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Optimization of the Pulp Mill Economical Efficiency; study on the behavior effect of the economically significant variables

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Abstract:

Aim for optimization of the Pulp Mill economical efficiency is to find the steady state, which maximizes the profit rate of the whole plant including energy costs, cooking and bleaching chemicals costs and taking into account delignification and brightness requirements. The construction of approximations of the economically significant variables as functions of the decision variables (setpoints) plays the key role in the optimization. In this paper, the linear and quadratic approximation approaches are studied, the optimization results compared and discussed.

Keywords: Plantwide optimization, Pulp and Paper production, response surface approximation.

1. INTRODUCTION

Due to the continuously increasing competition in the pulp and paper industry, there is a need to develop solutions that can increase the economical efficiency of the plants. However, the significant number of studies is still concentrated on the optimization of single unit operations in pulp mills: McDonough et al. (2008) studied the optimization of the ClO₂ requirements for the bleaching process; Tang et al. (2007) optimized the Pulp Washing process; Sidrak (1995) optimized the Kamyr Digester towards significant reduction in the amount of off-specification pulp; Santos and Durado (1999) optimized energy consumption and the plant's production.

Nowadays, the trend is to optimize the whole mill with respect to production and quality, minimization of energy, chemical consumption and effluents. In the recent paper Castro and Doyle (2004) have proposed the Pulp Mill benchmark model, having the standard architecture with a Kamyr digester, a bleaching plant and a chemical recovery (see Castro and Doyle (2004) for the details). The benchmark model is well suited for performing of a wide range of the Pulp Mill studies, including the optimization problem of economical efficiency. Recently Mercangöz and Doyle (2008) have performed the optimization of the benchmark model, which deals with the whole Pulp Mill and the optimization criterion is the plant's profit rate including energy costs, cooking and bleaching chemicals costs, final products sales (pulp and steam) and which takes into account delignification and brightness requirements to the final product. The optimization, performed in Mercangöz and Doyle (2008), is based on the approximation of the steady state values of the economically significant variables by linear functions of the decision variables.

Different pairs of the economically significant variables and decision variables have both linear and nonlinear dependences. In addition, some decision variables are interacting with each other. In this paper the benchmark problem is restudied in relation to the different approximation approaches of the economically significant variables as functions of the decision variables. The two following approaches are proposed: first approach of the sum of the one dimensional quadratic functions and the second approach of the multidimensional quadratic functions. As a result, the satisfactory quality of approximation will be achieved.

The article is structured as follows: the section two contains the brief description of the Pulp Mill Benchmark model. The MPC and low level controls structure and the Economical Efficiency of the plant are described in section three. In the section four two new approaches to the approximation are proposed

and discussed. Section 5 contains the results of computations for all approaches. In the section 6 the comparison of the approaches is presented. Finally, section 7 contains discussion and conclusion.

2. THE PULP MILL BENCHMARK PROBLEM

In the Pulp Mill Benchmark Model developed by Castro and Doyle (2004) the chemical process units are modeled as perfectly mixed vessels or distributed parameters systems (plug flow vessels). Mass and energy balances are combined with the empirical equations for physical properties or thermodynamic information. The model has been implemented in C language in the form of SIMULINK program.

The optimization of the economical efficiency is highly related to the control of the plant. The plant-wide control is provided with the Benchmark Pulp Mill model, which includes both the MPC and the SISO control loops. In the control strategy four MPC blocks control the most important variables in the Kamyr Digester (MPC1), Bleaching plant (MPC2), Chemical recovery (MPC3) and the Lime Kiln (MPC4). The general scheme is presented in Figure 1.

MPC controllers regulate 14 important quality and environmental variables. A number of SISO controllers are used to stabilize the open-loop unstable modes of the process and the rest are used to maintain the process conditions in various unit operations. The setpoints of the basic control loops are partly generated by the MPC and partly defined as inputs of the plant. In addition, 21 manipulated variables are not used for any process control purposes.

Fig 1. Scheme of the Pulp Mill control strategy

Hence, the setpoints that are given to the MPC, most of the setpoints of the basic controllers and some manipulated variables of the model, can be freely varied and used as decision variables for the economical optimization of the Pulp Mill. According to Mercangöz and Doyle (2008), the following 44 variables (denoted as U) are used for the optimization in the study:

Table 1

List of the decision variables

Short name	Variable Name	Related to	Lower bound	Nominal Value	Upper bound
U1	D2 prod. rate	MPC1	629.979	629.998	630
U2	Dig. Kappa	MPC1	12.5	19.9993	27.5
U3	Dig. upper EA	MPC1	2.399	9.59977	16.799
U4	Dig. lower EA	MPC1	2.25	9.00007	15.75
U5	Upp. Conduct.	MPC1	17.46	69.8423	12.224
U6	Low. Conduct.	MPC1	12.11	48.4702	84.822
U7	O kappa	MPC1	5	9.99972	15
U8	E kappa	MPC2	2.4	2.5	2.6
U9	E washer [OH]	MPC2	0.0004499	0.00085	0.0012499
U10	D2 brightness	MPC2	0.8	0.81	0.84
U11	Bl. liq. solids	MPC3	0.625	0.65001	0.675
U12	Slaker temp.	MPC3	669.012	674.012	679.012
U13	Kiln O2 exc. %	MPC4	0.0149	0.03497	0.0549
U14	Kiln CaCO3 residual %	MPC4	0.02487	0.02487	0.02487
U15	Dig. Liq. T	SISO	425.5	435.5	445.5
U16	O tower T.	SISO	368	371	373.55
U17	D1 tower T.	SISO	329	339	347.4999
U18	E0 tower T.	SISO	344	349	353.2499
U19	D2 tower T.	SISO	343	348	352.2499
U20	O tower cons.	SISO	0.05	0.1	0.1425
U21	Reca. CaOH2	SISO	3	18	30.75
U22	WL temp.	SISO	358	368	376.4999
U23	WL NaOH conc.	SISO	99.1498	100	150
U24	Steam flow 3	FREE	0.0373	0.103	0.18025
U25	Exc. WL split	FREE	0.9	1	1.1
U26	D1 water flow	FREE	68.5125	189	330.75
U27	E back-flush	FREE	240	484.06	847.105
U28	D2 caustic	FREE	0.07325	0.293	0.4797
U29	Split fraction 6	FREE	0.2936	0.31915	0.3408
U30	Split fraction 8	FREE	0.7176	0.78	0.83304
U31	Split fraction 9	FREE	0.38	0.4	0.417
U32	Split fraction 1	FREE	0.7202	0.9	1.575
U33	Effect 3 steam	FREE	821.92	881.893	952.44
U34	Filter lower	FREE	0.54	0.862	1.508
U35	Kiln prim. air	FREE	62.5	250	437.5
U36	Effect 5 exit	FREE	75	300	525
U37	Effect 4 exit	FREE	75	300	525
U38	Effect 3 exit	FREE	75	300	525
U39	Effect 6 exit	FREE	75	300	525
U40	Effect 2 exit	FREE	75	300	525
U41	Effect 1a exit	FREE	75	300	525
U42	Effect 1b exit	FREE	75	300	525
U43	Effect 1c exit	FREE	75	300	525
U44	Coolant 4 Slak	FREE	2.5	10	17.5

Since the set of values of the decision variables are defined in Table 1, the next step is to find the corresponding steady state of the Pulp Mill and compute its economical efficiency. The steady state (further nominal steady state that will be used as an initial approximation for the economical efficiency

optimization) can be easily found via performing the simulation with a long time horizon with the setpoints, defined by values of the decision variables. The nominal values of the following economically significant variables are thus defined and presented on Table 2.

Most of the economically significant variables (denoted as V) are the manipulated variables of the Pulp Mill (the only exceptions are the pulp production rate and steam production rate), expressed in a volumetric or mass flow rate. The costs of all the chemicals are presented in Table 3 (denoted as C).

Table 2

List of the economically significant variables

Short	Variable name	Related to	Lower bound	Nominal value	Upper bound	Costs
V1	Production	MPC1	629.97	629.998	630	- c_pulp
V2	Steam 1	SISO	0	0.175	0.398	c_dig_st
V3	Steam 2	MPC1	0	0.204	0.5306	c_dig_st
V4	Steam 4	MPC1	0	0.126	0.3172	c_dig_st
V5	Steam 5	MPC1	0	0.101	0.3184	c_dig_st
V6	Water fl	SISO	0	675.293	1 324.6	c_water
V7	O steam 1	SISO	0	53.603	134	c_bl_st
V8	O steam 2	SISO	0	53.603	134	c_bl_st
V9	O steam 3	SISO	0	234.761	392	c_bl_st
V10	D1 ClO2	MPC2	0	674.316	3 316	c_ClO2
V11	D1 wash	SISO	0	6 025.59	1 237	c_water
V12	D1 steam	SISO	0	238.954	990.6	c_bl_st
V13	E caustic	MPC2	0	129.354	536	c_NaOH
V14	E steam	SISO	0	132.929	505.6	c_bl_st
V15	D2 ClO2	MPC2	0	496.924	1 904	c_ClO2
V16	D2 wash	SISO	0	6 164.76	6 263.1	c_water
V17	Coolant fl	MPC1	0	1 249.44	1 264.1	c_cooling
V18	Oxygen fl	SISO	0	13.221	50	c_O2
V19	Salt-cake	SISO	0	56.096	168.5	c_Na2SO4
V20	Coolant 1	MPC3	0	1.938	8	c_cooling
V21	Wash 1	SISO	0	0.041	0.1	c_cooling
V22	Fresh lime	SISO	0	1.848	8	c_CaO
V23	Coolant 2	SISO	0	0.160	2	c_cooling
V24	Caustic 3	SISO	0	0.014	0.068	c_NaOH
V25	Wash 2	SISO	0	2.227	4.466	c_cooling
V26	Wash 3	SISO	0	0.683	1.508	c_cooling
V27	Wash 4	SISO	0	0.193	0.298	c_cooling
V28	Wash 4	SISO	0	0.229	0.454	c_cooling
V29	Kiln fuel	MPC4	0	42.367	74	c_nat_gas
V30	Water fl	SISO	0	0.399	0.944	c_water
V31	Steam pr	FREE	0	4 158.71	∞	-c_recov
V32	Wood ch	MPC1	0	2.55	5.2286	c_wood
V33	O caustic	MPC1	0	29.000	87	c_NaOH
V34	Effect 1 st	MPC3	0	942.401	1 847.2	c_steam
V35	Effect 2 st	MPC4	92.36	923.600	1 385.4	c_steam
V36	Steam 3	Free	0	0.103	0.206	c_dig- st
V37	D1 water	Free	0	189.000	378	c_water
V38	E back-fl	Free	0.06	484.060	968.06	c_water
V39	D2 caustic	Free	0	0.293	0.586	c_NaOH
V40	Effect 3	Free	0	881.893	1 763.8	C_steam

Table 3

Costs and prices of the chemicals and products

Cost item	Cost (US\$/kg)
c bl st	0.0038
c CaO	0.0629
c ClO ₂	1.0000
c cooling	0.0530
c dig st	0.0065
c Na ₂ SO ₄	0.1020
c NaOH	0.2000
c nat gas	0.1702
c O ₂	0.1000
c pulp	-0.3300
c recov	-0.0051
c steam	0.0025
c water	0.0001
c wood	0.0390

3. FORMULATION OF THE PROBLEM

The objective of the optimization in the benchmark problem is simply to maximize the profit rate from the operation of the Mill. There are two sources for revenues: the bleached pulp product and the steam production in the recovery boiler. The expenses consist of woodchips costs as the raw material, the steam, chemicals and energy costs. There are three grades of the steam with different pressures used in various parts of the Pulp Mill. The bleaching chemicals are O₂, ClO₂ and NaOH. In the chemical recovery loop salt-cake, fresh lime and caustic make-up flows contribute to the costs together with the natural gas, which is used in the limekiln. In the benchmark problem, the profit of the plant given by (1) is used as the objective function for the optimization:

$$P = -(C_1V_1 + C_2V_2 + \dots + C_{40}V_{40}) \quad (1)$$

where V_i are values of the economically significant variables and C_i are their prices that are given in Table 3. Using the prices the profit of the Pulp Mill is about 105.9 USD/min at the nominal steady state.

The economical efficiency optimization is a typical constrained optimization problem: first, it includes the lower and upper non-equality constraints both to the Economically Significant and Decision variables and second the problem includes the equality constraints related to the dependences of the Economically Significant variables on the Decision variables. There are following two cases:

The case of the linear approximations

The objective function is a linear function of the Economically Significant variables and dependences between the Decision variables and Economically Significant variables are linear in the frames of the linear approximation method, the linear programming technique can be employed in order to find the optimal setpoint (see Mercangöz and Doyle (2008) for details). However, the real dependences between economically significant variables and the decision variables may be nonlinear. As a result, the insufficient reliability of the approximations is the essential drawback of the approach.

The case of the quadratic approximations

In the case of quadratic approximation, the second type constraints contain quadratic functions and the problem is not a Linear Programming problem anymore. However, the optimality conditions can be checked using the Lagrange function:

$$L(x, \lambda) = f(x) + \sum \lambda_i g_i(x) \quad (2)$$

where constraints have the following form: $g_i(x) \leq 0$. The Lagrange function is optimized using the Sequential Quadratic Programming method.

The only problem is that the non-convex search space is possible. Hence the problem can have several local maximums. In order to find the global optimum, the optimization has to be repeated with a row of random initial approximations.

4. APPROXIMATION METHODS OF THE ECONOMICALLY SIGNIFICANT VARIABLES

The behavior in the steady states of the economically significant variables with respect to the Decision variables can be the linear or nonlinear. As an example of the linear behavior, the values of the steady states of Economically Significant variables 12 (D1 steam flow), 13 (E Caustic flow), 15 (D2 ClO2 flow) are presented for different values of the Decision variable 9 (E Washer [OH]) in Figure 2. Since the main reagent in the E tower is NaOH, the Decision variable 9 (E Washer [OH]) affects to the rate of reactions in the tower.

Fig 2. Dependence of the steady states of the economically significant variables 12 (d1 steam flow), 13 (E Caustic flow), 15 (D2 ClO2 flow) on the decision variable 9 (E Washer [OH]) value

Thus, the linear approximation approach gives satisfactory approximation for such pairs of the economically significant and decision variables (D1 steam flow, E Caustic flow, D2 ClO2 flow dependence on E Washer [OH] setpoint value).

However, sometimes the behavior is essentially nonlinear. In this case, the quadratic approximation is needed. In Figure 3, the dependence of the Economically Significant variable 9 (O steam flow 3) on the Decision variable 9 (E Washer [OH]) is presented together with its quadratic and linear approximations.

In addition, some decision variables are highly interacting with each others in the terms of the steady states of the Economical Significant variables behavior (for example, seven decision variables, which are used for MPC1 control are highly interacting with each other).

Fig 3. Dependence of the steady states of the economically significant variable 9 (O steam flow 3) on the decision variable 9 (E Washer [OH]) value

1.1 The linear approximation method

The most pairs of the economically significant variables and decision variables produce dependences that can be described satisfactory with the simple linear approximation model as follows:

$$V_i \approx V_i^0 + \sum_j K_{i,j}(U_j - U_j^0) \quad (3)$$

where V_i , U_j are the i-th economically significant variable and j-th decision variable, and V_i^0 , U_j^0 are their nominal values.

The elements of the matrix $K_{i,j}$ are defined as partial derivatives of the economically significant variables with respect to the decision variable:

$$K_{i,j} = \partial V_i / \partial U_j .$$

The values of the matrix $K_{i,j}$ elements are identified using a number of the plant tests and setpoints changes. Let consider the following setpoints: the nominal steady state with the j-th component that has been stepped up by +5% and the nominal steady state with the j-th component that has been stepped down by -5%.

The values of the Economically Significant variables at these steady states are denoted as V_i^u and V_i^l . At the next step, the finite difference estimation is used:

$$K_{i,j} = (V_i^u - V_i^l) / 0.1 \quad (4)$$

4.2 The one dimensional quadratic approximation of the economically significant variables

As an alternative to the linear technique, one can use the following approximation of an economically significant variable:

$$V_i \approx V_i^0 + \sum_j K_{i,j}(U_j - U_j^0) + 0.5 \sum_j L_{i,j}(U_j - U_j^0)^2 \quad (5)$$

This way promises increase of the accuracy of the approximations. A quadratic function can have an optimum inside of the region in contrast with a linear one. Consequently there is a chance to obtain a good approximation of the optimum, even if the true optimum is placed inside of the region. Moreover, the quadratic approximation requires the same number of computations as the linear one: two simulations for every decision variable are still enough. For further this approach is denoted as the one dimensional quadratic approach.

4.3 The multidimensional quadratic approximation of the economically significant variables

Finally, the third way can be used: every economically significant variable can be approximated by a multidimensional quadratic function of all decision variables:

$$V_i \approx V_i^0 + \sum_j K_{i,j}(U_j - U_j^0) + 0.5 \sum_{j,k} M_{i,j,k}(U_j - U_j^0)(U_k - U_k^0) \quad (6)$$

Actually, the approach includes all terms from the previous one, since the one dimensional quadratic terms are covered by the case j equals k . This way promises the most accurate approximation, however it requires a lot of computations to perform. For further let denote this approach as the multidimensional quadratic approach.

5. SIMULATION RESULTS

5.1 The linear approximation approach

Since our objective function (1) is a linear function of the economically significant variables, the optimization problem is actually a LP problem. The LP solution gives us the optimal setpoint and the forecast of the values of the economically significant variables both. The LP solution forecasts the profit increase about 26.7 USD/min. Finally, the simulation with the optimal setpoint, derived from the LP solution is performed. The results of the simulation are presented below in the scaled values (most of economically significant and decision variables can change their scaled values between -1 and 1).

It is clearly seen from Figure 5, that some setpoints are not achieved. In fact, setpoints 5 and 6 (Upper extract conductivity and Lower extract conductivity) do not affect anything, so in the future let exclude them from the set of decision variables. As a consequence, further there are only 42 decision variables. Setpoints 7 (O kappa) and 12 (Slaker temperature) are achieved for small variations, but if a variation is large enough, the correspondent control loop exhausts its regulation capabilities due to nonlinearity.

From Figure 4 it is obvious, that the quality of the approximation of many economically significant variables (V5 (Steam flow 5), V9 (O Steam flow 3), V10 (D1 ClO2 flow), V30 (Water flow), V34 (Effect 1 Steam flow), V35 (Effect 1 Steam flow) variables) is insufficient. As a result, the simulation shows only 12.2 USD/min increase for the optimal setpoint instead of the 26.7 USD/min that is predicted by the linear approximation.

Fig 4 and 5. Results of the simulation of the optimal solution

5.2 The sum of one dimensional quadratic functions Approach

Due to random factors that have been included to the model and errors of numerical computations, the values of the steady states can be defined only approximately with the simulations. Consequently, if one uses the set of variations with steps +5% and -5% from the nominal values of the decision variables, the errors of the second derivatives estimations will be too large (although these variations allow to get quite accurate estimations of the first derivatives). This is a reason why the simulations with the +25% and -25% variations from the nominal values of the decision variables have been used in order to compute the matrix L .

The rest of optimization was performed using the 'fmincon' function from the optimization MATLAB toolbox. Since the matrix L has a lot of negative elements, the optimization problem is not concave anymore, so few local optimums can exist. In order to find all local optimums, 100 of random points were used as initial points of the 'fmincon' function.

The results of the simulation are presented below:

Fig 6 and 7. Results of the simulation of the optimal solution

The forecast for the profit increase is about 18.4 USD/min, at the same time the real profit increase is only about 14.1 USD/min. Although the one dimensional quadratic approximations are still too optimistic, they are significantly more accurate than the linear ones. As a consequence, this approach is able to obtain the higher profit during the simulation (although the forecast of the profit is smaller compared with the linear approximation approach).

5.3 Multidimensional quadratic approach

Due to problems with the stability of the plant, the variables U23 (WL NaOH concentration), U25 (Excess WL split), U29 (Split fraction 6), U32 (Split fraction 1 MEE) were excluded from the matrix M computation (the second order mixed partial derivatives are put to be zero when one of the directions is U23, U25, U29 or U32).

The MATLAB 'fmincon' function has been used in order to complete the optimization. The results of simulation are presented below:

Fig 8 and 9. The results of the simulation of the optimal solution

The approximation forecasts a profit increase about 17.3 USD/min that is slightly less than the forecast of the 1-dimensional quadratic approximation approach. The simulation shows, that the real increase of the profit is about 16.0 USD/min. Although the errors of the economically significant variables estimations in Figure 8 are quite high, the difference between the predicted and real profit of the plant is only about 1.3 USD/min.

6. COMPARISON OF THE RESULTS AND QUALITIES OF THE APPROXIMATION APPROACHES

In this section the comparison of the results, derived by the different approximation methods, is given.

First, let examine the quality of different approximations while moving the setpoint smoothly from the nominal setpoint to the optimal one. Let us consider the setpoints, which are the linear combinations of the one dimensional quadratic approximation optimal setpoint and the nominal setpoint with coefficients α and $(1 - \alpha)$ correspondently.

The real profit and its forecasts are printed in Figure 10. The figure shows, that the multidimensional quadratic approximation is much better, than the linear one. However, the real profit is smooth for α in the interval $(-0.5, 0.75)$. At $\alpha = 0.75$ the behavior of the system changes non-smoothly, this leads to the significant increase of the errors of all approximations at the optimal point ($\alpha = 1$).

Fig 10. Dependence of the real profit and its forecasts on the coefficient α

Finally, the comparison of the quality of approximations on the three presented solutions in the L1 sense is given in Table 4.

Table 4

Comparison of the errors of different approximation approaches in the L1 sense

	LA solution	QA 1D solution	QA MD solution
Profit increase	12.2 USD/m	14.1 USD/min	16.0 USD/min
Linear app. error	3.59	3.64	7.38
1D quad. app. err	2.94	2.65	6.72
MD quad. app. err	2.95	2.50	8.25

The one dimensional quadratic approach always performs better, than the linear one. The multidimensional quadratic approach has a quite good quality of approximation for the first and the second solutions, but its quality falls seriously for the third setpoint.

7. CONCLUSION

The most important and the most difficult part of the economical efficiency optimization is the construction of approximations of the economically significant variables as functions of the decision variables. Three types of approximations have been discussed in the paper: linear, one dimensional quadratic approach and a multidimensional quadratic approach. It is clear from the previous sections that the approximation methods proposed in the paper are more accurate compared with the linear technique.

The multidimensional quadratic approximation requires a lot of computations to do. However, some of the decision variables do not have a lot of serious interactions with the other decision variables (i.e.

corresponding elements of the matrix M are close to zero). This leads to the idea, that some decision variables could be excluded from the multidimensional quadratic approximation without any serious loss of the accuracy. Probably, the mixed approach is the most suitable for practical needs: the approximation consists of the multidimensional quadratic function of a part of decision variables and the sum of one dimensional quadratic functions of others decision variables. At the same time the reliability of the three dimensional matrix M , that is used by the multidimensional quadratic approach, could be sufficiently improved by a price of additional simulations.

In the paper, the insufficient reliability of the linear approximations of the economically significant variables as functions of the decision variables has been observed. As a consequence, the results of the economical efficiency optimization could be quite inaccurate. In order to overcome the problem, the more precise approaches were proposed: the first the sum of the one dimensional quadratic functions approach combines the increase of the accuracy with the low volume of required computations. The second the multidimensional quadratic functions approach provides the good approximation precision by the cost of significant volume of computations.

As a result, an additional increase of the profit has been derived for the both suggested approximations compared with the linear approximation approach.

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