This is an electronic reprint of the original article.
This reprint may differ from the original in pagination and typographic detail.

Kong, Beibei; Yla-Oijala, Pasi; Sihvola, Ari

A discontinuous Galerkin surface integral equation method for scattering from IBC targets

Published in:
INTERNATIONAL JOURNAL OF NUMERICAL MODELLING: ELECTRONIC NETWORKS DEVICES AND FIELDS

DOI:
[10.1002/jnm.2650](https://doi.org/10.1002/jnm.2650)

Published: 01/03/2020

Document Version
Peer reviewed version

Please cite the original version:

Kong, B., Yla-Oijala, P., & Sihvola, A. (2020). A discontinuous Galerkin surface integral equation method for scattering from IBC targets. *INTERNATIONAL JOURNAL OF NUMERICAL MODELLING: ELECTRONIC NETWORKS DEVICES AND FIELDS*, 33(2), [e2650]. <https://doi.org/10.1002/jnm.2650>

This material is protected by copyright and other intellectual property rights, and duplication or sale of all or part of any of the repository collections is not permitted, except that material may be duplicated by you for your research use or educational purposes in electronic or print form. You must obtain permission for any other use. Electronic or print copies may not be offered, whether for sale or otherwise to anyone who is not an authorised user.

ARTICLE TYPE

A Discontinuous Galerkin Surface Integral Equation Method for Scattering from IBC Targets

Beibei Kong* | Pasi Ylä-Oijala | Ari Sihvola

¹Department of Electronics and Nanoengineering, School of Electrical Engineering, Aalto University, Finland

Correspondence

*Beibei Kong, Department of Electronics and Nanoengineering, Aalto University, 00076 Espoo, Finland. Email: beibei.kong@aalto.fi

Summary

A discontinuous Galerkin (DG) surface integral equation method is proposed for electromagnetic scattering from targets with the impedance boundary condition (IBC). We present electric field integral equation (EFIE), magnetic field integral equation (MFIE), and self-dual integral equation formulations for the problem, and study their numerical performance. Based on the results of these experiments, DG is developed for EFIE and MFIE (DG-EFIE and DG-MFIE). The convergence of the iterative solutions and the solution accuracy of DG-EFIE and DG-MFIE are further investigated for a wide range of surface impedances. The numerical performance of these formulations are found to be nearly complementary with respect to the surface impedance. The capability of the proposed DG solution strategy for calculating scattering from large multi-scale IBC targets is demonstrated.

KEYWORDS:

Discontinuous Galerkin method, impedance boundary condition, electric field integral equation, magnetic field integral equation, self-dual integral equation, multi-scale target.

1 | INTRODUCTION

Surface integral equation (SIE) methods are widely used in solving electromagnetic scattering from non-penetrable objects, in which the material-incident wave interaction can be characterized by boundary conditions defined on the surface of the structure^{1,2}. The perfect electric conductor (PEC) and the perfect magnetic conductor (PMC) are well-known examples of boundary conditions for non-penetrable targets. However, these boundary conditions are only available in ideal lossless cases. The impedance boundary condition (IBC)³, known also as the Leontovitch condition⁴, provides an approximative model for more general imperfectly conducting bodies. Recently, IBC has also been applied in modelling metasurfaces^{5,6}. In these structures, the surface impedance can possess very different values and thus requiring us to develop numerical solutions for a wide range of surface impedance.

Various SIE formulations for IBC targets have been presented in the literature^{7,8,9,10,11,12,13,14,15}. In the classical approach one of the surface currents is eliminated from the integral equations according to the IBC^{7,8,9}. However, when using Rao-Wilton-Glisson (RWG) functions as basis and test functions, hypersingular line integral are generated that are extremely difficult to evaluate numerically¹⁰. Moreover, the physics is not correctly modelled by expanding one of the currents with RWG functions, and the other one with rotated RWG functions¹¹. Recently, another approach is proposed to enforce IBC in the weak sense^{10,11,12,13,14,15}. In this approach IBC is combined with SIE as an extra equation and the highly singular line integrals caused by the rotation operator under the electric field integral operator can be avoided. The resulting equations can be discretized, either using RWG functions only^{12,13}, or as in¹⁰, utilizing both RWG and dual (Buffa-Christiansen) functions. The so called self-dual integral equation (SDIE) has been shown to have a good performance in solving electromagnetic scattering from large IBC objects¹³. The combined source integral equation, with a mixed testing scheme, is found to work well close to strong physical resonances¹⁵. However, these formulations do not support non-conformal meshes and may not be efficient in solving large multi-scale problems.

The discontinuous Galerkin (DG) SIE method is an attractive technique for solving multi-scale problems^{16,17}. The allowance of non-conformal meshes renders this method as being very flexible and effective in handling problems with different geometrical characteristics and details. In the DG method, the computational domain is first divided into smaller sub-domains. By independently meshing each sub-domain, and developing a preconditioner based on the domain decomposition, the mesh generation process can be significantly simplified and the condition of the matrix is improved^{17,18,19}. SIE-DG methods have been developed for PEC targets^{16,17,18,20} and for homogeneous dielectric objects¹⁹. However, objects with IBC have not been treated with this technique.

The goal of this paper is to introduce an efficient DG-SIE method for the scattering analysis of large multi-scale IBC targets. First, we perform a comparison of numerical performance of the tangential electric field integral equation (EFIE), magnetic field integral equation (MFIE), and SDIE. Though the EFIE and MFIE suffer from non-physical spurious internal resonances⁸, the accuracy of these formulations is found to be much better than that of SDIE, especially near the physical resonances²¹. Due to the good accuracy of EFIE and MFIE, DG is developed for EFIE and MFIE, giving DG-EFIE and DG-MFIE, respectively. An efficient block-diagonal preconditioner is developed by utilizing the matrix blocks of the sub-domains.

Our numerical experiments illustrate that DG-EFIE and DG-MFIE lead to good numerical performance (fast convergence and high accuracy) on nearly complementary values of the surface impedance. Hence, DG-EFIE and DG-MFIE can be jointly used in their respective ranges of the surface impedance. The capability of the proposed DG approach for the simulation of multi-scale IBC targets is also demonstrated.

2 | FORMULATION

Let us consider electromagnetic scattering by an arbitrarily shaped nonpenetrable object in free space with permittivity ε and permeability μ . The object is illuminated by an incident plane wave ($\mathbf{E}^{\text{inc}}, \mathbf{H}^{\text{inc}}$). Let $\mathbf{n}$ denote the exterior unit normal of the surface S pointing into the background. On the surface of the object, we assume the impedance boundary condition (IBC)

$$\gamma_t \mathbf{E} = Z_s \mathbf{n} \times \gamma_t (\eta \mathbf{H}). \quad (1)$$

Here Z_s is the normalized (dimensionless) surface impedance on S and $\gamma_t \mathbf{F} = (\mathcal{I} - \mathbf{nn} \cdot) \mathbf{F}|_S$ denotes tangential component of a vector field on the surface S , where $\mathcal{I}\{\mathbf{F}\} = \mathbf{F}$ is the identity operator.

2.1 | Surface Integral Equations

We continue by formulating three alternative SIE formulations for solving scattering by IBC targets. By utilizing the surface equivalence principle, tangential field integral equations can be formulated as follows

$$\begin{bmatrix} \gamma_t \mathcal{T} & -\gamma_t \mathcal{K} - \frac{1}{2} \mathbf{n} \times \mathcal{I} \\ \gamma_t \mathcal{K} + \frac{1}{2} \mathbf{n} \times \mathcal{I} & \gamma_t \mathcal{T} \end{bmatrix} \begin{bmatrix} \eta \mathbf{J} \\ \mathbf{M} \end{bmatrix} = \begin{bmatrix} -\gamma_t \mathbf{E}^{\text{inc}} \\ -\eta \gamma_t \mathbf{H}^{\text{inc}} \end{bmatrix}. \quad (2)$$

Here $\eta = \sqrt{\mu/\varepsilon}$ is the wave impedance of free space, $\mathbf{J} = \mathbf{n} \times \mathbf{H}$ and $\mathbf{M} = -\mathbf{n} \times \mathbf{E}$ are the equivalent electric and magnetic surface current densities on S , respectively. The integro-differential operators $\mathcal{T}$ and $\mathcal{K}$ are defined as

$$\mathcal{T}\{\mathbf{X}\}(\mathbf{r}) = \text{ik} \int_S \left[1 + \frac{1}{k^2} \nabla \nabla \cdot \right] \mathbf{X}(\mathbf{r}') G(\mathbf{r}, \mathbf{r}') dS' \quad (3)$$

$$\mathcal{K}\{\mathbf{X}\}(\mathbf{r}) = -\text{p.v.} \int_S \mathbf{X}(\mathbf{r}') \times \nabla G(\mathbf{r}, \mathbf{r}') dS' \quad (4)$$

where $G(\mathbf{r}, \mathbf{r}') = e^{ik|\mathbf{r}-\mathbf{r}'|}/4\pi|\mathbf{r}-\mathbf{r}'|$ is the Green's function of free space with wave number k , and p.v. stands for the principal value integral.

According to the IBC (1), the surface currents have the following relationship

$$\begin{bmatrix} \mathcal{I} & -\frac{1}{Z_s} \mathbf{n} \times \mathcal{I} \\ Z_s \mathbf{n} \times \mathcal{I} & \mathcal{I} \end{bmatrix} \begin{bmatrix} \eta \mathbf{J} \\ \mathbf{M} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \quad (5)$$

Combining the second row of (5) with the first row of Equation (2), gives the electric field integral equation (EFIE) for IBC. This equation can be written as¹²

$$\begin{bmatrix} \gamma_t \mathcal{T} & -\gamma_t \mathcal{K} - \frac{1}{2} \mathbf{n} \times \mathcal{I} \\ Z_s \mathbf{n} \times \mathcal{I} & \mathcal{I} \end{bmatrix} \begin{bmatrix} \eta \mathbf{J} \\ \mathbf{M} \end{bmatrix} = \begin{bmatrix} -\gamma_t \mathbf{E}^{\text{inc}} \\ 0 \end{bmatrix}. \quad (6)$$

Similarly, the magnetic field integral equation (MFIE) is obtained by combining the first row of (5) with the second row of Equation (2)¹⁵

$$\begin{bmatrix} \gamma_t \mathcal{T} & \gamma_t \mathcal{K} + \frac{1}{2} \mathbf{n} \times \mathcal{I} \\ -\frac{1}{Z_s} \mathbf{n} \times \mathcal{I} & \mathcal{I} \end{bmatrix} \begin{bmatrix} \mathbf{M} \\ \eta \mathbf{J} \end{bmatrix} = \begin{bmatrix} -\eta \gamma_t \mathbf{H}^{\text{inc}} \\ 0 \end{bmatrix}. \quad (7)$$

Another type of SIE for IBC is the self-dual integral equation (SDIE) introduced in¹³. In this equation, both the tangential EFIE and MFIE given in (2) are used and combined with the IBC (5). Scaling the equations with $\sqrt{Z_s}$, SDIE can be expressed with¹³

$$\begin{bmatrix} \gamma_t \mathcal{T} - \sqrt{Z_s} \mathcal{I} & -\gamma_t \mathcal{K} - \frac{1}{2} \mathbf{n} \times \mathcal{I} + \frac{1}{\sqrt{Z_s}} \mathbf{n} \times \mathcal{I} \\ \gamma_t \mathcal{K} + \frac{1}{2} \mathbf{n} \times \mathcal{I} - \sqrt{Z_s} \mathbf{n} \times \mathcal{I} & \gamma_t \mathcal{T} - \frac{1}{\sqrt{Z_s}} \mathcal{I} \end{bmatrix} \begin{bmatrix} \eta \mathbf{J} \\ \mathbf{M} \end{bmatrix} = \begin{bmatrix} -\gamma_t \mathbf{E}^{\text{inc}} \\ -\eta \gamma_t \mathbf{H}^{\text{inc}} \end{bmatrix}. \quad (8)$$

By discretizing the equations with RWG basis functions and by applying Galerkin's testing method, the discretized matrix equations for (6), (7) and (8) can be expressed as follows

$$\begin{bmatrix} \mathbf{T} & -\mathbf{K} - \frac{1}{2} \mathbf{R} \\ Z_s \mathbf{R} & \mathbf{G} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{x}}^J \\ \mathbf{x}^M \end{bmatrix} = \begin{bmatrix} -\mathbf{b}^E \\ 0 \end{bmatrix}, \quad (9)$$

$$\begin{bmatrix} \mathbf{T} & \mathbf{K} + \frac{1}{2} \mathbf{R} \\ -\frac{1}{Z_s} \mathbf{R} & \mathbf{G} \end{bmatrix} \begin{bmatrix} \mathbf{x}^M \\ \tilde{\mathbf{x}}^J \end{bmatrix} = \begin{bmatrix} -\tilde{\mathbf{b}}^H \\ 0 \end{bmatrix}, \quad (10)$$

$$\begin{bmatrix} \mathbf{T} - \sqrt{Z_s} \mathbf{G} & -\mathbf{K} - \frac{1}{2} \mathbf{R} + \frac{1}{\sqrt{Z_s}} \mathbf{R} \\ \mathbf{K} + \frac{1}{2} \mathbf{R} - \sqrt{Z_s} \mathbf{R} & \mathbf{T} - \frac{1}{\sqrt{Z_s}} \mathbf{G} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{x}}^J \\ \mathbf{x}^M \end{bmatrix} = \begin{bmatrix} -\mathbf{b}^E \\ -\tilde{\mathbf{b}}^H \end{bmatrix}. \quad (11)$$

Here $\mathbf{T}$, $\mathbf{K}$, $\mathbf{R}$ and $\mathbf{G}$ are the matrices corresponding to operators $\gamma_t \mathcal{T}$, $\gamma_t \mathcal{K}$, $\mathbf{n} \times \mathcal{I}$ and $\mathcal{I}$. Vectors $\tilde{\mathbf{x}}^J$ and $\mathbf{x}^M$ contain the coefficients of the RWG basis function approximations of $\eta \mathbf{J}$ and $\mathbf{M}$, and $\mathbf{b}^E$ and $\tilde{\mathbf{b}}^H$ are the excitation vectors due to the incident fields $\gamma_t \mathbf{E}^{\text{inc}}$ and $\eta \gamma_t \mathbf{H}^{\text{inc}}$, respectively.

2.2 | Discontinuous Galerkin Method

In the discontinuous Galerkin (DG) method, we first decompose the surface of the object into several nonoverlapping sub-domains $S_1, S_2, \dots, S_M$. Let C denote the contour boundary of these sub-domains. Each sub-domain can be independently meshed with planar triangular patches. The meshes on adjacent sub-domains can be nonconformal. Since full RWG functions require conformal mesh, half-RWG functions²² are applied on the edges on the contour boundary of the subdomains.

In the DG method, the elements of the matrices $\mathbf{K}$, $\mathbf{R}$ and $\mathbf{G}$ are similarly computed as in the traditional method of moments with full RWG functions. The elements of $\mathbf{T}$ can also be evaluated using traditional methods if both the basis and test functions are full RWG functions. However, since half-RWG functions do not satisfy current continuity from an element to another, the calculation of the elements of $\mathbf{T}$ requires special attention

when either a basis or a test function, or both, is defined on an edge at the contour boundary. Let $\mathbf{g}_i$ be a test function and $\mathbf{g}_j$ a basis function associated with an edge on the contour boundary. For the sake of simplicity here we consider only the case when both the basis and test functions are half-RWG functions. Then the elements of $\mathbf{T}$ read¹⁶

$$\begin{aligned} \mathbf{T}_{ij} = & ik \int_{S_i} \int_{S_j} \mathbf{g}_i \cdot \mathbf{g}_j \mathbf{G} dS' dS + \frac{1}{ik} \int_{S_i} \nabla \cdot \mathbf{g}_i \int_{S_j} \nabla' \cdot \mathbf{g}_j \mathbf{G} dS' dS - \frac{1}{ik} \int_{C_j} \mathbf{t}_j \cdot \mathbf{g}_j \int_{S_i} \nabla \cdot \mathbf{g}_i \mathbf{G} dS dC' \\ & - \frac{1}{ik} \int_{C_i} \mathbf{t}_i \cdot \mathbf{g}_i \int_{S_j} \nabla' \cdot \mathbf{g}_j \mathbf{G} dS' dC + \frac{1}{ik} \int_{C_i} \mathbf{t}_i \cdot \mathbf{g}_i \int_{C_j} \mathbf{t}_j \cdot \mathbf{g}_j \mathbf{G} dC' dC, \end{aligned} \quad (12)$$

where S_i and S_j denote the supports $\mathbf{g}_i$ and $\mathbf{g}_j$, i.e., patches with at least one edge on the contour boundary. These edges are denoted by C_i and C_j , respectively, and $\mathbf{t}_i$ denotes the unit vector normal to C_i , pointing away from S_i .

The challenge in (12) is that the last term on the right hand side is extremely difficult to evaluate numerically, when the field point ($\mathbf{r}$) approaches the source point ($\mathbf{r}'$). According to the current continuity on the contour boundary, the last term in (12) can be canceled by adding a penalty term corresponding to the field of the charge accumulated on the contour boundary¹⁶. Another boundary penalty term corresponding directly to the accumulation of the charge on the contour boundary reads¹⁶

$$\frac{1}{ik} \sum_{C_i \in C} \int_{C_i} \mathbf{t}_i \cdot \mathbf{g}_i (\mathbf{t}_i \cdot \mathbf{g}_i + \mathbf{t}_j \cdot \mathbf{g}_j) dC = 0. \quad (13)$$

In our approach, this term is added to $\mathbf{T}$ to enforce the current continuity across the contour boundary and to adjust the numerical performance of the DG method. With this additional penalty term the elements of $\mathbf{T}$ are expressed as

$$\begin{aligned} \mathbf{T}_{ij} = & ik \int_{S_i} \int_{S_j} \mathbf{g}_i \cdot \mathbf{g}_j \mathbf{G} dS' dS + \frac{1}{ik} \int_{S_i} \nabla \cdot \mathbf{g}_i \int_{S_j} \nabla' \cdot \mathbf{g}_j \mathbf{G} dS' dS - \frac{1}{ik} \int_{C_j} \mathbf{t}_j \cdot \mathbf{g}_j \int_{S_i} \nabla \cdot \mathbf{g}_i \mathbf{G} dS dC' \\ & - \frac{1}{ik} \int_{C_i} \mathbf{t}_i \cdot \mathbf{g}_i \int_{S_j} \nabla' \cdot \mathbf{g}_j \mathbf{G} dS' dC + \beta \frac{1}{ik} \int_{C_i} (\mathbf{t}_i \cdot \mathbf{g}_i) (\mathbf{t}_j \cdot \mathbf{g}_j) dC. \end{aligned} \quad (14)$$

Here β is an arbitrary coefficient added to further adjust the boundary penalty term (13). The optimal value of β will be determined with numerical experiments in Section 3.2.

2.3 | Block Diagonal Preconditioner

Next a block diagonal preconditioner is developed for the DG method. The discretized matrix of the entire structure is expressed with the sub-matrices corresponding to the sub-domains according to the domain decomposition. The preconditioned matrix equation can then be expressed as

$$\mathbf{P}^{-1} \begin{bmatrix} \mathbf{A}_1 & \mathbf{C}_{12} & \cdots & \mathbf{C}_{1M} \\ \mathbf{C}_{21} & \mathbf{A}_2 & \cdots & \mathbf{C}_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{C}_{M1} & \mathbf{C}_{M2} & \cdots & \mathbf{A}_M \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_M \end{bmatrix} = \mathbf{P}^{-1} \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \vdots \\ \mathbf{b}_M \end{bmatrix} \quad (15)$$

where the block-diagonal preconditioner is

$$\mathbf{P}^{-1} = \begin{bmatrix} \mathbf{A}_1^{-1} & 0 & \cdots & 0 \\ 0 & \mathbf{A}_2^{-1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{A}_M^{-1} \end{bmatrix}. \quad (16)$$

Here $\mathbf{A}_i$ is the matrix for a sub-domain S_i and $\mathbf{C}_{ij}$ is the interaction matrix between sub-domains S_i and S_j . Vectors $\mathbf{x}_i$ and $\mathbf{b}_i$ are the corresponding coefficient and excitation vectors of the sub-domains, respectively.

3 | NUMERICAL EXPERIMENTS

This section presents numerical experiments, which were carried out to investigate the performance of the proposed SIE formulations. Our calculations use both direct and iterative solvers to find solutions for the discretized matrix equations. The iterative solutions employ the generalized minimal residual (GMRES) iterative solver with the dimension of the Krylov subspace of 30. The residual error is 10^{-3} unless otherwise noted.

3.1 | Accuracy and Iteration Convergence

First we compare the accuracy of the solutions and convergence speed of the iterative solutions of EFIE, MFIE and SDIE without DG. We consider an IBC sphere with a radius of 0.5 m. The normalized surface impedance is $Z_s = R_s + C_s i$ ($C_s = -X_s$, where X_s is the surface reactance, and the surface is capacitive when $C_s > 0$), where R_s changes from 0 to 2 and C_s changes from -2 to 2. The sphere is illuminated by an incident plane wave with a frequency of 200 MHz. The mesh size is 100 mm.

To study the accuracy of the solutions, we compute the scattering and absorption efficiency versus surface impedance. First, the total absorbed and scattered power are computed from the solution vectors $\mathbf{x}^J (= \tilde{\mathbf{x}}^J / \eta)$ and $\mathbf{x}^M$ as²³

$$P^{\text{abs}} = \frac{1}{2} \text{Re} \left(\mathbf{x}^J \mathbf{R} \mathbf{x}^M \right) \quad (17)$$

$$P^{\text{sca}} = \frac{1}{2} \text{Re} \left(\mathbf{b}^{E\dagger} \mathbf{x}^J + \mathbf{b}^{H\dagger} \mathbf{x}^M \right) - P^{\text{abs}} \quad (18)$$

where $\mathbf{b}^H = \tilde{\mathbf{b}}^H / \eta$ and $\dagger$ denotes Hermitian transpose. Then the absorption and scattering cross sections are obtained as $\sigma^{\text{abs,sca}} = P^{\text{abs,sca}} / P^{\text{inc}}$, where $P^{\text{inc}} = |\mathbf{E}_0|^2 / (2\eta)$ is the incident power density and $\mathbf{E}_0$ is the polarization vector of $\mathbf{E}^{\text{inc}}$. The absorption and scattering efficiency, Q_{abs} and Q_{sca} , are obtained from the absorption and scattering cross sections by dividing them with the geometrical cross section of the sphere.

The number of iterations required for convergence are shown in Figure 1. In the calculations, the maximum number of iterations is set to 500. It can be seen that SDIE requires far less iteration steps than EFIE and MFIE. Since some of the results do not converge, we calculate Q_{sca} and Q_{abs} with the direct solver to investigate the accuracy of EFIE, MFIE and SDIE. Using the analytical Mie solution²¹ as a reference value (plotted in Figure 2) the relative error of Q_{sca} for EFIE, MFIE and SDIE are displayed in Figure 3(a), Figure 3(c) and Figure 3(d), respectively. Since Q_{abs} is zero for lossless IBC surfaces, Figure 3(b), Figure 3(d) and Figure 3(e) show the absolute error of Q_{abs} . It can be observed that EFIE and MFIE are much more accurate than SDIE practically for all considered values of Z_s . Near the physical resonances that appear for certain purely imaginary values of Z_s ²¹, EFIE is more accurate than MFIE when $\text{Imag}\{Z_s\}$ is positive, while MFIE provides more accurate results close to the resonances when $\text{Imag}\{Z_s\}$ is negative.

3.2 | Optimal Value of the Interior Penalty Term

Due to the good accuracy of EFIE and MFIE, the DG method is developed based for these two equations only, as described in Section 2.2. The resulting formulations are denoted by DG-EFIE and DG-MFIE, respectively. We continue by investigating the effect of the interior penalty term on the performance of DG-EFIE and DG-MFIE.

We consider an IBC sphere with a radius of 0.5 m and normalized surface impedance of $1.0 + 1.0i$. The sphere is decomposed into 8 sub-domains, as illustrated in Figure 4. The simulation frequency is 200 MHz. The sphere is meshed with different mesh sizes h . The scattering efficiency Q_{sca} for the sphere is calculated using DG-EFIE and DG-MFIE, and with different values of coefficient β . In this section the residual error of GMRES is set to 10^{-5} . The iteration numbers, and the relative error of Q_{sca} between the calculated value and the analytical value, are drawn in Figure 5. As can be seen from these results, the number of iterations for DG-EFIE and DG-MFIE rapidly increase as β is increased from zero, and then decline as β approaches value one. As the value of β is further increased, the number of iterations slowly increases. The minimum of the number of iterations is obtained when β is close to one. Relative error in Q_{sca} is clearly much less sensitive on the choice of β .

For a lossless surface impedance, i.e., as Z_s is purely imaginary, the convergence speed of the DG method is found to be more sensitive to the value of β , than if the surface is lossy. To demonstrate this, the number of iterations required in DG-EFIE and DG-MFIE methods for a sphere with $Z_s = 5.0i$ and mesh size of 100 mm are shown in Figure 6(a). The number of iterations of DG-MFIE dramatically changes as β is increased, and DG-EFIE does not converge within the maximum number iterations when β is small. On the other hand, DG-EFIE shows good convergence as β is larger than 0.5. We also observe that the minimum number of iterations is much smaller with DG-EFIE than with DG-MFIE. For DG-EFIE this minimum appears when $\beta \approx 1$. With $Z_s = -0.2i$, the results are opposite.

FIGURE 1 The number of iterations for solving the matrix equations iteratively in the case of an IBC sphere with radius 0.5 m and $Z_s = R_s + C_s i$ at a frequency of 200 MHz. (a) EFIE. (b) MFIE. (c) SDIE.

As a next example we investigate properties of the solutions as Z_s approaches zero ($Z_s = 10^{-5}$) or infinity ($Z_s = 10^5$). The iteration numbers of DG-EFIE and DG-MFIE are shown in Figure 6(b). When Z_s approaches zero, corresponding to PEC, DG-MFIE has a much better convergence speed than DG-EFIE. In that case the number of iterations of DG-MFIE is nearly independent of the value of β . When Z_s approaches infinity, i.e., PMC, the results are opposite, and DG-EFIE performs much better than DG-MFIE.

By repeating these experiments with different frequencies, we have noticed that the optimal value of β is rather insensitive on the operating frequency, excluding internal resonance frequencies. Therefore, in the following simulations, β is set to value one.

3.3 | Current Continuity on the Contour Boundary

To verify that the current computed with the DG method is continuous at the contour boundary, we consider a sphere with radius $r = 0.5$ m and size parameter $kr = 1.0$. The surface of the sphere is divided into two equal parts. On the upper hemisphere the surface impedance has value $1.0i$ and on the lower one it is $-1.0i$. The mesh size is 50 mm. In the DG method, the sphere is divided into two sub-domains corresponding to the regions where the surface impedance is constant, and the mesh is non-conformal, as illustrated in Figure 7. It is important to note that in order to calculate the boundary penalty term (13), the length of the common part of the source and field edges should be calculated²⁴. The surface currents obtained by the conventional EFIE and DG-EFIE methods with x-polarized, $-z$ -propagating plane wave excitation are shown in Figure 7. Currents computed with these formulations are practically identical. In this case, the results also obtained with DG-MFIE are very similar to the ones of DG-EFIE. We may conclude that the DG method, discretized with a non-conforming mesh and discontinuous half RWG functions, is able to give continuous currents across the contour boundary, even for discontinuous Z_s .

FIGURE 2 The analytical Mie solution for an IBC sphere with radius 0.5 m and $Z_s = R_s + C_s i$ at a frequency of 200 MHz. (a) Scattering efficiency Q_{sca} (dimensionless). (b) Absorption efficiency Q_{abs} (dimensionless).

3.4 | Comparison of DG-EFIE and DG-MFIE

To investigate the numerical performance of DG-EFIE and DG-MFIE, Q_{sca} and Q_{abs} are calculated for the sphere shown in Figure 4. The radius of the sphere is 0.5 m and the frequency is 200 MHz. The mesh size is 100 mm. The imaginary part of Z_s is varied from -2 to 2 and the real part of Z_s is varied from 0 to 2 . In the calculations, the maximum number of iterations is set to 500.

The iteration numbers of DG-EFIE and DG-MFIE are shown in Figure 8. Except for the resonances at certain purely imaginary values of Z_s ^{15,21}, DG-EFIE shows good convergence speed above the line $\text{Imag}\{Z_s\} = -2\text{Real}\{Z_s\}$. DG-MFIE, in turn, has very good convergence speed when $\text{Imag}\{Z_s\}$ is negative.

To further illustrate the nearly opposite performance of DG-EFIE and DG-MFIE with respect to $Z_s = R_s + C_s i$, the ratio of the number of iterations of DG-EFIE and DG-MFIE is plotted in Figure 9. When this ratio is larger than one, corresponding to red color in Figure 9, DG-MFIE has faster convergence speed, while for values smaller than one, blue color in Figure 9, DG-EFIE yields better convergence speed.

The relative error of Q_{sca} and the absolute error of Q_{abs} for DG-EFIE and that for DG-MFIE are displayed in Figure 10. It can be observed that the results obtained by DG-EFIE and DG-MFIE using iterative solver have almost the same accuracy with the results in Figure 3(a)-(b) and Figure 3(c)-(d), respectively.

We also note that the values of Z_s where DG-EFIE is more accurate than DG-MFIE, and vice versa, agree well with the values in which the same equation provides also better iteration convergence. Hence, DG-EFIE and DG-MFIE can be jointly used, and according to Figure 9 we may choose between them to obtain an efficient method for different surface impedances.

3.5 | Multi-scale IBC targets

Finally, we present two examples including multi-scale IBC targets to illustrate the capability of the proposed DG method for solving these kinds of problems. The first example includes a cone-shaped target with a height of 1.03 m and bottom radius of 0.5 m. The normalized surface impedance is $Z_s = 0.5 - 0.5i$. The object is illuminated by a plane wave at a frequency of 400 MHz propagating along the negative z direction. The incident electric field is x -polarized. The bistatic RCS (both in E and H planes), and Q_{abs} and Q_{sca} are calculated with the DG-MFIE method and with the traditional MFIE without preconditioner. For the DG method, the surface of the target is divided into 5 sub-domains. Each sub-domain is meshed with a different discretization size and the meshes between the adjacent sub-domains are non-conformal, as shown in Figure 11(a). The mesh sizes of the sub-domains are 20 mm, 30 mm, 40 mm, 50 mm, 60 mm, from top to bottom. For the MFIE method, the whole region is meshed with the a fixed mesh size of 20 mm. The solutions and the numerical performance of these two methods are compared in Figure 12(a) and in Table 1. Compared with the traditional MFIE method, DG-MFIE requires significantly lower number unknowns and iterations to obtain the same solution accuracy.

The second example is a missile-like object with a length of 4.21 m and radius of 0.4 m. The normalized surface impedance is set to value $Z_s = 2.0 + 0.1i$, and the operating frequency is 200 MHz. The incident electric field propagating along the $-z$ direction is polarized in the x direction. The DG-EFIE method is used to solve the problem, and the results are compared with the conventional EFIE without preconditioner. In the DG-EFIE, the surface of the target is divided into 5 sub-regions and meshed with the mesh size of 50 mm, 60 mm, 70 mm, 100 mm, 100 mm, from top to bottom. The meshes between different sub-domains are non-conformal, as shown in Figure 11(b). For the conventional EFIE method, the entire structure

FIGURE 3 The relative error of scattering efficiency (the unit is %) and the absolute error of absorption efficiency (which is dimensionless) between the analytical Mie solution and the results obtained with SIE methods. (a) The error of scattering efficiency of EFIE. (b) The error of absorption efficiency of EFIE. (c) The error of scattering efficiency of MFIE. (d) The error of absorption efficiency of MFIE. (e) The error of scattering efficiency of SDIE. (f) The error of absorption efficiency of SDIE.

is meshed uniformly with a 50 mm mesh. As shown in Figure 12(b) and in Table 1, the results of these two methods agree well with each other. Meanwhile, DG-EFIE shows a much better numerical performance in terms of the number of unknowns and number of iterations.

FIGURE 4 Domain decomposition for a sphere with 8 sub-domains.

FIGURE 5 The number of iterations and the relative error of the DG-EFIE and the DG-MFIE for calculating scattering by an IBC sphere with different mesh sizes versus β . (a) The number of iterations of DG-EFIE. (a) The number of iterations of DG-MFIE. (c) The relative errors, where the solid lines show the results for DG-EFIE, the dashed lines for DG-MFIE, and the colors correspond to different mesh sizes, as in Figure 5 (a) and Figure 5 (b).

FIGURE 6 The number of iterations of DG-EFIE and DG-MFIE in the case of an IBC sphere with radius 0.5 m, mesh size of 100 mm and frequency 200 MHz. (a) The number of iterations of DG-EFIE and DG-MFIE for a lossless IBC sphere. (b) The number of iterations of DG-EFIE and DG-MFIE for an IBC sphere with $Z_s = 10^{-5}$ and 10^5 .

FIGURE 7 Illustration of the current continuity on a sphere with discontinuous surface impedance, radius 0.5 m, mesh size of 50 mm and frequency 95.427 MHz. Top: Meshes used in EFIE (top-left) and DG-EFIE (top-right). Bottom: The real parts of an electric current using EFIE (bottom-left) and DG-EFIE (bottom-right).

FIGURE 8 The number of iterations in the case of an IBC sphere with radius 0.5 m and complex $Z_s = R_s + iC_s$ at a frequency of 200 MHz. (a) DG-EFIE. (b) DG-MFIE.

4 | DISCUSSION

From our numerical experiments several conclusions may be drawn. In this section we study the formulations more carefully to find explanations for these observations. First we note that using Galerkin's method with RWG basis and test functions, only the operators $\mathcal{T}$ and $\mathcal{I}$ are "well-tested"²⁵. Hence, it is expected that formulations in which these operators are dominant will lead to good numerical performance. It is also well-known that

FIGURE 9 The ratio of the iteration counts of DG-EFIE and DG-MFIE in the same case as in Figure 8.

TABLE 1 The numerical performance and results of different methods for calculating multi-scale targets shown in Figure 11.

Problem	Solution	Number of unknowns	Iteration counts	solution time	Q_{abs}	Q_{sca}
Figure 11(a)	MFIE	25857	594	910 s	1.162	1.328
	DG MFIE	5175	13	67 s	1.157	1.324
Figure 11(b)	EFIE	14028	802	214 s	5.184	5.053
	DG EFIE	5630	11	99 s	5.170	5.029

the electric field integral operator $\mathcal{T}$ provides more accurate results than the magnetic field integral operator $\mathcal{K}$, or the identity operator $\mathcal{I}$. On the other hand, the identity operator usually leads to very good iteration convergence.

First we note that the problem with SDIE is that, although the balance between the unknowns and equations of the matrix is improved by scaling with $\sqrt{Z_s}$, leading to good iteration convergence, combining operator $\mathcal{T}$ with $\sqrt{Z_s}\mathcal{I}$ or with $1/\sqrt{Z_s}\mathcal{I}$, reduces the solutions accuracy. The accuracy of SDIE has been found to be particularly poor close to the resonances of purely imaginary Z_s , as also reported in ¹⁵. If we remove either the first IBC equation or the second IBC in (8), the resulting two new formulations gives about the same accuracy as EFIE and MFIE, respectively.

The resonances at $C_s > 0$ are a magnetic type resonances ²¹, which are due to the electric currents $\mathbf{J}$. In EFIE, the field contribution of $\mathbf{J}$ is given by operator $\gamma_t \mathcal{T}$, while in MFIE, this is presented with operator $\gamma_t \mathcal{K}$. Hence, EFIE is expected to give more accurate results than MFIE at $C_s > 0$. At the negative values, i.e., when $C_s < 0$, the resonances are of the electric type. In those cases MFIE is more accurate than EFIE.

The iteration convergence speed of EFIE and MFIE is significantly improved with the DG method including block-diagonal preconditioner. When $Z_s = 1$, the number of iterations of DG-EFIE and DG-MFIE are equal. When Z_s is a pure real number ($C_s = 0$), and $R_s > 1$, the magnitude of the magnetic current is larger than that of the electric current according to the IBC. Then the identity operator $\mathcal{I}$ will have a dominant role in the EFIE formulation. The good condition of $\mathcal{I}$ improves the convergence speed of DG-EFIE compared to DG-MFIE. For the same reason, when $R_s < 1$, DG-MFIE converges faster than DG-EFIE. When the impedance surface is lossless, i.e., as $R_s = 0$, DG-EFIE converges faster than DG-MFIE for positive values of C_s , while DG-MFIE shows a better performance for negative values of C_s . Since these regions agree well with the values of Z_s in which EFIE is more accurate than MFIE, and vice versa, the DG-EFIE and DG-MFIE including the iterative solver, have good accuracy in their own respective ranges. The numerical performance of DG-EFIE and DG-MFIE are nearly complementary.

5 | CONCLUSION

A Discontinuous Galerkin (DG) surface integral equation (SIE) method is proposed for the scattering analysis of impenetrable objects equipped with the impedance boundary condition (IBC). Numerical performances of the electric field integral equation (EFIE), magnetic field integral equation

FIGURE 10 The relative error of scattering efficiency (the unit is %) and the absolute error of absorption efficiency (which is dimensionless) between the analytical Mie solution and the results obtained with SIE methods. (a) The error of scattering efficiency of DG-EFIE. (b) The error of absorption efficiency of DG-EFIE. (c) The error of scattering efficiency of DG-MFIE. (d) The error of absorption efficiency of DG-MFIE.

FIGURE 11 The non-conformal meshes for multi-scale targets: (a) The cone-shaped object. (b) The missile-like object.

(MFIE), and the self-dual integral equation (SDIE) are investigated. EFIE and MFIE are found to give much better accuracy than SDIE, especially near strong resonances corresponding to certain purely imaginary values of the surface impedance. Based on these results, DG method is developed for EFIE and MFIE, giving DG-EFIE and DG-MFIE. A simple block-diagonal preconditioner has been found to be crucial to obtain efficient iterative solutions. The solution accuracy and iteration convergence of these two formulations are studied, and compared, for a wide range of surface impedances. It is found that the regions of Z_s in which these two methods perform well are nearly complementary. Hence, DG-EFIE and DG-MFIE can be used

FIGURE 12 Bistatic RCS for multi-scale targets: (a) The cone-shaped object. (b) The missile-like object.

jointly, in their respective ranges. Furthermore, the proposed DG approaches are shown to have good capability for calculation scattering from multi-scale IBC targets.

ACKNOWLEDGMENTS

The authors would like to thank Dr. H. Wallén from Department of Electronics and Nanoengineering, Aalto University, Espoo, Finland, for helpful discussions.

References

1. Lindell IV, Sihvola A. Electromagnetic boundaries with PEC/PMC equivalence. *Prog. Electromagn. Res. Lett.* 2016; 61: 119-123.
2. Lindell IV, Sihvola A. Electromagnetic Wave Reflection From Boundaries Defined by General Linear and Local Conditions. *IEEE Transactions on Antennas and Propagation* 2017; 65(9): 4656-4663.
3. Lindell IV. Methods for electromagnetic field analysis. In: United States: Oxford University Press and IEEE Press. 1992, 1995.
4. Leontovitch MA. Approximate boundary conditions for the electromagnetic field on the surface of a good conductor. In: Moscow Russia: Printing House of the Academy of Sciences. 1948.
5. Minatti G, Maci S, De Vita P, Freni A, Sabbadini M. A Circularly-Polarized Isoflux Antenna Based on Anisotropic Metasurface. *IEEE Transactions on Antennas and Propagation* 2012; 60(11): 4998-5009.
6. Francavilla MA, Martini E, Maci S, Vecchi G. On the Numerical Simulation of Metasurfaces With Impedance Boundary Condition Integral Equations. *IEEE Transactions on Antennas and Propagation* 2015; 63(5): 2153-2161.
7. Rogers J. On combined source solutions for bodies with impedance boundary conditions. *IEEE Transactions on Antennas and Propagation* 1985; 33(4): 462-465.
8. Medgyesi-Mitschang L, Putnam J. Integral equation formulations for imperfectly conducting scatterers. *IEEE Transactions on Antennas and Propagation* 1985; 33(2): 206-214.
9. Sebak A, Shafai L. Scattering from arbitrarily-shaped objects with impedance boundary conditions. *IEE Proceedings H: Microwaves, Antennas and Propagation* 1989; 136(5): 371-376.
10. Ylä-Oijala P, Kiminki SP, Järvenpää S. Solving IBC-CFIE With Dual Basis Functions. *IEEE Transactions on Antennas and Propagation* 2010; 58(12): 3997-4004.

11. Zhang K, Jin JM. Parallelized multilevel fast multipole algorithm for scattering by objects with anisotropic impedance surfaces. *International Journal of Numerical Modelling: Electronic Networks, Devices and Fields* 2015; 28(1): 107–119.
12. Ismatullah, Eibert T. Surface Integral Equation Solutions by Hierarchical Vector Basis Functions and Spherical Harmonics Based Multilevel Fast Multipole Method. *IEEE Transactions on Antennas and Propagation* 2009; 57(7): 2084–2093.
13. Yan S, Jin JM. Self-Dual Integral Equations for Electromagnetic Scattering From IBC Objects. *IEEE Transactions on Antennas and Propagation* 2013; 61(11): 5533–5546.
14. Ylä-Oijala P, Lappalainen J, Järvenpää S. Characteristic Mode Equations for Impedance Surfaces. *IEEE Transactions on Antennas and Propagation* 2018; 66(1): 487–492.
15. Ylä-Oijala P, Kong B, Järvenpää S. Modeling of Resonating Closed Impedance Bodies With Surface Integral Equation Methods. *IEEE Transactions on Antennas and Propagation* 2019; 67(1): 361–368.
16. Peng Z, Lim KH, Lee JF. A Discontinuous Galerkin Surface Integral Equation Method for Electromagnetic Wave Scattering From Nonpenetrable Targets. *IEEE Transactions on Antennas and Propagation* 2013; 61(7): 3617–3628.
17. Peng Z, Hiptmair R, Shao Y, MacKie-Mason B. Domain Decomposition Preconditioning for Surface Integral Equations in Solving Challenging Electromagnetic Scattering Problems. *IEEE Transactions on Antennas and Propagation* 2016; 64(1): 210–223.
18. Bautista MAE, Vipiana F, Francavilla MA, Vasquez JAT, Vecchi G. A Nonconformal Domain Decomposition Scheme for the Analysis of Multiscale Structures. *IEEE Transactions on Antennas and Propagation* 2015; 63(8): 3548–3560.
19. Kong B, Sheng XQ. A Discontinuous Galerkin Surface Integral Equation Method for Scattering From Multiscale Homogeneous Objects. *IEEE Transactions on Antennas and Propagation* 2018; 66(4): 1937–1946.
20. Hou Y, Xiao G, Tian X. A Discontinuous Galerkin Augmented Electric Field Integral Equation for Multiscale Electromagnetic Scattering Problems. *IEEE Transactions on Antennas and Propagation* 2017; 65(7): 3615–3622.
21. Sihvola A, Tzarouchis DC, Ylä-Oijala P, Wallén H, Kong B. Resonances in small scatterers with impedance boundary. *Physical Review B* 2018; 98(23).
22. Wang X. *A domain decomposition method for analysis of three-dimensional large-scale electromagnetic compatibility problems*. PhD thesis. Ohio State University, 2012.
23. Reid MTH, Johnson SG. Efficient Computation of Power, Force, and Torque in BEM Scattering Calculations. *IEEE Transactions on Antennas and Propagation* 2015; 63(8): 3588–3598.
24. Kong B. *High Efficient Algorithms for the Solution of Surface Integral Equations of Complex Targets (in Chinese)*. PhD thesis. Beijing Institute of Technology, 2018.
25. Ylä-Oijala P, Taskinen M, Järvenpää S. Surface integræquation formulations for solving electromagnetic scattering problems with iterative methods. *Radio Science* 2005; 40(6). doi: 10.1029/2004RS003169.