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The effect of subsurface crack opening on the stress intensity factor under cyclic thermal loads

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Abstract

This study considers surface crack behaviour under periodic thermal loads using weight functions and finite element calculations. A surface and subsurface crack opening mode during the load cycle was observed. The subsurface mode where the crack mouth is closed while the crack tip remains open can be more critical for deeper cracks. Crack face contact is required to obtain the subsurface mode both with finite elements and weight functions. A combination of surface and subsurface weight functions can capture the crack behaviour throughout the load cycle. This behaviour is demonstrated using numerous simulations with periodic thermal loads with different extent and frequency.

Graphical abstract

Keywords

Thermal fatigue; Surface flaw; Crack closure; Stress intensity factor; Weight function

Highlights

- The possibility and effects of crack closure with thermal fatigue loads are studied
- The subsurface mode is found to be more critical for deeper cracks
- A solution method combining two weight function solutions is presented

Nomenclature

a	Crack length
c_p	Specific heat capacity
d	Crack face contact width
E	Elastic modulus
F	Thermal load frequency
H	Plate half-height
h_{sub}	Subsurface crack weight function
h_{surf}	Surface crack weight function
k	Thermal conductivity

K_0	Modified Bessel function of the second kind
K_I	Mode I stress intensity factor
R	Thermal load area half-width
$T, \Delta T, T_0$	Temperature, range and mean temperature
t	Time
W	Plate width
x, y	Cartesian coordinates along and normal to the crack line
α	Thermal expansion coefficient
κ	Thermal diffusivity
ν	Poisson ratio
ρ	Density
σ	Crack opening stress
ω	Thermal load angular velocity
CMOD	Crack mouth opening displacement
SIF	Stress intensity factor
WF	Weight function

1. Introduction

Components subjected to repeating thermal loads are susceptible to damage and eventually surface cracking. This phenomenon has been observed in nuclear piping components such as elbows and flow mixers where there is localized turbulence that causes high frequency temperature oscillations in the fluid near the pipe wall. The main requisite for thermal fatigue is that the free thermal expansion is completely or partially constrained and the thermal strains are compensated by mechanical strains in the structure. After a sufficient number of cycles, a surface crack may initiate in the thermally cycled location. The crack will be opened by the tensile stresses during the cooling phase as the material contracts due to negative thermal expansion and the crack will be closed during the heating phase due to compressive stresses induced by the positive thermal expansion. There can be a phase difference in the crack face loading along the crack length such that the crack is partially subjected to tensile loads and partially to compressive loads. This is the case if the crack size is sufficiently large with respect to the characteristic dimensions of the temperature fluctuation. The compressive loads at the crack faces may cause partial or full crack face closure.

Under linear elastic conditions, the criticality and growth potential of cracks are commonly assessed using the stress intensity factor (SIF). The SIF can be calculated numerically for a given crack using the finite element method or using weight function methods together with the stress distribution of the uncracked component. As shown in this work, the traditional weight function methods are applicable for the SIF calculations for thermal fatigue loads in case of small surface cracks, but the commonly used surface crack solution is not applicable at time instants when the thermal load induces crack mouth closure while keeping the tip of the crack open. The crack geometry will then correspond to a subsurface crack, requiring the use of a different, subsurface, crack weight function solution to obtain accurate SIF results.

We show that, for deeper cracks under the studied harmonic thermal loads, the subsurface mode (i.e. the crack mouth is closed) may produce a considerably larger stress intensity factor during the thermal load cycle than the surface mode (the crack mouth is open) for a given crack depth. This is because the traditional linear weight function approach using the surface crack geometry fails to capture the subsurface mode as it does not take into account crack face contact and the linearly calculated stress intensity factor can even be negative when the crack tip is actually open.

In the previous studies of crack behaviour under cyclic thermal loads, the possibility of crack closure during the thermal load cycle is mostly neglected. Hayashi [1] presents a straightforward assessment of surface crack arrest based on the analytical temperature and thermal stress solutions together with a stress intensity factor calculation based on a polynomial stress fitting. Possible crack closure behaviour is not taken into account in the study. Jones [2] argues that the weight function solutions based on load boundary conditions produce inaccurate results if the crack size is not small compared to the other dimensions of the plate, due to the displacement controlled nature of the thermal induced stresses. Jones [2] verifies the displacement controlled weight function stress intensity factor calculations with thermal loads using finite element analyses. The analyses seem to neglect the crack face contact and the surface crack weight function predictions closely agree with the finite element results. Green and Munz [3] utilize an effective stress based weight function approach for high-strain cases of thermal cycling. The effective stress is linearly converted from the plastic strain distribution and crack closure is modelled through a parameter q between 0.5 and 1.0 that reduces the stress range acting on the crack, based on the work by Haigh and Skelton [4]. Green and Munz [3] justify their reduction parameter based on the observation that normal linear fracture mechanics methods under predict the through-thickness growth and accounting for the crack closure improves the prediction. Gardin et al. [5] studied numerically an experimental thermal fatigue set-up where a surface crack propagates due to cyclic thermal loading. The SIFs calculated numerically using Abaqus finite element software without crack face contact predict crack growth behaviour consistent with the experimental findings. The reported results end shortly after the maximum SIF is obtained and do not reach a point where the subsurface crack opening mode would be significant.

There has been a significant contribution from France to the research of thermal fatigue both experimentally and computationally [6–10]. The modelling of the phenomenon has been twofold. The component lifetime has been explained using the amplitude of plastic strain or the amount of energy dissipation during the load cycle (e.g [9,10]). Also, finite element studies on thermal fatigue crack growth have been reported by the same group of authors [11]. These studies employ fracture mechanical tools for predicting the initiation and growth of a single or multiple thermal fatigue cracks. For example, Haddar et al. [12] use a linear superposition based SIF calculation to determine the stress intensity of multiple cracks under cyclic thermal loads. They use the stress difference over the period when the crack is open at the surface in the stress intensity factor calculations. An extensive study of the mechanisms leading to cracking and failure of a mixing tee was presented by Chapuliot et al. [13], who modelled the rapid propagation and arrest of surface cracks using the stress intensity factor calculated using polynomial stress fitting.

Although including crack face contact seems natural when performing finite element analysis of crack behaviour under cyclic thermal loads, and some studies (e.g. [14]) have included contact to obtain better J-integral approximations and approximate methods to take crack closure effects into account in crack growth calculations have been presented [4], it appears that the subsurface crack opening mode behaviour presented here has not been reported previously.

The case used to demonstrate the behaviour is a two-dimensional large plate under external localized forced temperature fluctuations. The periodic heating and cooling over a small surface area causes localized thermal expansion at and below the heated surface, which results in mechanical strains as the surrounding material partially prevents the expansion. The mechanical stresses and strains are cyclic and oscillate between their minimum and maximum values during the thermal load cycle. We study the behaviour of small surface cracks located in the middle of the heated area aligned perpendicular to the free surface. The studied case is visualized in Figure 1. A large number of thermal load parameter combinations is studied over a crack length range from 0.4 mm to 38 mm. The analysis cases and other parameter values utilized in the study are given in Table 1.

The focus of this work is in the application and suitability of the weight function methods used to determine the stress intensity factor for cyclic thermal loads. The weight function approach is often preferable over finite element analyses due to computational efficiency. The weight function methods are based on linear superposition and thus are only applicable under linear elasticity or limited plasticity. Therefore, all computations in this work assume linear elasticity and heat transfer behaviour.

Fig. 1. A schematic drawing of the surface cracked plate thermally cycled over a small surface segment at the free surface. The crack size ($a < 40$ mm) is small compared to the other dimensions in the model ($W=2H=1000$ mm).

The work is performed in three parts. First, finite element analyses are performed to determine the temperature and stress cycles in the assumed crack line due to the thermal load. The results are presented in Chapter 3.1 Second, cracks of different sizes are introduced in the simulation models and the SIFs are calculated using the contour integral technique. These results are considered as an accurate baseline to which the weight function predictions are compared. Third, the stresses of the uncracked component are utilized to determine the SIFs using the weight function method using both the surface and subsurface crack weight functions. The stress intensity factor results are presented in Chapter 3.2

Table 1. Parameters utilized in the thermal stress and stress intensity factor calculations.

Parameter	Symbol	Value(s)	Unit
Elastic modulus	E	190	GPa
Poisson ratio	ν	0.3	
Thermal conductivity	k	18.0	W/m/K
Specific heat capacity	c_p	500	J/kg/K
Density	ρ	7900	kg/m ³
Thermal diffusivity	κ	4.56×10^{-6}	m ² /s
Thermal expansion coefficient	α	16×10^{-6}	1/K
Thermal load range	ΔT	300	°C
Thermal load mean	T_0	220	°C
Thermal load frequency	F	0.05, 0.2, 0.5, 1, 2, 5	Hz
Heat area half-width	R	2, 4, 8, 12, 16	mm
Crack length	a	From 0.4 mm to 10 mm in 0.4 mm increments and from 10 mm to 38 mm in 2 mm increments	

2. Methods

The study was performed by studying the behaviour of surface cracks under cyclic thermal loads using finite element analyses and applying applicable weight functions to predict the crack behaviour analytically. The finite element calculations are performed using Abaqus FE software version 2018 and the weight function results are calculated using Matlab version 2018a and its numerical *integral* function that uses a global adaptive quadrature for the integration.

2.1 Heat transfer and stress simulations

Linear uncoupled heat transfer and thermal stress analyses were performed to determine the temperature and stress distributions in the crack line over the thermal load cycle. The studied case is visualized in Figure 1. Both the heat transfer properties and material behaviour are assumed to be fully linear, i.e. there are no temperature dependent material properties and the material response is linear. The simulations are performed sequentially coupled such that the temperature fields from the heat transfer simulations are used in the stress analyses.

A structured rectangular mesh in the crack location shown in Figure 2 is used in all simulations regardless of the heating area radius or heat load frequency. A constant element size of 0.08 mm is used near the heating area. Quadratic eight node heat transfer elements (DC2D8 in Abaqus) and plane strain structural elements (CPE8) are used in the simulations. The extent of the model is 1000 mm by 1000 mm and the element size is increased outside of the region of interest. The forced temperature excitation is modelled as a time dependent temperature boundary condition on the surface nodes. Convective and other heat losses in the system are assumed to be zero. Inertial effects are not considered in the stress or crack opening simulations. The initial temperature of the model is set as the mean temperature of the temperature fluctuation and tens of load cycles are applied to obtain a stabilized temperature and stress cycles in the crack line. Each thermal load cycle is solved using 20 time increments.

Fig. 2. A close-up of the finite element mesh used in the heat transfer and stress computations. The element size is 0.08 mm. The heating area centre point is in the lower left corner of the figure and the crack is located along the bottom edge which is also a symmetry plane. The contour colouring gives an example of the temperature field (°C) at the maximum surface temperature time instant during the stabilized temperature cycle. The extent of the heating area is $R=2$ mm and heat frequency is $F=1$ Hz in the figure.

2.2 Crack opening and stress intensity factor simulations

To determine the reference values for the SIFs caused by the repeated thermal loads, surface cracks of different sizes are introduced in the simulation models by releasing nodes on the symmetry plane. The cyclic temperature fields will open and close

the crack periodically. To model crack face contact, a rigid surface is included in the symmetry plane of the model against which the crack face is able to compress. The default “hard” contact properties of Abaqus with frictionless tangential behaviour are used.

The same quadratic structured mesh used to determine the uncracked stress distributions is used in the cracked models. The SIF over the load cycle for each cracked model is calculated using the pre-built Abaqus contour integral evaluation routine. Due to the dense structured mesh and linear material behaviour, the results are path independent after only a few contours. The average of contours 5-10 are used for the finite element SIF value. An example of the cracked mesh and nodes used in the contour integration routine is shown in Figure 3.

Fig. 3. The mesh used in the cracked simulations and the nodes used in the contour integral evaluation. The bottom surface is the symmetry plane where a rigid support is included to model crack face contact.

2.3 Weight function calculations

Two separate weight function solutions are employed to calculate the stress intensity factor; a surface crack weight function and a subsurface crack weight function.

For an extended surface flaw of length a in an finite width plate loaded by the stress distribution $\sigma(x)$, the mode I stress intensity factor is given by the integral [15,16]:

$$K_I = \int_0^a h_{surf}(x, a, W) \sigma(x) dx \quad (1)$$

where the weight function for the surface crack is defined as

$$h_{surf}(x, a, W) = \frac{1}{\sqrt{2\pi a}} \sum_1^5 F_i \left(\frac{a}{W} \right) \left(1 - \frac{x}{a} \right)^{i-3/2} \quad (2)$$

The constants F_i describing the effect of the finite width plate are given tabulated as a function of the relative crack length in several handbooks (e.g. [15,16]) and are not repeated here. The tabulated values and a polynomial fitting as a function of a/W are given in a recent paper by Wu et al. [17: Appendix A and Eqs. (10)]. This solution is essentially equivalent to the solution of Zheng et al. [18] included in API-579 [19], for example.

Based on the work by Fett [20], for a subsurface crack near the free surface in a half-space, the mode I stress intensity factor for the two crack tips is given by

$$K_I = \int_{-a}^a h_{sub}(x, a) \sigma(x) dx \quad (3)$$

where a is now the crack half-length and the coordinate x is relative to the crack midpoint. Separate weight functions are defined for the crack tips A and B, where the tip A is closer to the free surface:

$$h_{sub,A}(x, a) = \frac{1}{\sqrt{\pi a}} \sqrt{\frac{1-x/a}{1+x/a} \left[1 + 2(F_A - 1) \left(1 + \frac{x}{a} \right) - C_A \frac{x}{a} \left(1 + \frac{x}{a} \right) \right]} \quad (4a)$$

$$h_{sub,B}(x, a) = \frac{1}{\sqrt{\pi a}} \sqrt{\frac{1-x/a}{1+x/a} \left[1 + 2(F_B - 1) \left(1 - \frac{x}{a} \right) - C_B \frac{x}{a} \left(1 + \frac{x}{a} \right) \right]} \quad (4b)$$

where the coefficients F_A , F_B , C_A and C_B are functions of the crack proximity to the free surface $a/(d+a)$, where d is the distance between the tip A and the free surface. The equations for the coefficients are given by Fett [20: C6.4]. The equation is reported to be valid up to $a/d=2.33$.

2.4 Determination of the subsurface crack configuration

As shown below, the surface crack may partially close due to the compressive stresses acting near the free surface of the surface cracked plate. Adapting the idea presented by Beghini et al. [21,22], the remaining open section of the surface crack is modelled as a subsurface crack. The width of the closed section of the crack is determined via the stress intensity factor value calculated for the virtual crack tip located at the end of the closed zone. Beghini et al. [21,22] demonstrate that the SIF at the virtual crack tip is equal to zero, otherwise the virtual crack tip would be open or overlapping. This concept is visualized in Figure 4.

Fig. 4. The geometry of the subsurface crack configuration and the definition of the actual and virtual crack tips for the subsurface crack.

To determine the width of the crack mouth contact zone d for the studied time instant, an iterative algorithm was developed. Initially, the algorithm assumes a small value of the subsurface crack length $2a$ and calculates the stress intensity factor at the virtual crack tip (K_{IA} for point A of Figure 4) using Equations (3) and (4a) and the stresses acting on the open section of the crack. If $K_{IA} > 0$, the crack length is increased until $K_{IA} = 0$. The correct stress intensity factor for the actual crack tip, K_{IB} , is calculated using Equations (3) and (4b) once the contact zone width d is found at the end of the iterative process. The process is repeated for each studied time instant with unique stress distributions.

The crack opening displacement can be used to assess whether the surface crack will be closed at the crack mouth. It can be calculated using the weight function and stress intensity factor distribution. For the surface crack, the crack mouth opening displacement (CMOD) is obtained with [22]:

$$CMOD = \frac{1}{E/(1-\nu^2)} \int_{x=0}^a K_I(\alpha) h_{surf}(x, \alpha) d\alpha, \quad (5)$$

where a plane strain state is assumed. A negative CMOD value indicates that the crack mouth is closed and the surface crack weight function cannot accurately describe the crack tip conditions.

3. Results

This section presents the main results obtained in the heat transfer and stress analysis and SIF calculations. All variations of the loading parameters and crack lengths makes the total number of studied cases to exceed 1800. The results are discussed in more detail in Chapter 4.

3.1 Temperature and strain fields

As an example, the temperature and stress distributions at two time instants during the thermal cycle are shown in Figure 5. The overall attenuation trends in the temperature and stress ranges over the crack line are shown in Figure 6.

Fig. 5. The temperature (a) and stress distributions (b) at the surface maximum temperature (red lines) and surface minimum temperature (blue lines) time instants. Subfigure (c) is a close-up of (b). The results are taken from calculations with heat area width $R=2$ mm, $R=8$ mm and $R=16$ mm and heat frequency of $F=1$ Hz. The analytical solution for surface heated semi-infinite continuum is shown with dots.

Fig. 6. Depth locations of $\Delta T > 100\text{ }^{\circ}\text{C}$ (a) and $\Delta\sigma > 300\text{ MPa}$ (b) in the crack plane. The analytical solution for surface heated semi-infinite continuum is shown with dots.

3.2 Stress intensity factors

For each case listed in Table 1 and for each time instant of the finite element simulations, the stress intensity factors at the actual crack tip are calculated using the weight functions (Chapter 2.3) and numerically (Chapter 2.2). The iterative procedure described in Chapter 2.4 is used to determine the extent of crack face contact needed in the subsurface crack SIF evaluation. Thus, the SIF time histories over the thermal load cycle are obtained for each unique combination of the heat load extent and frequency and the crack length. In Figures 7-10, the maximum SIF values at the actual crack tip occurring over the thermal load cycle are plotted as a function of the surface crack length calculated with all three methods. The results calculated with $R=8\text{ mm}$ are omitted as they show essentially the same behaviour as the presented results.

● Finite element; - WF, surface crack; - WF, subsurface crack

Fig. 7. The maximum SIFs at the actual crack tip during the load cycle for cases with $R=2\text{ mm}$.

● Finite element; - WF, surface crack; - WF, subsurface crack

Fig. 8. The maximum SIFs at the actual crack tip during the load cycle for cases with $R=4\text{ mm}$.

● Finite element; - WF, surface crack; - WF, subsurface crack

Fig. 9. The maximum SIFs at the actual crack tip during the load cycle for cases with $R=12$ mm.

● Finite element; - WF, surface crack; - WF, subsurface crack

Fig. 10. The maximum SIFs at the actual crack tip during the load cycle for cases with $R=16$ mm.

The Figures 7-10 depict the maximum SIF during the load cycle. The FE calculated SIF is seen to agree with the surface or subsurface weight function solution, whichever is greater. The evolution of the stress intensity factor during the load cycle is shown in Figure 11 for cases with $R=12$ mm. Contrary to the preceding figures, the lines denote the finite element results and the markers denote the weight function predictions.

- Finite element, $a=5.2$ mm; - Finite element, $a=10$ mm; - Finite element, $a=20$ mm;
●, ●, ● WF, Surface crack; ●, ●, ● WF, subsurface crack

Fig. 11. The stress intensity factors at the actual crack tip during the thermal load cycle for cases with $R=12$ mm and $a=5.2$ mm, 10 mm and 20 mm. The horizontal axis τ is the relative time over the thermal load cycle.

The time instants when the crack mouth is closed and the crack behaves as a subsurface crack can be assessed based on the crack opening displacement calculated at the crack mouth using Equation 5. As an example, the evolution of the CMOD during the load cycle is shown in Figure 12 for cases with $R=12$ mm, calculated both using finite elements and the surface crack weight function. The negative dotted values in the figure denote crack mouth closure, as predicted by the weight function. For these time instants, the subsurface crack solution is invoked. The contact width d that is solved during the subsurface crack SIF solution is compared with the finite element results in Figure 13.

- Finite element, $a=5.2$ mm; - Finite element, $a=10$ mm; ●, ● WF, Surface crack

Fig. 12. The crack mouth opening displacement during the thermal load cycle for cases with $R=12$ mm and $a=5.2$ mm and 10 mm. The horizontal axis τ is the relative time over the thermal load cycle.

- Finite element, $a=5.2$ mm; - Finite element, $a=10$ mm; ●, ● WF, Subsurface crack

Fig. 13. Relative contact width d/a during the thermal load cycle for cases with $R=12$ mm and $a=5.2$ mm and 10 mm. The horizontal axis τ is the relative time over the thermal load cycle.

4. Result analysis

4.1 Temperature and strain fields

Figure 6 shows that the extent of the thermal load at the surface affects the range of the temperature cycles below the heated surface and, more pronouncedly, the stress ranges at and below the heated surface. The studied thermal load can be modelled as a sum of a semi-infinite continuum heated periodically. The steady-state temperature field caused by a periodic line heat source in a continuum is [22: Ch. 10.4]:

$$T(r, t) = \frac{1}{2\pi d} K_0(r\sqrt{i\omega/\kappa}) \exp(-i\omega t), \quad (6)$$

where K_0 is the modified Bessel function of the second kind, $r = \sqrt{x^2 + y^2}$ is the shortest distance from the observation point to the heat source line, ω is the angular frequency of the heat source, κ is the thermal diffusivity and t is time. The temperature field in the case of a finite heating area is obtained from the integral $\int_{-R}^R T dy$.

If the heating extends over the full free surface of the plate, the temperature field in the crack region would correspond to a semi-infinite medium heated periodically at the surface with a forced temperature of $A \cos(\omega t)$. This is a basic heat transfer problem with the steady-state analytical temperature solution readily available [22: Ch. 2.6]:

$$T(x, t) = A \exp(-x\sqrt{i\omega/\kappa}) \cos(i\omega t) \quad (7)$$

The above harmonic solutions show that the amplitude of the temperature fluctuations diminish as governed by the modified Bessel function of the second kind (K_0), in the case of the finite heat source, or exponentially, in the case of the infinite heat source, as a function of the distance from the heat source and even more rapidly if the load frequency ω is increased. The modified Bessel function of the second kind attenuates faster than the exponential function, which is consistent with the temperature range decays in Figure 5. Additionally, for small heating areas, the temperature fluctuations attenuate as a function of the coordinate perpendicular to the crack line, y in Equation (6), an effect that does not occur with infinite heating areas.

The rapid decay of the temperature fluctuations cause the temperature range to diminish to a small fraction at the wavelength depth, indicating that only the first wave just below the surface will generate significant thermal stresses. The thermal stress distributions are at their maxima at the heated surface. For each studied heating area size, there is a different heating frequency that produces the highest surface stress range. For small heating areas ($R < 10$ mm), the frequency producing the maximum surface stress range is above the upper limit of the studied frequency range, 5 Hz. For $R = 12$ mm and $R = 16$ mm, the thermal load with $F = 2$ Hz causes the maximum surface stress range of the studied frequencies. The thermal stress amplitudes at and below the heated surface are strongly dependent on the extent of the periodic heating. Under temperature control, the amount of energy input to the system is proportional to the heating area. Larger heating areas cause a stronger lateral constraint to the heating area centre point and a

greater portion of the thermal strains need to be compensated by mechanical strains. In case of edge restrained plate, the stress field perpendicular to the crack line in the depth direction is [24]:

$$\sigma(x,t) = \frac{E\alpha}{1-\nu}T(x,t), \quad (8)$$

where E and ν are the elastic modulus and Poisson ratio and α is the coefficient of thermal expansion. The temperature distribution of Equation 6 is compatible with this equation. This equation was used to plot the analytical temperature and stress solutions in Figures 5 and 6, where an exponential decay as function of loading frequency was observed.

The above noted trends in the temperature and stress fields explain the attenuation behaviour of the SIF results discussed next.

4.2 Stress intensity factor results

Figures 7-10 show that there are two distinct regions in the maximum SIFs during the load cycle for different crack lengths. Initially at smaller crack lengths, the maximum SIFs during the thermal load cycle are caused by the surface crack opening mode during the cooling phase and the weight function predictions (the red lines in the figures) agree with the finite element results (dots). There is a clear transition point seen in all maximum SIF result plots at the intersection of the red line (the surface crack SIFs) and the blue line (the subsurface crack SIFs). From this point on, the maximum SIFs during the cycle are caused by the subsurface crack opening mode instead of the surface opening mode. The surface opening mode still occurs with a positive SIF value during the thermal load cycle but the SIF caused by the subsurface mode is greater and the maximum finite element result SIFs begin to agree with the subsurface weight function predictions. The surface and subsurface crack opening modes are clearly seen in the SIFs over the thermal load cycle plotted in Figure 11. With smaller load frequencies and crack sizes (e.g. the red lines for $a=5.2$ mm in Figure 11a,b,c) the second peak in the curves, which is caused by the surface crack opening mode, is greater than the first peak caused by the subsurface crack opening mode.

The other mode not causing the maximum SIF during the thermal load cycle is still active. Figure 11 shows with separate marker types that the finite element results are first predicted by the subsurface crack geometry weight function (the square markers at $\tau < 0.5$) and then by the surface crack geometry weight function (the circles at $\tau > 0.5$).

Consistent with the intersection points seen in Figures 7-10, the subsurface mode becomes dominant at smaller and smaller crack lengths as the thermal load frequency increases. In particular cases, such as with $R = 12$ mm, $F = 1$ Hz and $a=5.2$ mm (shown with the red dots in Figure 11d), the surface and subsurface modes are nearly equal and the crack tip experiences two mechanical load cycles during one thermal load cycle. For this case, the crack opening displacements at the time points corresponding to the maximum surface and subsurface crack opening are drawn in Figure 14. The figure shows that, while there is a considerable difference in the crack opening profiles, the crack tip opening displacement and angle are nearly equal in both modes, consistent with the SIF results in Figure 11d for the same case.

Fig. 14. The crack opening displacement shapes (a) and crack opening stress distributions (b) for the case with $R=12$ mm, $F=1$ Hz at the time instants (τ , see Figure 11d) when the surface and subsurface mode SIFs are at their maxima. In (a), the lines are the external boundaries of the cracked FE model. The vertical displacements are scaled by a factor of 200 and the lower half is drawn symmetrically. The crack length (the distance from the free surface to the crack tip) is 5.2 mm. The corresponding crack opening stress distributions calculated with the uncracked model are shown in the same scale (b).

The subsurface mode SIF does not decay strongly with increasing crack lengths like the surface mode SIF does. This is because the subsurface mode SIF is caused by a stress distribution that is mostly tensile over the open portion of the crack. The increase in

the crack length counters the decrease of the stresses and thus the maximum SIFs remain nearly constant at larger crack lengths, as can be seen in Figures 7-10.

The inclusion of the crack face contact allows to capture the subsurface crack opening mode in the finite element computations. Excluding the contact causes that a negative SIF is calculated at time instants when the crack mouth is strongly compressed but tensile stresses act near the tip of the crack. Without contact, the maximum SIFs over the load cycle are always caused by the stress distributions that open the crack mouth. This behaviour was confirmed with finite element analyses without crack face contact. As shown in Figure 15, the cycle-wise SIF maxima at smaller crack lengths are identical to those calculated with crack face contact. As expected, the transition to subsurface mode dominated behaviour is not present and the finite element results and surface crack weight function results continue to diminish in unison, incorrectly.

Fig. 15. The maximum SIFs during the thermal load cycle calculated with and without crack face contact and using the weight functions for the case with $R=12$ mm and $F=1$ Hz.

While Figures 7-10 show that there is a clear transition point after which the subsurface mode becomes dominant for the maximum SIF, the subsurface mode SIF increases gradually from zero at small crack lengths. These values comprise the initial part of the subsurface crack SIF results shown with the blue lines in Figures 7-10. The slight oscillation in some of the blue lines is caused by the approximate numerical determination of the crack mouth contact length. There is a moderate error in the SIFs between the finite element results and the weight function predictions at higher load frequencies (most notable at Figure 7e and Figure 11f). This is due to the high stress gradient caused by the high frequency loads and additional weight function calculations using the stress distributions from a refined element mesh yielded a better agreement between the finite element and weight function SIFs.

Determined from the intersection points of the surface and subsurface crack weight function results in Figures 7-10, the crack length when the subsurface mode becomes dominant for the maximum SIF is shown in Figure 16 together with the cycle-wise maximum SIF at the transition crack length. The figure visualizes how the subsurface mode becomes dominant at smaller and smaller crack lengths as the load frequency increases. At larger heating areas, the transition point location appears to be less dependent on the heating frequency.

Fig. 16. The crack length values for points when the subsurface mode becomes dominant for the maximum SIF values over the thermal load cycle (a) and the corresponding cycle-wise maximum SIF values (b).

5. Discussion

We have argued that there is a clear possibility of surface crack face closure for larger surface cracks subjected to cyclic thermal loads. The subsurface crack opening mode becomes dominant for the maximum SIF value over the thermal load cycle and this behaviour is not captured using linear analysis methods such as finite elements calculations without crack face contact or surface crack weight functions. When the crack faces are not in contact, the normal surface crack weight function calculations agree with the finite element results. The SIFs including crack face contact can be solved using a subsurface crack weight function but the length of the crack face contact needs to be calculated first. We have used the criterion of zero SIF at the near-surface virtual crack

tip of the subsurface crack, as suggested by Beghini et al. [21,22], to determine the extent of crack face contact and the results agree with finite element calculations with crack face contact. The surface crack weight function solution by itself would be sufficient if the crack face contact pressures were known. In this case, the correct crack opening shape would be obtained as a superposition of the actual stress distribution caused by the external loads and the additional stress distribution correcting for the crack face contact. The difficulty with this approach is that the contact stress distribution is not directly deductible from the stress distribution from the external loads nor the linear crack opening displacement shape that would contain crack face overlapping. Beghini and Bertini [21] argue that the contact stresses are almost the crack line stress distribution negated, but not exactly. To maintain accuracy, an iterative solution procedure would be required to solve for the contact stresses and the solution to be sought would be more complex than simply solving for the location of the near-surface crack tip with the subsurface crack solution.

Our results show that the subsurface mode SIF exceeds the surface mode SIF at reasonably small crack lengths, from 1 mm to 12 mm, when the frequency of the cyclic load is in the range relevant to the thermal fatigue phenomena [13,25]. The larger extent of the surface thermal cycling increases the crack length after which the subsurface crack opening mode causes larger SIFs than the surface opening mode. For stress distributions that are both compressive and tensile over the crack face, the use of surface crack weight function alone is potentially unsafe as the SIF solution gives no indication if the crack faces overlap and the solution is not valid.

In the calculations, we have selected a temperature range which causes stresses that would exceed the yield limit for normal stainless steels. Because of the full linearity in the heat transfer and stress calculations, the SIF values are proportional to the applied temperature range. However, the crack face contact extent and the transition from the surface crack dominated behaviour to the subsurface crack dominated behaviour is not dependent on the magnitude of the applied load.

At shorter crack depths, the surface opening mode causes the maximum SIF over the load cycle and the maximum obtainable SIF as a function of crack length with the studied load cases is always caused by the surface mode. During the thermal load cycle, the subsurface mode exceeds the surface mode only at deeper crack lengths when the SIF magnitude has significantly attenuated, and the stress intensity factors due to the subsurface mode with deeper cracks are in general small compared to the overall maximum SIFs caused by the surface mode at smaller crack lengths. Therefore, the subsurface mode will not change the trend that the SIF will attenuate as the crack length increases. Stainless steels are known to exhibit a fatigue crack growth threshold and the subsurface SIFs may fall below this threshold and thus the cracks would be arrested before the effects of subsurface mode would be observed in the crack growth behaviour. However, the subsurface crack opening behaviour adds to the understanding of the loading imposed by, e.g., turbulent mixing and stratification. Such effects have been reported to contribute to the initiation and growth of through-wall cracks, together with the effect strong global loads [13].

6. Conclusions

- For cyclic thermal loads, the inclusion of crack face contact in the finite element computations allows to capture a subsurface crack opening behaviour that is more severe than the surface crack opening mode at larger crack lengths.
- Linear surface and subsurface weight functions can be used to predict this behaviour. First, the possibility of crack face closure needs to be determined using crack opening calculations and second, the extent of the crack face closure is determined from the subsurface crack weight function and the condition that there is a zero SIF at the border of the closed zone. This combination of two simpler analytical methods eliminates the need to perform complex simulations with contact elements to capture the crack closure behaviour.
- The crack length after which the subsurface crack becomes dominant for the maximum SIF depends of the extent and frequency of the thermal load.

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