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Influence of crack tip plasticity on fatigue behaviour of laser stake-welded T-joints made of thin plates

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Abstract

The paper investigates the crack tip plasticity of laser stake-welded T-joints when the thickness of the face plate is varied. For thin joints, mixed-mode condition is dominant and the Minimum Plastic Zone Radius (MPZR) criterion is employed to estimate the crack initiation direction. Subsequently, the same first-order plastic zone size r_y as defined by Irwin is kept equal for the selected load levels in the MPZR direction, and finite element (FE) analyses are carried out to investigate the stress gradient over r_y comparing different thicknesses. The results allow us to explain why the fatigue slope m is different for thin and thick joints. Moreover, a new method is proposed that permits the number of cycles to failure of thin joints to be derived directly from the fatigue curve of thick joints, employing an effective J-integral. The results prove that the method is able to give an estimation of the number of cycles to failure for thin joints with acceptable error.

Keywords: plastic zone; crack; laser stake-welds; fatigue assessment; slope of fatigue curve

1. Introduction

Weight reduction is a crucial design challenge in the automotive and aerospace industries, but also in shipbuilding. In marine design, for instance, thick monolithic plates can be replaced with sandwich panels made from thin plates to achieve a weight reduction. These lightweight structures made of thin plates pose new engineering challenges: the current methods to estimate the fatigue life of steel structures were developed for thick plates, and their application to thin plates raised a new set of challenges [1–5], such as proper consideration of joint geometry and stress gradient at critical weld notch [6,7], in addition to other well-known difficulties [8–10]. In steel sandwich structures, the thin plates are assembled together by means of a laser welding process, and the sandwich panel consists of laser stake-welded T-joints, as shown in Fig. 1. When the laser penetrates the thickness of the face plate, the latter is not fully melted and crack-like notches [11,12] are created on both sides of the joint. These may have different lengths, and create a gap between the face and web plate. Under service loading conditions, the panel deformation behavior is very complex and it generates a compound bending of the joints (see Fig. 1c). Therefore, a systematic investigation of the joints under more simple loading conditions (e.g. tension) is of primary importance for the fundamental understanding of the phenomenon involved in the fatigue behavior of panel and joints. Nevertheless, experimentally verified design procedures for laser stake-welded T-joints are only few in number and these are not included in current design standards.

Fig. 1. Illustration of (a) web-core steel sandwich panel, (b) geometrical details of the laser stake-welded T-joint, (c) example of loading condition in service and (d) schematic geometry of the T-joint

The first fatigue tests of laser stake-welded T-joints were reported in the late '90s [13–15]. Moreover, fatigue experiments conducted on web-core sandwich panels were reported in Refs. [16–18]. Those authors found that the slope of the fatigue resistance curves for laser stake-welded T-joints in sandwich panels depends on the loading condition and web/face plate thickness. In all these experimental works, the slope value of the fatigue resistance curves was larger than the value commonly observed for other steel joints. Fricke et al. [19], for example, showed a fatigue slope varying between 3 and 22. The difference in the slope of the fatigue resistance curve leads to an overestimation of the fatigue strength at high load levels, and to an underestimation at low load levels when the International Institute of Welding (IIW) design recommendations are applied to thin welded joints [3]. However, possible reasons for the difference in the slope were not given in these studies.

Only a few studies have tried to explain why the slope of the fatigue resistance curve changes with the loading conditions and the web plate thickness. Preliminary but fundamental comments on the slope issue were provided by Berto and Lazzarin [20–22] who used the Strain Energy Density concept to address the fatigue of welded joints. They showed that loading has, indeed, an impact on the slope. Some fundamental contributions on the topic are provided by Frank and collaborators [23–26], who proposed fatigue resistance curves of laser stake welded T-joints (as-welded) loaded under bending (similar to in service loading condition, see Fig. 1c) and additionally under tension in terms of J-integral. These experimental loading conditions permitted to investigate the fatigue behavior of the T-joints and gave interesting results. Specifically, important considerations concerning the effects of the thickness are reported in Ref. [26]. These authors showed an interesting analysis of the stress state ahead of the crack tip and proved that the mixed-mode ratio is strongly influenced by the face plate thickness: when the thickness approaches large values, the stress state becomes mode I dominant; when the thickness decreases mode II starts to be relevant and a mixed-mode condition occurs. A thickness of 16 mm can be considered as a qualitative transition value between mode I and mixed mode (I-II), but the change seems to be gradual rather than sudden [26]. Further developments were presented in Ref. [23], where a stress gradient at the notch tip was employed to explain the changes of the slope. This parameter showed a higher gradient of elastic stresses in bending than in tension and in thin than thick plates (similar findings were reported by Lazzarin and Berto [27]). Gallo et al. [28] showed theoretically that the reason for the difference in the slope in the case of laser-stake welded T-joints is indeed related to the stress state ahead of the crack tip, and that in bending the plasticity at the weld notch develops faster than in tension. On the basis of those findings, the authors proposed a method to compute the bending J-integral fatigue resistance curve from the corresponding tension one. The idea to employ the J-

integral for fatigue assessment of laser stake welded T-joints was proposed by Frank and collaborators [26] as reported earlier, but it should be mentioned the contribution given by Lazzarin and Berto in several publications dealing with notches and cracks [29,30].

Thus, the short literature review reported above shows that the fatigue strength is affected by the plate thickness and the loading mode. While the latter has been addressed recently [23,28], the effect of the face thickness is still an open topic. The present paper concentrates on the crack tip plasticity of laser stake-welded T-joints when the thickness of the face plate varies. The method originally proposed by Gallo et al. [28] for thick joints loaded under tension and bending is here extended to include the effect of variation in the thickness and the resulting mixed-mode stress state. This is achieved by evaluating the crack initiation direction as defined by the Minimum Plastic Zone Radius criterion by means of FE analyses. At the end, a new method based on the results obtained is proposed for the fatigue assessment of laser stake-welded T-joints made of thin plates if the thick fatigue data is known. The method is then validated against experiments [23,24] synthesized by the J-integral. The present work aims to give a first theoretical interpretation of the slope issue and to provide a useful practical tool for fatigue strength assessment of laser stake-welded T-joints.

2. Crack tip plasticity of thin laser stake-welded T-joints

2.1 Crack tip plastic zone according to Irwin

In the 1960s, Irwin [31,32] proposed a method for the quantification of the size of the crack tip yielding zone. To summarize Irwin's work, he combined a mode I singular stress field in the crack plane with a yielding criterion, i.e. $\sigma_{yy} = \sqrt{3}\sigma_{YS}$ for plane strain and $\sigma_{yy} = \sigma_{YS}$ for plane stress, and assumed an elastic perfectly plastic behaviour of the material. On the basis of these assumptions, he obtained the so-called *first-order* estimation of the plastic zone size r_y (see Fig. 2) and, by simple force balance within the forces F_1 and F_2 depicted in Fig. 2b, the *second-order* estimation of the plastic zone size r_p [33] (see Fig. 2):

$$r_y = \frac{1}{6\pi} \left(\frac{K_I}{\sigma_{YS}} \right)^2, \quad r_p = \frac{1}{3\pi} \left(\frac{K_I}{\sigma_{YS}} \right)^2 \quad \text{plane strain} \quad (1)$$

$$r_y = \frac{1}{2\pi} \left(\frac{K_I}{\sigma_{YS}} \right)^2, \quad r_p = \frac{1}{\pi} \left(\frac{K_I}{\sigma_{YS}} \right)^2 \quad \text{plane stress} \quad (2)$$

Fig. 2. (a) Qualitative representation of the first-order r_y and second-order r_p plastic zone size, and (b) load redistribution and force equilibrium; plane strain.

It should be noted that the factor of $\sqrt{3}$ for plane strain condition was originally proposed by Irwin in his original formulation [32] and it is based on Irwin's observations. He stated, indeed, that the effect of plane strain state was somehow smaller than the typical value of 2.5 obtained by combining Hooke's equations and maximum shear stress criterion. For the sake of coherence with the original formulation, factor of $\sqrt{3}$ is assumed in the present study. It is also worth noting that even if Irwin's approach seems to neglect any contribution of the cyclic plasticity, Irwin's plastic radius and cyclic plastic zone are correlated to each other. At the tip of a growing fatigue crack, indeed, each loading cycle generates a monotonic plastic zone during increased loading, and a much smaller reversed (cyclic) plastic zone. The reversed plastic zone is approximately $\frac{1}{4}$ of the size of the monotonic plastic zone and therefore very small in the cases considered here. Moreover, its contribution to high fatigue load levels (that is the range under consideration in the present work) becomes even less relevant [34]. Therefore, Irwin's method permits to analyze and compare the crack tip plasticity of different cases easily. Indeed, in the present paper the final aim is not a precise estimation of the size and shape of the plastic zone, but the changes in that. In Gallo et al. [28], the first-order plastic zone r_y as defined in [31,32] was assumed as the comparison condition between thick joints loaded under bending and tension. The force F_1 depicted in Fig. 2 was then considered to be the *crack driving force* and was used to quantify the difference between tension and bending. With those joints loaded under mode I, the force F_1 was effectively representative of the crack driving force acting on the plane of crack initiation, which is along the bisector for dominant mode I. However, the assumption of a pure mode I singular stress field holds only when thick laser stake-welded T-joints are being considered [26,28]. When mode II becomes relevant and

thus when thin joints are being considered [26], the crack initiation direction does not lie along the bisector anymore but rotates, as does the plastic radius. For this reason, it is necessary to define the correct initiation direction in which the crack driving force F_I has to be quantified. If this correction is taken into account, Irwin's approach still provides a simple and effective tool to quantify the difference between thin and thick joints. Among the theories available to define the crack initiation angle, the Minimum Plastic Zone Radius criterion has been employed and it is briefly summarized in Section 2.2.

2.2 The Minimum Plastic Zone Radius (MPZR) criterion

There are several theories concerning crack initiation orientation under mixed-mode loadings [35–39], and they are mainly based on the singular elastic field, which may become inaccurate when the crack tip plasticity increases. Recently, methods for the determination of crack initiation angles based on the shape of the plastic zone have been developed, such as those of Golos and Wasiluk [40], Wasiluk and Golos [41], Kahn and Khraisheh [42], and Sharanaprabhu and Kudari [43]. The idea behind the Minimum Plastic Zone Radius (MPZR) criterion is that the crack initiates in the direction of the minimum distance between the crack tip and elastic-plastic boundary [40,41] or in the direction of the local or global minimum distance, depending on the loading condition [42]. The shape of the plastic zone, and thus the elastic-plastic boundary, is defined by combining stress field equations at the crack tip and a yielding criterion. For an isotropic cracked body under mixed-mode (I-II) conditions, the singular elastic stress field at the crack tip is given by superimposing the mode I and II contributions:

$$\begin{aligned}
 \sigma_x &= \frac{1}{\sqrt{2\pi r}} \left[\left\{ K_I \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \right\} - \left\{ K_{II} \sin \frac{\theta}{2} \left(2 + \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \right) \right\} \right] \\
 \sigma_y &= \frac{1}{\sqrt{2\pi r}} \left[\left\{ K_I \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \right\} + \left\{ K_{II} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \right\} \right] \\
 \tau_{xy} &= \frac{1}{\sqrt{2\pi r}} \left[\left\{ K_I \cos \frac{\theta}{2} \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right\} + \left\{ K_{II} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \right\} \right] \\
 \sigma_z &= \nu(\sigma_x + \sigma_y) \quad \text{For plane strain} \\
 \sigma_z &= 0 \quad \text{For plane stress}
 \end{aligned} \tag{3}$$

where ν is the Poisson's ratio, r and θ are the polar coordinates, and the origin is at the crack tip as shown in Fig. 3a.

Fig. 3. (a) Stresses at the crack tip; (b) schematic representation of MPZR parameters.

The von Mises [44] yield function for a general stress state at the crack tip assumes the following form:

$$\sigma_{YS}^2 = \frac{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6\tau_{xy}^2}{2} \quad (4)$$

where σ_{YS} is the tensile yield strength of the material. By substituting Eq. (3) into Eq. (4), an explicit relation for the elastic-plastic boundary is obtained [45]:

$$r = \frac{1}{2\pi\sigma_{YS}^2} \left[K_I^2 \cos^2 \frac{\theta}{2} \left\{ \kappa^2 + 3 \sin^2 \frac{\theta}{2} \right\} + K_I K_{II} \sin \theta \{ 3 \cos \theta - \kappa \} + K_{II}^2 \left\{ 3 + \sin^2 \frac{\theta}{2} \left(\kappa^2 - 9 \cos^2 \frac{\theta}{2} \right) \right\} \right] \quad (5)$$

$\kappa = 1 - 2\nu$ For plane strain
 $\kappa = 1$ For plane stress

where r is the radius of the core region. By minimizing Eq. (5), the crack initiation angle θ_0 can be obtained, i.e.:

$$\left(\frac{\partial r}{\partial \theta} \right)_{\theta=\theta_0} = 0, \quad \left(\frac{\partial^2 r}{\partial \theta^2} \right)_{\theta=\theta_0} > 0. \quad (6)$$

In general, the solution allows a global and a local minimum. Golos and Wasiluk [40] and Wasiluk and Golos [41] consider the minimum distance to the plastic zone, while Kahn and Khraisheh [42] consider the crack initiation angle to be the direction of the local or global minimum, depending on the loading condition. The proposed criterion has been found to be more accurate than other methods under small-scale yielding [46], but in the case of a large-scale yielding regime, the best determination of the elastic-plastic boundary is based on numerical methods such as elastic-plastic finite element analysis rather than theoretical formulation [46,47]. In fact, the traditional estimate of the elastic-plastic boundary presented here is reasonable only for very low nominal load-yielding stress ratios. Moreover, Eq. (3) does not consider, for example, the T-stress, which becomes

relevant when dealing with thin plates. Even if further theoretical developments are possible [48], e.g. including the T-stress or the high-order terms of Williams' series expansion, or assuming a Hutchinson-Rice-Rosengren (HRR) field [49], it is shown that there is no satisfactory improvement in the estimation of the elastic-plastic boundary when compared with FE analysis, whether elastic or elastic-plastic [50,51].

For all these reasons, in the present study the crack initiation direction is evaluated directly by linear elastic FE analysis. This includes an accurate definition of the geometry and also of the boundary conditions, including any effect of the T-stress on the stress state. The crack initiation angle is defined as the minimum distance to the crack tip, i.e. in the direction of a relative MPZR where the circumferential stress is tensile according to Refs. [40,41]; see Fig. 3b. The elastic-plastic boundary is defined according to the von Mises yield criterion, as shown in the brief theoretical overview. It is shown later in the results that the crack initiation angle θ_0 that is obtained is close to the direction reported experimentally in the literature [26].

2.3 Definition of the representative crack driving force ratio under mixed mode (I-II) loading

When dealing with thin joints, as already stated before and by other authors [26], a mixed-mode (I-II) loading condition occurs, and the crack does not propagate along the bisector as assumed in Ref. [28]. Through the concept of the MPZR criterion, briefly introduced in Section 2.2, the crack initiation angle θ_0 under mixed-mode (I-II) loading can be evaluated. Considering the new direction, the same first-order plastic radius r_y along θ_0 is assumed as a comparison condition between thin and thick joints; see Fig. 4. The force F_1 depicted in Fig. 4 is then considered to be the *crack driving force*, and can be used to quantify the difference between thin and thick laser stake-welded T-joints, similarly to what is defined by Gallo et al. (2017) and close to the concept of the force method presented by Raju [52] and others [53–59].

Fig. 4. Schematic representation of the crack driving force F_1 and first-order plastic zone size r_y in the MPZR direction.

The crack driving force evaluated over r_y , but in the θ_0 direction (see Fig. 4), permits the implicit consideration of the mixed-mode effect and it is theoretically defined assuming a plane strain condition by the following integral, both for thin and thick joints:

$$F_1 = \int_0^{r_y} \sigma_{\theta\theta} dr - \sqrt{3} \sigma_{ys} \cdot r_y \quad (7)$$

where $\sigma_{\theta\theta}$ is the generic tensile elastic stress distribution, in polar coordinates. It should be noted that for pure mode I and thus when very large thicknesses are being considered, the angle θ_0 becomes 0 and the approach corresponds to what is defined in Ref. [28]. The ratio between the force F_1 of the thin and thick joints, assuming the same r_y along the MPZR direction, is defined as the *representative crack driving force ratio*, F_R :

$$F_R = \frac{F_{1 \text{ thin}}}{F_{1 \text{ thick}}} \quad (8)$$

The analytical calculation of F_R is possible but difficult since it leads to a set of equations to be solved numerically [53,56]. In the present paper, the crack driving force ratio is instead easily derived numerically with the aid of finite element analysis and used as a correction parameter to define an effective J-integral $\sqrt{J_{\text{eff}}}$. This is shown in detail in the next section.

3. Estimation model for the number of cycles to failure of thin joints

Preliminarily, in this study the focus is on joints subjected to a tension load only. A statistical analysis of the fatigue data [23,26] showed that the fatigue curves of thick and thin joints loaded under tension are linked to each other. Indeed, at two million cycles, all the fatigue curves present the same J-Integral value [23,26], i.e. $0.37 \text{ kJ}^{0.5}/\text{m}$, regardless of the thickness involved; in the medium- and high-cycle range, however, the two geometries differ, since the thin joints present a higher fatigue curve slope, m ; see Fig. 5. The joints with a face plate thickness of 8 and 16 mm are considered together, and are here referred to as “thick”, while the joints with a face plate thickness of 2.5 mm have been analyzed and labelled as “thin”. Additionally, the statistical analysis permitted the value of the slopes m to be clearly defined as 4.23 for thin joints and 3.76 for thick joints.

Fig. 5. Fatigue resistance curves for laser stake-welded T-joints of different thicknesses; tension load data re-analyzed from [23,26]; $P_s=50\%$.

The present authors assume that the fatigue curve of the thin joints can be derived from the curve of the thick joints and that the slope effect is taken into account through the crack driving force ratio F_R presented previously. On the basis of these assumptions, the new procedure for the fatigue assessment of the thin joints is developed below according to Gallo et al. [28].

The fatigue curves of the thin and thick joints, in terms of the square root of the J-Integral, are defined by these Wöhler equations [23,25]:

$$\left(\sqrt{J}\right)^{m_T} \cdot N_{f,T} = C_T \quad \text{for thick joints} \quad (9)$$

$$\left(\sqrt{J}\right)^{m_t} \cdot N_{f,t} = C_t \quad \text{for thin joints,} \quad (10)$$

where $\sqrt{J}$ is a generic load, m is the slope of the curve, N_f is the number of cycles to failure, and C is a constant; the subscript T stands for “Thick”, while the subscript t stands for “thin”. In order to take into account the slope effect and the relationship between the fatigue resistance curves of the thick and thin joints, the latter is expressed as a function of the parameters of the curves of the thick joints (m_T , C_T) through an *effective* J-integral, defined as follows [28]:

$$\sqrt{J}_{eff} = \sqrt{J} \cdot F_R, \quad (11)$$

where F_R is the crack driving force ratio defined in Section 2.3. As a consequence, the fatigue curve of the thin joints, i.e. Eq. (10), assumes the following form:

$$\left(\sqrt{J_{eff}}\right)^{m_T} \cdot N_{f,t} = C_T \quad (12)$$

Equation (12) summarizes an important concept: if the effective J-integral is employed in the fatigue curve of the thick joints, the number of cycles to failure of the thin joints is easily derived.

On the basis of the crack driving force ratio concept F_R , the following procedure for the fatigue assessment of thin laser stake-welded T-joints is proposed:

- A. the first step is the determination of the crack initiation direction, $\theta_{0,T}$ for thick and $\theta_{0,t}$ for thin joints, employing the MPZR criterion and finite element analysis;
- B. once the desired $\sqrt{J}$ is selected, evaluation through finite element analysis of the corresponding force $F_{1 \text{ thick}}$ and first-order plastic radius r_y in the $\theta_{0,T}$ direction (see Fig. 4); it should be noted that $\theta_{0,T}$ is close to 0 approaching large thicknesses;
- C. through finite element analysis, evaluation of the equivalent force $F_{1 \text{ thin}}$ for the thin joints in the $\theta_{0,t}$ direction, assuming the same first-order plastic radius as evaluated in the previous step (the external load is determined by trial and error until the target r_y is obtained);
- D. evaluation of F_R as defined by Eq. (8);
- E. evaluation of the effective $\sqrt{J_{eff}}$ according to Eq. (11);
- F. the new effective J-Integral is then used as the input parameter in the fatigue curve equation of the thick joints and the number of cycles to failure of the thin joints is obtained; Eq. (12).

A simplified diagram of the procedure is proposed in Fig. 6.

Fig. 6. Simplified diagram of the proposed procedure for the evaluation of the number of cycles to failure of thin laser stake-welded T-joints.

By repeating the steps from B to F and varying $\sqrt{J}$, the fatigue curve in the case of thin joints can be derived and experimental testing avoided.

As shown in Ref. [28], the experimental correction factor, equivalent to the crack driving force ratio F_R , can also be derived for comparison. This is obtained by equating the number of cycles to failure

of Eqs. (10) and (12). By simple mathematical manipulations, the experimental correction factor H is:

$$H = \left(\frac{C_T}{C_t} \right)^{\frac{1}{m_r}} \cdot (\sqrt{J})^{\frac{m_t-1}{m_r}} \quad (13)$$

Details on the derivation of H are reported extensively by Gallo et al. [28] and are omitted here for the sake of brevity. It has to be said that the factor expresses the ratio within $\sqrt{J}$ of the thick and of the thin fatigue curve when the same number of cycles to failure is considered. Experiments also showed that run-out specimens are obtained for very high numbers of cycles [23,26], and the fatigue curve can be approximated with a straight line over two million cycles. Therefore, the parameter H should assume the value of 1. In the light of the above, the final equations are [28]:

$$\begin{aligned} F_R &\cong H = \left(\frac{C_T}{C_t} \right)^{\frac{1}{m_r}} \cdot (\sqrt{J})^{\frac{m_t-1}{m_r}} \quad \text{when } \sqrt{J} > \sqrt{J}_A \\ F_R &\cong H = 1 \quad \text{when } \sqrt{J} \leq \sqrt{J}_A \end{aligned} \quad (14)$$

where $\sqrt{J}_A$ is the J-integral at the endurance limit and crossing points of the curves, that is two million cycles for the joints considered according to Frank et al. [23]. If the procedure is generalized to different joints and/or weld processes, the crossing point may be located at a different number of cycles. However, the difference in the plasticity at the crack tip between the joints vanishes at the endurance limit, as does the difference between the crack driving forces. Thus, if the crossing point is unknown, it is determined as the load and corresponding number of cycles at which the crack driving force ratio $F_R=1$. At any rate, small differences in the location of the crossing point are expected in general since the fatigue strength at two-five million cycles is very similar for a large variety of weld geometries and processes.

4. Case study

4.1 Geometry and mechanical properties

The verification of the proposed method was carried out by analysing a case study from Refs. [23,26]. Specifically, the analysis was limited to thick and thin specimens loaded under tension. This simple loading configuration, even if it cannot be directly related to the complex loading condition in service depicted in Fig. 1c, it is fundamental to understand the strength of the joints and related phenomena [23,26]. The analysis of bending load of thick joints was presented instead in [28]. With respect to a real T-joint, the following simplifications were assumed: symmetry along the web plate axis and cracks of the same length; a null web-face gap based was assumed since as

reported in Ref. [23], it is very small, being only 9 μm on average. The negligibility of the influence of such a small gap was also found by other authors [19,60]. In detail, with reference to Fig. 7, a face plate thickness t_f equal to 2.5 mm and a web plate thickness t_w equal to 4 mm were chosen for the thin joints. A crack length $a = 1.27$ mm was assumed. Two geometries were considered instead for the thick joints: the first geometry had a t_w and a t_f equal to 8 mm with a crack length a of 2.5 mm; the second one had a t_w equal to 16 mm and a t_f equal to 14 mm with a crack length $a = 4.25$ mm; see Fig. 7a for the geometric terms. The length of the face plate l_f and of the web plate l_w were assumed to be equal to 100 mm and 60 mm, respectively, for all the joints considered. The geometric parameters are summarized in Table 1. The crack lengths considered here are a medium value of those reported in Ref. [23]. The gap-tip was modelled as a crack, since the notch-gap radius $\rho \ll a$ [11,23]. The mechanical properties, also reported in Table 1, refer to the base material in accordance with Frank et al. [23,25]. This means that the elastic-perfectly plastic behavior of base material and weld, inside the plastic zone, are supposed to be the same. This choice does not directly affect the subsequent linear elastic FE analyses since Young's moduli of base material and weld are effectively similar as reported by Boroński [61]. However, remarkable differences may arise in the value of the yield strength and therefore in the first order plastic zone size evaluated a posteriori. On the other hand, Boroński [61] showed that the static yield strength of the base material and the cyclic yield strength of base material and weld have negligible differences. Furthermore, accurate definition of the mechanical properties of the weld is difficult and very questionable, because of the reduced thicknesses involved and of the particular welding process [61]. For all these reasons the base material mechanical properties are used in the subsequent analyses since very close to the cyclic weld mechanical properties, and additional comments are provided in the Discussion section.

Fig. 7. (a) Geometric details of laser stake-welded T-joint and (b) constraints configurations.

Table 1. Mechanical properties and geometric parameters of the case study.

σ_{YS} [MPa]	σ_{UTS} [MPa]	E [GPa]	t_f [mm]	l_f [mm]	t_w [mm]	l_w [mm]	a [mm]	D ₁ [mm]	D ₂ [mm]
235	400	206	2.5	100	4	60	1.27	4.8	44
			8	100	8	60	2.50	3.2	44
			16	100	14	60	4.25	0	44

4.2 Finite Element (FE) analysis

The geometry presented in Fig. 7 was modelled by means of the ANSYS® APDL15.0 finite element software package and several numerical simulations under linear elastic conditions were carried out. The 2D 8-node element type PLANE183 was employed with unit thickness, while plane strain condition was assumed. Because of the symmetry of the load and geometry, only half of the joint was modelled, and symmetry constraints were imposed along the web plate axis. A uniform nominal stress was applied along the web plate, as shown in Fig. 7b. The definition of the constraints on the face plate is very important, and an expedient was used to preserve the local stress state. In detail, the distances between the constraint and the web, D₁ and D₂ (see Fig. 7b), were defined to obtain the same mixed-mode ratio as that reported by Frank et al. [26]. The values of D₁ and D₂ are reported in Table 1, while the mixed-mode ratios achieved appear in Table 3. Therefore, the configuration of load and constraints proposed in Fig. 7 correctly replicates the experimental conditions of the thick and thin welded joints loaded under tension as reported in Ref. [23,26]. As the joints were loaded only under tension, the contact within the web and the face along the crack surfaces had no influence on the results and was neglected. A very accurate mesh, i.e. with elements of a size 2.5E-05 mm close to the crack tip, was realized, and sensitivity analysis was carried out on the stress singularity, the stress intensity factor, and the first-order plastic radius. Moreover, particular attention was paid to the distribution of the nodes in the circumferential direction. This facilitated the numerical detection of the minimum r and thus of the MPZR direction. The concentration “keypoint” option [62] was employed to model the crack tip. The model had around 25600 elements, mainly located close to the crack tip. The 2D model employed here neglects a relevant aspect related to thin plates, i.e., the so-called mode O (3D effects) [9,10]. In Frank et al. [23], indeed, it is shown that the fracture mode O does not occur in the cases investigated in the paper. This was verified through accurate 3D FE analyses by including the contact between the web-face plates. The results showed that the shear stress that causes the mode O is always negligible in the critical cross-section.

The following procedure was used when performing the analysis for each load level: first, the external load was applied to the thick joints and the crack initiation direction was found according

to the MPZR criterion. Then the numerical stress $\sigma_{\theta\theta}$ distributions in $\theta_{0,T}$ were stored. Following Irwin's formulation, the first-order plastic radius r_y was determined by considering the distance from the crack tip in the MPZR direction at which $\sqrt{3}\sigma_{YS}$ was detected (in accordance with the yield criterion proposed by Irwin in his original formulation [32]), directly from the numerical stress distribution. Moving on to consider the thin joints, the crack initiation $\theta_{0,t}$ was determined first. The external load was subsequently gradually varied until the same first-order plastic radius as that obtained for the thick case was detected in the $\theta_{0,t}$ direction. Then the stress distribution of the thin joints $\sigma_{\theta\theta}$ over r_y was stored. At this point, the force F_1 could easily be determined by a trapezoidal method algorithm according to Eq. (7). The J-integral was evaluated through the FE analysis at the same time for each load case. Several FE simulations were carried out but only five load levels have been selected and presented here for the sake of brevity. The load levels are classified as *low* and *high* as a function of r_y , and are listed in Table 2 together with the corresponding numerical J-integrals. These have been verified and are in agreement with the theoretical elastic mixed-mode (I-II) J-integrals [63,64]. The nominal net stress $\sigma_{nom,net}$ referred to the actual cross-section area of the weld (2b in Fig. 1) is also reported in the same table for the sake of clarity.

Table 2. Load levels, corresponding numerical J-integral, and nominal loads applied; $\sigma_{nom,net}$ refers to the actual cross-section area of the weld (2b in Fig. 1).

Load Level	r_y [mm]	$t_f = 2.5$ mm		$t_f = 8$ mm		$t_f = 16$ mm	
		J [kJ/m ²]	$\sigma_{nom,net}$ [kJ/m ²]	J [kJ/m ²]	$\sigma_{nom,net}$ [MPa]	J [MPa]	$\sigma_{nom,net}$ [MPa]
1	0.051	0.1647	136	0.2171	154	0.2309	153
2	0.082	0.2752	175	0.3481	195	0.3682	193
3	0.104	0.3600	201	0.4431	220	0.4739	219
4	0.135	0.4862	233	0.5721	250	0.6091	249
5	0.171	0.6414	268	0.7177	280	0.7568	277

5. Results

5.1 Crack initiation direction

Figure 8 shows the numerical evolution of the plastic radius as the mixed-mode ratio changes when the thickness is reduced. As already stated earlier, the mode II contribution has effects: it changes the shape of the plastic radius and it rotates the plastic radius. The crack initiation directions according to the MPZR criterion are reported in Table 3, together with the mixed-mode ratios achieved. It must be noted that from the numerical simulation a range of nodes rather than a single

one results in the MPZR direction, but the solution can be improved by imposing strict tolerance on the minimum r and realizing an accurate mesh with a large number of circumferential nodes. More importantly, only the correct direction provides the maximum F_1 . This is addressed in detail in the discussion section.

Fig. 8. Numerical evolution of the plastic radius for different thicknesses and mixed-mode ratios: (a) $t_f = 16$ mm, (b) $t_f = 8$ mm, (c) $t_f = 2.5$ mm

Table 3. Mixed-mode ratios achieved and crack initiation direction.

Specimen	K_{II}/K_I [mm]	θ_0 [deg]	θ_0 exp. [26] [deg]
$t_f = 2.5$ mm	0.63	55	45-50
$t_f = 8$ mm	0.18	20	30
$t_f = 16$ mm	0.06	7	-

Frank et al. [26] experimentally evaluated the crack propagation angles as shown in Fig. 9, and these are in good agreement with those obtained through the MPZR criterion. For the thin joints, $t_f = 2.5$ mm, the experimental crack propagation angle falls between 45° and 50° . When the face plate thickness increased to 8 mm, an angle of 30° was reported. Information on a face plate thickness of 16 mm was not reported. However, these were evaluated considering only two specimens and thus statistical analysis was not carried out [26]. The slight overestimation found here is in agreement with other results reported in the literature [46].

Fig. 9. Experimental crack propagation angles for specimens with (a) $t_f = 8$ mm and (b) $t_f = 2.5$ mm; reproduced from [26].

5.2 Crack driving force ratio F_R

The results obtained in terms of F_1 as a function of the first-order plastic radius are depicted in Fig. 10 and summarized in Table 4 to Table 6, depending on the face plate thickness. The K_I and K_{II} are reported in the same tables for the sake of completeness. For a low value of r_y , all the thicknesses tend to the same crack driving force value, with F_1 being equal to 17.76 N/mm, 18 N/mm, and 18.60 N/mm respectively for $t_f = 16$, 8, and 2.5 mm. From the results, it is clear that there is no difference in terms of the local stress gradient and re-distribution between all the thicknesses for low load levels. With r_y increasing, the thick (i.e. 8- and 16-mm) and thin joints start to differ. For example, for the load level 5, the crack driving force F_1 is 62.68 N/mm for $t_f = 16$ mm, 63.67 N/mm for $t_f = 8$ mm, and 73.82 N/mm for the thin joints $t_f = 2.5$ mm. The stress intensity factor ratio K_{II}/K_I , on the other hand, is constant for each geometry, regardless of the load levels.

Fig. 10. Evolution of the force F_1 as a function of the first-order plastic radius.

Table 4. Force F_1 for the selected load levels and stress intensity factors of the thin joint $t_f = 2.5$ mm.

Load Level	r_y [mm]	F_1 [N/mm]	K_I [MPa mm ^{0.5}]	K_{II} [MPa mm ^{0.5}]
1	0.051	18.60	163	102
2	0.082	31.73	211	132
3	0.104	41.70	241	151
4	0.135	56.29	280	176
5	0.171	73.82	321	202

Table 5. Force F_1 for the selected load levels, crack driving force ratio, and stress intensity factors for $t_f = 8$ mm; $F_R = F_1 \text{ } t_f=2.5 \text{ mm} / F_1 \text{ } t_f=8 \text{ mm}$.

Load Level	r_y [mm]	F_1 [N/mm]	F_R	K_I [MPa mm ^{0.5}]	K_{II} [MPa mm ^{0.5}]
1	0.051	18.00	1.033	220	39
2	0.082	29.77	1.066	278	50
3	0.104	38.40	1.086	314	56
4	0.135	50.22	1.121	357	64
5	0.171	63.67	1.159	400	72

Table 6. Force F_1 for the selected load levels, crack driving force ratio, and stress intensity factors for $t_f = 16$ mm; $F_R = F_1 \text{ } t_f=2.5 \text{ mm} / F_1 \text{ } t_f=16 \text{ mm}$.

Load Level	r_y [mm]	F_1 [N/mm]	F_R	K_I [MPa mm ^{0.5}]	K_{II} [MPa mm ^{0.5}]
1	0.051	17.83	1.043	231	15
2	0.082	29.45	1.077	292	19
3	0.104	37.99	1.097	329	21
4	0.135	49.65	1.134	373	24
5	0.171	62.68	1.178	417	27

The difference between the thin and thick joints is synthesized by the crack driving force ratio F_R . It is the ratio between the F_1 of the thin joint, i.e. $t_f = 2.5$ mm, and the crack driving force F_1 of $t_f = 8$ mm (see Table 5) or $t_f = 16$ mm (see Table 6). Figure 11a shows the trend of the crack driving force ratio as a function of the number of cycles to failure. This was evaluated by considering the J-integral reported in Table 2 and the fatigue curve for thick joints. Therefore a slope $m = 3.76$ was considered (see Fig. 5). The values are summarized in Table 7. For a high number of cycles, the

ratio tends to assume the value of 1, while when N_f approaches low values, the ratio increases. The crack driving force ratio shows perfectly what happens when the number of cycles to failure varies, underlining the fact that the joints with different thicknesses are not equivalent. A similar trend is observed in Fig. 11b, where the crack driving force ratio is plotted versus r_y . The figure shows that the two parameters are linearly dependent, and in agreement with comments provided by Morais [65]. The load levels present a good alignment in general, and the dependence can be expressed mathematically by the equations reported in the picture:

$$\begin{aligned} F_R &= 1.1181r_y + 0.9845 & \text{for } t_f = 16 \text{ mm} \\ F_R &= 1.0509r_y + 0.979 & \text{for } t_f = 8 \text{ mm} \end{aligned} \quad (15)$$

Fig. 11. (a) Evolution of the crack driving force ratios against the number of cycles to failure of the thick joints and (b) F_R against r_y .

Table 7: Evolution of F_R as a function of the number of cycles to failure of the fatigue curve for the thick joints. *fatigue curve slope $m = 3.76$, fatigue strength at two million cycles = $0.37 \text{ kJ}^{0.5}/\text{m}$.

Load Level	$t_f = 8 \text{ mm}$				$t_f = 16 \text{ mm}$		
	r_y [mm]	$\sqrt{J}$ [kJ ^{0.5} /m]	N_f (thick)*	F_R	$\sqrt{J}$ [kJ ^{0.5} /m]	N_f (thick)*	F_R
1	0.051	0.4659	8.41E+05	1.033	0.48146	7.43E+05	1.043
2	0.082	0.5900	3.46E+05	1.066	0.60866	3.08E+05	1.077
3	0.104	0.6656	2.20E+05	1.086	0.68623	1.96E+05	1.097
4	0.135	0.7564	1.36E+05	1.121	0.77888	1.22E+05	1.134
5	0.171	0.8472	8.88E+04	1.159	0.86996	8.03E+04	1.178

5.3 Evaluation of the proposed model

The method was first evaluated by proving the equivalence between the experimental correction factor H and the crack driving force ratio F_R . Later, the proposed procedure was applied to experimental data taken from the literature [23,26].

The parameter H was derived for several arbitrary $\sqrt{J}$ through Eq. (13), and plotted against the number of cycles to failure of the fatigue curve of the thick joints in Fig. 12, in semi-log scale, together with the force ratio F_R . Two million cycles are indicated by a vertical red line. The parameters needed for the estimation of H are reported in the figure for the sake of clarity. Figure 12 shows that the trend of the force ratio F_R was found to be in good agreement with the trend of the correction factor H between the medium- and high-cycles range. The trend of the crack driving force is also very clear: it tends to a value of 1 when approaching two million cycles.

Fig. 12. Trends of the correction factor H and crack driving force F_R ratio as a function of the number of cycles to failure; subscript T refers to the thick joints, t refers to the thin joints.

The proposed approach was tested with consideration being given to the available load levels of the thick joints (see Table 2) since for these values the exact F_R was known. These values were used as the design target for the thin joints. The estimated number of cycles to failure for the thin joints evaluated according to Eq. (12) was then compared with the expected experimental value shown in Fig. 5 [23,26]. The results are reported in Table 8 and Table 9. The results prove that the method gives an estimation of the number of cycles to failure of the thin joints with a very good approximation. The errors, if compared to the common scatter band of fatigue data, are acceptable.

It must be noted that the number of cycles to failure is always slightly underestimated. In detail, the error varies from -1% to -15% when a thick joint with a thickness of 8 mm is used as a reference, and from -3% to -19% when a joint with a thickness of 16 mm is used instead.

Table 8. Comparison between the estimated and expected number of cycles to failure for the thin joints, evaluated from FE analyses of the thick joints $t_f = 8 \text{ mm}$; $F_R = F_{1 \text{ tf}=2.5 \text{ mm}}/F_{1 \text{ tf}=8 \text{ mm}}$.

Load Level ($t_f = 8 \text{ mm}$)	F_R	$\sqrt{J}$ [kJ/m ²]	$\sqrt{J}_{\text{eff}}$ [kJ/m ²]	N_f estimated ($t_f = 2.5 \text{ mm}$)	N_f exp. ($t_f = 2.5 \text{ mm}$)	$\Delta\%$
1	1.033	0.4659	0.4813	7.44E+05	7.54E+05	-1%
2	1.066	0.5900	0.6289	2.72E+05	2.78E+05	-2%
3	1.086	0.6656	0.7228	1.61E+05	1.67E+05	-3%
4	1.121	0.7564	0.8477	8.86E+04	9.71E+04	-9%
5	1.159	0.8472	0.9822	5.09E+04	6.01E+04	-15%

Table 9. Comparison between the estimated and expected number of cycles to failure for the thin joints evaluated from FE analysis of the thick joints $t_f = 16 \text{ mm}$; $F_R = F_{1 \text{ tf}=2.5 \text{ mm}}/F_{1 \text{ tf}=16 \text{ mm}}$.

Load Level ($t_f = 16 \text{ mm}$)	F_R	$\sqrt{J}$ [kJ/m ²]	$\sqrt{J}_{\text{eff}}$ [kJ/m ²]	N_f estimated ($t_f = 2.5 \text{ mm}$)	N_f exp. ($t_f = 2.5 \text{ mm}$)	$\Delta\%$
1	1.043	0.4815	0.5020	6.35E+05	6.57E+05	-3%
2	1.077	0.6087	0.6557	2.33E+05	2.44E+05	-4%
3	1.097	0.6862	0.7531	1.38E+05	1.47E+05	-6%
4	1.134	0.7789	0.8830	7.60E+04	8.58E+04	-11%
5	1.178	0.8700	1.0246	4.34E+04	5.38E+04	-19%

6. Discussion

The results obtained in the present study confirm the strong influence on the fatigue behaviour of the face plate thickness, shown in the past by other authors [18,23,26]. When the stress state ahead of the crack tip is analyzed in detail, it is shown that mixed mode (I-II) occurs for thin plates; see Fig. 8 and Table 3. By taking into account this effect, and by further developing the approach recently proposed by Gallo et al. [28], a comparison between different joints was carried out, with the thickness being varied. The results show that for high external load levels, the thin and thick joints are not equivalent, and that for the thin joints the plasticity develops faster than for the thick joints. The first-order plastic zone in the MPZR direction, in which it is assumed that the fatigue phenomena take place, is subjected to a bigger crack driving force F_I and so to a more critical stress field when thin joints are being considered; see Fig. 10. This is true despite the fact that the same r_y is considered and then equivalence is expected following Irwin's formulation. This is in agreement

with what was stated by Sousa and Figueiredo [51]. Those authors proved that the common assumption of equivalence between the same plastic zone and same stress intensity factor and thus fracture behaviour is not verified, even when very low load levels are considered and the T-stress effect is taken into account.

If an interpretation from the fatigue viewpoint is given to our results, the bigger crack driving force F_1 in the case of the thin joints generates a higher level of damage in the representative fatigue volume, which results in a lower number of cycles. The physical meaning of this statement is shown in Fig. 13. The low load values situation and corresponding stress distribution are labelled as number 1, the high load values situation as number 2. In case 1, the thin and thick joints return the same stress redistribution; for case 2 (high load levels) the crack driving force F_1 resulting from the thin plates ($F_{1,t}$ and the area depicted in red) is higher than the tension case ($F_{1,T}$ and the area depicted in black). Therefore, for a chosen number of cycles to failure, this phenomenon is taken into account by a lower value of $\sqrt{J}$ for the thin joints needed to generate the same amount of damage.

Fig. 13. High load-level (2) and low load-level (1) crack driving force, and matching with fatigue curves; the subscript t stands for thin, T for thick.

The parameter F_R defined by Eq. (8) proved to estimate very well the ratio within $\sqrt{J}$ for the thin and thick joints at the same number of cycles to failure. For this reason, it resulted in being equal to the parameter H , which mathematically expressed the relationships between the experimental fatigue curves; see Fig. 12. Following the proposed procedure, the fatigue assessment of a laser stake-welded T-joint made of thin plates can be carried out only on the basis of the thick joints and the corresponding fatigue curve. The results are in good agreement with the experimental ones (see Table 8 and 9). Finite element analysis is still necessary for the evaluation of F_R , because of the theoretical limitation in the definition of the crack driving force as shown by [28], and on the minimum plastic zone size [50,51]. It is worth noting that all the analyses are carried out in linear elastic condition, and that few analyses are needed if the relationship between F_R and r_y is derived as in Fig. 11b [65]. If the steps of the procedure proposed in Section 3 and Fig. 6 are repeated for limited numbers of $\sqrt{J}$, Eq. (15) is easily obtained for the geometry of interest. At this point, the F_R can be evaluated for any given r_y , avoiding any additional analysis. This expedient drastically reduces the amount of analysis that is needed.

Compared to the original method proposed by Gallo et al. [28] for thick joints subjected to tension and bending loads, there are some differences. In that work, the joints considered were subjected to pure mode I while having the same geometry. Therefore, the comparison based on the Irwin's plastic zone size was carried out along the bisector. Moreover, even if the hypothesis of plane strain was assumed from the numerical point of view, the criterion for the definition of r_y was, for simplification, that of the plane stress, i.e. $\sigma_{yy} = \sigma_{YS}$. These assumptions are not valid when the aim is a comparison between thin and thick joints. Indeed, as the geometry is different, and the thin joints have a small thickness, the overestimation of r_y may generate a large discrepancy in the final results, as well as the fact that the first-order plastic zone size r_y can easily be larger in size than the thickness of the thin joints that are being considered. For these reasons, plane strain condition must be imposed on the determination of r_y , i.e. $\sigma_{yy} = \sqrt{3}\sigma_{YS}$ [32]. Regarding mode I, instead, it has been proved that when dealing with thin joints, a mixed-mode condition occurs. Therefore a comparison cannot be carried out simply along the bisector, since that direction is not representative of the crack driving force for the thin joints. Consequently, the minimum plastic zone radius (MPZR) criterion was employed to define the crack initiation direction under mixed-mode (I-II) loading. As shown by the results in Table 3, the method seems to give good estimations. The MPZR direction was therefore evaluated by means of linear elastic FE analyses, as explained in Section 5.1, and the procedure is very simple. On the other hand, a few nodes, rather than a single one, seem to be good candidates for the definition of the MPZR direction, and they may have several degrees of difference. However, some expedients can assure the determination of the right direction: first,

by realizing an accurate mesh, second, by verifying the values of F_I and r_y . The crack driving force, indeed, assumes its maximum value only in the MPZR direction. This is shown in Fig. 14. The crack driving force, the Irwin's first-order plastic zone size r_y , and the generic distance between the crack tip and the elastic-plastic boundary (PZR) are plotted in different directions, defined by the angle θ between the crack bisector and the general direction. When $\theta=0$, the bisector is considered. It is clear from the figure that both the crack driving force and r_y have their maximum values only when the MPZR direction is considered. If pure mode I joints are studied, the present method returns to being similar to what was defined in Gallo et al. [28]. Theoretical developments on MPZR are certainly possible, and mainly depend on the accuracy of the theoretical definition of the plastic boundary. However, this is beyond the scope of the present paper, and for the sake of brevity extensive comments and developments are not provided here. Among the references provided, it is necessary to mention the interesting work of Gao et al. [48], who provided analytical solutions for the crack tip plastic zone under several loading conditions and small-scale yielding, as well as the deep and accurate analysis of Khan and Khraisheh [42]. In addition to the references given earlier, [66–69] give additional food for thought.

Fig. 14. Numerical variation of the crack driving force F_I , plastic zone size (PZR), and Irwin's first-order plastic zone size r_y in different directions from the crack tip.

The fatigue curve slope issue, as briefly summarized in the introduction, has been considered by different authors who provided some recommendations but did not address the phenomenological aspect. The interpretation given here can easily be extended to those cases. The conclusion drawn in the present paper also found good agreement with Frank et al. [23], who provided an interpretation of the slope issue in terms of a relative stress gradient. It was shown that the local stress state was related to the slope of the fatigue curves and affected by the mixed mode (I-II). These results are in

agreement with the conclusion reached in the present paper, where a deeper and more detailed analysis gave a more detailed explanation which takes the local plasticity into consideration. It is legitimate to ask if other factors may generate the slope issue as well. Residual stresses, for example, have a relevant effect on the fatigue behavior of welded joints and comments on this aspect are provided in Ref. [23,25,26] where the fatigue curves of Fig. 5 are derived experimentally. However, the residual stresses would affect the fatigue curves at low load levels, e.g. at the fatigue limit. Figure 5 clearly shows, instead, that the fatigue curves of thin and thick joints have a crossing point at two million cycles. This means there is no relevant difference in the residual stresses of the two geometries, and that the slope issue is not due, in this case, to residual stresses. This is implicitly assumed by the procedure proposed here that hypothesized as general condition that the fatigue curves have a crossing point in the range of two-five million cycles. The method is then derived for cases in which the slope issue is due to stress gradient and directly modifies, through the crack driving force ratio, the fatigue curve of as-welded thick specimens (loading ratio $R=0$). In general, the effects of residual stresses are very complex to be considered and quantified. Indeed there is no guarantee that the process always generates the same residual stresses, not to mention thermal treatments and other phenomena. It is worth mentioning the interesting discussion and contributions given in [70–74].

The approach based on the crack driving force ratio was presented here as a method to derive the number of cycles to failure of thin joints from the fatigue curve of thick joints, but it can be considered as a method to evaluate the number of cycles to failure between joints of any thicknesses. The results presented here, combined with the previous work by Gallo et al. [28], show that any combination of thickness and loading condition (tension and/or bending) can be successfully treated with the method. Moreover, it has the potential to be extended to notches by employing the stress concentration factor K_t instead of the stress intensity factor and developing the theory accordingly. However, the approach may break down for low values of K_t because of the low effects of the stress gradient, low plasticity involved, and thus the small crack driving force ratio. On the other hand, even when the effect of the stress gradient would be certainly relevant, the method may break down for cases in which the plastic zone size is not able to quantify a significant crack driving force ratio or simply when the slope issue is not due to the stress gradient. Lastly, for the reasons explained earlier in Section 4.1, yielding strength of the base material has been employed in this study to derive the first order plastic zone size since this was a good approximation of the cyclic weld yield strength. However, there may be cases where the difference between base material and weld cyclic elastic-plastic behavior is so large that the assumption of the base material or weld mechanical properties produces a significant discrepancy in the final crack

driving force ratio. Employment of the high yield strength of the weld, for example, may generate a plastic radius too small to quantify the stress gradient effect. This is not the case of the laser stake-welded T-joints analyzed here, as explained in Section 4.1, while further investigation and analyses should be done for other welding processes and joints.

7. Conclusions

In this paper, the influence of the face plate thickness, local plasticity, and mixed-mode ratio on the fatigue behavior of laser stake-welded T-joints was investigated, and a new model for the fatigue life estimation of a thin joint was presented. The present work gives a new theoretical interpretation of the slope issue and provides a useful practical tool for the fatigue assessment of laser stake-welded T-joints when the thicknesses of the face plate are varied.

Finite element analyses were carried out on laser stake-weld T-joints of varying thicknesses, and the first-order plastic zone size was kept equal for selected load levels in the crack initiation direction. The latter varies, as the mixed-mode ratio does, and was derived for each geometry by employing the MPZR criterion. For small plastic zones (i.e. low values of r_y), the thin and thick joints are equivalent and return the unified fatigue strength shown in Fig. 5. For large plastic zones (i.e. high values of r_y) the thin joint presents a higher crack driving force called F_1 . The difference is quantified through a crack driving force ratio, F_R , which increases as the load does (Fig. 11). The F_R that was obtained was employed as a correction factor for $\sqrt{J}$ to define an effective J-integral $\sqrt{J}_{\text{eff}}$. The effective J-integral is later employed in the fatigue curve of the thick joints to obtain the number of cycles to failure of the thin ones. The results are in agreement with the experimental tests (see Table 8 and Table 9).

Future work should explore the accuracy of the proposed method for different notch geometries and for more complex loading conditions.

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