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Numerical simulation of the THz lasing in the cavity with graphene-based hyperbolic medium

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ABSTRACT

THz wave emission in the cavity which contains the graphene-multilayer asymmetric hyperbolic metamaterials is investigated numerically using transfer matrix method. It was assumed that the gain saturation appears as the decrease of imaginary part of effective dielectric permittivity. Gain–loss balance predicts the intensity of THz lasing.

Keywords: hyperbolic metamaterials, terahertz emission, transfer matrices, eigenwaves, saturation

1. INTRODUCTION

Terahertz (THz) frequencies have suggested a wide range of highly prospective applications from harmless security systems and noninvasive medical treatment to time-domain spectroscopy [1]. In spite of recent advances in the investigations in THz region the creating of compact and effective source of a coherent radiation at THz frequencies remains a problem. There are some ways to develop THz sources, from quantum cascade laser structures [2] and semiconductor multi-well lattice structures [3] to plasma wave instabilities in semiconductor heterostructures [4]. The other route for development generators in THz frequencies is employing metamaterials, new structures with preset properties. One of the promising types of the metamaterials is hyperbolic metamaterial (HMM).

It is known that the hyperbolic medium exhibits hyperbolic-type dispersion in space of wave-vectors and has the diagonal extremely anisotropic permittivity tensor [5]. The dispersive properties of the hyperbolic metamaterials are inherent to uniaxial materials whose axial and tangential permittivity components are of different signs [6]. Recently, it has been proposed that indefinite permittivity can even be observed in anisotropic natural materials as, for example, graphite [7]. From the other hand, graphene is one of the challenging materials to observe THz waves amplification.

As was shown [8], at sufficiently strong optical pumping the real part of dynamic conductivity of graphene becomes negative in the terahertz (THz) range due to the interband population inversion that results in amplification of the THz plasmons. The hyperbolic properties (at THz region) of multilayer periodic structure composed of graphene layers were present in [9].

Earlier we have shown the existence of amplification of the THz waves in periodically arranged graphene and semiconductor layers, titled with respect to outer boundary. However, gain saturation was not taken into account. Here we continue to explore asymmetrical hyperbolic metamaterial (AHMM) which have been proposed in [10] due to its promising properties. In this work we investigate based characteristic of electromagnetic waves propagation inside complex cavity contains asymmetrical hyperbolic medium (AHM) at THz frequencies, accounting the saturation of the gain.

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2. METHOD AND RESULTS

We have investigated electromagnetic waves propagation in the complex cavity which contains asymmetrical hyperbolic medium (AHM), see Fig.1. The total length of the ring cavity is L , AHM has thickness h . The optical axis of the AHM is perpendicular to graphene sheets shown by red lines. Incident wave vector is $k=(k_x,0,k_z)$ in the coordinate system connected with the AHM. x -component of the incidence wave vector $k_x=K\sin\alpha$, $k_z=K\cos\alpha$. AHM consists of periodically arranged layers in a host media, tilted relatively to outer boundary. The orientation of optical axis is determined by Euler angles (θ, φ, ψ) according to the AHM interface plane [10]. When $\varphi=\pi/2$ then optical axis belongs to xz plane and ψ can be 0.

The effective medium model (Maxwell–Garnett homogenization) is appropriate for analyzing the periodic graphene lattice. The transverse permittivity of the AHM with saturation is given as follow [11,12,13]

$$\varepsilon_{\perp} = \varepsilon_{\parallel} + \frac{i}{d\omega\varepsilon_0} \left[\frac{\sigma'(\omega)}{1+S} + i\sigma''(\omega) \right], \quad (1)$$

where $\sigma(\omega)=\sigma_{\text{intra}}+\sigma_{\text{inter}}$ is the dynamic conductivity of graphene and d is the period of structure, $\varepsilon_{\parallel}=\varepsilon_h$ is the permittivity of the host medium, S is the dimensionless field intensity, prime and double prime denotes the real and imaginary parts of conductivity. We assume that the saturation occurs similarly to laser media. Possible presence of line enhancement factor [14] may be easily included into (1).

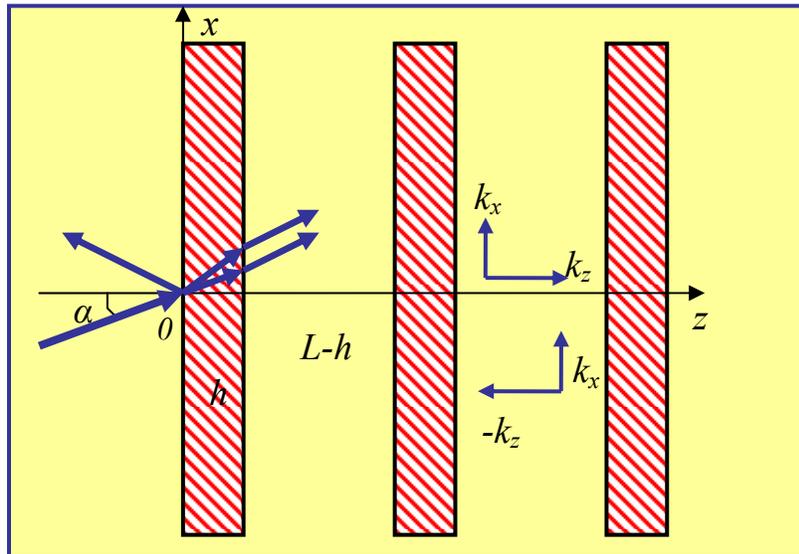


Figure 1. Structure under investigation. A cavity partially filled with AHM slab (rectangle, tilted planes symbolized graphene sheets) is modeled as an infinite number of AHM slabs, periodically placed in the isotropic lossy medium. Cavity modes are eigenwaves of this structure. Wave vectors of eigenwaves are shown schematically by dark blue arrows.

The field in the cavity shown in Fig.1 is supposed to be transverse plane wave with wavenumber K in the lossy parts of the cavity. To avoid the use of boundary conditions we simulate mirrors transmission by inner losses in the cavity. For z -dependence of the field inside AHM we used the solution of Maxwell equation based on the Berreman 4x4 matrix \mathcal{A} which is convenient for the investigation of the propagation of polarized light in anisotropic media [18, 19]. The components of the electric field E and the magnetic field H in the plane of the slab can be written as $\Psi \exp(ikr-i\omega t)$, where Ψ is a column vector and where the angular frequency $\omega=cK=2\pi c/\lambda$, $K=\omega/c=2\pi/\lambda$ is the wavenumber; $r=(x,y,z)$, $k=(k_x,k_y,k_z)$. The column vector Ψ satisfies the equation:

$$\frac{\partial}{\partial z} \Psi = \frac{i\omega}{c} \Delta \Psi \quad (2)$$

Berremian 4x4 matrix Δ includes the matrix elements Δ_{ij} which generally determined by main components of the dielectric tensor $\{\varepsilon_{\perp}, \varepsilon_L, \varepsilon_{\parallel}\}$, the angles θ, φ, ψ , which describes the orientation of optical axis, according to the interface plane, $\Psi = \{E_x, H_y, E_y, -H_x\}$ [10,18,19]. If the medium have the losses (or gain) the components ε_{\perp} and ε_{\parallel} are complex. Electromagnetic fields at the left interface plane of the AHM slab with the thickness h and at the right interface plane are related as follows

$$\Psi(R) = P(h)\Psi(L) \quad (3)$$

where $P(h)$ is the propagation matrix for layer with thickness h [18], Ψ_L and Ψ_R are column vectors of the left and right waves.

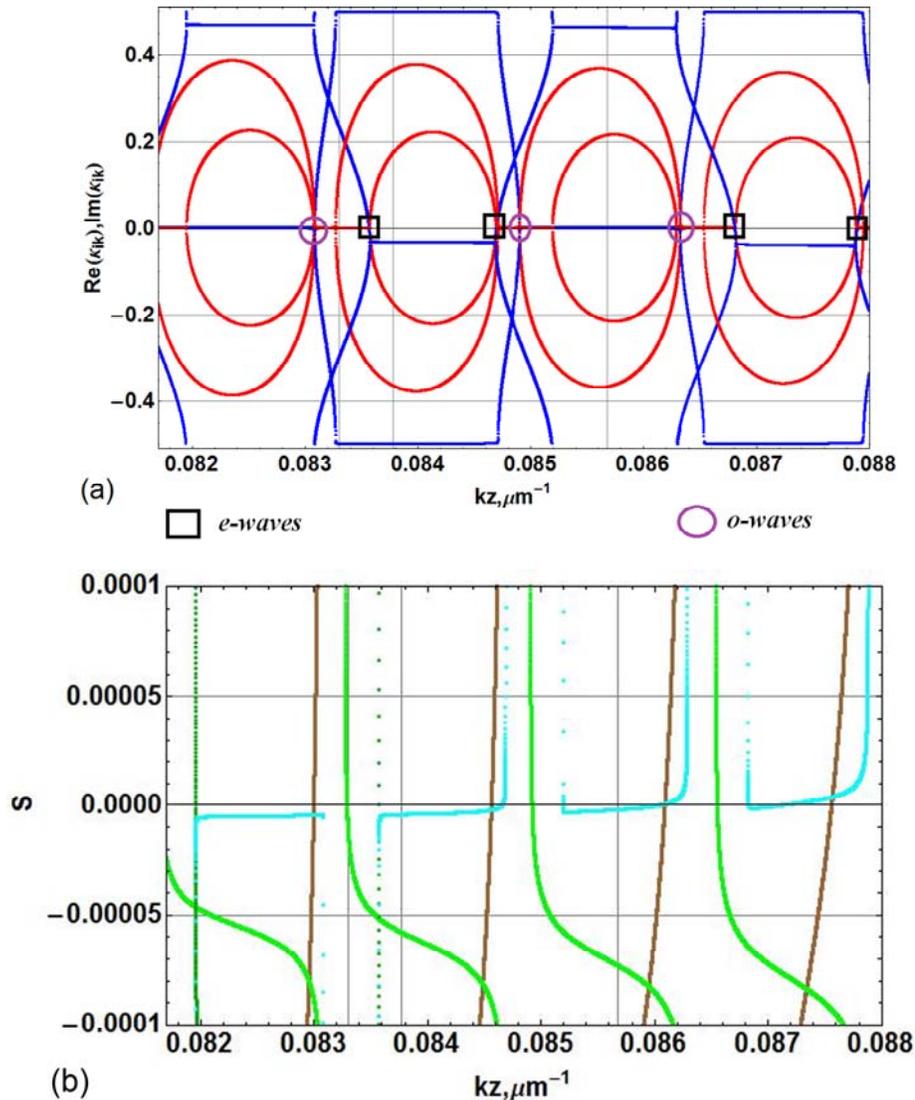


Figure.2. (a) Real and imaginary parts of eigenvalues vs k_z ; ordinary $\kappa_{1,2}$ and extraordinary $\kappa_{3,4}$ waves. (b) z-component of Pointing vector. $l_1=600\mu\text{m}$, $l_2=1320\mu\text{m}$, $h=5\mu\text{m}$. $\varphi=\pi/2$, $\theta=55^\circ$, $\alpha=15^\circ$.

Using this approach we can calculate four eigenvalues of the matrix Δ which correspond to the longitudinal (z) components of the wave vector of eigenwaves in the hyperbolic medium. Two of them correspond to forward and backward ordinary waves and two others to forward and backward extraordinary waves [20].

The eigenwaves of the structure in which there are periodically repeated blocks (free space gap+AHM slab) can be calculated similarly to the eigenwaves in 1D active photonic crystals [15]. If $P_0(L)$ is the matrix describing the propagation in air gap with the length $(L-h)$, then total transfer matrix of the structure is $P_t=P_0(L-h)P(h)$. Due to small thickness of hyperbolic medium we can put $S \approx \text{const}(z)$ in (1).

Due to linearity of the eigenproblem at given S , the eigenvalues $A_i = \exp(i\kappa_i L)$ of the total round-trip transfer matrix P_t determine the gain and phase delay of the corresponded eigenwaves at one pass. Thus $\text{Re}(\kappa_i) = 2\pi m$, $m=0, \pm 1, \pm 2, \dots$ determines the eigenwaves. Four eigenvalues κ_i correspond to left/right propagated two extraordinary (e) and two ordinary (o) plane waves. At given incidence angle α these eigenvalues are dependent from longitudinal wavenumber k_z , and the condition $\text{Arg}[\kappa_i] = 2\pi m$, m is integer, gives the oscillation frequency of the laser [20]. The cavity losses were simulated via complex permittivity of the host medium part of the cavity. Using the eigenwaves we have calculated time-averaged z -components of Pointing vector P_z which is dependent on z due to gain/losses. Only waves with $|\kappa_i| > 1$, $P_z > 0$ or $|\kappa_i| < 1$, $P_z < 0$ are experienced net gain. We have found the regimes with all, 3, 2, or only one wave may contribute to laser oscillations. However as a rule only one eigenmode exists having definite k_z , other modes have its own k_z and corresponding frequencies. For sufficiently long cavities its frequencies are within the gain frequency region and these modes can compete for the gain. Mode competition depends from mode overlapping on the gain section of the cavity. In present study we treat only single mode regime.

For the calculation of graphene conductivity we have used Kubo formula [16] with parameters values $E_f = 25 \text{ meV}$, $\tau = 10^{-12} \text{ s}$, $T = 300 \text{ K}$. For these parameters gain occurs in the range 3...5 THz. Host medium was assumed SiC, and its permittivity was calculated using [17].

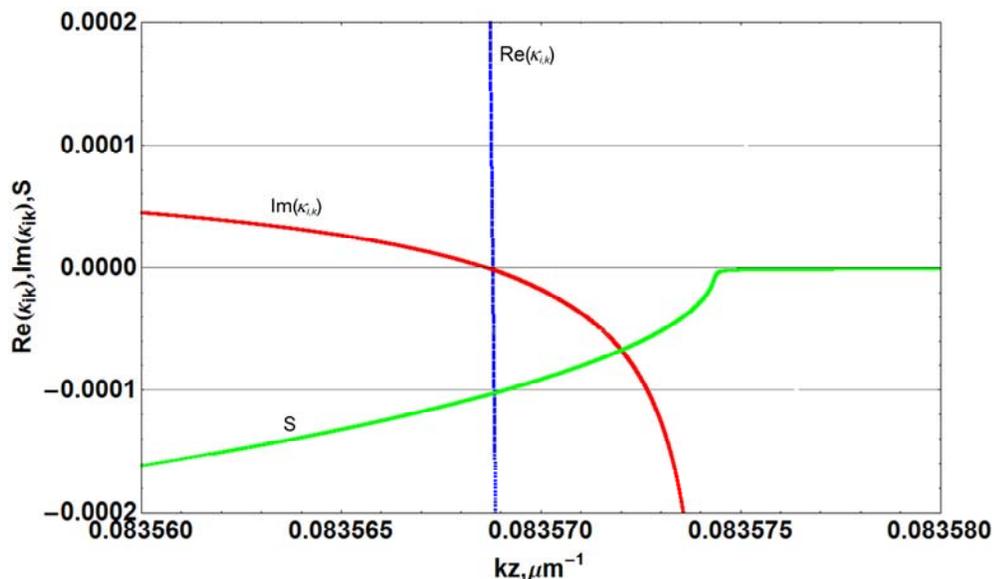


Figure.3. Real (blue curve) and imaginary (red curve) parts of eigenvalues of the extraordinary wave vs k_z and Pointing vector for this wave (green curve) for one of extraordinary wave. $l_1=600\mu\text{m}$, $l_2=1320\mu\text{m}$, $h=5\mu\text{m}$. $\varphi=\pi/2$, $\theta=55^\circ$, $\alpha=15^\circ$.

We calculated eigenvalues of the matrix P_t at given S and plotted them vs k_z . The eigenvalues of the ordinary $\kappa_{1,2}$ and extraordinary $\kappa_{3,4}$ waves vs k_z are present on Fig.2(a). z -components of Pointing vectors for these eigenvalues are shown on Fig.2(b). The intersection of dispersion curves for real parts of κ_i in the spectral range considered are shown by circles (ordinary waves) and rectangles (extraordinary waves). Starting with $S=0$, and calculating the eigenvalues we find one of the modes with k_z , which is determined by the equation $\text{Re}[\kappa_i(k_z)]=0$. Variation of S leads to small variation in

mode position k_z , and to more pronounced variation in $Im[\kappa_i(k_z)]$, which determines net gain. When saturated gain is equal to losses, then net gain becomes 0. Thus we should change S till $Im[\kappa_i(k_{z0}, S)] = 0$. The result is shown in Fig.3. Mode frequency and mode intensity which is proportional to S should be the solution of the equations:

$$\operatorname{Re}[\kappa_i(k_z, S)] = 0, \quad \operatorname{Im}[\kappa_i(k_z, S)] = 0 \quad (4)$$

These equations may be solved numerically giving k_{z0} and S_0 . Due to small variation of k_{z0} from S_0 it is convenient to find k_{z0} from first equation (4) and S_0 from second equation (4).

In this work the theory of single mode THz wave lasing in the cavity with asymmetric hyperbolic active media was presented. Model of saturation was similar to laser media and equations for field were the same as for 1D active photonic crystals. The frequency of oscillation and field intensity can be calculated from the solution of equations for real and imaginary parts of log of eigenvalues of total transfer matrix of one period of the structure, which models the cavity with asymmetric hyperbolic medium.

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