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A MILP Model for Incorporating Reliability Indices in Distribution System Expansion Planning

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***Abstract*— Reliability cost is considered as an inevitable criterion in expansion planning studies of distribution systems. However, nonlinear expressions of reliability indices aggravate complexity of planning studies. To address this issue, this letter proposes a novel method to linearize mathematical model of the reliability-based distribution expansion planning problem. Using this variant of reliability indices, reliability costs can easily be involved in mixed-integer linear programming (MILP) model of distribution expansion planning. Validity of the derived expressions is tested by simulation results.**

***Index Terms*— Distribution network reliability, distribution system expansion planning, mixed-integer linear programming (MILP).**

I. Introduction

Reliability level of distribution network is a key factor in distribution systems expansion planning (DSEP) studies. This is due to the major contribution of distribution system failures to customer interruptions as well as the increased demand for continuity of supply, either by the customers or regulatory authorities. However, complexity of calculating reliability indices without knowing the exact network topology (since optimal topology is the objective of the DSEP problem) makes it difficult to incorporate this important factor into the standard mathematical models of DSEP. Therefore, many authors have resorted to solve this issue employing metaheuristic methods in which the network topology is known in each iteration. Another solution is proposed in [1] and further employed in [2], where a pool of low-cost expansion plans is obtained from solving a mixed-integer linear programming (MILP) model without considering reliability metrics. Subsequently, reliability indices and interruption cost are calculated for each of these plans to determine the most convenient expansion plan. However, none of these works can guarantee optimality of the obtained solution.

A groundbreaking method for incorporating reliability costs into the standard mathematical programming models of DSEP has been recently published in [3]. However, the proposed model is case-dependent, so that it can only be applied to traditional distribution network switch arrangement in which there is a non-reclose circuit breaker at the beginning of each feeder and all feeder sections are also equipped with disconnects or isolators. Moreover, this method excessively increases dimension (i.e. number of decision variables) of the mathematical problemwhich in turn negatively affects the MILP solver efficiency.

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Motivated by the aforementioned points, this letter intends to propose an efficient method to incorporate reliability indices and the associated costs into the MILP model of DSEP. Using this method, optimal solution of the reliability-based DSEP can be efficiently found in a single optimization step.

II. Problem Description

DSEP study generally aims to determine the optimal sets of newly added feeders and substations as well as reinforcement of the existing ones in each year to efficiently (with cost and reliability considerations) meet the forecasted electricity demand. Equation (1) expresses object function of the presented DSEP problem, which should be minimized.

$$OF^{EDP} = \sum_{t=1}^{T} \left(\mathrm{PVF}_{\mathrm{t,Inv}} Inv_t + \mathrm{PVF}_{\mathrm{t,Op}} (Op_t + IC_t) \right) \tag{1}$$

where, Inv_t and Op_t are investment and operating costs of adding new feeders and substations or reinforcing the existing assets. IC_t stands for interruption cost and is a function of reliability indices. Furthermore, $\mathrm{PVF}_{\mathrm{t,Inv}}$ and $\mathrm{PVF}_{\mathrm{t,Op}}$ are present value factors for the investment and operating costs. As in [2], a perpetual or infinite planning horizon is assumed for investment and operating costs, i.e. each asset will be replaced by the same one after reaching the end of its lifetime, and the operating and interruption costs of year T, will be repeated in the following years.

This problem is generally a mixed-integer nonlinear programming (MINLP). In this letter, the MILP approximate model which has been proposed in [4], and further used successfully in recent publications [1], [2], [5] is employed. Note that all of these studies have ignored the effects of interruption costs in the MILP model. This is due to the fact that involving reliability indices as the basic part of interruption cost changes this MILP model to a nonlinear one. Detailed discussion about the MILP model is out of the scope of this letter and can be found in the aforementioned papers. In the next section, linearized model of reliability indices that can be added to this MILP model in order to obtain the reliability indices will be derived and explained.

III. Proposed Methodology

The proposed method to obtain the most common distribution network reliability indices, i.e. EENS, SAIFI, and SAIDI [1], [6] are presented in the following. It is worth noting that in all the equations, the italic letters represent variables, while non-italic ones are associated with parameters.

A. Expected Energy not Supplied (EENS)

This index is generally expressed as follows [6]:

$$EENS^t = \sum_{i=1}^{\mathrm{N_{lp}}} \sum_{ll=1}^{\mathrm{N_{ll}}} \nu_i \delta_i \frac{\mathrm{Dur_{ll}}}{8760} \mathrm{P}_{\mathrm{i,ll}}^{\mathrm{t}} \tag{2}$$

where $\mathrm{N_{lp}}$, $\mathrm{N_{ll}}$ are number of load points and load levels, ν_i, δ_i are average number and duration of yearly interruptions of load point i, Dur_{ll} is duration of load level ll, and $\text{P}^{\text{t}}_{\text{i,ll}}$ is power demand at load point i. The most important issue regarding the EENS calculation by (2), is the calculation of v_i, and δ_i, since both of these variables are functions of network topology. However, as mentioned before, network topology is output of the optimization problem, and is unknown during the optimization process. In order to address this issue, we can take the advantage of radial operation of distribution networks. It is further assumed that each line has a disconnecting device (e.g. fuse, circuit breaker, or sectionalizer) at the power supply side which can isolate the downstream network in the case of failures. Moreover, as in [3], in order to make the problem tractable, the operation of normally-open backup switches (i.e. tie switches) is neglected. Hence, one can conclude that the failure of a given line results in the interruption of downstream customer demand which is equal to the power flow through that line. Therefore, equation (2) can be replaced by (3):

$$EENS^t = \sum_{j=1}^{\text{N}_{\text{br}}} \sum_{ll=1}^{\text{N}_{\text{ll}}} \lambda_{\text{j}} \text{r}_{\text{j}} \frac{\text{Dur}_{\text{ll}}}{8760} f^t_{j,ll} \tag{3}$$

where, N_{br} is number of network branches, λ_{j}, r_{j} are failure rate and repair time of line j, respectively, and $f^t_{j,ll}$ is power flow through branch j.

However, in planning problem, various candidate alternatives are available for each branch and also network topology, i.e. the existence as well as power flow direction of each branch, is unknown during the solving process. Hence, reliability parameters (i.e. failure rate and/or repair time) of each line depends on the chosen alternative. As an example, considering two various alternatives of overhead line and underground cable for construction of a given branch, one can say that the former has higher failure rate and lower repair time. Moreover, since the direction of flow through branches depends on the network topology, $f^t_{j,\,ll}$ can take negative values. Hence, equation (3) should be written in the general form using binary variables and absolute value function as follows:

$$EENS^t = \sum_{j=1}^{\text{N}_{\text{br}}} \sum_{k=1}^{\text{N}_{\text{alt,j}}} \sum_{ll=1}^{\text{N}_{\text{ll}}} \lambda_{\text{j,k}} \text{r}_{\text{j,k}} y^t_{j,k} \frac{\text{Dur}_{\text{ll}}}{8760} \left| f^t_{j,ll} \right| \tag{4}$$

where, $\text{N}_{\text{alt,j}}$ is number of candidate alternatives for branch j, and $y^t_{j,k}$ is a binary variable which is unity if k^{th} alternative of line j is in service in year t. As can be seen, this equation is nonlinear, owing to the absolute value calculation and the product of $y^t_{j,\,k}$ and $f^t_{j,\,ll}$. To address this issue, the nodal power balance equations of the MILP model introduced in [4] should be replaced by the following equations:

$$\sum_{j\in\Psi^m} \sum_{k=1}^{\text{N}_{\text{alt,j}}} \chi_{\text{m,j}} (f'^t_{j,k,ll} - \hat{f}'^t_{j,k,ll}) + \text{D}^{\text{t}}_{\text{m,ll}} = 0 \quad , \ \forall \text{m} \in \Omega^n \tag{5.a}$$

$$\sum_{j\in\Psi^m} \sum_{k=1}^{\text{N}_{\text{alt,j}}} \chi_{\text{m,j}} (f'^t_{j,k,ll} - \hat{f}'^t_{j,k,ll}) - G^t_{m,ll} = 0 \quad , \ \forall \text{m} \in \Omega^{sn} \tag{5.b}$$

$$f'^t_{j,k,ll}, \hat{f}'^t_{j,k,ll} \le y^t_{j,k} \text{f}^{\max}_{\text{j,k}} \tag{5.c}$$

$$\frac{1}{\text{M}} \sum_{k=1}^{\text{N}_{\text{alt,j}}} f'^t_{j,k,ll} \le \sigma'^t_j \le \text{M} \sum_{k=1}^{\text{N}_{\text{alt,j}}} f'^t_{j,k,ll}, \ \text{M is a big number} \tag{5.d}$$

$$\frac{1}{\text{M}} \sum_{k=1}^{\text{N}_{\text{alt,j}}} \hat{f}'^t_{j,k,ll} \le \hat{\sigma}'^t_j \le \text{M} \sum_{k=1}^{\text{N}_{\text{alt,j}}} \hat{f}'^t_{j,k,ll} \tag{5.e}$$

$$\sigma'^t_j + \hat{\sigma}'^t_j \le 1 \tag{5.f}$$

where, Ψ^m is set of branches connected to node m, Ω^n and Ω^{sn} are sets of demand and substation nodes, and $\chi_{\text{m,j}}$ is element of node-branch incidence matrix of the network, which is -1 or +1 if branch j is connected to node m and the predetermined current or flow direction is toward or away from node m, respectively, and is 0, otherwise. Furthermore, $\text{D}^{\text{t}}_{\text{m,ll}}$ is power demand, $G^t_{m,ll}$ is a positive variable denoting power injection at substation nodes, $f'^t_{j,k,ll}, \hat{f}'^t_{j,k,ll}$ are positive variables associated with power flows through k^{th} alternative of j^{th} branch at load level ll, in predetermined direction and the opposite one, and $\text{f}^{\max}_{\text{j,k}}$ is the maximum allowable flow. Using binary variables σ'^t_j, $\hat{\sigma}'^t_j$, equations (5.d)-(5.f) denote that only one of the two positive variables $f'^t_{j,k,ll}$, $\hat{f}'^t_{j,k,ll}$ can take non-zero value at a time. Hence, by the use of linear constraints (5), the nonlinearity of product of $y^t_{j,\,k}$ is addressed through the introduction of $f'^t_{j,k,ll}, \hat{f}'^t_{j,k,ll}$ variables. Furthermore, considering (5), the absolute value of power flows can be expressed as the sum of $f'^t_{j,k,ll}$, and $\hat{f}'^t_{j,k,ll}$. Hence, equation (4) can be linearized as (6). Note that in case the optimization algorithm minimizes the EENS value, it automatically sets one of the $f'^t_{j,k,ll}, \hat{f}'^t_{j,k,ll}$ to zero, so that (5.c)-(5.d) can be eliminated.

$$EENS^t = \sum_{j=1}^{N_{br}} \sum_{k=1}^{N_{alt,j}} \sum_{ll=1}^{N_{ll}} \lambda_{\text{j,k}} \text{r}_{\text{j,k}} \frac{\text{Dur}_{\text{ll}}}{8760} (f'^t_{j,k,ll} + \hat{f}'^t_{j,k,ll}) \tag{6}$$

B. System Average Interruption Frequency Index (SAIFI)

This index is generally calculated as below [6]:

$$SAIFI^t = \sum_{i=1}^{\text{N}_{\text{lp}}} v_i \text{N}^{\text{t}}_{\text{i}} \Big/ \sum_{i=1}^{\text{N}_{\text{lp}}} \text{N}^{\text{t}}_{\text{i}} \tag{7}$$

where $\text{N}^{\text{t}}_{\text{i}}$ is number of customers at load point i. Similar to explanations about (2), in order to tackle the issue regarding calculation of v_i, this equation can be rewritten as (8):

$$SAIFI^t = \sum_{j=1}^{\text{N}_{\text{br}}} \sum_{k=1}^{\text{N}_{\text{alt,j}}} \lambda_{\text{j,k}} n^t_{j,k} \Big/ \sum_{i=1}^{\text{N}_{\text{lp}}} \text{N}^{\text{t}}_{\text{i}} \tag{8}$$

where $n^t_{j,k}$ is number of customers affected by the outage of branch j if the k^{th} alternative has been chosen. Again, since the network topology is unidentified during the problem optimization, some auxiliary variables must be introduced to the model for calculation of $n^t_{j,k}$. In this respect, inspired by the nodal power balance equations, i.e. (5.a)-(5.f), the following equations can be derived (In the case that optimization problem minimizes SAIFI value, (9.e) can be eliminated):

$$n^t_{j,k} = n'^t_{j,k} + \hat{n}'^t_{j,k} \tag{9.a}$$

$$\sum_{j\in\Psi^m} \sum_{k=1}^{\text{N}_{\text{alt,j}}} \chi_{\text{m,j}} (n'^t_{j,k} - \hat{n}'^t_{j,k}) + \text{N}^{\text{t}}_{\text{m}} = 0 \quad , \ \forall \text{m} \in \Omega^n \tag{9.b}$$

$$\sum_{j\in\Psi^m} \sum_{k=1}^{\text{N}_{\text{alt,j}}} \chi_{\text{m,j}} (n'^t_{j,k} - \hat{n}'^t_{j,k}) - ng^t_m = 0 \quad , \ \forall \text{m} \in \Omega^{sn} \tag{9.c}$$

$$n'^t_{j,k}, \ \hat{n}'^t_{j,k} \le \text{M} y^t_{j,k}, \ \text{M is a big number} \tag{9.d}$$

$$n'^t_{j,k} \le \text{M} \sigma'^t_j, \ \hat{n}'^t_{j,k} \le \text{M} \hat{\sigma}'^t_j \tag{9.e}$$

where, $n'^t_{j,k}$, $\hat{n}'^t_{j,k}$, are the number of customers affected by outage of line j, when power flow is respectively in the predetermined direction and the opposite one. Moreover, $\text{N}^{\text{t}}_{\text{m}}$ is number of customers at load point m, and ng^t_m is a positive variable indicating total number of customers served by the substation located at node m. For better understanding of these equations, a numerical example on a small network is presented in Fig. 1. The distribution network in the base case, Fig.1 (a), is comprised of a substation node (Point 1), three load points (Points 2-4), and five candidate feeders for construction. The arrows in this figure demonstrate the predetermined direction for candidate lines. Hence, the node-branch incidence matrix is:

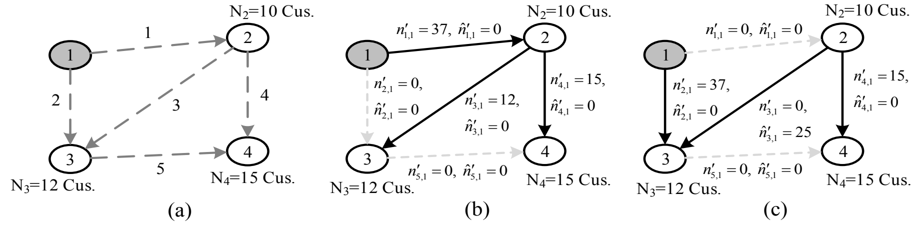

Fig. 1. An Illustrative example of the proposed method for calculating the number of customers supplied through each line.

$$X=[\chi_{m,j}]=\begin{bmatrix}1 & 1 & 0 & 0 & 0\\ -1 & 0 & 1 & 1 & 0\\ 0 & -1 & -1 & 0 & 1\\ 0 & 0 & 0 & -1 & -1\end{bmatrix}$$

For simplicity, only one alternative is considered for construction of each branch, i.e. $N_{alt,j}=1$. Now, assuming a candidate solution in which feeders 1, 3, 4 are in service, i.e. $y_{1,1}=y_{3,1}=y_{4,1}=1$ and $y_{2,1}=y_{5,1}=0$, the results illustrated in Fig. 1 (b) are achieved. In fact, the inequality constraint (9.d), forces $n'^{t}_{j,k}$, $\hat{n}'^{t}_{j,k}$, associated with lines 2, 5 to zero. Subsequently, equality constraint (9.b) together with (9.e) at the three load points give the values of $n'^{t}_{j,k}$, $\hat{n}'^{t}_{j,k}$, variables for the other lines. As shown in Fig. 1 (b), since the predetermined direction of all in-service branches are in the direction of the paths from source node 1 to the load nodes, all $\hat{n}'^{t}_{j,k}$ are equal to zero. Finally, as depicted in Fig. 1 (b), the $n^{t}_{j,k}=n''_{j,k}+\hat{n}'^{t}_{j,k}$ value indicates the number of customers supplied through line j.

As another example, assuming the topology illustrated in Fig. 1 (c), the predetermined direction for branch 3 is not consistent with the flow direction. Hence, according to (9.b), and (9.e), $n'^{t}_{3,1}$ would be zero, and $\hat{n}'^{t}_{3,1}$, becomes 25. It is worth noting that based on the equality constraint (9.c), in both cases the value of ng^{t}_{1} is 37, since all customers are supplied by this substation.

C. System Average Interruption Duration Index (SAIDI)

Considering the constraints derived in (9), SAIDI can be readily calculated as below:

$$SAIDI^{t}=\sum_{j=1}^{N_{br}}\sum_{k=1}^{N_{alt,j}}\lambda_{j,k}r_{j,k}n^{t}_{j,k}\Big/\sum_{i=1}^{N_{lp}}N^{t}_{i} \tag{10}$$

IV. Illustrative Example

The proposed method is implemented on modified 18-bus test distribution grid with a planning horizon of three years [4]. In order to better demonstrate applicability of the proposed formulation, four different Cases are defined. While *IC* is neglected in the objective function for Case I, it is included in the other cases as the product of EENS and value of lost load (4000 \$/MWh). Moreover, in cases III and IV, it is assumed that the company is obliged to maintain the SAIDI index below 15 hours per customer per year. It is further assumed in case IV that the SAIFI index must be lower than 7 interruptions per customer per year.

As can be traced in Table I, total investment and operation costs increase as the company tries to achieve a more reliable network. Moreover, Fig. 2 illustrates various cost terms versus 3-year average EENS of investigated Cases. As shown, cost of reliability improvement increases more rapidly as the network reliability enhances (i.e. EENS decreases). This is due to the fact that cost of reliability improvement for a reliable network is higher than that of a network with lower reliability level [6].

Table I: Reliability Indices and Cost Terms in Different Cases (M$)

Case	Year	I	II	III	IV
EENS (MWh)	t=1	46.876	42.412	40.039	40.041
	t=2	76.388	76.388	68.961	64.671
	t=3	79.182	79.182	72.545	66.594
SAIDI (h/Cust./Year)	t=1	17.410	15.846	14.925	14.995
	t=2	16.570	16.570	14.883	13.930
	t=3	16.409	16.409	14.995	13.693
SAIFI (Int./Cust./Year)	t=1	8.095	7.269	6.952	6.844
	t=2	8.355	8.355	7.615	6.971
	t=3	8.362	8.362	7.803	6.976
Inv+Op		17.817	17.826	20.725	23.923
IC		3.345	3.327	3.049	2.817
Total Cost		21.162	21.152	23.774	26.740

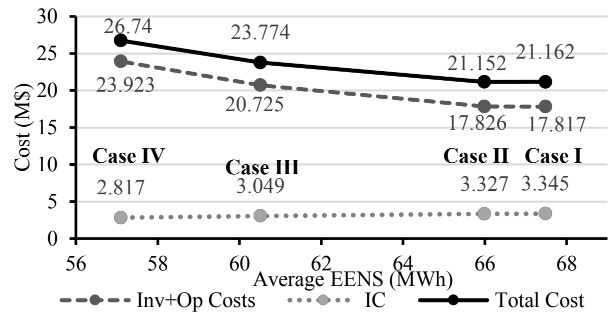

Fig. 2. Various cost terms versus average EENS in different Cases.

V. Concluding Remarks

This letter proposes linearized yet effective variant of reliability indices to be used in DSEP studies. To achieve this goal, mathematical model of the most well-known systematic reliability indices, i.e. EENS, SAIFI and SAIDI are presented and it is shown how defining some new variables can help us in reaching linearized formulations for these indices. This provides capability to involve reliability costs in mathematical model of DSEP studies. Some applications of the proposed method are investigated through various case studies. As a future work, we are going to develop a new planning model for considering the impacts of distributed generations as well as storage devices on the distribution system reliability measures.

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