance Y_d is different in both cases. The admittance Y_d in the CCM case cannot be affected using only the loop gain as that in the GCM case.

III. CURRENT CONTROL

Fig. 5 shows the overall block diagram of the current control system. The current controller operates in PCC-voltage coordinates, where $u_g = u_g + j0$. The DC-link voltage u_{dc} is measured for the PWM and the PCC voltage is measured for the phase-locked loop (PLL). An AC-voltage controller can also be used for operation in weak grids in order to keep the PCC voltage at 1 p.u. [10].

Fig. 6 shows observer-based state-space current control in more detail. Firstly, full-state feedback control is designed assuming all the states are available, and secondly, the reduced-order observer is designed separately based on the separation principle [12].

A. Grid Current Measurement (GCM)

The reduced-order observer-based current controller based on the GCM is presented in detail in [10]. In accordance with Fig. 6, the control law is

$$u_{c,ref}(k) = k_t i_{g,ref}(k) + \frac{k_i}{z-1} [i_{g,ref}(k) - i_g(k)] - K \hat{x}_d(k) \quad (12)$$

where $\hat{x}_d = [i_g, \hat{i}_c, \hat{u}_f, u_c]^T$ is the state vector combined with the measured state, estimated states, and delayed voltage reference, k_t is the feedforward gain, k_i is the integral gain, and K is the state-feedback gain. The integral state eliminates the steady-state error between the grid current reference and measured grid current, i.e., $i_{g,ref} = i_g$. The reference feedforward improves the reference-tracking performance [12]. The gains are calculated based on the direct pole-placement method after defining the desired locations of the closed-loop poles. A reduced-order observer estimates the unmeasured states $x_r = [i_c, u_f]^T$ as explained in [10].

B. Converter Current Measurement (CCM)

1) *State-Space Current Controller*: The design of the current controller based on the CCM is presented in detail in [5]. According to Fig. 6, the control law is

$$u_{c,ref}(k) = k_t i_{c,ref}(k) + \frac{k_i}{z-1} [i_{c,ref}(k) - i_c(k)] - K \hat{x}_d(k) \quad (13)$$

where $\hat{x}_d = [i_c, \hat{u}_f, \hat{i}_g, u_c]^T$. Here, the integrator forces the steady-state error to be zero between the converter current reference and measured converter current, i.e., $i_{c,ref} = i_c$. The gains are calculated analogously to the GCM case based on the direct pole-placement method [5].

The full-order observer in [5] is replaced with a reduced-order observer in order to make the current controllers comparable, i.e., both cases have equal number of closed-loop poles. The observer estimates the unmeasured states $x_r = [u_f, i_g]^T$. The observer design for the CCM case is analogous to that

Fig. 6. Observer-based state-space current control based on the GCM and CCM. The rated grid voltage is denoted by u_{gN} . For the GCM, $\hat{\mathbf{x}}_r = [\hat{i}_c, \hat{u}_f]^T$ and for the CCM, $\hat{\mathbf{x}}_r = [\hat{u}_f, \hat{i}_g]^T$. A simple realizable-reference anti-windup is used in the current control [22], but not shown in the figure for simplicity.

Fig. 7. Pole locations under nominal conditions ($L_g = 0$): open-loop poles (blue); control poles (green); observer poles (red). The control parameters are: $\alpha_c = 2\pi \cdot 400$ rad/s and $\zeta_r = \zeta_o = 1$. The sampling period is $T_s = 100$ μ s.

in the GCM case as explained in [10]. The reduced-order observer can be formulated as

$$\begin{aligned} \hat{\mathbf{x}}_r(k) = & \Phi_{22}\hat{\mathbf{x}}_r(k-1) + \Phi_{21}\mathbf{i}_c(k-1) + \Gamma_{c2}\mathbf{u}_c(k-1) \\ & + \mathbf{K}_o[\mathbf{i}_c(k) - \phi_{11}\mathbf{i}_c(k-1) \\ & - \gamma_{c1}\mathbf{u}_c(k-1) - \Phi_{12}\hat{\mathbf{x}}_r(k-1)] \end{aligned} \quad (14)$$

where $\mathbf{K}_o = [\mathbf{k}_{o1}, \mathbf{k}_{o2}]^T$ is the observer gain. The matrix elements ϕ_{11} , Φ_{12} , Φ_{21} , and Φ_{22} are the submatrices of Φ and γ_{c1} and Γ_{c2} are the submatrices of Γ_c .

2) *Reference Calculation*: As already mentioned, the grid current is chosen as the controlled variable in both cases. For the CCM, the grid current reference $i_{g,\text{ref}}$ is translated to the converter current reference $i_{c,\text{ref}}$ as shown in Fig. 6. According to Fig. 2, the converter current in the steady state under strong grid conditions ($L_g = 0$ or $\mathbf{u}_g = \mathbf{e}_g$) is

$$\mathbf{i}_c = (1 - \omega_g^2 C_f L_{fg})\mathbf{i}_g + j\omega_g C_f \mathbf{u}_g. \quad (15)$$

C. Selection of Poles Under Nominal Grid Conditions

1) *Parameter Selection*: The controller gains k_t , k_i , $\mathbf{K}$, and $\mathbf{K}_o$ are calculated according to the pole-placement method

TABLE I
NOMINAL PARAMETERS OF A 12.5-KVA CONVERTER SYSTEM

Parameter	Value	Value (p.u.)
<i>LCL filter</i>		
Converter-side inductance L_{fc}	3.3 mH	0.081
Grid-side inductance L_{fg}	3.0 mH	0.074
Capacitance C_f	8.8 μ F	0.036
<i>Grid</i>		
Inductance L_g	0	0
Angular frequency ω_g	$2\pi \cdot 50$ rad/s	1
Voltage (phase-neutral, peak)	$\sqrt{2/3} \cdot 400$ V	1
<i>Converter</i>		
Rated current (peak)	$\sqrt{2} \cdot 18$ A	1
DC-bus voltage u_{dc}	650 V	2
Sampling frequency f_s	10 kHz	200

[5], [10]. In order to calculate the controller gains, the system parameters have to be known and the closed-loop poles of the system have to be selected. The nominal parameters of the converter and the LCL filter are typically known at the design stage. However, the grid inductance L_g is often unknown. If the grid inductance is considered as a part of the grid-side inductance of the LCL filter, the actual inductance on the grid side becomes $L_{fg} + L_g$, cf. Fig. 2. In this paper, the nominal parameters of the LCL filter are used in the gain calculation, i.e., a strong grid is assumed ($L_g = 0$).

2) *Control Poles*: Fig. 7 shows the three poles of the open-loop system (3) under the strong grid assumption, located at

$$\begin{aligned} p_{ol,1,2} &= \exp[-j(\omega_g \pm \omega_p)T_s] \\ p_{ol,3} &= \exp(-j\omega_g T_s). \end{aligned} \quad (16)$$

The computational delay and the integral action add two more poles in the closed-loop system. The closed-loop control pole locations are parametrized as [5]

$$\begin{aligned} p_{1,2} &= \exp\left[\left(-\zeta_r \pm j\sqrt{1-\zeta_r^2}\right)\omega_p T_s\right] \\ p_{3,4} &= \exp(-\alpha_c T_s) \quad p_5 = 0 \end{aligned} \quad (17)$$

where ζ_r is the damping factor of the resonant poles and α_c is the approximate bandwidth of the current control. The

Fig. 8. Loci of the closed-loop poles (GCM: blue and CCM: red) when the grid inductance is increased in the range $L_g = 0 \dots 1$ p.u.

undamped natural frequency ω_p of the resonant pole pair $p_{ol,1,2}$ is not altered, since the radial projection minimizes control effort by adding damping to the undamped or lightly-damped open-loop poles [12]. The open-loop pole $p_{ol,3}$ and the pole originating from the integral state are placed to determine the dominant dynamics. Here, the dominant dynamics are determined by the pair $p_{3,4}$ of double real poles [5], [10]. The pole originating from the computational delay is not moved (p_5 is already in the optimal location). The referenced-feedforward zero is placed to cancel one of the control poles at $p_{3,4}$ [5]. Fig. 7 shows the resulting control poles under nominal conditions for $\zeta_r = 1$ and $\alpha_c = 2\pi \cdot 400$ rad/s.

3) *Observer Poles:* The observer pole locations are parametrized as [5]

$$p_{ol,1,2} = \exp \left[\left(-\zeta_o \pm j\sqrt{1 - \zeta_o^2} \right) \omega_p T_s \right] \quad (18)$$

where ζ_o is the damping factor of the observer poles. Fig. 7 also shows the observer poles for $\zeta_o = 1$. This selection gives a pair of real poles at the same location as $p_{1,2}$. It is worth mentioning that the closed-loop poles are placed at the same locations in both current measurement cases under nominal conditions. The closed-loop poles are the union of the control poles and the observer poles [12].

IV. PERFORMANCE ANALYSIS

The nominal system parameters, given in Table I, are used in the gain calculation and in the observer matrices. The grid is assumed to be strong ($L_g = 0$) under nominal conditions, as explained in Section III-C1. The same design parameters are used in both current measurement cases: $\zeta_r = \zeta_o = 1$ and $\alpha_c = 2\pi \cdot 400$ rad/s. The resonance frequency of the system under nominal conditions is 1.34 kHz, which is below the critical frequency of 1.67 kHz. In this paper, the grid is referred to as strong if $L_g = 0$ and very weak if $L_g = 1$ p.u.

Fig. 9. Frequency response of the transfer function $G_{Re}(z)$ in (20): (a) strong grid; (b) very weak grid.

Fig. 10. Frequency response of the transfer function from the grid voltage e_g to the grid current i_g in synchronous coordinates: (a) closed-loop response under strong grid conditions; (b) very weak grid conditions. Green line shows the open-loop transfer function Y_{gg} .

A. Loci of the Closed-Loop Poles

Fig. 8 shows the loci of the closed-loop poles for both current measurement cases as the grid inductance varies in the range $L_g = 0 \dots 1$ p.u. The green crosses show the pole locations under nominal conditions, i.e., $L_g = 0$, corresponding to Fig. 7. When the grid inductance increases, the poles move toward the unit circle. The blue crosses show the pole locations under very weak grid conditions for the GCM and the red crosses for the CCM. All the closed-loop poles remain inside the unit circle in the both cases, i.e., the systems are stable.

Fig. 11. Stability maps for the variations of L_g and the LCL-filter capacitance C_f : (a), (b), and (c) GCM; (d), (e), and (f) CCM. Between the unity contour (red lines), all the eigenvalues of the closed-loop systems are inside the unit circle. Dashed lines show the contours of the lowest damping ratios of the eigenvalues. The real LCL-filter inductances are: (a) and (d) nominal; (b) and (e) 20% larger than the nominal values; (c) and (f) 20% smaller than the nominal values.

B. Reference Tracking

According to (9) and (11), the closed-loop reference-tracking transfer function for both cases is denoted by

$$\frac{i_g(z)}{i_{g,\text{ref}}(z)} = G_r(z) = G_{\text{Re}}(z) + jG_{\text{Im}}(z). \quad (19)$$

The transfer function G_r is equal in both cases under nominal conditions. When the disturbances are omitted, the d -component of the grid current is

$$i_{gd}(z) = G_{\text{Re}}(z)i_{gd,\text{ref}}(z) - G_{\text{Im}}(z)i_{gq,\text{ref}}(z). \quad (20)$$

Fig. 9(a) shows the frequency responses of the transfer function G_{Re} in (20) for both cases under nominal conditions. The closed-loop systems are identical and critically damped. Fig. 9(b) shows G_{Re} under very weak grid conditions. The closed-loop systems become underdamped in weak grids, cf. Fig. 8. As shown in Figs. 8 and 9(b), the CCM provides better damping for the resonance-frequency poles. In addition, it also provides larger realized bandwidth of the closed-loop system [see Fig. 9(b)].

C. Disturbance Rejection

Fig. 10(a) shows the frequency responses of the closed-loop admittance Y_d for both measurement cases under nominal conditions, cf. (9) and (11). The magnitude is zero at the zero frequency in both cases, which indicates perfect disturbance rejection at the zero frequency (in synchronous coordinates). The GCM also rejects the disturbance at the antiresonance frequency of the open-loop admittance Y_{gg} . As a consequence, the admittance Y_d has a higher peak value in this case below the antiresonance frequency. Hence, the GCM case becomes more sensitive to the disturbances at lower-order harmonic

frequencies (5th, 7th, and 11th) under strong grid conditions. In addition, it has a lower disturbance-rejection bandwidth¹ (151 Hz) as compared to the CCM (311 Hz). A smaller bandwidth corresponds to slower disturbance rejection.

Fig. 10(b) shows the frequency responses of the closed-loop admittance Y_d for both cases under very weak grid conditions. The disturbance rejection becomes slower in both cases due to the low realized disturbance-rejection bandwidth. Since the CCM case has higher disturbance-rejection bandwidth under very weak grid conditions, it provides better disturbance rejection in comparison with the GCM.

D. Parameter Sensitivity

In both current measurement cases, the sensitivity of the control methods to the parameter errors in the LCL filter is studied by computing the eigenvalues of the closed-loop systems. The nominal LCL filter parameters are used in the control system. The following three cases are considered in the real plant: 1) L_{fc} and L_{fg} are nominal; 2) L_{fc} and L_{fg} are 20% larger than the nominal values; and 3) L_{fc} and L_{fg} are 20% smaller than the nominal values.

Fig. 11 shows the regions where all the eigenvalues are inside the unit circle for both cases, i.e., the systems are stable. The real filter capacitance C_f^r is varied in the range $C_f^r = 0 \dots 3C_f$ and the grid inductance is varied in the range $L_g = 0 \dots 1$ p.u. The figure also shows the contours for the lowest damping ratios of the closed-loop poles. Even though the lowest damping ratios are small in the case of very weak grids, the closed-loop poles remain inside the unit circle. This example indicates that the controller provides stable operation

¹A disturbance-rejection bandwidth is the frequency where the sensitivity function first crosses 0.707 (−3 dB) from below [23].

Fig. 12. Measured step responses of the grid current components i_{gd} and i_{gq} : (a) strong grid; (b) very weak grid.

in both measurement cases for a wide range of the parameter variations in the plant.

The closed-loop system is better damped with the CCM as compared to the GCM for the whole range of the grid inductance variations when the real filter inductances are nominal, as shown in Figs. 11(a) and 11(d). The GCM provides better damping under strong grid conditions (e.g. $L_g = 0.1$ p.u.) if $C_f^r \approx C_f$ and the real filter inductances are larger [Figs. 11(b) and 11(e)] or smaller [Figs. 11(c) and 11(f)] than the nominal values. As can be seen from the overall analysis, the GCM provides similar damping in all the cases of the parameter variations. In the CCM case, the lowest damping ratios depend significantly on the parameter errors in the LCL filter.

V. EXPERIMENTAL RESULTS

Both current measurement cases are verified by means of experiments using a three-phase 12.5-kVA 50-Hz grid-connected converter (Table I). The switching frequency of the converter is 5 kHz and synchronous sampling (twice per carrier) is used. The design parameters of the current controller are: $\alpha_c = 2\pi \cdot 400$ rad/s and $\zeta_r = \zeta_o = 1$ and the strong grid is assumed in control tuning for both cases. A slow PLL having the bandwidth of $2\pi \cdot 2$ rad/s is used in order to avoid the coupling between the current control and PLL dynamics [15].

Fig. 13. Measured responses of i_{gd} and i_{gq} when a grid-voltage dip of 0.5 p.u. is applied at 0.01 s: (a) strong grid; (b) very weak grid. The grid voltages (phase-neutral) are denoted by e_{ga} , e_{gb} , and e_{gc} .

Two different grid conditions were tested: strong grid ($L_g = 0$) and very weak grid ($L_g = 1$ p.u.).

Fig. 12 shows the measured responses of the grid current components i_{gd} and i_{gq} , when a step of 0.2 p.u. is applied to the current reference $i_{gd,ref}$. Fig. 12(a) shows the responses under strong grid (nominal) conditions. Both cases provide equal dynamics, since they have equal closed-loop poles and equal reference-tracking performance, cf. Figs. 7 and 9(a). As expected, the responses under strong grid conditions are critically damped. The realized dynamics are close to the desired dynamics. The rise time of the measured response i_{gd} is approximately 1.1 ms, which is slightly greater than the rise time (0.9 ms) of a first-order system with the same nominal bandwidth, due to the effect of the extra closed-loop poles.

Fig. 12(b) shows the measured responses under very weak grid conditions ($L_g = 1$ p.u.). The closed-loop responses are underdamped, cf. Figs. 8 and 9(b). The realized dynamics are slower than the specified nominal dynamics in both cases. Due to the larger realized bandwidth, as shown in Fig. 9(b), the CCM provides a shorter rise time of 2.1 ms as compared to the rise time (3.7 ms) of the GCM case. However, the CCM leads to a steady-state error of 3.6% under very weak grid conditions. The steady-state error originates from the inaccurate inductance estimate in (15), since the grid inductance is assumed to be zero under nominal conditions.

Fig. 13 shows the measured responses of the grid current components i_{gd} and i_{gq} , when a balanced grid-voltage dip of 0.5 p.u. is applied. The grid-voltage dip was generated using a 50-kVA three-phase four-quadrant power supply (Regatron TopCon TC.ACS). The converter supplies the active power of 0.3 p.u. to the grid. Fig. 13(a) shows the measured responses under strong grid conditions and Fig. 13(b) under very weak grid conditions. As can be seen, both measurement cases reject the grid-voltage dip well under both grid conditions. The CCM provides faster disturbance rejection in comparison with the GCM case, as expected based on Fig. 10. In the case of very weak grids, slower dynamics are encountered as compared to the strong grid conditions. The measured settling times with the CCM are 2.6 ms and 6.8 ms under strong and very weak grid conditions, respectively. The GCM case rejects the disturbances with the settling time of 4.8 ms under strong grid conditions and of 26.6 ms under very weak grid conditions.

VI. CONCLUSION

This paper presented a comparison between the converter and grid current measurements in discrete-time state-space control. The performance of both measurement cases are evaluated by means of analysis and experiments under strong and very weak grid conditions. Equal reference-tracking performance is designed for both cases under strong grid conditions. The presented design example, where the LCL-filter resonance frequency is below the critical frequency, provides stable operation even under very weak grid conditions, without changing the tuning of the control systems. The CCM provides faster reference tracking, faster disturbance rejection, and better resonance damping than the GCM under very weak grid conditions. Finally, it is worth mentioning that the robustness against the grid strength variations is different when the filter resonance frequency is above the critical frequency, which will be studied in the future research.

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