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PRICE-AWARE RENEWABLE ENERGY MANAGEMENT WITH TRANSMISSION LOSSES

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ABSTRACT

In this paper we propose a genie-aided strategy to optimize the use of renewable energy (RE) in a community of households with shared access to storage and RE generation facilities. The households are spread over a limited geographical area, and are subject to different time-varying power consumption profiles, and energy prices. We consider a finite number of RE generators and energy storage devices (ESDs), which are deployed in specific locations. The proposed strategy seeks to minimize the energy cost incurred by the participating households by optimizing the rate at which RE is consumed over time. Our model takes into account the power loss incurred in the transmission of energy from the generators to the loads. The optimization problem is cast as a non-convex quadratically constrained quadratic program, which is simplified in order to derive an approximate solution. Numerical results show that transmission losses and differences across price and load can significantly affect the optimal RE allocation among the households. The proposed strategy offers valuable insights for energy planning purposes and can be used to devise real-time RE management algorithms by incorporating the necessary forecasting techniques.

Index Terms— Renewable energy management, quadratic programming, transmission losses.

1. INTRODUCTION

The sharing of RE generation and storage facilities is becoming an attractive alternative for households with space limitations. Community solar (CS), for example, is an energy development model that is gaining popularity in the United States [1]. CS encompasses various models in which households cooperate to meet their energy requirements at lower economical and environmental costs.

Among the various CS models, one that has attracted considerable attention is Shared Solar, which consists of deploying solar PV panels at designated locations to power a number of households in their vicinities [2]. This configuration is attractive to households with restricted rooftop space or to the ones who prefer sharing capital investment.

In this paper we develop a mathematical model for energy management in a Shared Solar environment. Our framework can be used to optimize the rate at which the households consume energy from each RE production and storage center (REPSC). We take into account the power loss incurred by the wires connecting the REPSCs and the households. The power lines are modeled as real impedances, and the loss incurred in the transmission of power is modeled by using Ohm’s law. With these considerations, we formulate a non-convex quadratically constrained quadratic programming problem, for which we propose a simplification. The proposed simplification shrinks the original feasible space, and hence, the solution obtained satisfies all the constraints of the original formulation.

The proposed model leads to an RE management strategy, which can be used for energy planning purposes and to derive real-time energy management algorithms. Specifically, our framework allows us to devise clustering schemes to reduce capital investment by deploying a smaller number of connections between REPSCs and households. Moreover, in a real-time setting, the proposed strategy can be used to recompute the RE consumption schedules in response to updated estimates of future RE production and load. Thus, forecasting techniques can be incorporated in our framework to implement the optimization strategy in real time.

To the best of our knowledge, there are no works in the literature proposing energy management strategies for households in a Shared Solar environment, which are also aware of time-varying electricity prices, and power loss dissipated through ESDs and connecting wires. Related works in the literature include [3–6], where greedy RE management strategies are proposed at an individual level. Cooperative RE management strategies have been proposed in [7–22]. However, most of these strategies do not take into consideration electricity price variations across time and location, as well as the energy loss incurred by power lines connecting RE generators and loads.

Unlike existing works, we take into consideration the power loss incurred by the ESDs, and the wires connecting the REPSCs and the loads (households). As shown by our numerical results, the power loss incurred in the energy transmission can be used as a criterion for the households to choose some REPSCs over others. This result motivates the clustering of households to avoid the deployment of some of the connecting wires in the network. Transmission line power losses have been accounted for in works such as [11] and [23]. However, these works either disregard the time-varying nature of the electricity prices, or do not account for the losses incurred in the operation of ESDs charged with RE.

2. SYSTEM MODEL

2.1. Loads, Planning Horizon, and RE Consumption Schedules

We consider a set of $M$ households connected to the same power grid, and deployed across a finite area, as shown in Fig. 1. The planning horizon is $[0, S]$ where $S > 0$ is an arbitrary positive real number. The power consumed by the $m$th household at time $\tau \in [0, S]$, with $S > 0$, is denoted by $L_m(\tau) : [0, S] \rightarrow [0, L_{\text{max}}]$, where $L_{\text{max}}$ denotes the maximum power that the household can consume. We consider a set of $N$ REPSCs, each one with an RE generator and ESD. The REPSCs are deployed across different locations. The
power drawn by the \( m \)-th household from the \( n \)-th RE generator is
\[
D_{m,n}(\tau) = \int_0^\tau P_m(t) \left[ L_m(t) - \sum_{n=1}^N \left[ D_{m,n}(t) - \rho \text{dis}_{m,n} \left( \frac{D_{m,n}(t)}{V} \right)^2 \right] \right] dt,
\]
where \( J_n(0) \geq 0 \) is the energy initially available in the battery, and \( R_n(\tau) \) is the renewable power charged to the ESD, which is assumed to be within the ESD’s allowed charging rate.

### 2.3.3. Limited storage capacity

The capacity of the \( n \)-th ESD is denoted by \( \Psi_n \in \mathbb{R}^+ \). Therefore, the \( D_{m,n}(\tau) \)'s must be such that
\[
0 \leq J_n(\tau) \leq \Psi_n, \forall \tau.
\]

### 2.3.4. Limited discharging rates

Each ESD has a limited discharging rate, expressed as the maximum amount of energy that can be drawn from the ESD in each time slot. The maximum discharging rate that the \( n \)-th ESD can handle is \( q_{D,n} \) power units. Therefore,
\[
\sum_{m=1}^M D_{m,n}(\tau) \leq q_{D,n}, \forall \tau, \forall n.
\]

### 3. Problem Statement

#### 3.1. Decision Variables and Constraints

The decision variables are the discharging schedules \( D_{m,n}(\tau) \) which will determine the optimal RE consumption patterns. There are thus two kinds of constraints that need to be satisfied in the formulated optimization problem. The first set of constraints derives from the bounded storage capacities of the ESDs and the causality condition, according to which only RE readily available in the ESDs can be dispatched. These constraints can be stated mathematically as follows:

\[
0 \leq J_n(0) + \int_0^\tau \left[ \alpha_n R_n(t) - \frac{1}{\beta_n} \sum_{m=1}^M D_{m,n}(t) \right] dt \leq \Psi_n, \forall \tau, \forall n, \quad \text{(4)}
\]

where \( \Psi_n \) denotes the storage capacity of the ESD at the \( n \)-th REPSC. Constraints (4) were obtained by using the definition of \( J_n(\tau) \), presented in (2), and are introduced to enforce that each \( J_n(\tau) \) is within the range \([0, \Psi_n]\). A second type of constraints arises if we assume that distributed RE generation needs to be used only locally,\(^2\) which means that the consumption of RE is upper bounded by the load in each household:

\[
\sum_{n=1}^N D_{m,n}(\tau) - \rho \text{dis}_{m,n} \left( \frac{D_{m,n}(\tau)}{V} \right)^2 \leq L_m(\tau), \forall \tau, \forall m.
\]

The left-hand side of (5) represents the effective power drawn from the \( m \)-th household from all the \( N \) REPSCs, i.e., the power obtained after transmission losses. The right-hand side of (5) denotes the power required by the \( m \)-th household at time \( \tau \).

#### 3.2. Formulation and Considerations

We formulate a mathematical problem to optimize the discharging schedules as follows:

\[
\text{P0: } \min \sum_{m=1}^M \sum_{n=1}^N D_{m,n}(\tau), \quad \text{s.t. } (3), (4), \text{ and } (5).
\]

\(^2\)This constraint follows when no RE can be injected into the grid. In a net metering scenario, this constraint can be updated to include subscriber limits.
P0 is a very challenging problem because of the following reasons: its objective is not a function, but a sum of functionals. Its decision variables are not scalar or vectors, but trajectories (functions defined in continuous time). Equations (4) and (5) represent and infinite number of constraints, which must hold in all the realizations of the stochastic processes $L_1(\tau), \ldots, L_M(\tau)$ and $R_1(\tau), \ldots, R_N(\tau)$.

3.3. Genie-Aided Solution

We propose a method to solve P0 by assuming full\footnote{This assumption does not limit the application of the proposed strategy, as forecasts can be used to replace information unavailable in a practical setting.} knowledge of the loads and RE generation profiles across time. This genie-aided solution can be used to benchmark online strategies, and to devise real-time RE management algorithms based on forecasts. For tractability, we will introduce discretization, and determine the optimal $D_{m,n}(\tau)$ only at a finite number of points. After introducing discretization, the problem can be cast as a quadratically constrained quadratic programming problem as will be shown next.

We sample the functions $P_m(\tau), D_m(\tau), L_m(\tau)$ and $R_m(\tau)$, $\forall m, \forall n$, at $T > 1$ equally-spaced points, and thus divide the planning horizon into $T - 1$ subintervals. Let $\Delta t$ denote the sampling interval, $t \in \{1, \ldots, T\}$ be the slot index, and $x \in \mathbb{R}^{MNT}$ denote the $MNT$ variables to optimize over the entire planning horizon as follows: $x = [x_1, x_2, \ldots, x_M]^T$, where $x_n = [y_{m,1}, \ldots, y_{m,T}]$ and $y_{m,t} = [D_{m,1}(t\Delta t), D_{m,2}(t\Delta t), \ldots, D_{m,N}(t\Delta t)]$. Moreover, we denote the pricing vectors obtained after the sampling by $p_m$, i.e., $p_{m,t} = P_{m}(t\Delta t)$, and simplify notation by introducing the following definitions $K_{m,n} \triangleq p_{m,n} \xi^T$, and $k_m = 1_T \otimes [K_{m,1}, K_{m,2}, \ldots, K_{m,N}]$. Then, the objective function can be written as the following quadratic form:

$$
\xi \approx \Delta t \left[ 1, M \begin{bmatrix} p_1^T \xi_1 \\ \vdots \\ p_M^T \xi_M \end{bmatrix} - 1, M P x + x^T Q K x \right],
$$

with $P$, $Q$ and $K$ written as follows:

$$
P = d_g\left((p_1 \otimes 1_N), (p_2 \otimes 1_N), \ldots, (p_M \otimes 1_N)\right),
$$

$$
Q = d_g\left(d_g(p_1 \otimes 1_N), d_g(p_2 \otimes 1_N), \ldots, d_g(p_M \otimes 1_N)\right),
$$

$$
K = d_g\left((k_1, k_2, \ldots, k_M)\right),
$$

where $d_g(v)$ denotes a matrix with vector $v$ in its main diagonal, and zeros elsewhere. Constraints (3) and (4) can be written in terms of $x$ as follows:

$$
M_1 x \preceq v_1, \quad M_2 x \preceq v_2, \quad M_3 x \preceq v_3,
$$

for matrices $M_1$, $M_2$ and $M_3$, and vectors $v_1$, $v_2$ and $v_3$, which can be derived from (3) and (4), respectively. Constraints (5) can be written using the following quadratic form

$$
1_M U_{m,t} x - x^T V_{m,t} x \leq L_{m}(t\Delta t), \quad \forall m, \forall t,
$$

for appropriate matrices $U_{m,t}$ and $V_{m,t}$. A discrete-time version of P0 can then be written as:

$$
P1: \max_x 1_M P x - x^T Q K x \\
\text{s.t.} \quad (7) \text{ and } (8).
$$

In P1 we have removed the term $\Delta t \sum_{m=1}^M \sum_{t=1}^T p_m(t) l_m(t)$, because it does not depend on the design variable $x$. Similarly, for simplicity, the constant $\Delta t$ has been removed from the objective function, as it does not affect the result of the optimization. By solving P1 we can determine the optimal discharging schedules at equally-spaced time instants. The higher the sampling rate, the more information about the optimal solution can be obtained through the discrete-time formulation.

**Remark:** P1 is a quadratically constrained quadratic programming problem in which we want to maximize a concave function of $x$. However, P1 is not a convex optimization problem because the matrices $V_{m,t}$ are positive definite and preceded by a negative sign in constraints (8). To show that $V_{m,t}$ is a positive definite matrix, we note that it is a diagonal matrix, since no cross-terms appear in (8), and it only has non-negative numbers because the coefficients of the quadratic terms $[D_{m,n}(t)]^2$ are all positive, i.e., the resistance per unit length $\rho$, the distance between the generators and the households $d_{m,n}$, and the voltage $V$, are all non-negative quantities.

3.4. Simplification

To tackle P1, we propose the following simplified formulation:

$$
P2: \max_x 1_M P x - x^T Q K x \\
\text{s.t.} \quad (7) \text{ and } (8),
$$

which replaces (8). Note that (9) is a more stringent constraint than the original (8). As a result, this simplification will shrink the feasible space, and hence, the solution obtained for P2 will satisfy the original constraint (8).

**Proposition 1.** $P2$ can be written as a convex optimization problem.

**Proof.** The quadratic form $1_M P x - x^T Q K x$ is concave in $x$ because $Q K$ is positive semi-definite. In P2 we thus seek to maximize a concave function. Moreover, the constraints (7) and (9) are all affine. Therefore P2 can be written as a convex optimization problem simply by expressing it as a minimization problem by multiplying its objective function by -1.

4. NUMERICAL RESULTS

We provide numerical results to analyze the proposed strategy. Several simulation scenarios (load, RE generation profiles and prices) are considered in this section in order to illustrate the characteristics of the proposed strategy. System parameters that are used throughout this section are presented in Table 1. The optimization problems are solved by using CVX on Matlab. Throughout this section, storage capacity is measured in energy units [EU] and energy expenditure in monetary units [MU]. To simplify notation, let $D_{m}(t)$ denote the total renewable power drawn by the $m$th household at time $t$, i.e., $D_{m}(t) = \sum_{n=1}^{N} D_{m,n}(t)$, $\forall t$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$, $\Delta t$, $M$, $N$</td>
<td>$[20, 1, 3, 2]$</td>
</tr>
<tr>
<td>$q_{D,n}$</td>
<td>$\Delta t \sum_{t=1}^{T} r_n(t), \forall n$</td>
</tr>
<tr>
<td>${a_n, \beta_n, \Psi_n, J_n(0)}$</td>
<td>$[1, 1, \Delta t \sum_{t=0}^{T} r_n(t), 0, \forall n]$</td>
</tr>
</tbody>
</table>

We start by illustrating the characteristics of the RE consumption schedule. We thus consider time-varying prices and loads, and
fixed resistances across connecting wires of equal length. Specifically, we assume the price and load profiles illustrated in Fig. 2, and let $K_{1,1} = K_{1,2} = 0.01, K_{2,1} = K_{2,2} = 0.01$, and $K_{3,1} = K_{3,2} = 0.01$. In Fig. 2 we can see that, when the wires connecting the households and the RE generators have the same characteristics (length and resistance), the RE consumption rate is only influenced by the price variations across time and the load. When the load is above the RE generation at all times, then the optimal schedule only responds to price variations across time. When the load is below the RE generation at all times, then the optimal schedule is determined by the loads, since the consumption of RE is upper bounded by the load in each household.

**Fig. 2:** Time-variation of RE consumption rate. Top: Influenced by price, when the load is above the RE generation at all times.

We now investigate the effect of the price loss factor in the RE use pattern. We thus consider constant prices, loads and RE generation profiles, but varying distances and wire resistances. Specifically, we assume the price, load and RE generation profiles illustrated in Fig. 3, and let $K_{1,1} = K_{1,2} = 0.005, K_{2,1} = K_{2,2} = 0.008$, and $K_{3,1} = K_{3,2} = 0.01$. In Fig. 3 we illustrate both, the RE consumption rates across time, and the result of the RE allocation strategy. As seen, the RE consumption rates are nearly constant. Interestingly, since $K_{m,1} = K_{m,2}, \forall m$, in this scenario the share of RE that goes to the $n$th household from the $m$th generator is given by the ratio $\frac{\prod_{j=1}^{m} K_{n,j}}{\prod_{j=1}^{N} K_{n,j}}$.

**Fig. 3:** Effect of power loss in RE allocation across households. Share of RE is given by $\frac{\prod_{j=1}^{m} K_{n,j}}{\prod_{j=1}^{N} K_{n,j}}$. More RE is allocated to the households with better connecting power lines.

When the power loss is high, e.g., when $K_{11} = K_{12} = K_{21} = K_{22} = K_{31} = K_{32} = 0.02$, prices will have a reduced impact on the RE allocation policy. As seen in Fig. 4, all households get the same share of RE use, despite the price differences across locations. This result follows because the load is above the RE generation at all times, the wires connecting households and RE generators have the same characteristics (length and resistance), and the quadratic term dominates the objective function in P0.

As expected, prices will dominate the RE allocation policy when households and RE generators have similar characteristics, and when the power loss is not as dominant as in the scenario considered in Fig. 4. This can be seen in Fig. 5, where we considered a scenario in which prices vary across locations and connecting lines offer less than significant resistance, i.e., $K_{11} = K_{12} = K_{21} = K_{22} = K_{31} = K_{32} = 0.002$.

**Fig. 4:** Despite price differences across households, the RE allocation is even, following poor power connecting lines.

**Fig. 5:** With highly efficient connecting lines, the RE allocation policy is mainly determined by the price differences across households.

## 5. CONCLUSIONS

We have proposed an energy management strategy which seeks to minimize the cost incurred by a cooperating group of households over a finite planning horizon. The households share access to a group of RE generators and ESDs. In our framework we have considered the distance-dependent power loss incurred when transmitting energy from the RE generators to the loads. We have cast the optimization problem as a non-convex quadratically constrained quadratic programming problem and proposed a solution through discretization and relaxation. We have presented numerical results to illustrate the characteristics of the proposed solution. Through simulations, we have shown that the RE consumption rate depends on the price variations across time, the loads, and the characteristics of the power lines connecting the households and the RE generators. The proposed strategy can be used for energy planning purposes and to benchmark and devise real-time energy management algorithms by incorporating forecasting techniques to estimate future RE generation and power consumption. It can also be used for establishing energy cooperation clusters of households so as to reduce capital expenditure, i.e., the cost incurred in the deployment of transmission lines and RE production centers.
6. REFERENCES


