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Carbon Footprint Management: A Pathway toward Smart Emission Abatement

Mahdi Pourakbari-Kasmaei, Member, IEEE, Matti Lehtonen, Javier Contreras, Fellow, IEEE, and José Roberto Sanches Mantovani, Member, IEEE

Abstract—There is an increasing concern about controlling and reducing carbon emissions in power systems. In this regard, researchers have focused on managing emissions on the generation side, which is the main source of emissions. Considering emission limits on the generation side results in an increase in locational marginal prices that negatively affects social welfare. However, carbon emissions are a by-product of electricity generation that is used to satisfy the demands on the consumer side. Consequently, demand side emission control may not be achieved if only generation is taken into account. In order to fill this existing gap, in this paper, a demand-side management approach aiming at carbon footprint control is proposed. First, the carbon footprint is allocated among the consumers using an improved proportional sharing theorem method. Each consumer learns about their real-time carbon footprint, excess carbon footprint, and the incurred surcharge tax. Then, demands are adjusted via a proper adjustment procedure. This provides enough information for consumers where demand management may result in carbon footprint and demand reductions as well as the exemption of incurred taxes. Profit analysis for both generation and demand sides is carried out to show the effectiveness of the proposed framework using two illustrative case studies. The results obtained, compared with existing policies such as carbon cap, cap-and-trade, and carbon tax, prove the fairness and the advantages of the proposed model for both the demand and the generation sides.

Index Terms—Carbon footprint allocation, carbon abatement, demand side management, power tracing, tax exemption.

NOMENCLATURE

A. Indexes and Sets

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<thead>
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<td>i, j, n</td>
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<td>ij</td>
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<td>( ECP_{g_i \rightarrow ij} )</td>
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<td>Demand contribution of unit g to the energy consumption price at bus i.</td>
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<td>( DECP_{g_i \rightarrow i} )</td>
<td>Demand energy consumption price at bus i.</td>
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<tr>
<td>( DCF_{i} )</td>
<td>Demand carbon footprint surcharge cost at bus i.</td>
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Generators’ contribution on demand at bus $i$. 
Excess system carbon footprint on generation side. 
System carbon footprint surcharge. 
Loss carbon footprint in line $ij$. 
Loss energy consumption price at line $ij$. 
Active/reactive power generation of unit $i$. 
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I. INTRODUCTION

The progress and evolution of a country highly depends on electricity [1]. However, each technology used to generate electricity has undesired by-products such as emissions and other harmful environmental impacts [2].

To cope with climate change, it will be necessary to reduce global greenhouse gas (GHG) emissions up to 90% between 2040 and 2070 [3]. Among the primary energy sources used to generate electric power, coal is the cheapest and most popular in developing countries, where 41% of electricity comes from coal-fired power plants. On the other hand, coal accounts for 73% of GHG emissions in the electric power sector [4]. Therefore, this sector poses huge environmental concerns as a major source of carbon emissions and the future control of climate change will depend on the electric power generation policies [5]. In order to slow down this phenomenon, curbing GHG emissions is very important as well as establishing main targets and proposing appropriate tools for carbon emission abatement [6], [7].

Until now, most papers have focused on the generation side, where the technologies can be categorized as: i) CO$_2$ control devices, such as carbon capture and storage (CCS) [8], [9], ii) renewable-energy-based technologies (RET) [10], [11], iii) emission-constrained models (ECM) [12], [13], and iv) demand-response (DR) based models [14], [15]. The CCS technology is used to capture CO$_2$ from the power plant depositing it in a place where it is not able to enter the atmosphere. RETs are sources of energy with quasi-zero emissions that are widely used to abate them. ECM models, by restricting generation from producing emissions, control emission levels and this usually results in a cost increase. In DR-based models the economic incentives result in a demand decrease and, consequently, emission reduction may be the result. Several countries, by using environmental-based policies aim at abating carbon emissions. The existing policies are Carbon Cap (CC), Cap and trade (C&D) and Carbon Tax (CT). In the CC and C&T models, a cap is a binding constraint that greatly affects the optimal dispatch and increases the total costs. Much of the cost of such limits is passed on to households. These are simply unfair policies that the costs associated with adapting such changes are borne by the consumers [16]. Therefore, such methods would reduce emissions while decreasing social welfare, by making the cost of energy much higher for the consumer [17]. Therefore, cap-based policies will raise the bills of all the consumers, even those with very low demands [18]. The other shortcoming of cap-based models is the free permits given to companies, which use energy intensively and are big polluters, where the vast majority of these valuable emission permits are awarded for free. The more an industry has been polluting, the more permits it will get. This way, these polluters and such industries, as well as the suppliers, made extra profits, because of raising the energy costs of consumers [19]. The carbon tax is an environmental tax that is levied on the carbon emitted to the atmosphere by the [20], [21]. However, the CT policy does not achieve a certain level of emissions reduction because: 1) emissions vary from year-to-year and 2) there are no clear incentives for the consumers and, above all, it raises the bill of all the consumers, even those with very low demands [18]. Neither CC-based policies nor the CT policy are capable of providing useful information for the consumers (carbon footprint, excess footprint, corresponding surcharge, etc..) and demand side management is done directly, that is, by using the CC, C&T and CT policies, all the consumers face a rise in their bills and they may decide to reduce the consumption. This shows the unfair nature of these policies in which the consumers with low demands (normally those in difficult financial situation) are charged as well, and this may release the burden of such surcharges from the shoulder of those consumers with high-energy consumption, which are the prime responsible for carbon emissions and put on the shoulder of low demands. In addition, a demand decrease negatively affects the net profit of those power plants that use these policies. This means that the more a demand decreases, the lower their profit becomes. Consequently, these methods cannot provide effective incentives to the generation side; on the contrary, there exists a deterrent that prevents them from encouraging the consumers to decrease their demands. It is true that emissions are a by-product of power generation; however, generation is driven by demand, hence, consumers should be considered as the prime responsible agents for carbon emissions.

In recent years, some actions have been adopted for consumers to reduce their energy consumption as well as carbon emissions, such as the UK Carbon Reduction Commitment Energy Efficiency Scheme, the Tokyo Emission
Trading Scheme, and the Perform Achieve Trade Scheme in India [22]. However, such obligations need to provide proper information to the demand side. The influence of a change on the generation or demand sides has been investigated via marginal carbon intensity [23], but it neither provides suitable information for the generation side nor for the demand side. In [24], two issues such as the carbon accounting at the regional level and locational carbon intensity assessment at the user level were addressed. It is assumed that the generators have the priority to supply the load at the same bus, and this is not always applicable, as more often than not the load at a bus may use the supply of adjacent units due to the system topology, operators’ objectives, etc., [25]. In [5], a network-based model was proposed to accumulate the carbon emissions produced by various units on the demand side via the Proportional Sharing Theorem (PST) [26], [27], which is basically adequate for lossless systems. This approach works based on matrix operations that makes it inappropriate for large-scale networks since it requires the calculation of the inverse of large-scale sparse matrices. The same strategy was developed in [28] to quantify the carbon emission in multiple energy systems. In [29], a graph-based carbon emission tracing method was proposed. Although the paper addressed the main drawback of graph-based models (inapplicability for systems without loops), it uses the same assumption in [24] that makes it unsuitable in real systems.

None of the aforementioned works provides enough information for the demand side to manage carbon emissions. In comparison with existing methodologies, the contributions of this paper are threefold.

1) An improved PST model [30] as a tracing tool is used to provide useful information for the consumers, such as carbon footprint, excess carbon footprint (ECF), carbon footprint surcharge, energy consumption price, and hourly-based bill (before and after demand adjustment). Our tracing model, unlike [5] and [28], is not based on inverse matrix rules. The drawbacks in [24] and [29] are also addressed, in which the priority assumption (each unit has the priority to supply the load at the same bus) makes them overly simplified models inappropriate in the real world.

2) A demand adjustment process is proposed in which the consumers can adjust their demands by exchanging information with the utility side to eliminate carbon footprint surcharges.

3) A profit analysis on generation and demand sides is carried out; this analysis is used as an incentive for both sides.

The rest of this paper is organized as follows. Section II contains the proposed carbon footprint allocation and management. Case studies and results are presented in Section III. Section IV shows concluding remarks and future works.

II. CARBON FOOTPRINT MANAGEMENT

To manage the carbon footprint in the power system, we should see the problem from the demand perspective. This paper proposes an allocation-based solution approach to handle this situation. It is worth mentioning that all the models have been implemented on AMPL and a commercial solver KNITRO has been used to solve them.

A. Solution Approach

As can be seen from the carbon footprint management flowchart shown in Fig. 1, an optimal power flow (OPF) is used to minimize the total system cost. Then, the carbon footprint is allocated among the demands via an iterative process presented in Fig. 2. This process continues until all the powers from the generators are allocated to the demands and transmission losses. A detailed formulation of the allocation process is presented in subsection b. Then, the predefined carbon footprint limits (CFLs) of the demands are checked and in case that a CFL has been violated, the excess demand that yields the CFL violation is calculated by an adjustment process and the demand is adjusted. In this paper, the CFL violation check and the adjustment process is carried out by numerical order of demands instead of using a parallel adjustment or descending order of the demand carbon footprint (DCF). The parallel adjustment results in asking the consumers to decrease their demands much more than
necessary, while the adjustment in DCFs’ decreasing order changes the power flow calculation. Since heavy loads are located in different places/areas of the system, it may result in more violations of the CFL of normal loads. If the DCF limit of demand \(i\) is violated, the adjustment process starts, otherwise the program continues until it checks the DCFs of all the demands. The adjustment process obtains the adjusted demands by using (40) and (41), see item 9 in subsection A and sub-subsection b, and these adjusted demands are updated in the OPF tool for recalculations. Due to peculiar structure of the emission function (in some situations, a decrease in generation may lead to increasing emissions), and the nonlinear and nonconvex nature of the OPF (running the OPF after adjusting a demand may lead to a different dispatch), the adjustment or readjustment of a demand may occur, then, the readjustment process is performed again. Note that the CFL, similar to the cap size of the polluters, is defined using several parallel analyses such as weather conditions, emission concentrations, etc. The management process is explained as follows.

\[ \min_{P_c} \quad F_c = \sum_{i=1}^{n} C_i(P_{c,i}) \]  
\[ \text{s.t.} \quad P_{g,i} - P_{p,i} - g_i v_i^2 - \sum_{j \in \Omega} P_{j,i} - \sum_{j' \in \Omega} P_{j',i} = 0; \forall i \in \Omega \]  
\[ Q_{g,i} - Q_{p,i} + b_i v_i^2 - \sum_{j \in \Omega} Q_{j,i} - \sum_{j' \in \Omega} Q_{j',i} = 0; \forall i \in \Omega \]  
\[ \|f_l(v, \theta, \phi)\| \leq \|f_l\|; \forall i \in \Omega \]  
\[ P_{g,i} \leq P_{\text{U},g,i}; \forall i \in \Omega \]  
\[ P_{p,i} \leq P_{\text{L},p,i}; \forall i \in \Omega \]  
\[ Q_{g,i} \leq Q_{\text{U},g,i}; \forall i \in \Omega \]  
\[ Q_{p,i} \leq Q_{\text{L},p,i}; \forall i \in \Omega \]  

The fuel cost of unit \(i\) is approximated by a quadratic function of the active power in (9), [32]. The upper limit of the active generation in (6) is obtained from the capability curve. The interested readers may refer to [31] for details.

\[ C_i(P_{c,i}) = a_i(P_{c,i})^2 + b_i P_{c,i} + c_i; \forall i \in \Omega \]  

As for the cost function, the emission function of unit \(i\) is approximated by a quadratic function in (10), [21].

\[ E_m(P_{g,i}) = \alpha_i(P_{g,i})^2 + \beta_i P_{g,i} + \gamma_i; \forall i \in \Omega \]  

Note that the emission limit on the generation side (11) is considered in the OPF problem [33]. However, its main drawback is the cost that may be incurred by the demand. Therefore, in this paper, this is disregarded and, in case of an emission excess, a surcharge cost is paid by the producers. See Section II, subsection D.

\[ \sum_{i=1}^{n} E_m(P_{g,i}) \leq \mathcal{SE} \]  

\(\text{b. Carbon footprint and price allocation}\)

After performing the power flow calculation and using the units’ output power, transmission losses, and transmission line flows, the carbon footprint is allocated among the demands. Thus, a tracing-based allocation approach that can be seen in Fig. 2 is used. The basic concepts and the procedure are explained as follows.

1) Bus Absorbed Power (BAP)

The BAP is the amount of active power absorbed by bus \(i\), \(\bar{P}_i\). This power consists of the input power flow to bus \(i\) from an adjacent bus \(j\), \(p_{ji}\), and the power generated at bus \(i\), \(P_{gi}\), (12).

\[ P_i = \sum_{j \in \Omega} (p_{ji} - P_{ji}) + P_{gi} = \sum_{j \in \Omega} (-p_{ji} + P_{ji}) \quad \forall i \in \Omega, p_{ji} > 0 \]  

where \(P_{ji}\) stands for the transmission loss at line \(ij\).

As shown in Fig. 3, \(p_{ji} > 0\) guarantees that bus \(i\) is receiving power from bus \(j\), however, the power sent from bus \(j\) as a result of transmission loss, will not be received completely at bus \(i\). Therefore, transmission loss must be subtracted from it. As can be seen from Eq. (12), instead of taking into account \(p_{ji} - P_{ji}\), the equivalent term \(p_{ji}\) can be used. The transmission loss of line \(ij\) is obtained as follows:

\[ P_{ij} = |p_{ji} - p_{ji}|; \forall ij \in 

On the other hand, the BAP at bus \(i\) is split into two terms, traced power, \(\bar{P}_i^T\), and untraced power, \(\bar{P}_i^U\), in (14).

\[ \bar{P}_i = \bar{P}_i^T + \bar{P}_i^U; \forall i \in \Omega \]  

2) Bus Traced Power (BTP)

The BTR is the traced active power produced by unit \(g\) and absorbed by bus \(i\). Note that, initially, only the output power of committed units at bus \(i\), \(P_{g,i}\), can be traced.

\[ \bar{P}_i^{T(i)} = \sum_{g \in \Omega, g \in \Omega_g} P_{g,i}; \forall i \in \Omega \]  

3) Bus Untraced Power (BUP)

The BUP is the untraced active power absorbed by bus \(i\). Note that, initially, all power coming from the adjacent buses is untraced.

\[ \bar{P}_i^{U(i)} = \sum_{j \in \Omega, j \in \Omega_g} (p_{ji} - P_{ji}); \forall i \in \Omega, p_{ji} > 0 \]  

At each iteration, a part of the untraced power is traced. In order to find out the traced power at each bus \(i\), the generating unit contribution (GUC) to this bus, \(P_{g,i}^{GUC}\), is used, (17), and the untraced power is defined in (18).
\[ \tilde{P}_i = \sum_{j \in \Omega} (p_j - P_{ij}) \forall i \in \Omega, p_j > 0 \]  \hspace{1cm} (18)

To start the tracing process, two ratios, namely traced and untraced ratios, are defined.

4) Traced and Un traced Power Ratios

In order to trace the untraced power, two auxiliary ratios, traced power ratio (TPR) for traced power from unit \( g \) to bus \( i \), and untraced power ratio (UPR) for untraced power from bus \( j \) to bus \( i \) are used, as shown in (19) and (20), respectively.

\[ TPR_{g_{i \rightarrow w}} = \frac{P_{g_{i \rightarrow w}}}{P_i} \forall i \in \Omega, g \in \Omega \]  \hspace{1cm} (19)

\[ UPR_{j_{i \rightarrow w}} = \frac{p_j - P_{j_{i \rightarrow w}}}{P_i} \forall j \in \Omega, i \in \Omega, p_j > 0 \]  \hspace{1cm} (20)

Since initially only the traced powers at the generation buses are known, the corresponding initial TPR is calculated in (21), while the UPR is calculated via (20) as (22).

\[ TPR^{(0)}_{g_{i \rightarrow w}} = \frac{P_{g_{i \rightarrow w}}}{P_i} \forall i \in \Omega, g \in \Omega \]  \hspace{1cm} (21)

\[ UPR^{(0)}_{j_{i \rightarrow w}} = \frac{p_j - P_{j_{i \rightarrow w}}}{P_i} \forall j \in \Omega, i \in \Omega, p_j > 0 \]  \hspace{1cm} (22)

According to the PST rule, the ratio of the input power at bus \( j \) is equal to the ratio of the constituent components of \( p_j \). Consequently, via a distribution process, \( UPR_{j_{i \rightarrow w}} \) can be taken away, by adjusting it to zero, and distributing it over \( \tilde{P}_i \) according to the component ratio of \( P_i \) using (23) and (24). In (23), the UPR at iteration \( k + 1 \) from each bus \( h \) to bus \( i \) is equal to the UPR at the previous iteration \( k \) plus the UPR from bus \( h \) to \( j \) (for each bus \( h \) adjacent to \( i \)) multiplied by the UPR from bus \( j \) to bus \( i \). It is worth mentioning that this distribution is done if there is any bus \( h \) that satisfies \( UPR_{h_{i \rightarrow j}} \geq 0 \). In (24), the TPR at iteration \( k + 1 \) from each generating unit \( g \) to bus \( i \) is equal to the TPR at the previous iteration \( k \) plus the TPR from unit \( g \) to bus \( j \) multiplied by the UPR from bus \( j \) to bus \( i \); however, the distribution is done if there is any bus \( m \) that satisfies \( TPR_{m_{i \rightarrow j}} \geq 0 \). Note that, to take away \( UPR_{h_{i \rightarrow j}} \) in a power system without loops, this value is set to zero, otherwise a small value, \( \varepsilon \), is assigned.

\[ UPR^{(k+1)}_{j_{i \rightarrow w}} = UPR^{(k)}_{j_{i \rightarrow w}} + UPR^{(k)}_{j_{h \rightarrow i}} \cdot UPR^{(k)}_{h_{j \rightarrow i}}, \forall hj \in \Omega, h \neq j \]  \hspace{1cm} (23)

\[ TPR^{(k+1)}_{g_{i \rightarrow w}} = TPR^{(k)}_{g_{i \rightarrow w}} + TPR^{(k)}_{g_{j_{i \rightarrow w}}} \cdot UPR^{(k)}_{j_{i \rightarrow w}}, \forall g \in \Omega, i \in \Omega, g \neq j \]  \hspace{1cm} (24)

The other elements that are not involved in the distribution process of bus \( i \) remain unchanged.

\[ TPR^{(k+1)}_{j_{i \rightarrow w}} = TPR^{(k)}_{j_{i \rightarrow w}}, \forall j \in \Omega, \forall n \in \Omega, n \neq i \]  \hspace{1cm} (25)

\[ UPR^{(k+1)}_{j_{i \rightarrow w}} = UPR^{(k)}_{j_{i \rightarrow w}}, \forall jn \in \Omega, n \neq i \]  \hspace{1cm} (26)

The distribution process is terminated when all the untraced elements, \( UPR_{j_{i \rightarrow w}} \) are taken away by repeating (23) and (24). The stopping criterion is as follows:

\[ UPR_{j_{i \rightarrow w}} \leq \varepsilon \forall ji \in \Omega \]  \hspace{1cm} (27)

Interested readers may refer to [34] for more details via a didactic example.

5) Generators' Contribution on Demand (GCD), Demand Energy Consumption Price (DECP) and Demand Carbon Footprint (DCF)

DECP and DCF are the price of the consumed energy and the corresponding carbon emitted to the atmosphere, respectively. After tracing the power at each bus, the carbon footprint is allocated among the demands. The contributions of unit \( g \) to calculate DECP and DCF are shown in (28) and (29), respectively, while the total DECP and DCF values of demand \( i \) are shown in (31) and (32), respectively.

\[ DECP_{g_{i \rightarrow w}} = \lambda_g \cdot TPR_{g_{i \rightarrow w}} \cdot P_{g_{i \rightarrow w}}, \forall g \in \Omega, i \in \Omega \]  \hspace{1cm} (28)

\[ DCF_{g_{i \rightarrow w}} = Em_g \cdot TPR_{g_{i \rightarrow w}} \cdot P_{g_{i \rightarrow w}}, \forall g \in \Omega, i \in \Omega \]  \hspace{1cm} (29)

\[ GCD_{i_{j \rightarrow w}} = TPR_{j_{i \rightarrow w}} \cdot P_{j_{i \rightarrow w}}, \forall j \in \Omega, i \in \Omega \]  \hspace{1cm} (30)

\[ DECP_i = \sum_{g \in \Omega} DECP_{g_{i \rightarrow w}}, \forall i \in \Omega \]  \hspace{1cm} (31)

\[ DCF_i = \sum_{g \in \Omega} DCF_{g_{i \rightarrow w}}, \forall i \in \Omega \]  \hspace{1cm} (32)

6) Loss Energy Consumption Price (LECP) and Loss Carbon Footprint (LCF)

LECP and LCF are the price of consumed energy due to transmission loss and the corresponding carbon emitted to the atmosphere, respectively. The contributions of unit \( g \) to calculate LECP and LCF are presented in (33) and (34), respectively, while the total LECP and LCF of demand \( i \) are obtained in (35) and (36).

\[ LECP_{g_{i \rightarrow w}} = \lambda_g \cdot TPR_{g_{i \rightarrow w}} \cdot |p_j - p_{ij}|, \forall g \in \Omega, p_j > 0 \]  \hspace{1cm} (33)

\[ LCF_{g_{i \rightarrow w}} = E_m \cdot TPR_{g_{i \rightarrow w}} \cdot |p_j - p_{ij}|, \forall g \in \Omega, p_j > 0 \]  \hspace{1cm} (34)

\[ LECP_i = \sum_{g \in \Omega} LECP_{g_{i \rightarrow w}}, \forall p_j > 0 \]  \hspace{1cm} (35)

\[ LCF_i = \sum_{g \in \Omega} LCF_{g_{i \rightarrow w}}, \forall p_j > 0 \]  \hspace{1cm} (36)

c. Demand side management

After the allocating process is done, each demand can observe its energy consumption price, carbon footprint, excess carbon footprint (ECF), and carbon tax. Providing useful information to the consumers about how much of a reduction in their demand may result in tax exemption and bill savings is used as an incentive. However, demand adjustment to eliminate the ECF on the demand side is a complicated task. The adjustment procedure is explained in detail as follows.

1) Excess Demand Carbon Footprint and Surcharge

In order to control the carbon footprint from the demand side, first, a predefined threshold for the demands is considered. If the threshold is reached, the demands receive a surcharge corresponding with the ECF. Carbon surcharges offer a potentially cost-effective tool for carbon footprint
reduction. The ECF of a demand and its incurred surcharge tax are calculated in (37) and (38), respectively. Note that the carbon footprint tax, $\tau$, is defined, see the European Union Emission Trading System [35].

$$DCF_{E_C}^{\text{acc}} = DCF_{i} - DCF_{i}, \forall i \in \Omega_o$$  
(37)

$$DCF_{i}^{\text{exc}} = DCF_{i}^{\text{acc}} \cdot \tau/\forall i \in \Omega_o, DCF_{i}^{\text{exc}} > 0$$  
(38)

2) Finding the Contribution of Each Unit to the Excess Carbon Footprint (ECF)

By using the traced power ratios in (19) and the demand ECFs, the contribution of each unit to the ECF is obtained in (39).

$$DCF_{i}^{\text{acc}} = DCF_{i}^{\text{acc}} - DCF_{i}^{\text{exc}}, \forall g \in \Omega_o, \forall i \in \Omega_o$$  
(39)

3) Demand Adjustment

To approximate the adjustment of the demand, each unit’s generation is modified and this adjustment is transferred to the demand side. Due to contribution theory and considering loss reduction (as a consequence of generation reduction), ECF reduction is guaranteed. By using (40) and (41), the contribution of each unit to the demand adjustment and total demand adjustment is obtained.

$$P_{g,ij}^\text{adj} = \frac{P_{g,ij}^\text{adj} - (Em_j(P_{g,ij}^\text{adj}) - DCF_{g,ij}^{\text{acc}}) - DCF_{g,ij}^{\text{exc}}}{P_{g,ij}^\text{adj} - P_{g,ij}^{\text{adj} \rightarrow \text{adj}}}$$  
(40)

$$P_{g,ij}^\text{adj} = \sum_{i \epsilon \Omega_o} P_{g,ij}^\text{adj}, \forall i \in \Omega_o$$  
(41)

B. Demand Side Benefit Analysis

The consumers may only reduce their demands if there is an incentive such as eliminating the incurred ECF tax and lowering the energy usage bill. Hence, the demand energy bill (DEB), before and after adjustment, is provided for the consumers. This payment is calculated in (42).

$$DEB_{i} = DECP_{i} + DCF_{i}^{\text{exc}}, \forall i \in \Omega_o$$  
(42)

C. Generation Side Profit Analysis

Demand side management disregarding the profit on the generation side does not reveal the advantages and drawbacks of the management system. Therefore, a profit analysis for the generation side is performed.

By solving (1)–(8), the vector of marginal prices, $\lambda$, is formed by the Lagrange multipliers associated with the active power balance in (2). Finding the Lagrange multipliers in a linear system is an easy task; however, in order to find these multipliers in a nonlinear model, an iterative process is used. At each iteration, the demand at a bus is increased by 1 MW and the difference between the optimal solutions before and after that 1 MW increase produces the Lagrange multiplier at that bus [36]. Using the marginal prices at generating bus $g$, the generator’s profit is obtained in (43), [37].

$$P_{r,ij} = \lambda_{g,ij} P_{g,ij} - C_{g}(P_{g,ij})$$  
(43)

The excess system carbon emissions (44) and the corresponding surcharges (45) should be considered to calculate the net profit on the generation side, as shown in (46).

$$SCF_{E_C}^{\text{acc}} = \sum_{g \in \Omega_o} Em_j(P_{g,ij}^\text{adj}) - SE$$  
(44)

$$SCF_{E_C}^{\text{exc}} = SCF_{E_C}^{\text{acc}} \cdot \tau, \forall SCF_{E_C}^{\text{exc}} > 0$$  
(45)

$$\Pr_{r}^\text{adj} = \sum_{i \epsilon \Omega_o} (\lambda_{g,ij} P_{g,ij} - C_{g}(P_{g,ij}^\text{adj})) - SCF_{E_C}^{\text{exc}}$$  
(46)

III. CASE STUDIES AND RESULTS

In this section, two case studies are presented: a 5-bus system and the IEEE 118-bus system. The proposed model is implemented on AMPL and solved via the KNITRO solver on a Windows-based workstation with two processors clocking at 3.3 GHz and 8 GB RAM. The carbon tax for excess carbon emissions on both generation and demand sides is $23/ton [38].

A. 5-Bus System

This system consists of 7 transmission lines, 3 generating units (G1, G2, and G5), and 3 demand buses (L2, L3, and L4) with a total demand of 320 MW, where data comes from [33].

![5-bus test system without line flow limits](image)

Fig. 4. 5-bus test system without line flow limits.

**TABLE I**

<table>
<thead>
<tr>
<th>Demand</th>
<th>G1</th>
<th>G2</th>
<th>G5</th>
<th>Carbon Footprint (ton/h)</th>
<th>GUC (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L2</td>
<td>14.267</td>
<td>12.048</td>
<td>115.428</td>
<td>31.485</td>
<td>298.005</td>
</tr>
<tr>
<td>L3</td>
<td>73.268</td>
<td>61.872</td>
<td>192.185</td>
<td>31.485</td>
<td>298.005</td>
</tr>
<tr>
<td>L4</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Total</td>
<td>88.263</td>
<td>74.535</td>
<td>298.005</td>
<td>74.535</td>
<td>298.005</td>
</tr>
</tbody>
</table>

1) Case 1: Without Line Flow Limits

This case is used to provide the results of the allocation process and the GUC on carbon footprint, loss, and energy consumption price, and also to explain the adjustment process in detail. CFL values for L2, L3, and L4 are 100, 180, and 120 ton/h, respectively, while the desired system emission limit is 120.185 ton/h and a 2.454 ton/h violation of the
CFLs for L3 and L4, respectively. Therefore, the operator may manage them by providing enough information, warnings, and incentives for these demands. The warnings are the current DCF, DECP, ECF, and DCF surcharge values. The incentives are the demand adjustment results, such as money savings in the electricity bill by lowering the demand and the elimination of the ECF surcharge. After providing such information, the adjustment process is taken into account, which is explained in detail using demand L3, as follows.

Table II shows the information provided for the consumers as well as total power savings (TPS), demand carbon footprint change (DCFC), and total cost savings (TCS). Such information is used to encourage the consumers to manage their demands.

From (37) and (38), the ECF and the surcharge of L3 are 12.185 ton/h and $280.26/h, respectively, as seen in Table II. From Table I and using (39), the contributions of G1, G2, and G5 to ECF are obtained: 3.923, 0.944, and 7.318 ton/h, respectively, subject to the CFLs for L3, L4, and L3 decrease by $1.3/h and $2.6/h, respectively, this negatively affects the DCFs of both loads, with increases of 0.052 ton/h and 0.077 ton/h, respectively. This is due to the nonconvex characteristic of the emission function that may have a positive effect in some cases and a negative effect in others. However, for L2, which does not require an adjustment, the DECP decreases by $13.3/h corresponding with a DCF increase of 0.444 ton/h. Note that, in this case, such an increase in the DCF of L2 does not result in a violation of its CFL. However, if the adjustment of the demands results in a violation of the CFL of other demands, the corresponding demands are adjusted/readjusted to eliminate the violation. This situation is explained by using the 5-bus system with transmission line limits.

### Table II

**INFORMATION PROVIDED TO DEMANDS, BEFORE AND AFTER ADJUSTMENT—WITHOUT FLOW LIMITS FOR THE 5-BUS SYSTEM, CASE 1**

<table>
<thead>
<tr>
<th>Demand (MW)</th>
<th>L2</th>
<th>L3</th>
<th>L4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before</td>
<td>After</td>
<td>Before</td>
<td>After</td>
</tr>
<tr>
<td>65.0</td>
<td>65.0</td>
<td>165.0</td>
<td>154.539</td>
</tr>
<tr>
<td>DCF (ton/h)</td>
<td>87.202</td>
<td>87.646</td>
<td>192.185</td>
</tr>
<tr>
<td>DECP (ton/h)</td>
<td>0.0</td>
<td>0.0</td>
<td>12.183</td>
</tr>
<tr>
<td>DECP (S/h)</td>
<td>2.72565</td>
<td>2.71235</td>
<td>6.90834</td>
</tr>
<tr>
<td>TPS (MW/h)</td>
<td>0.0</td>
<td>0.0</td>
<td>280.26</td>
</tr>
<tr>
<td>DCFC (S/h)</td>
<td>13.30</td>
<td>13.435</td>
<td>747.64</td>
</tr>
<tr>
<td>TCS ($/h)</td>
<td>6441.0</td>
<td>6443.6</td>
<td>122.454</td>
</tr>
</tbody>
</table>

### Table III

**DEMAND ADJUSTMENT PROCEDURE STEPS FOR THE 5-BUS SYSTEM, CASE 1**

<table>
<thead>
<tr>
<th>Adj</th>
<th>L2</th>
<th>L3</th>
<th>L4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before</td>
<td>After</td>
<td>Before</td>
<td>After</td>
</tr>
<tr>
<td>P0</td>
<td>65.0</td>
<td>87.202</td>
<td>2725.6</td>
</tr>
<tr>
<td>DECP</td>
<td>12.185</td>
<td>2.454</td>
<td>0.0</td>
</tr>
<tr>
<td>DECP</td>
<td>3.923</td>
<td>0.944</td>
<td>7.318</td>
</tr>
<tr>
<td>DECP</td>
<td>280.26</td>
<td>56.44</td>
<td>0.0</td>
</tr>
<tr>
<td>DCFC</td>
<td>13.30</td>
<td>13.435</td>
<td>747.64</td>
</tr>
<tr>
<td>TCS</td>
<td>6441.0</td>
<td>6443.6</td>
<td>122.454</td>
</tr>
</tbody>
</table>

### Table IV

**PROFIT, EXCESS CARBON FOOTPRINT AND SYSTEM NET PROFIT FOR THE 5-BUS SYSTEM BEFORE AND AFTER ADJUSTMENT, CASE 1**

<table>
<thead>
<tr>
<th>G1</th>
<th>Before adjustment</th>
<th>After adjustment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit ($/h)</td>
<td>75.589</td>
<td>61.862</td>
</tr>
<tr>
<td>G2</td>
<td>14.164</td>
<td>11.123</td>
</tr>
<tr>
<td>G5</td>
<td>2395.855</td>
<td>2349.483</td>
</tr>
<tr>
<td>Total</td>
<td>2485.608</td>
<td>2422.408</td>
</tr>
<tr>
<td>Carbon footprint</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SCF (ton/h)</td>
<td>19.025</td>
<td>3.302</td>
</tr>
<tr>
<td>SCFexc (S/h)</td>
<td>437.575</td>
<td>75.946</td>
</tr>
<tr>
<td>Net Profit ($/h)</td>
<td>2048.033</td>
<td>2346.522</td>
</tr>
</tbody>
</table>

The total emissions for steps S1, S2, and S3 are 404.025 ton/h (401.841 ton/h of DCF and 2.184 ton/h of LCF), 389.645 ton/h (387.620 ton/h of DCF and 2.025 ton/h of LCF), and 388.302 ton/h (386.281 ton/h of DCF and 2.021 ton/h of LCF), respectively. It can be seen that, by managing the loads at each step, total emissions decrease. Table IV shows the effects of this decrease on the generation side.

Table IV presents some useful information on the generation side, before and after the demand adjustment process, such as the profit of each unit and the total profit, carbon footprint (the generation side excess footprint, $SCF_{exc}$, and its incurred surcharge cost, $SCF_{sur}$), and the generation side net profit. As it can be seen, the profit of units G1, G2, and G5 after demand adjustment decreases by $13.727/h, $3.041/h, and $46.372/h, and, consequently, the total profits of the units decrease by $63.14/h. At first glance, it may seem that this adjustment negatively affects the profit on the generation side and, therefore, which incentive should the producers be willing to accept? Before the adjustment, an ECF of 19.025 ton/h causes a surcharge cost of $437.575/h. Therefore, the net profit on the generation side before the adjustment is $2048.033/h. After demand adjustment, the carbon footprint on the generation side is reduced so that the surcharge is decreased. This shows that, although the profits for the generating units decrease, the generation net profit increases by $298.489/h, which is an effective incentive.
2) Case 2: With Line Flow Limits

This case is used to explain the situations in which the adjustment of the demand results in a violation of the CFL of other demands. In this case, the CFL values of demands at buses 2, 3, and 4 are 100, 180, and 115 ton/h, respectively.

Other demands. In this case, the CFL values of demands at adjustment of the demand results in a violation of the CFL of 2)

Fig. 5. Line flows of the 5-bus system before and after the adjustment, case 2.

<table>
<thead>
<tr>
<th>TABLE VI</th>
<th>DEMAND ADJUSTMENT PROCEDURE FOR THE 5-BUS SYSTEM, CASE 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Process</td>
<td>L2</td>
</tr>
<tr>
<td>S1:</td>
<td>65.0</td>
</tr>
<tr>
<td>S2:</td>
<td>126.9</td>
</tr>
<tr>
<td>S3:</td>
<td>269.954</td>
</tr>
<tr>
<td>S4:</td>
<td>269.954</td>
</tr>
</tbody>
</table>

Total system emissions obtained from running the OPF before adjustment are 387.345 ton/h, showing an excess emission of 2.345 ton/h, with a surcharge of $53.935/h. However, this surcharge is eliminated after the adjustment and the net profit increases by $62.415/h.

B. IEEE 118-Bus System

This system contains 10 demands with CFLs set at buses 59, 70, 74, 75, 76, 77, 78, 90, 92, and 116 with 310, 210, 170, 150, 150, 190, 220, 400, 160, 450 ton/h, respectively. The data of this system is retrieved from [39].

Table VIII presents the carbon footprint of each demand, initially and after adjustment. Each row presents Sx (x is an integer) is the step counter, Bus # is the bus whose demand must be adjusted, and (AD) is the adjusted demand in MW. It can be seen that all the CFLs have been violated initially. Therefore, all the corresponding demands need to be adjusted.
In some steps, S2 and S4, the adjustment of demands 59 and 74 satisfies the carbon footprint of other demands such as 75, 77, and 78, while in some steps, such as S6 and S8-S12, the adjustments result in a carbon footprint of other demands and, consequently, the readjustment process is applied. Note that boldface values represent violations of the carbon footprints in Table VIII. On the other hand, those consumers located in critical regions or the ones that must decrease a noticeable fraction of their demands, may incur in a violation after adjustment and running of the OPF. This situation can be observed in demand 116 at step 8 (S8). In order to consider the adjustment/readjustment procedure of this demand in detail, the generating units’ contribution and the corresponding carbon footprints before adjustment, S7, and during the adjustment process are reported in Table IX. As can be seen from this table, the adjustment of demand 116 is done in five iterations. Considering S7 shows a DCF limit violation of demand 116 with 45.5798 ton/h of excess emissions. In S8(1), the adjustment process asks the consumers for a decrease of 16.9230 MW to satisfy the DCF limit. However, after this demand decrease, when running the OPF, the optimal dispatch and, consequently, the unit contributions are changed and this results in a smaller DCF limit violation, with excess emissions of 3.0219 ton/h. In S8(2), to address this violation, a decrease of 1.1145 MW is requested and managed but, again, after running an OPF, a smaller violation than the previous violation in DCF limit results. Eventually, at S8(5) the readjustment process satisfies the DCF limit at this bus. However, this causes a violation of the DCF at bus 74, which is addressed at S9. This process continues until all DCF limits are satisfied. Since, in this system, load buses 70, 74, 76, 90, 92, and 116 are located in the same region, running the OPF after adjusting one of these demands may result in a new violation of the recently-adjusted demand or other demands.

Table X shows that there are initial carbon footprint violations of 91.4 ton/h and 648.9 ton/h on the demand and generation sides, respectively, which cause surcharges of $2,102.2/h and $14,924.7/h. However, after applying the proposed approach, the consumers decrease their demands by 46.4 MW/h resulting in a significant decrease of their energy bills of $4,225.9/h. Although this decrease does not totally eliminate the carbon footprint surcharge on the generation side, it positively affects the net profit, increasing it by $3,233.2/h, compared with the case before adjustment.

### IV. Conclusions

The framework presented in this paper is a novel solution for the existing gap in this area of research in which only the generation side used to be considered as the main source of emissions. However, generation is driven by demand, and consequently, consumers should be deemed as the prime responsible agents for carbon emissions. In this paper, the carbon footprint is allocated among the consumers using the improved Proportional Sharing Theorem, and then the demands are adjusted to satisfy their carbon footprint limits via an adjustment procedure. Warning signals, such as an excess carbon footprint and its incurred surcharge tax, and incentive information, such as tax exemption and electricity bill reduction, may convince consumers to decrease their demands, and, therefore, reduce carbon footprints. Results show that the proposed framework is beneficial for both the demand and generation sides. The demand side, by supporting the carbon policy and participating in the demand adjustment process, pays a lower electricity bill. On the other hand, from the generation standpoint, unlike the existing policies such as Carbon Cap (CC), Cap and Trade (C&T) and Carbon Tax (CT) where a decrease in demand reduces the benefit, the proposed approach provides a considerable increase in the net profit. The proposed model, compared to the aforementioned policies, shows a lower but reasonable computational efficiency, so the model is fast enough to be applied in online-based problems such as demand side management, market-based problems, etc. A little increase in CPU time is the price to be paid to obtain a fair model that can make effective incentives for both the generation and demand sides without imposing too much cost on society. Future work will apply this framework to a full-fledged carbon market environment.

### Appendix

In this Appendix, the models of commonly used policies to reduce greenhouse gas emissions such as Carbon Cap (CC), Cap and trade (C&T) and Carbon Tax (CT) are presented. These approaches have been tested in two power systems: 5-bus and IEEE 118-bus. Different loading conditions have been
provided, and the outcomes of each approach before and after demand reduction provide useful information to reveal the potentials and shortcomings of each approach.

A. Models of Greenhouse Gas Emissions Reduction Policies

1) Carbon Cap (CC) Model

The CC model minimizes the total system cost while considering the constraint set (48) and the constraint corresponding to the cap, (49), as follows [40].

\[
\min_{p_{i_1}, q_{i_1}, q_{i_1}, p_{i_1}} F^C = \sum_{i \in \Omega_1} C_i(P_{i_1}) \quad (47)
\]

s.t.

constraints (2)-(8) \hspace{1cm} (48)

\[
\sum_{i \in \Omega_2} E_{m_i}(P_{i_2}) \leq \text{Cap} \quad (49)
\]

2) Carbon Cap-and-Trade (C&T) Model

To obtain the C&T model, the objective function considers not only the system total cost but also the term related to carbon permit trade, (50). The deficit in the emissions permit, \(E_{m_{ defect}}\), should be considered in the cap constraint (52). Since the trade mechanism among several systems is not considered in this paper, finding the best trade, the optimal value, \(E_{m_{ defect}}\), for this single system is used; this way we can compare the best outcome of the C&T policy with the proposed model.

\[
\min_{p_{i_1}, q_{i_1}, q_{i_1}, p_{i_1}} F^C = \sum_{i \in \Omega_1} C_i(P_{i_1}) - \tau \cdot E_{m_{ defect}} \quad (50)
\]

s.t.

constraints (2)-(8) \hspace{1cm} (51)

\[
\sum_{i \in \Omega_2} E_{m_i}(P_{i_2}) + E_{m_{ defect}} \leq \text{Cap} \quad (52)
\]

3) Carbon Tax (CT) Model

The carbon tax is an environmental tax that is levied on the carbon content of fuels. The CT model minimizes the system total cost and the tax-based emission penalty [20].

\[
\min_{p_{i_1}, q_{i_1}, q_{i_1}, p_{i_1}} F^C = \sum_{i \in \Omega_1} C_i(P_{i_1}) + \sum_{i \in \Omega_2} \tau \cdot E_{m_i}(P_{i_2}) \quad (53)
\]

s.t.

constraints (2)-(8) \hspace{1cm} (54)

B. Comparisons and Discussion

In this subsection, the approaches are considered from both demand and generation sides. To reveal the potentials and shortcomings of the aforementioned approaches, for the 5-bus system, two loading conditions such as 1) initial loading and 2) load increase in bus 2, L2, are studied, while the IEEE 118-bus system is only studied under the initial loading condition. The demand of L2, under the load increase condition, is set to 95 MW. To compare the results of the approaches for the after-adjustment case, the adjusted demands (obtained by the proposed model) are set as the inputs of the three other approaches.

1) 5-bus system

The cap of the 5-bus system is set to 385 ton/h. The cost of emissions trading is considered to be equal to the carbon tax, $23/ton [20].

**Table A1**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>CC</td>
<td>13,890.35</td>
<td>13,040.19</td>
<td>17,758.69</td>
<td>13,651.43</td>
<td></td>
</tr>
<tr>
<td>C&amp;T</td>
<td>24,337.81</td>
<td>22,910.48</td>
<td>28,808.54</td>
<td>23,960.60</td>
<td></td>
</tr>
<tr>
<td>CT</td>
<td>24,337.87</td>
<td>22,910.53</td>
<td>28,808.58</td>
<td>23,960.66</td>
<td></td>
</tr>
</tbody>
</table>

a) Demand side:

Table A1 provides useful information on the total DEBs of the different approaches under different loading conditions to show which approach reduces the emissions by imposing lower costs on society.

As can be seen from Table A1, the lowest DEBs are highlighted in boldface. This shows that, among all, the proposed approach performs far better than commonly used policies (i.e., C&T and CT) and better than the CC model. Before adjustment, under the first loading condition, the CC, C&T and CT approaches result in respectively $147.5/h, $10,594.97/h, and $10,595.03/h higher costs than the proposed model, while, after adjustment, they show more or less the same outcomes but $175.68/h, $10,045.97/h and $10,046.02/h higher costs than the proposed approach, respectively. This shows the unacceptable costs that the commonly used policies such as the C&T and CT impose to reduce carbon emissions, while the CC approach is far better than these policies. However, considering the CC approach under the second loading condition and before adjustment, shows that this approach under this loading condition imposes too high costs on society, $1,541.54/h more than the proposed model. The CPU times of the proposed model for the first and second loading conditions are 0.088 s and 0.239 s, respectively. CPU times of CC, C&T, and CT for the first loading condition are 0.061 s, 0.079 and 0.024s while, for the second loading condition are 0.198 s, 0.283 s, and 0.140 s, respectively. CPU time stands for the whole process (finding LMCs and running OPFs before and after adjustment).

The other important issue to consider is to investigate the impacts of increasing a demand on the DEBs of other demands. In this regard, the changes of DEBs of all demands due to a load increase in bus 2 is depicted in Fig. A1.

**Fig. A1**

DEB changes due to demand increase in L2, before adjustment.
As can be seen from Fig. A1, by increasing the demand in bus 2, L2, all the approaches result in an increase in the DEBs of the consumers at this bus. For this bus, the outcomes of the proposed approach and the CC approach are more or less similar, while the other two commonly used policies impose much higher costs to this demand. The superiority of the proposed method is revealed when the DEBs of other buses, 3 and 4, are considered with respect to the other approaches. By using the proposed model, the bills of other demands increase only by $260.54/h and $251.87/h, respectively, while the other approaches impose much higher costs on these demands. The DEB changes of demand L3 using the CC, C&T, and CT are 364.7% ($950.18/h), 297.6% ($775.38/h) and 297.6% ($775.31/h) higher than the DEB changes obtained by the proposed model, respectively. For L4, the DEB changes of the aforementioned approaches are 161.4% ($460.54/h), 121.5% ($360.09/h) and 161.4% ($360.08/h) higher than the bills provided by the proposed model, respectively. For further analysis, the changes in DES after increasing the demand at L2 for the case after adjustment is portrayed in Fig. A2. As can be seen, after the adjustment process, the proposed approach performs far better than the existing policies. In the proposed approach, the demand increase in L2 results in a higher bill for the consumers at L2 while the impact on the bills of other consumers, by a $7.43/h increase at L3 and by a $16.79/h decrease at L4, is negligible. However, the three other approaches cannot provide such fair bills since increasing the demand at L2 not only affects the DEB of the corresponding bus but also the increases on the DEBs of other buses are considerable.

Therefore, it can be concluded that the only approach that is fair from the consumers’ point of view is the proposed model, since: 1) reduces carbon emissions by imposing the lowest possible costs on society, and 2) responds fairly to likely demand changes at a bus; meaning that if one consumer asks for more energy, it is the prime responsible for this increase and it has to pay more. The DEBs of other demands, as a result of re-dispatching using OPF, always faces changes.

From the consumers’ standpoint, simply showing them that their DEBs decrease after a demand adjustment is not an incentive. Actually, incentives are meaningless for those approaches that, first, impose a high cost on the society (C&T and CT) or cannot provide a fair DEB under load-changing conditions (CC, C&T, and CT), asking the consumers to manage their demands to decrease their bills. Therefore, from the demand standpoint, the only approach that can provide an effective and fair incentive for the consumers is the proposed approach.

b) Generation side:

To show which approach provides a better incentive for the suppliers, the suppliers’ net profit changes (due to demand adjustment) under both loading conditions are depicted in Fig. A3, i.e., the bars show the difference between the net profit after demand adjustment and before adjustment condition.

From Fig. A3, it can be concluded that, for the three existing policies (CC, C&T, and CT), not only there is no incentive for the suppliers that motivates them to encourage consumers to decrease their demands, but there is also a deterrent that prevents them from doing so. Using these approaches, the more the demands decrease, the lower their benefits becomes. For example, applying the C&T policy, a load decrease results in a net profit decrease of $930.29/h and $3,395.15/h for the first and second loading conditions, respectively.

Consequently, among all the approaches, the only approach that provides effective incentives for the generation side is the proposed approach, in which the demand adjustments result in net profit increases of $298.49/h and $817.86/h, respectively.

Fig. A2. DEB changes due to demand increase in L2, after adjustment.

Fig. A3. Suppliers’ net profit changes due to demands’ adjustment

2) 118-bus system

The cap of the 118-bus system is set to 6800 ton/h. The cost of emissions trading is considered to be equal to the carbon tax, $23/ton [20].

Table A2 shows the DEBs of all the consumers as well as the net profits of all the generating units obtained by the different approaches. Boldface figures on the demand side present the approaches that impose lower costs, while, on the generation side, we show the approach that provides an effective incentive to the suppliers. The results show that the CC, C&T, and CT approaches, first, impose too high costs and then provide some incentives. Among the existing policies, the best outcomes are related to the CC approach which, for the before- and after- adjustment conditions, imposes an increase in cost of $4,518.85/h and $3,955.91/h, compared to the proposed model. Compared with the proposed approach, C&T imposes $93,875.69/h and $91,237.46/h, and CT imposes $93,875.55/h and $91,237.44/h higher costs. On the other hand, the proposed approach is the only one that can provide an effective incentive for the suppliers, for which decreasing their demands results in an increase in their net profits of $3,233.27/h, while the other approaches are deterrent, preventing the suppliers from encouraging consumers to
decrease their demands. The CPU time of the proposed approach for this system is 65.39 s, while the CC, C&T and CT policies require 36.98 s, 58.02 s, and 53.28 s, respectively. The reason why the CPU time of the proposed approach is slightly higher than the other approaches is the time consumed in the adjustment process. This increase in CPU time can be considered as the price of having a fair framework that can provide effective incentives for both consumers and suppliers.

**Table A2**

<table>
<thead>
<tr>
<th>Approach</th>
<th>DEBS (S/h)</th>
<th>Suppliers’ Net Profit (S/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CC</td>
<td>168,184.15</td>
<td>165,395.25</td>
</tr>
<tr>
<td>C&amp;T</td>
<td>257,540.99</td>
<td>252,676.81</td>
</tr>
<tr>
<td>CT</td>
<td>257,540.85</td>
<td>252,676.81</td>
</tr>
<tr>
<td>Proposed</td>
<td>163,665.30</td>
<td>161,439.37</td>
</tr>
</tbody>
</table>

**REFERENCES**


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