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Abstract—This paper proposes a trilateral bilevel stochastic mixed integer bilevel linear programming (MILP) model to handle a joint master-slave operation-planning problem. This model considers the interactions among an integrated community energy system (ICES), prosumers, and the wholesale electricity market (WM). The ICES, in the upper level, makes a joint optimal photovoltaic (PV) and energy storage system plan taking into account the concurrent interactions with the WM and prosumers to maximize its profit. On the other hand, in the lower level, the prosumers aim at minimizing their bills via an optimal PV-package sizing while interacting with the ICES and the WM simultaneously. The strong duality theorem is used to recast this MILP model into an equivalent single-level model. Since the lower level is also an operation-planning problem, this recast results in a mixed integer nonlinear programming (MINLP) model that prevents finding a satisfactory solution. To remedy this issue, sensitivity analysis and a separation-based linearization technique are applied. Numerical results illustrate the effectiveness of the proposed business model while providing valuable information for the ICES and consumers.

Index Terms—Energy storage device, integrated community energy system, planning problem, prosumers, renewable energy.

NOMENCLATURE

A. Indexes and Sets

\( b/i/s/t \) Indexes for load block/prosumers/scenarios/time stages.

\( ch/dis \) Script referring to charging/discharging.

\( D \) Script referring to demand.

\( ICES \) Script referring to the ICES.

\( l \) Index for load level split in quarter \( q \), working day/weekend \( r \), day/night \( n \), and load block \( b \).

\( N_l(q/r/n/b) \) Sets of load level splits (quarter \( q \), working day/weekend \( r \), day/night \( n \), and load block \( b \)).

\( PD \) Script referring to peak demand.

\( PV \) Script referring to photovoltaic (PV) system.

\( S \) Set of scenarios, \( S = \{WMS, PVS, DS\} \) where WMS, PVS, and DS stand for wholesale energy market, photovoltaic, and demand scenarios.

\( ST/WM \) Script referring to storage devices/WM.

\( T \) Set of time stages \( t \), \( 1, \ldots, N_T \).

\( \xi_{aux} \) Set of auxiliary variables used in the linearization process.

\( \xi_{ICES} \) Set of decision variables for ICES.

\( \xi_{prosumers} \) Set of decision variables for prosumers.

B. Parameters

\( c_{ST,ICES} \) Installation costs of the ICES storage devices.

\( c_{OM,ST,ICES} \) O&M costs of storage devices of the ICES.

\( c_{ch/dis,ICES/i} \) Charging and discharging costs of storage devices of the ICES/prosumer \( i \).

\( c_{PV,ICES/i} \) Installation costs of PV units of prosumer \( i/ICES \).

\( c_{OM,PV,ICES/i} \) O&M costs of PV units of prosumer \( i/ICES \).

\( c_{PV,ICES/i} \) Production cost of the PV units of ICES/prosumer \( i \).

\( D_{i,t,s} \) Depth of discharge of storage device of the ICES/prosumer \( i \) at stage \( t \) in scenario \( s \).

\( \Delta_t \) Annual investment rate.

\( p_{ST,i} \) Maximum investment cost of the ICES/prosumer \( i \) at stage \( t \).

\( p_{WM,ICES} \) Maximum capacity of storage device of prosumer \( i \).

\( p_t \) Contracted power of the ICES at stage \( t \).

\( \gamma \) Parameter for discretization.

\( \delta_{l,t} \) Duration of each load level \( l \) (in hours).

\( \eta_{ch,ICES/i} \) Charging and discharging efficiency of the energy storage device of the ICES/prosumer \( i \).

\( \theta_t^{PD,ICES} \) Network charge \( (\varepsilon/\text{kW}) \) of the ICES requesting more energy than the contracted peak demand at stage \( t \).

\( \lambda_{t,1,s} \) Energy price \( (\varepsilon/\text{kW}) \) sold to the WM by prosumer \( i \) at stage \( t \) for load level \( l \) in scenario \( s \).

\( \lambda_{t,1,s} \) Energy price \( (\varepsilon/\text{kW}) \) bought from the WM by prosumer \( i \) at stage \( t \) for load level \( l \) in scenario \( s \).

\( \lambda_{t,1,s} \) Energy price \( (\varepsilon/\text{kW}) \) sold from/sold to the WM by the ICES at stage \( t \) for load level \( l \) in scenario \( s \).

\( \mu_{i,t} \) Average per unit demand factor at time \( t \), load level \( l \), and scenario \( s \).

\( \mu_{PV,i,t} \) Vector of maximum per unit levels of PV generation at load level \( l \) and scenario \( s \).

\( \Omega_s \) Probability of scenario \( s \) for load level \( l \).

\( \phi_{i,t} \) Lifetime of storage devices of the ICES.

\( \phi_{i,t,1} \) Lifetime of PV devices of the ICES/prosumer \( i \).

\( \omega \) Price adjusting coefficient.

C. Variables

\( c_t^{PV,ICES} \) Amortized PV investment cost of prosumer \( i/ICES \).
Recent developments in modular low-carbon resources are reshaping power systems, posing new operational and planning challenges for distribution networks [1, 2]. To this end, the evolution in power delivery has resulted in sophisticated structures with generation close to consumers, advanced metering, distributed control, and active distribution networks. On the other hand, developments in renewable energy-based distributed generation (DG) and energy storage systems (ESSs) delivered a more effective control over the generation and consumption of communities and individuals. By 2030, behind-the-meter storage systems are expected to represent over half of the total installed capacity [3]. Such a significant transformation in the energy sector requires the development of a new paradigm where the consumers or prosumers will be at the core of this transformation [4]. Consequently, the adoption of ESSs is expected to increase rapidly in the coming years and become a significant portion of the total energy storage deployment.

The evolution from passive to active distribution grids has required new operating and planning actions [5]. Such requirements resulted in conducting profound studies on operational, planning, and market strategies, mainly at the distribution level. A plethora of energy system integration based on coalitional solutions, comprising community microgrids (CMGs), virtual power plants (VPPs), energy hubs (EHs), and ICESs, among others, has evolved over the past years to adequately cope with the challenges presented by the energy system transformation. The backbone of a CMG consists of renewable or other distributed energy resources (DER), such as demand response (DR), ESSs, etc., which is an effective way for coordinated on- or off-grid systems to meet their demands locally ensuring their cost-effectiveness and reliability [6], [7]. A VPP offers the joint coordination of DGs and flexible loads to mitigate unwelcome power fluctuations of renewables and demands [8]. EHs, by integrating electricity, thermal energy, natural gas, etc., provide high flexibility for the electricity consumers to choose the best source of energy, which guarantees the environmental cost-effectiveness [9], [10]. However, an ICES, combining existing energy infrastructure and available resources in a community, maximizes the energy efficiency via the most ecological, technical, and economic way [11]. ICESs not only address issues regarding the network side such as security, energy efficiency, and volatility of renewables and demands but also provide more flexibility to the consumers while mitigating climate change [12], [13]. Through article 16 of the Directive on Common Rules for the Internal Energy Market (COM [2016] 864) [14], the European Commission requires member states to adopt a legal framework that ensures the possibility for local energy communities to own, establish, or lease community networks and to autonomously manage them. These communities can access all organized markets either directly or through aggregators or suppliers. Such a degree of freedom empowers innovative energy service companies to offer smart energy solutions to consumers in new ways that were not possible before. Among the options stated above, ICESs provide better collaboration and services as well as take into account self-provision and self-sufficiency offering a secure infrastructure that supports the future energy and climate objectives [15].

On the other hand, the increasing share of renewable energy sources (RES) will transform the very nature of electricity grid operation in the short term, and will determine how upgrades should be planned in the medium and long terms. Therefore, it may pose a tremendous challenge in the efficient management of the grids to maintain system security and reliability, mainly due to the high variability of distributed RES. This highlights the significant role of DR and energy management systems, electricity and thermal storage, interactions with other systems, and solar PV business models in planning and operating problems [16]. The expansion planning problem applied to energy community services has been adequately addressed in [17], [18]. In [19] and [20], different problems in energy trading and cooperation among microgrids equipped with RES and ESSs were studied. The impact of strategic behavior of an independent trader operating ESSs in a nodal electricity market was evaluated in [21]. In [22], the interactions and energy trading decisions of geographically distributed resources were studied using a game theory-based framework. In [23], coupling neighboring microgrids were proposed for overload management while addressing several
issues such as reliability, supply security, power losses, electricity costs, and CO₂ emission. An optimization model for a load aggregator with ESS to determine its net imported power in electricity markets was presented in [24]. A comprehensive literature review on siting and sizing of DG units was presented in [25], while, in [26], a bilevel DR-based planning framework for distribution network and renewable energy expansion was shown. In [27], aiming at addressing the future grid issues, a bilevel program in which the upper-level minimizes the total generation cost, and the lower-level maximizes the prosumers’ aggregate self-consumption while no interaction between the WM and the aggregator was considered. In [6], flexibility in power systems planning in the presence of flexible resources such as conventional units, ESSs, heat storage devices, and electric boilers was studied. A robust bilevel approach was proposed in [28] while modeling the interaction between the electricity retailer and prosumers. It was assumed that the prosumers in the lower-level were equipped by renewables, therefore, no planning action was considered.

The increasing penetration of these technologies, together with the deployment of advanced metering solutions facilitating an active role for consumers/prosumers calls for the development of advanced planning solutions. This issue is addressed more effectively via interactions among the wholesale market (WM), the prosumers and communities [29]. In [30], the authors analyzed the value of ESSs for the local communities in the presence of ESSs. The impacts of strategic coordination of households, as well as their interaction with the distribution service operator, were presented in [31]. In [32], the prosumers interacted with the WM in the presence of VPPs via a joint power and gas system to obtain their optimal scheduling. The possibility and effects of sharing energy among neighboring PV-based prosumers via ESS-equipped energy sharing provider was investigated in [33]. Transactions among energy users owning DERs and ESSs from both non-cooperative and cooperative perspectives were presented in [34]. A cooperative game between households aiming at balancing and flattening the load of the community was proposed in [35]. To have a more effective structure, a robust programming model was proposed in [36]. The model performed a day-ahead interaction strategy for the ICES in the joint energy and ancillary service markets considering ESS, thermal storage and renewables.

The literature review reveals that only bilateral interactions among the WM, prosumers, and the ICES exist where community-based interactions, ESS or renewable energy sizes are predefined, and no planning action is considered. In works that studied the siting and/or sizing of storage devices and renewables, the planning action was only performed in the master problem in a game theory-based environment.

This paper addresses these gaps by proposing a trilateral interaction in an operation-planning environment. In the proposed model, prosumers have the capability of simultaneous interacting with the ICES and the WM to make an optimal PV sizing plan over the planning horizon that ensures their minimum bills. On the other hand, the ICES maximizes its profit taking into account the optimal planning of ESSs and PV modules while trying to keep its contract with the WM; if the contract is violated, a surcharge is applied. The original problem is a mixed-integer bilevel linear programming (MIBLP) that can be handled via an iterative process that, depending on the stopping criterion, might not converge to the optimal solution or the convergence can be subject to a low computational efficiency [37]. Therefore, in this paper, the strong duality theorem is used to recast this MIBLP model as an equivalent single-level model, see Section II. The ICES, in the upper level, sends two signals to the prosumers: 1) the energy that is willing to be purchased from the prosumers and 2) the offered prices corresponding to the prosumers’ demand. The prosumers, in the lower level, in return, submit two signals: 1) the amount of energy they intend to buy from the ICES and 2) the prices corresponding to the energy demanded by the ICES. Both the ICES and the prosumers simultaneously interact with the WM to find the optimal plan and operating actions. In this model, to optimize the present value of expected profit, operational costs, and electricity bills, a scenario-based stochastic programming framework, as done in [25], [38]–[40] is used, while taking into account the correlated uncertainty of renewable-based generation and demand. The model is a single-leader multi-follower one in which the followers (consumers) are willing to cooperate with the leader (ICES) that tailors the optimistic bilevel programming nature [41], [42]. A sufficient amount of literature review on existing bilevel optimization models can be found in [43], while useful information on stochastic programming, e.g., quality metrics, risk analysis, etc., are available in [44]. Therefore, the main contributions of this work are:

- Proposing a bilevel operation-planning model to handle ESSs and PV planning while considering operational conditions. Unlike existing works, where only planning is considered in the upper level, in the proposed model not only the ICES considers ESS and PV sizing but also the prosumers handle a PV-package planning problem.
- Proposing a tractable scenario-based stochastic programming framework to take into account the correlated uncertainty characterizing demand and renewable-based energy production.
- Applying an excess-demand penalty to supervise the contract between the ICES and the WM. Since the ICES is a bigger dealer in the WM compared to the prosumers, it may request more power from the WM than its pre-contracted power while paying a higher price, which is followed by an excess-demand penalty.
- Proposing linearization techniques to recast the bilinear terms corresponding to the multiplication of bounded and unbounded continuous variables into linear terms.

The rest of this paper is organized as follows. Section II presents the problem formulation and the solution approach. Assumptions and case studies are provided in Section III. Numerical results and analysis are presented in Section IV. Section V presents the concluding remarks and prospects for future works.

II. PROBLEM FORMULATION

A trilateral interaction among the ICES, the WM, and the prosumers is portrayed in Fig. 1. Each household (HH) is a
According to [38], the stochastic programming model can be mathematically formulated as a scenario-based model, as follows.

### A. Upper Level

The goal of the upper-level problem is to maximize the total profit of the ICES determining the optimal size of the ESS and PV modules while interacting with the prosumers and the WM via a stochastic programming model. The profit is the revenue from selling/buying energy to/from the WM and the prosumers. The set of decision variables is $\mathcal{ICES} = \{\mathcal{ICES}_{i}, \mathcal{ICES}_{p}\}$, $\mathcal{ICES}_{t}$, $\mathcal{ICES}_{s}$, $\mathcal{ICES}_{d}$, $\mathcal{ICES}_{e}$, $\mathcal{ICES}_{dis}$, $\mathcal{ICES}_{pv}$, $\mathcal{ICES}_{PV}$, $\mathcal{ICES}_{PST,ICES}$). In this level, the optimal energy exchange between the ICES and the prosumers ($p_{t,ls}^{\mathcal{ICES}_{i-\mathcal{ICES}_{i}}}$) and the price to be paid if the ICES buys electricity from a prosumer ($\lambda_{t,ls}^{\mathcal{ICES}_{i-\mathcal{ICES}_{i}}}$) are the optimal values of the lower-level variables, namely exchange variables, see Fig. 1.

Maximize $\sum_{t,s} \tau_{t,s} \Delta$ \left\{ $A_{t} \left[ \sum_{i} A_{i}^{\mathcal{ICES}_{i}} p_{t,ls}^{\mathcal{ICES}_{i}} - \sum_{i} A_{i}^{\mathcal{ICES}_{i}} p_{t,ls}^{\mathcal{ICES}_{i}} \right]$
+ $B_{t} \left[ \sum_{i} A_{i}^{\mathcal{ICES}_{i}} p_{t,ls}^{\mathcal{ICES}_{i}} - \sum_{i} A_{i}^{\mathcal{ICES}_{i}} p_{t,ls}^{\mathcal{ICES}_{i}} + \sum_{i} A_{i}^{\mathcal{ICES}_{i}} p_{t,ls}^{\mathcal{ICES}_{i}} \right]$
+ $C_{t} \left[ \sum_{i} A_{i}^{\mathcal{ICES}_{i}} p_{t,ls}^{\mathcal{ICES}_{i}} + \sum_{i} A_{i}^{\mathcal{ICES}_{i}} p_{t,ls}^{\mathcal{ICES}_{i}} \right]$
\right\}

s.t.

$\sum_{t} (p_{t,ls}^{\mathcal{ICES}_{i}} - p_{t,ls}^{\mathcal{ICES}_{i}}) + p_{t,ls}^{\mathcal{ICES}_{i}} - p_{t,ls}^{\mathcal{ICES}_{i}} + p_{t,ls}^{\mathcal{ICES}_{i}} = 0, \forall (t,ls)$

$\sum_{t} (\Delta_{t}^{\mathcal{ICES}_{i}} p_{t,ls}^{\mathcal{ICES}_{i}} - \frac{1}{\eta_{t}^{\mathcal{ICES}_{i}}} p_{t,ls}^{\mathcal{ICES}_{i}}) = 0, \forall (t,ls)$

$0 \leq p_{t,ls}^{\mathcal{ICES}_{i}} \leq \frac{\eta_{t}^{\mathcal{ICES}_{i}}}{\eta_{t}^{\mathcal{ICES}_{i}}} P_{t,ls}^{\mathcal{ICES}_{i}}, \forall (t,ls)$

$0 \leq \lambda_{t,ls}^{\mathcal{ICES}_{i}} \leq P_{t,ls}^{\mathcal{ICES}_{i}}, \forall (t,ls)$

where:

$c_{t}^{\mathcal{ICES}_{i}} = \sum_{i} \tau_{t,s} p_{t,ls} \Delta (C_{t}^{\mathcal{ICES}_{i}} p_{t,ls}^{\mathcal{ICES}_{i}} + C_{t}^{\mathcal{ICES}_{i}} p_{t,ls}^{\mathcal{ICES}_{i}})$

$c_{t}^{\mathcal{ICES}_{i}} = C_{t}^{\mathcal{ICES}_{i}} P_{t,ls}^{\mathcal{ICES}_{i}}$

PV-based prosumer that can participate in the WM, interact with the ICES, and manage its generation to minimize its electricity bill. On the other hand, the ICES, via a joint interaction with the prosumers and the WM, performs an optimal ESS and renewable energy resource plan that guarantees the maximum profit. For the sake of tractability and according to the scales of consumers and ICES, they are considered to be price-takers from the market side, not affecting the overall wholesale electricity price. This interaction is formulated as a bilevel optimization problem.

Tractability is a significant issue in handling operating-planning models due to the plethora of decisions to be made via the decision-making tool. In this paper, with the aim of finding an appropriate tradeoff between modeling accuracy and computational tractability, load and PV duration curves are used to obtain a tractable number of scenarios, as portrayed in Fig. 2. The idea is to split the load and solar irradiation curves into quarter $q$, weekday/weekend $r$, and day/night $n$, where each curve contains 4 blocks $b$, and 3 levels of demands and irradiations. This way, the demand factor, $\mu_{t,n}^{\mathcal{ICES}_{i}}$, and the maximum level of PV power generation, $\mu_{t,n}^{\mathcal{ICES}_{i}}$, are obtained (see [45] and [40]).
Load balance of the ICES is stated in (2); charging and discharging of the ESSs is presented in (3) to integrate chronological information [40]; PV capacity to be installed at hour $t$ is defined in (4) and (5); (6) is used to check if the ICES requests more energy than in the contract, $\bar{p}^{PV,ICES}_t$, or stays within the limit, consequently, the corresponding cost is determined using a binary term in (1). Charging and discharging limits of the ESSs of the ICES are presented in (7) and (8), respectively; (9) and (10) stand for the interaction limits with the prosumers and the WM; (11) shows that the price of electricity that the ICES may sell to prosumers should remain lower than a price threshold proportional to the price of electricity that it buys from the WM; (12)-(13) are the maximum investment costs on PV modules of the ICES and ESSs at hour $t$, respectively; investment, charging and discharging, and maintenance costs of the ESSs of the ICES are presented in (14), (15), and (16); investment, generation, and maintenance costs of the PV units are presented in (17), (18), and (19), respectively; (20) stands for the excess demand surplus cost; and (21) and (22) are the coefficients to obtain the present values of investment and operating costs.

**B. Lower Level**

In this level, each prosumer minimizes its bill, which is equivalent to maximizing its utility (23), by interacting with the WM and the ICES while determining the size of the PV package, PV panels and ESS together, to be installed. For the sake of simplicity and due to technical reasons, prosumers cannot request energy from the WM above their contract, but they may compensate their deficit via interaction with the ICES. The set of decision variables of this level is $\mathbf{\pi}_{\text{Prosumers}} = \{ \lambda^\text{ICES,j}_t, p^\text{WM,j}_t, p^\text{PV,j}_t, p^\text{ESS,j}_t, \text{PV}_t, \text{PC}_t \}$, and $\mathbf{\rho}^{\text{dis,j}_t}$. The ICES, in the upper level, submits the optimal values of its energy exchange with the prosumers, $p^\text{ICES,j}_t$, and the requested price $\lambda^\text{ICES,j}_t$, in the case a prosumer buys electricity from the ICES, to the lower level, and accordingly, the prosumers maximize their profit. The dual variable of each constraint is indicated following a colon. Practically, at the lower level, we have several individual optimization problems corresponding to multiple followers. However, the prosumers play simultaneously while there are no binding constraints among them and the objective functions are all separated, therefore, without loss of generality, the follower problem can be mathematically modeled as a single-objective problem [46]. It is worth mentioning that the lower level does not have any non-convexity thanks to the assumptions of the model, see Section III-A-f.

**Solution Approach**

The aforementioned model can be solved via an iterative process, which, on the one hand, can be computationally expensive.
inefficient, while, on the other hand, defining a stopping criterion is a challenging issue [37]. To facilitate finding the solution, the original bilevel model is recast into an equivalent single-level problem. In this regard, the lower-level optimization problem is transformed to a set of constraints of the upper-level problem by using the strong duality theorem [47]. Note that $\alpha_{i,t,s}^{ICES^{+}}$ and $\lambda_{i,t,s}^{ICES^{-}}$, which are variables in the upper-level problem, are sent to the lower-level problem as parameters $\pi_{i,t,s}^{ICES^{+}}$ and $\lambda_{i,t,s}^{ICES^{-}}$ in (23) and (24). Therefore, the lower level is linear and the application of the strong duality theorem is justified. By doing so, a single-level problem is obtained where the aforementioned parameters become the decision variables for the single-level problem. It is worth mentioning that this is only a reformulation that facilitates handling the bilevel problem via a commercial solver. It does not mean that, in reality, only one agent will optimize both the upper- and lower-level problems. In fact, it is a game, and cross-border information will be exchanged where no agent, in the upper or the lower levels, will come to know the objectives, constraints, etc., of the other one(s).

Although in such reformulations privacy is questionable, it is a very useful technique to get rid of the iterative/heuristic-based process in handling bilevel programming problems.

$$\text{Maximize } \sum_{i,t,s} \pi_{i,t,s}(A) \left[ \sum_{j,s} \pi_{j,s}^{ICES^{+}} - \sum_{j,s} \pi_{j,s}^{ICES^{-}} + \lambda_{j,s}^{ICES^{+}} p_{j,s}^{ICES^{-}} - \lambda_{j,s}^{ICES^{-}} p_{j,s}^{ICES^{+}} \right]$$

$$+ B \left[ \sum_{j,s} \pi_{j,s}^{ICES^{+}} p_{j,s}^{ICES^{+}} - \sum_{j,s} \pi_{j,s}^{ICES^{-}} p_{j,s}^{ICES^{-}} + \lambda_{j,s}^{ICES^{+}} p_{j,s}^{ICES^{-}} - \lambda_{j,s}^{ICES^{-}} p_{j,s}^{ICES^{+}} \right]$$

$$- \lambda_{j,s}^{ICES^{+}} p_{j,s}^{ICES^{+}} - \pi_{j,s}^{ICES^{-}} p_{j,s}^{ICES^{-}}$$

(38)

$$s.t. \sum_{j,s} (p_{j,s}^{ICES^{+}} - p_{j,s}^{ICES^{-}}) + p_{j,s}^{ICES^{+}} - p_{j,s}^{ICES^{-}} - p_{j,s}^{ICES^{+}} + p_{j,s}^{ICES^{-}} = 0; \forall (i,l,s)$$

$$p_{j,s}^{ICES^{+}} - p_{j,s}^{ICES^{-}} + p_{j,s}^{ICES^{+}} - p_{j,s}^{ICES^{-}} - p_{j,s}^{ICES^{+}} + p_{j,s}^{ICES^{-}} = D_{j,s}; \forall (i,t,l,s)$$

$$\sum_{j,s} \left[ A \pi_{j,s} \lambda_{j,s} \left( \sum_{i,t,s} \pi_{i,t,s} - \sum_{i,t,s} \pi_{i,t,s} \right) - \lambda_{j,s}^{ICES^{+}} p_{j,s}^{ICES^{-}} - \lambda_{j,s}^{ICES^{-}} p_{j,s}^{ICES^{+}} - p_{j,s}^{ICES^{+}} + p_{j,s}^{ICES^{-}} \right]$$

$$+ B \left[ \sum_{j,s} \pi_{j,s}^{ICES^{+}} p_{j,s}^{ICES^{+}} - \sum_{j,s} \pi_{j,s}^{ICES^{-}} p_{j,s}^{ICES^{-}} + \lambda_{j,s}^{ICES^{+}} p_{j,s}^{ICES^{-}} - \lambda_{j,s}^{ICES^{-}} p_{j,s}^{ICES^{+}} \right]$$

$$- \lambda_{j,s}^{ICES^{+}} p_{j,s}^{ICES^{+}} - \pi_{j,s}^{ICES^{-}} p_{j,s}^{ICES^{-}}$$

(41)

$$= \sum_{i,t} \chi_{i,t,s}^{ICES^{+}} D_{i,t,s}^{ICES^{+}} + p_{i,t,s}^{ICES^{+}} + p_{i,t,s}^{ICES^{-}} + p_{i,t,s}^{ICES^{+}} + p_{i,t,s}^{ICES^{-}}$$

$$+ \sum_{i,t} \chi_{i,t,s}^{ICES^{+}} D_{i,t,s}^{ICES^{+}} + p_{i,t,s}^{ICES^{+}} + p_{i,t,s}^{ICES^{-}} + p_{i,t,s}^{ICES^{+}} + p_{i,t,s}^{ICES^{-}}$$

$$+ \sum_{i,t} \chi_{i,t,s}^{ICES^{+}} D_{i,t,s}^{ICES^{+}} + p_{i,t,s}^{ICES^{+}} + p_{i,t,s}^{ICES^{-}} + p_{i,t,s}^{ICES^{+}} + p_{i,t,s}^{ICES^{-}}$$

$$+ \sum_{i,t} \chi_{i,t,s}^{ICES^{+}} D_{i,t,s}^{ICES^{+}} + p_{i,t,s}^{ICES^{+}} + p_{i,t,s}^{ICES^{-}} + p_{i,t,s}^{ICES^{+}} + p_{i,t,s}^{ICES^{-}}$$

$$= 0$$

$$\sum_{i,t} \chi_{i,t,s}^{ICES^{+}} + \sum_{i,t} \chi_{i,t,s}^{ICES^{-}} = 0$$

(42)

$$\sum_{i,t} \chi_{i,t,s}^{ICES^{+}} + \sum_{i,t} \chi_{i,t,s}^{ICES^{-}} = 0$$

(43)

$$\chi_{i,t,s}^{ICES^{+}} + \sum_{i,t} \chi_{i,t,s}^{ICES^{-}} = 0$$

(44)

$$\chi_{i,t,s}^{ICES^{+}} + \sum_{i,t} \chi_{i,t,s}^{ICES^{-}} = 0$$

(45)
the bilinear term $\chi_{t,LS}^j p_{t,LS}^{ICES,i+}$, $\chi_{t,LS}$ is the marginal value corresponding to (24). This variable, due to the optimality condition, is an unbounded variable that makes the linearization difficult. From a mathematical standpoint, the bounds for a marginal value can be obtained via sensitivity analysis. The details of this method, which adds a set of auxiliary variables, $\Xi^{aux}$, are presented in the electronic companion [52]. The equivalent single-level mixed integer linear programming (MILP) model is presented in (59)-(61) where the set of decision variables is $\Xi^{joint} = (\Xi^{ICES} \cup \Xi^{prosumers} \cup \Xi^{aux})$.

Maximize $\sum \pi_A \left( (\Psi^{ICES,i} - \Psi^{2O,ij,t}) \right)$

- $\sum_{i,j} \left( (\Psi^{ICES,j,i} - \Psi^{2O,ij,t}) \right) + \sum_{i,j} (\Psi^{ICES,j,i} - \Psi^{2O,ij,t})$

+ $B_n \left( (\Psi^{ICES,j,i} - \Psi^{2O,ij,t}) \right)$

s.t. (3)-(13), (25)-(33), (40), (41), and (43)-(58)

III. ASSUMPTIONS AND CASE STUDIES

The proposed trilateral model is tested on a system containing three groups of households and an ICES, which are actively interacting with the WM. This section presents the assumptions, and the case studies used to validate the model. Technical and economic information are provided in the electronic companion [52].

A. Assumptions

The main assumptions to provide a proper analysis for the proposed model are summarized as follows.

a) The price of the contracted energy of the ICES with the WM, as a large electricity customer, is mostly the same as the wholesale price of electricity. Consequently, the households, as small customers, should pay higher prices for their contracts with the WM than the ICES.

b) Only cross-border information is exchanged between the two active players, the prosumers and the ICES, and none of them is aware of the priorities, objectives, and constraints of the others. This is valid for the original bilevel model, however, in the equivalent single-level model, keeping privacy is indeterminate.

c) The ICES may demand more energy than its contract but at a higher price. In practice, this price is determined in the balancing market, however, a penalty is applied to handle this situation in this work.

d) The WM is willing to buy from or sell to the prosumers and the ICES at the day-ahead price.

e) The energy selling price to the households by the ICES should remain below the price of the contract between the households and the WM at the same spot.

f) The households are not eligible to ask for more energy than the contracted energy either from the ICES or from the WM.

g) The costs of the devices to be installed by the ICES, as a big dealer, are lower than the costs of the households.

h) The households, under the condition of being prosumers, are provided with ESSs.

i) The capacity of the storage device of the ICES should be at least 25% of the maximum output of the installed renewable energy. This helps keep PV curtailment to an acceptable level.

B. Case Studies

To analyze the proposed model properly, two different cases: a system with conventional consumers, namely passive consumers, and a system with modern consumers, namely active prosumers, are studied. These studies are carried out in three stages where the time span of each stage is one year.

- **Case I:** Considering passive consumers

In this case, the households are conventional consumers, prosumers that do not own any PV technology or ESS and
only consume energy. These households aim at decreasing their costs toward smart demand satisfaction via requesting energy from the WM and the ICES.

- **Case II: Considering active prosumers**
  In this case, the households own a small ESS and actively interact with the WM and the ICES to manage their costs by making an appropriate PV installation plan.

C. **Technical and Economic Information**

The demand of three groups of households for the first stage is depicted in Fig. 3. Generic domestic demand data from the Spanish energy system are considered in this work. As can be seen, each stage is represented by 192 scenarios that have been generated using duration curves [53]. Each stage is divided into different quarters standing for four seasons. The most effective procedure at this step is picking up samples related to the weekdays and weekends and selecting at each sample day two representative hours for day and night; due to the demand or generation outputs. These historical data are adequately scaled and, consequently, the duration curves obtained are divided into four different blocks, and each block contains three levels of demand and PV generation.

The annual expected load growth rates are assumed to be random values between 3% and 6% where a normal-looking probability distribution is used for this purpose, see the sample codes in the electronic companion [52]. The annual capital investment budgets for the active prosumers to install PV package, and for the ICES to install PV systems and ESS, technical data for the ICES and the prosumers, installation, and operation costs of the PV system and ESSs of the ICES, and bilateral contracts among the ICES, the prosumers, and the WM are provided in [52].

Bilateral contracts among the ICES, the prosumers, and the WM. Such contracts enable competitiveness among them that finally may result in an electricity price reduction as long as the predefined objectives are met. In this work, the contracts have been defined in such a way that the households may satisfy their demands under critical circumstances in which there are no renewable energy and energy storage units, by interacting with the WM and/or the ICES. Therefore, the prosumers have enough freedom in making the most appropriate decisions for their interactions and PV package installation plans. On the other hand, the ICES, due to its high budget for the PV and ESS installations, has a lower contract amount with the WM than the prosumers. This is mainly because, firstly, the prosumers may provide cheaper energy to the ICES than the WM during some periods, and secondly, because the ICES has permission to request more energy than its contract with the WM under critical circumstances. Although such unpredicted energy request from the WM is subject to a high penalty, it provides enough flexibility to the ICES to optimally manage its interactions with the WM and the prosumers while making the best PV and ESS installation plans.

IV. **Numerical Results and Analysis**

This section analyzes the trilateral framework via two case studies, introduced in Section III and subsection B while different interactions among the consumers, the ICES and the WM are studied. The first case is used as a benchmark for further analysis of the proposed model. To this end, the resulting operating costs and interactions with the ICES and the WM in critical hours, the investment decisions of the ICES and the corresponding operating conditions are considered in the passive consumer-based environment. The second case shows the effectiveness of having smart prosumers instead of passive consumers for both the demand and ICES sides. The proposed models have been implemented on an HP Z240 Tower Workstation with eight Intel Xeon E3-1230 v5 processors at 3.4 GHz and 48 GB of RAM using CPLEX 12.8 [54] under GAMS 25.1.2 [55].

1) **Case I: Considering passive consumers**

The results obtained by the proposed framework for the case in which the households are conventional consumers are presented in Table I and Table II. The investment decisions for the ICES are represented in Table I, while the overall investment, O&M, and energy costs for the ICES are
summarized in Table II. The outcomes of case I are used to analyzing the impacts on the ICES and the demand side of being a smart prosumer in case II.

From Table I, it can be seen that the PV and storage installations at each stage are lower than the previous stages. However, Table II reveals that the PV installation, at the first stage, and the ESS installation, at the second stage, meet the allocated budgets. Moreover, Table II shows that the O&M and energy costs are negligible compared to the investment costs. The total O&M costs of PV and ESS for all the three stages are $30/\text{MWh}$ and $20/\text{MWh}$, respectively. To investigate more on the consumer’s interactions, Fig. 4 is used.

Fig. 4 depicts the interactions between each consumer and the ICES as well as the overall interaction at stage 3. In this figure, positive and negative bars stand for the power bought from and sold to the ICES, respectively. From the subplots related to the consumers, it can be seen that, in most of the scenarios, the consumers interact with the ICES, either by buying or selling power. The subplot related to the overall interaction provides the possibility of observing the extreme interactions: the highest amount of power that the consumers sell to the ICES (scenario #23), the highest amount of power that the ICES sells to the consumers (scenario #73), and the net interaction (scenario #84). Therefore, for didactic purposes and for the sake of comprehension, these interactions are illustrated in Fig. 5 in more detail with the following format: Power in MW (price in €/MWh). This figure not only provides power interactions among the consumers, the ICES, and the WM, but also presents the corresponding prices.

Fig. 5, considering the net interaction between the consumers and the ICES in scenario #84, shows that all three groups of households are buying 0.168 MW, 0.326 MW, and 0.315 MW from the WM at the price of €37.879/MWh. The main reason for this net interaction is that the market price is much lower than the interaction price between the consumers and the ICES; for all the three groups of households, the interaction price is €56.819/MWh. In scenario #23, the consumers are selling to the ICES, buying from the WM at the same time. By studying more scenarios like this, it can be concluded that this situation happens when: 1) the market price is lower than the interaction price between the consumers and the ICES, 2) the ICES does not have enough PV generation and power stored in the ESS, and 3) the ICES is purchasing the maximum contracted power from the WM. For example, in this scenario, the market price is €24.381/MWh, therefore the consumers and the ICES are buying their maximum contracted powers with the WM, 0.35 MW and 1.10 MW, respectively. At the same time, the consumers are selling 0.025 MW, 0.035 MW, and 0.098 MW to the ICES at the prices of €36.572/MWh, €36.572/MWh, and €34.576/MWh, respectively. On the other hand, PV production of the ICES is zero and the ESS is charging, 1.258 MW, to store energy for the upcoming situations. In scenario #73, the ICES is selling different amounts of power to the consumers at different prices. The purchase price for the first consumer, HH1, that is buying more power than the others is lower, and vice versa, i.e., the consumers are buying 1.2 MW, 1.054 MW, and 1.106 MW, respectively, at €55.559/MWh, €55.706/MWh, and €55.653/MWh. This proves that the model is smart enough to provide better offers for bigger consumers.

![Fig. 5. Interactions among the ICES, HHs, and the WM, case I, stage 3.](image)

**Table I**

<table>
<thead>
<tr>
<th>Cost</th>
<th>Stage 1</th>
<th>Stage 2</th>
<th>Stage 3</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>PV (MW)</td>
<td>3.698</td>
<td>1.779</td>
<td>0.533</td>
<td>6.010</td>
</tr>
<tr>
<td>ESS (MW)</td>
<td>0.925</td>
<td>0.4448</td>
<td>0.1333</td>
<td>1.5031</td>
</tr>
</tbody>
</table>

**Table II**

<table>
<thead>
<tr>
<th>Cost</th>
<th>Stage 1</th>
<th>Stage 2</th>
<th>Stage 3</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investment</td>
<td>263,000</td>
<td>127,450</td>
<td>38,175</td>
<td>430,626</td>
</tr>
<tr>
<td>ESS</td>
<td>72,727</td>
<td>35,000</td>
<td>10,483</td>
<td>118,206</td>
</tr>
<tr>
<td>O&amp;M and PV</td>
<td>4.69</td>
<td>6.87</td>
<td>7.46</td>
<td>19.02</td>
</tr>
<tr>
<td>Energy</td>
<td>341.46</td>
<td>537.61</td>
<td>596.24</td>
<td>1,475.31</td>
</tr>
<tr>
<td>Total</td>
<td>338,068</td>
<td>162,995</td>
<td>49,262</td>
<td>550,326</td>
</tr>
</tbody>
</table>

**INVESTMENT DECISIONS FOR THE ICES OVER THE PLANNING HORIZON—CASE I**

**INVESTMENT, O&M, AND ENERGY COSTS (€) FOR THE ICES—CASE I**
Moreover, the ICES is selling 1.1 MW to the WM, which is the contract limit. Therefore, for this scenario, the ICES is selling 4.46 MW to the consumers and the WM. To analyze this situation in more detail, the total RES-based power generation and the SOC of the ESS are taken into account.

From the results provided in Table I, the total installed capacities of PV and ESS at the end of stage 3 are 6.010 MW and 1.5031 MW, respectively. However, the results obtained show that for scenario #73, the PV system is generating about 5.53 MW and, at the same time, the ESS is charging about 1.07 MW. Consequently, from the ICES perspective, the total power (generation plus import, 5.53 MW) is equal to the total demand (ESS charging, 1.07 MW, plus export, 4.46 MW). The total profit of the ICES is €1,082,805.03, while the total cost of all consumers in all stages is €11,975,925.55. Details regarding the costs corresponding to the consumers’ interactions with the ICES and the WM are presented in Table III. It can be seen that, for all consumers, above 60% of the energy has been bought from the ICES, while, on average, 66.6% of the energy comes from the ICES. This means that the ICES provides more economic benefits to the consumers. Among them, consumer 3 with 70.6% and consumer 2 with 66.6% of the electricity purchased present the highest interaction with the ICES and the WM, respectively.

In order to see whether the investment decisions are worth being applied or not, in this paper, the return on investment (RoI) ratio is used, (62) [56]. The RoI shows the effectiveness of the prosumers’ investments, where a zero RoI means that the investment is not worth it.

\[ \text{RoI} = \frac{\text{GoI} - \text{CoI}}{\text{CoI}} \]  

(62)

where GoI and CoI stand for the gain of investment and cost of investment, respectively.

The RoI of the ICES for this case is 0.968: (1,082,805.03 – 550,326.93)/550,326.93, which reveals the high effectiveness of the investment; it not only covers all the expenses but also results in a very high profit, €532,478.62.

On the other hand, results show that for case I, after making the best operation-planning decision, there are several situations in which the ICES decides to request more energy from the WM than its contract by paying the imposed penalty. This happens in five scenarios at stage 3, once in fall, by requesting 1.743 MWh and four times in winter by requesting 1.466 MW, 1.540 MW, 1.428 MW, and 1.393 MW. To study such conditions, the scenario corresponding to the fall is considered in more detail. Considering that the contract between the ICES and the WM is 1.2 MWh [52], this sample scenario shows that the ICES is requesting 0.543 MW more than its contract amount. Since this scenario takes place at night, there is no PV production. The ESS is charging 1.277 MW while, at the same time, the ICES is selling 0.466 MW to the consumers (0.458 MW, 0.000 MW, and 0.008 MW to each consumer, respectively). Consequently, the ICES is buying 1.743 MW and selling 0.466 MW, and the rest is stored in the ESS. All the scenarios in which extra power has been requested occur during the night when there is no PV production. Then, by requesting extra power, the ICES does not give a negative response to those consumers that are willing/asking to buy from the ICES instead of from the WM, therefore making a profit.

2) Case II: Considering active prosumers

This case shows the benefit of being an active prosumer owning a renewable energy package (PV and ESS). The prosumers make their own planning decisions via an operation-planning model to minimize the energy and investment costs.

Table IV and Table V present the investment decisions and the investment and energy costs related to the ICES for case II. Compared to the investment decisions obtained for the first case in Table I, it can be seen that, for the first two stages, the installed PV and ESS are similar, while the ICES has installed less PV and ESS at stage 3 in case II compared to the first case. The O&M of the ESSs for all the stages are different from those obtained in the first case, see Table II. The main reason for such differences is that the operating costs of the storage devices are proportional to the charging and discharging powers, see (14). Moreover, in this case, compared to the first case, the ICES requests extra power than its contract more frequently. This occurs 3 times at stage 2 in winter and 5 times at stage 3 (twice in spring, once in fall, and twice in winter). This situation happens because 1) the prosumers request more power from the ICES to store it in their ESSs; in this situation the prosumers have reached the contract limit with the WM and cannot request more power, and 2) the ICES decides to install lower ESS capacity at stage 3 (see Table IV), compared to the first case. The total cost for the ICES is €533,606.4, while these investment decisions result in €1,069,081.84 of profit. Although the profit of the ICES for this case is €13,723.71 lower than its profit in the first case, the total cost is also €16,720.53 lower than in the
first case. Therefore, the RoI ratio for this case is 1.003, which is 3.59% higher than the first case. Overall, results reveal that, although the ICES requests more extra power to compensate for a power lack and the net profit decreases, unlike expected, a smart prosumer-friendly ICES is much more efficient than an ICES with passive consumers. The impacts of smart prosumers on the demand side are provided as follows.

Table VI shows the investment decisions and the corresponding investment and O&M costs for each prosumer at different stages. The effectiveness of the investment decisions is investigated by interpreting the RoI. To obtain the total benefit of the prosumers, their total payments for this case are compared with the total payments in the first case, where they are passive consumers. Table VII presents the total payments of each prosumer to the ICES and the WM. This table, compared to Table III, indicates that the payment of the three prosumers decreases by €386,263.19, €152,495.63, and €323,161.16, respectively. These values reveal how a small investment and being a smart prosumer make a big difference in the consumers’ energy bills. Comparing the latter tables also shows that the overall prosumers’ payments to the ICES and the WM decrease by €486,544.33 and €375,375.65, respectively.

The results obtained for case II reveal the profitability of changing the old passive consumer to a smart prosumer. It not only positively affects the ICES, by decreasing the investment cost and increasing its RoI, but also results in huge decreases in the electricity bills of the consumers while the very large

**Table IX**

<table>
<thead>
<tr>
<th>Size</th>
<th>Order of Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td># of SOS2 variables</td>
<td>(6(N_a-N_b-N_c)N_1T-S)</td>
</tr>
<tr>
<td># of binary variables</td>
<td>((N_a-N_b-N_c)T)</td>
</tr>
<tr>
<td># of positive continuous variables</td>
<td>(4(N_a-N_b-N_c)+N_1T-S)</td>
</tr>
<tr>
<td># of negative continuous variables</td>
<td>(10(N_a-N_b-N_c)S+N_1T)</td>
</tr>
<tr>
<td># of free continuous variables</td>
<td>((N_a-N_b-N_c)+2S+N_1T-S)</td>
</tr>
<tr>
<td># of equality constraints</td>
<td>(1+N_1T+(N_a-N_b-N_c)T+S+27(N_a-N_b-N_c)S+N_1T-S)</td>
</tr>
<tr>
<td># of inequality constraints</td>
<td>(2T+2N_1T+10(N_a-N_b-N_c)S+N_1T-S+12(N_a-N_b-N_c)+N_1T-S)</td>
</tr>
</tbody>
</table>

**Table VI**

<table>
<thead>
<tr>
<th>Investment Decisions and Costs, O&amp;M, and Energy Costs (€) for the Prosumers—Case II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>HH 1</td>
</tr>
<tr>
<td>Investment</td>
</tr>
<tr>
<td>HH 2</td>
</tr>
<tr>
<td>Investment</td>
</tr>
<tr>
<td>HH 3</td>
</tr>
<tr>
<td>Investment</td>
</tr>
<tr>
<td>Total Cost</td>
</tr>
</tbody>
</table>

**Table VII**

<table>
<thead>
<tr>
<th>Interaction Costs (€) of Prosumers with the ICES and the WM at All Stages—Case II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prosumer 1</td>
</tr>
<tr>
<td>Payment to ICES</td>
</tr>
<tr>
<td>Payment to WM</td>
</tr>
<tr>
<td>Total</td>
</tr>
</tbody>
</table>

**Table VIII**

<table>
<thead>
<tr>
<th>Computing Times Required (s)</th>
<th>Optimality Gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case I</td>
<td>108.39</td>
</tr>
<tr>
<td>Case II</td>
<td>178.95</td>
</tr>
</tbody>
</table>

**B. Computational Analysis**

This section provides the computational analysis regarding the presented case studies. Although medium- and long-term planning decisions are not considered as prompt actions, which occur in a day, week, or even months, the computational efficiency is always an issue to bear in mind.

Table VIII presents the computational behavior for different optimality gaps. In literature, a gap lower than 5% is sometimes an acceptable gap for large-scale optimization problems, however, we consider only those gaps lower than 3%. As can be observed from this table, for the first case, the computational time exponentially increases when the gap decreases, while, for the prosumer-based case, case II, the effect of a gap decrease is not that much. The prosumers offer more flexibility to this complex planning problem by providing more freedom regarding storage, power generation, and effective interactions. As a result, convergence is much faster for the lower gaps than in the first case, e.g., for gaps lower than 1%, the computational efficiency of case II is 463.3% higher than in the first case. Hence, considering the prosumers in the proposed model resulted in an enhancement in computational efficiency.

However, to elaborate more on the computational complexity, the size and order of complexity is considered. Table IX summarizes the computational complexity of the proposed model. The second column stands for the size of the equivalent single-level MILP model, while the third column shows the order of complexity for large-scale problems. We assume that the number of discrete SOS2 variables, and the number of quarters q, working day/weekend r, day/night n, and the number of load blocks N_b are the same for all scales. Consequently, the order of complexity is proportional to the number of prosumers N_a, the number of time periods T, and the number of scenarios S.

**V. CONCLUDING REMARKS**

This paper presents a bilevel programming model to address the operation-planning problems of the ICES and PV-based prosumers while interacting with the WM. The ICES, in the upper level, aims at maximizing their profit, while the prosumers, in the lower level, intend to minimize their bills. Appropriate strong duality theorem-based recast techniques allow us to convert the bilevel model into an equivalent single-level model. However, the recasting process results in a highly nonconvex MINLP model as a consequence of bounded and unbounded bilinear terms. To get rid of these bilinear terms, SOS2 techniques are used concurrently with...
sensitivity analysis. Moreover, in order to have a tractable model, demand and PV factors are obtained via duration curve analysis. The ICES and the prosumers, by simultaneously interacting with each other and the WM make the most effective decisions to maximize their profit and minimize their bills, respectively. Results show that considering smart prosumers not only decreases the energy bills of end-users but also, surprisingly, positively affects the ICES profit. From the computational standpoint, smart prosumers highly increase the computational efficiency by providing more freedom to the trilateral model.

Future research will model flexible loads such as heating, ventilation and air conditioning (HVAC), electric water heaters (EWHs), etc., as well as critical loads to activate the demand response options.

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