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Solid-liquid circulating fluidized bed: a way forward

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Abstract: Solid-liquid circulating fluidized beds (SLCFBs) offer several attractive features over conventional solid liquid fluidized beds such as efficient liquid-solid contact, favorable mass and heat transfer, reduced back-mixing of phases, and integrated reactor and regenerator design. These unique features have stimulated theoretical and experimental investigations over the past two decades on transport phenomena in SLCFBs. However, there is a need to compile and analyze the published information with a coherent theme to design and develop SLCFB with sufficient degree of confidence for commercial application. Therefore, the present work reviews and analyzes the literature on hydrodynamic, mixing, heat transfer, and mass transfer characteristics of SLCFBs comprehensively. Suitable recommendations have also been made for future work in concise manner based on the knowledge gaps identified in the literature. Furthermore, a novel multistage SLCFB has been proposed to overcome the limitations of existing SLCFBs. The proposed model of SLCFB primarily consists of a single multistage column which is divided into two sections wherein both the steps of utilization *viz.* loading (adsorption, catalytic reaction, etc.) and regeneration of solid phase could be carried out simultaneously on a continuous mode.

Keywords: heat transfer; hydrodynamics; mass transfer; mathematical modeling; solid-liquid circulating fluidized bed.

1 Introduction

Fixed bed mode of solid-liquid contact essentially possesses three major constraints: (i) batch-wise overall operation, (ii) high pressure drop, and (iii) utilization of bed is restricted to the small fraction of total bed height, called active zone. These constraints make overall operation time and energy intensive. Moreover, if feed containing suspended solid is charged into the fixed bed, it may clog the space between the solid particles and the whole operation becomes difficult due to loading, flooding, and channeling. Expanded bed mode of operation overcomes the constraints of fixed bed related to high pressure drop and clogging of bed if feed contains suspended solid. However, it is necessary to increase the fraction of total bed height as an active zone and throughput.

Solid-liquid circulating fluidized bed (SLCFB) overcomes the constraints associated with fixed bed and expanded bed. SLCFB primarily consists of riser column, main column, solid-liquid separator, and other auxiliary devices. The riser is operated at superficial liquid velocity higher than terminal settling velocity of solid particle, and main column is operated at low superficial liquid velocity either in fixed or in expanded bed mode. The two liquid streams are supplied separately in the riser and main column without intermixing. The solid and liquid phases are contacted co-currently and countercurrently in the riser and main column, respectively. SLCFBs have a number of prominent features such as an ability to accommodate continuous operation with respect to solid phase for higher throughputs which include both the steps of utilization loading (catalytic reaction, adsorption, etc.) and regeneration, efficient mass transfer, and reduced back-mixing. These unique features of SLCFB make them suitable for various industrial processes such as production of linear alkyl benzene (Liang et al. 1995, Liang and Zhu 1997a, Han et al. 2003, Xu et al. 2004), continuous recovery of fermentation products (Lan et al. 2000, 2002a,b, Lau et al. 2013), removal and recovery of cesium from liquid radioactive nuclear waste streams (Feng et al. 2003), wastewater treatment (Cui et al. 2004, Chowdhury et al. 2008, 2010, Li et al. 2012, Nirmala and Muruganandam 2013), and continuous enzymatic polymerization of phenol in bio-refining process (Trivedi et al. 2006).

SLCFBs studied so far are mainly divided into two sections: (i) riser section wherein superficial liquid velocity is maintained above the terminal settling velocity of solid particle (known as circulating fluidization regime) and (ii) conventional fluidized bed section wherein superficial liquid velocity is maintained between minimum fluidization velocity and terminal settling velocity of the solid particle. This hardware, in particular, gives rise to several limitations like (i) a proper pressure balance is needed for a stable operation, (ii) a proper dynamic seal is necessary between the two sections, (iii) higher liquid phase mixing and solid phase mixing is expected in the riser section since it operates at superficial liquid velocities greater than terminal settling velocities of solid particle, and (iv) it may fail when loading/regeneration of solid phase is time intensive, demanding an enormous height of the riser section. The multistage SLCFBs have also been studied with an emphasis on hydrodynamic (Singh et al. 2008, Chavan et al. 2009, Verma et al. 2009), mixing (Singh et al. 2008, Kalaga et al. 2012), and mass transfer (Kishore and Verma 2006, Kalaga et al. 2014) aspects. The stage configuration used, however, is similar to that of conventional distillation column stage wherein solid particles flow across the stage and move to the subsequent stage through a downspout provided near the periphery of the column. In such a design, the location of a downspout is eccentric causing non-uniform flow of solid particles across the stage with a fraction of solid particles short-circuiting from one side to the other side of the stage, leading to reduction in overall mass transfer rate. Therefore, it is essential to propose a new SLCFB which would overcome the disadvantages associated with the existing configuration of SLCFBs.

In order to design and develop SLCFB, it is necessary to follow the rational design procedure. In particular, (i) We should investigate the effect of physical properties of liquid and solid phases, superficial liquid velocity, solid circulation rate, and geometrical parameters on the flow structure of SLCFB. (ii) We should be able to get the flow behavior of solid and liquid phases as close to plug flow as possible. For this purpose, we need information regarding the extent of axial dispersion in solid and liquid phases with respect to physical properties of liquid and solid phases, liquid velocity, number of stages, and the other geometrical details of each stage. (iii) We need the estimation procedure for heat transfer coefficient (HTC) and solid-liquid mass transfer coefficient (MTC). The present work comprehensively reviews hydrodynamic,

mixing, heat transfer, mass transfer characteristics, and modeling strategies of SLCFB. Further, a new configuration of SLCFB has also been proposed to overcome the limitations of the present configurations of SLCFB.

2 SLCFB configurations

There are mainly three SLCFB configurations that exist in the literature. The information pertaining to geometrical and operational details of these configurations available in the literature is given below at a glance.

2.1 SLCFB configuration 1

This configuration has been proposed and well studied for hydrodynamic characteristics by Liang and coworkers (Liang et al. 1995, 1996, 1997, Liang and Zhu 1997a,b). The SLCFB mainly consists of a plexiglass riser column of 140 mm. i.d. and 3 m in height, a liquid-solid separator, a device for measuring the solids circulation rate, a storage vessel serving as the solids reservoir, and a solids feed pipe connecting the storage vessel and the base of the riser. At the base of the riser, there are two distributors: (i) the main liquid distributor, made up of 60 stainless steel (SS) tubes of 10 mm i.d., occupying 31% of the total bed area, and (ii) the auxiliary liquid distributor, a porous plate with a 5% opening area at the base of the bed. Similar configuration has been used by Zheng and coworkers with a little change in the dimensions of riser column and liquid distribution system at the bottom of the riser column to investigate hydrodynamic behavior of SLCFB (Zheng et al. 1999, 2002, Zheng and Zhu 2000a,b, 2001, Zhu et al. 2000, Zheng 2004). The riser column of 76.2 mm i.d. and 3 m length has been used. The main distributor consists of seven SS tubes occupying 19.5% of total area and the auxiliary liquid distributor, a porous plate with 4.8% opening at the base of the riser column. Figures 1 and 2 show the SLCFB configurations proposed by Liang and coworkers and Zheng and coworkers, respectively.

The liquid pumped from the reservoir is divided into two streams at the bottom of the riser column with the main stream entering the bed through the pipe distributor and another from the porous plate distributor. The liquid and solid particles move co-currently upwards in the riser and separate at the top of the riser. The solid-liquid separator is a cone-shaped large cylindrical vessel that allows the solid particles to settle out of the liquid flow. The liquid is then returned to the liquid reservoir, and solid particles are returned to the solid particle storage vessel. By closing the valve at the bottom of the solid particle metering vessel, the volume of particles circulated in a given time interval can be determined, giving the solid particle circulation rate. The pressure drop at the test section has been measured using U-tube manometer to measure average bed voidage. The electrical conductivity probes have been used to measure local bed voidage and liquid velocity. The local voidage and liquid velocity have been averaged over the bed cross section to compare the average bed voidage and liquid velocity obtained from pressure gradient method and flow meter, respectively.

Figure 1: SLCFB configuration proposed by Liang and coworkers: 1, liquid reservoir; 2, water pump; 3, rotameter; 4, liquid distributor; 5, pressure taps; 6, solid-liquid separator; 7, riser; 8, solid flow measuring device; 9, valve; 10, particle storage vessel.

Figure 2: SLCFB configuration proposed by Zheng and coworkers: 1, liquid-solids separator; 2, riser; 3, pressure taps; 4, main liquid distributor section; 5, auxiliary liquid distributor section; 6, feed pump; 7, storage vessel; 8, solids flow measuring device; 9, solids return pipe; 10, rotameter; 11, liquid reservoir.

2.2 SLCFB configuration 2

The SLCFB configuration proposed by Lan and coworkers is one of the well-studied configurations in the literature. This configuration is mainly employed to demonstrate the application potential of SLCFB *viz.* continuous recovery of fermentation products (Lan et al. 2000, 2002a,b, Mazumder et al. 2009a,b, 2010, Prince et al. 2012), wastewater treatment (Islam et al. 2009, Eldyasti et al. 2010), and enzymatic polymerization of phenol in bio-refining process (Trivedi et al. 2006). The hydrodynamic modeling studies are also available in the literature (Roy and Dudukovic 2001, Gaikwad et al. 2008, Razzak 2012, Razzak et al. 2012a,b). The major components include the riser column, a downcomer, a solid-liquid separator, and two solid

particle returning pipes, one connecting the base of the separator and upper part of the downcomer and other connecting the base of the downcomer and the riser. The riser column has 38 mm i.d. and 3 m length, and the downcomer has 120 mm diameter and 2.5 m length. The distributor for the riser is designed to divide the incoming stream into two sub-streams: the primary and the auxiliary streams. The primary stream enters through a tubing of 11 mm i.d. and auxiliary stream through porous plate at the bottom of the riser column. Figure 3 shows the schematic of the SLCFB.

Figure 3: SLCFB configuration proposed by Lan and coworkers: 1, riser; 2, liquid-solids separator; 3, top washing section; 4, top solids- return pipe; 5, solids flow measuring device; 6, downcomer; 7, distributor of the downcomer; 8, bottom washing section; 9, bottom solids-return pipe; 10, liquid distributor of the riser; 11, auxiliary stream; 12, primary stream; 13, raffinate; 14, extract; 15, feed.

The riser column is operated in circulating fluidization regime wherein solid particles are carried up in a co-current mode by combination of the primary liquid and auxiliary liquid streams. The outlet of primary distributor is located right above the solids entrance at the bottom of the riser. This design helps to increase the pressure drop across the particle returning pipe at the bottom, which is very important for the dynamic seal between the riser and the downcomer and the stabilization of the whole system. The inlet of the auxiliary stream is at the bottom of the riser. The purpose of the auxiliary stream is to mobilize the particles at the bottom, which are then carried up in the riser by the combination of the primary and the auxiliary streams. The solid

particles are transferred into the downcomer from the top of the riser through a solid-liquid separator. A downcomer is a conventional fluidization column and is operated in a countercurrent fashion. The solid particles travel the downcomer length and enter the riser column via solid particle returning pipe, connecting the downcomer and the riser column at the bottom.

2.3 SLCFB configuration 3

Joshi and coworkers have modified the SLCFB configurations proposed by Liang and coworkers and Zheng and coworkers by replacing the solid particle storage vessel with multistage column. The hydrodynamic (Chavan et al. 2009), mixing (Kalaga et al. 2012), and mass transfer (Kalaga et al. 2014) characteristics have been well investigated using the modified SLCFB configuration. The SLCFB system mainly comprises a glass riser column (50 mm i.d. and 2 m height), a liquid-solid separator (130 mm i.d. and 0.85 m long), a top solids return pipe (25 mm i.d.) connecting the riser and the multistage column, a glass multistage column (100 mm i.d. and 0.95 m height), calming section at the bottom of multistage column, and a bottom solids return pipe (25 mm i.d.) connecting the riser and the multistage column at the base. Figure 4 shows the schematic of SLCFB.

Figure 4: SLCFB configuration proposed by Joshi and coworkers: 1, riser column; 2, top solid return pipe; 3, settler; 4, multistage column; 5, sieve plate; 6, downcomer; 7, calming section; 8, bottom solid return pipe; VL1, riser column inlet stream; VL2, calming section inlet stream; VL3, multistage column inlet stream.

The solid and liquid phases contact countercurrently in the multistage column and concurrently in the riser column. The superficial liquid velocity in the riser is maintained higher than the terminal velocity of the solid particles so that the solid particles are carried upward by the upflowing water. The solid particles and the liquid are separated in the cone-shaped solid-liquid separator. The solid particles then move to the multistage column. The configuration of the multistage column is similar to that of the sieve trays distillation column used for vapor-liquid contacts. The fluidized solid particles move across the stage to the next stage through a downcomer (10 mm i.d. and 50 mm long, SS tube), as water flows upward through the mesh openings. The solid particles return to the riser column through the solid-liquid return pipe. The arrangement is made to measure pressure drop over a given length of the riser as well as multistage column using U-tube manometer. The local superficial solid velocity and voidage have been measured using ultrasonic velocity profiler (UVP) and γ -ray tomography, respectively.

3 Hydrodynamics

Rational design of multiphase contactors, SLCFB in our case, demands thorough knowledge of hydrodynamic behavior for the prediction of the design parameters such as MTC and the extent of mixing as a function of system, geometric, and operating parameters. The system parameters include physical properties of liquid and solid phase, geometric parameters consider dimensions of the hardware and internals, and operating parameters consider superficial liquid velocity and solid circulation rate. The solid-liquid fluidization falls in fixed bed or conventional (expanded bed) regimes when superficial liquid velocity is less than particle terminal settling velocity while in circulating and transport regimes when superficial liquid velocity is higher than particle terminal settling velocity. It has long been recognized that the flow structure prevailing in solid-liquid fluidization is uniform in both the axial and radial directions in conventional fluidization regime. This homogeneous behavior forms the basis of theoretical background of conventional solidliquid systems as reported by Richardson and Zaki (1954) and Kwauk (1963, 1964). Experimental results also confirm that conventional solid-liquid fluidization is indeed homogeneous (Wilhelm and Kwauk 1948, Richardson and Zaki 1954, Mertes and Rhodes 1955, Lapidus and Elgin 1957, Foscolo et al. 1983, Khan and Richardson 1987, Chavan and Joshi 2008). The flow pattern, however, becomes nonuniform when superficial liquid velocity exceeds particle terminal velocity (Liang et al. 1996, 1997, Liang and Zhu 1997b, Roy et al. 1997, Zheng et al. 1999, 2002, Roy and Dudukovic 2001, Zheng 2004, Razzak et al. 2008, 2012a,b, Cheng and Zhu 2008, Chavan et al. 2009, Shilapuram et al. 2011, Sang and Zhu 2012, Dadashi et al. 2014a). It is clear that the flow structure in SLCFB differs greatly as compared to the conventional solid-liquid fluidized bed (SLFB). The uniform flow pattern in SLFB abruptly becomes non-uniform in SLCFB in axial as well as radial directions when conventional fluidization regime transforms to circulating fluidization regime. Moreover, the flow pattern and holdup profiles are greatly affected by physical properties of solid and liquid phases, superficial liquid velocity, and solid circulation rate. A summary of previous work pertaining to hydrodynamic characteristics of SLCFBs is given in Tables 1 and 2.

Table 1: Summary of previous work on hydrodynamic characteristics of SLCFB

Investigator	Solid Phase				Liquid Phase			Geometrical Parameters		Liquid velocity and SCR*	Measurement technique: voidage and SCR*	Important observations
	Material	d_p , mm	ρ_s , kg.m^{-3}	$V_{S\infty}$, mm.s^{-1}	Material	ρ_L , kg.m^{-3}	μ_L , $\text{kg.m}^{-1}.\text{s}^{-1}$	Riser, mm	Main column/storage vessel, mm			
Liang et al., (1996)	Glass beads	0.405	2460	53	Water	1000	0.001	D=140 H=3000	-	34- 108 mm.s^{-1} ; 0- 2 mm.s^{-1}	Pressure gradient and electrical conductivity methods; Solid accumulation method	1, 2, 3
Liang et al., (1997)	Glass beads and silica gel beads	0.405 0.385	2460 1360	53 18	Water	1000	0.001	D=140 H=3000	-	34- 108 mm.s^{-1} ; 0- 2 mm.s^{-1}	Pressure gradient and electrical conductivity methods; Solid accumulation method	1, 2, 3, 4, 5, 6
Roy et al., (1997)	Glass beads	2.5	-	-	Water	1000	0.001	D=150 H=2100	-	120- 230 mm.s^{-1}	γ ray tomography; Calibration of SCR with motive liquid velocity	7, 8
Zheng et al., (1999)	Plastic beads, glass beads, steel shot	0.526 0.508 0.580	1100 2490 7000	10 59 216	Water	1000	0.001	D=76.2 H=3000	-	10- 500 mm.s^{-1} 0.8- 7.3 mm.s^{-1}	Pressure gradient method; Solid accumulation method	9, 10, 11
Zheng and	Plastic	0.526	110	10	Water	100	0.001	D=76.	-	42- 150	Fibre optics;	1, 2, 3, 12

Zhu, (2000b)	beads, glass beads	0.508	0 249 0	59		0		2 H=300 0		mm. s ⁻¹ 1- 15 kg.m ⁻² .s ⁻¹	Solid accumulation method	
Lan et al., (2000)	Polystyrene	0.32	108 0	4.5	Bovine Serum Album in solution and dilute salt solution	-	-	D=38 H=300 0	D=120 H=2500	11- 22.4 mm. s ⁻¹ 1- 1.42 kg.m ⁻² .s ⁻¹	Pressure gradient method; Solid accumulation method	6, 13, 14
Zheng and Zhu, (2000a)	Glass beads	0.508	249 0	59	Water	100 0	0.001	D=76 H=270 0	D=203	75-480 mm. s ⁻¹ 8.7- 21.5 kg.m ⁻² .s ⁻¹	Pressure gradient method; Solid accumulation method	15, 16
Roy and Dudukovic, (2001)	Glass beads	2.5	250 0	-	Water	100 0	0.001	D=150 H=210 0	-	150- 230 mm.s ⁻¹	γ ray tomography; Calibration of SCR with motive liquid velocity	1, 3, 6, 17, 18, 19. 20
Zheng and Zhu, (2001)	Plastic beads, glass beads I, glass beads II, steel shots	0.526 0.508 1.0 0.58	110 0 249 0 254 1 700 0	10 59 144 216	Water	100 0	0.001	D=76. 2 H=270 0	-	10- 500 mm.s ⁻¹	Pressure gradient method; Solid accumulation method	21, 22, 23, 24, 25

Zheng et al., (2002)	Plastic beads Glass beads	0.526 0.508	110 0 249 0	10 59	Water	100 0	0.001	D=76. 2 H=270 0	-	28-420 mm. s ⁻¹ 0.4- 6.0 mm. s ⁻¹	Fibre optics; Solid accumulation method	1, 2, 3, 6, 26, 27
Lan et al., (2002a)	Polystyrene	0.32	108 0	4.5	Bovine Serum Album in solution and dilute salt solution	-	-	D=38 H=300 0	D=120 H=2500	Riser: 7.35 mm.s ⁻¹ ; Main: 1.4- 2.8mm. s ⁻¹ ; SCR: 1.42 kg.m ⁻² .s ⁻¹	Pressure gradient method; Solid accumulation method	13, 14, 28
Feng et al., (2003)	Glass beads Exchanger	0.415 0.425	247 5 153 5	52 26	Water	100 0	0.001	D=6 H=210 0	D=50 H=1550	26- 75 mm.s ⁻¹ ; 1- 32 mm.s ⁻¹	SCR: slurry collection wrt time at the riser outlet	9, 29, 30
Zheng, (2004)	Glass beads	0.5	250 0	29.7 - 73.4	Water and glycerol solutions	100 0- 109 0	0.001- 0.0048	D=76 H=200 0	D=100	80- 180 mm.s ⁻¹ ; 4- 12 kg.m ⁻² .s ⁻¹	Electrical conductivity;	1, 2, 3, 31
Trivedi et al., (2006)	Immobilized particles	-	-	-	Phenol - hydrogen peroxide solution and	-	-	D=38 H=400 0	D=120 H=3500	13- 37 mm.s ⁻¹ ; 7.8 kg.m ⁻² .s ⁻¹	SCR: Solid accumulation method	3, 14, 18, 32

					ethanol							
Shilapuram et al., (2008)	Glass beads	1.36	2468	-	Water	1000	0.001	D=80 H=2800	-	14-447 mm.s ⁻¹ ; 1- 38 kg.m ⁻² .s ⁻¹	Pressure gradient method; Solid accumulation method	5, 29, 33, 34
Singh et al., (2008)	Ion exchange resin	0.55 0.65 0.925	1330	21.80 22.70 36.50	Water	1000	0.001	-	D=100 H=500	0.5-12.73 mm.s ⁻¹ ; 0.083-0.75gm.s ⁻¹	Pressure gradient method	35, 36, 37
Islam et al., (2009)	Lava rock	0.7	2560	-	Waste water	-	-	D = 20	D= 76	-	-	38
Chavan et al., (2009)	Polymeric resins	0.365 0.605 0.725	1100	4.70 9.00 13.00	Water	1000	0.001	-	D=50 H=2000	0.8- 3.6 mm.s ⁻¹ ; 0.4- 2.30 gm.s ⁻¹	Pressure gradient method and γ ray tomography; Ultrasound velocity profiler	1, 3, 14, 35, 39
Verma et al., (2009)	Ion exchange resin	0.325 - 0.65	-	2-10	Water	1000	0.001	-	D=100 H=500	2- 5.3	Pressure gradient method; Solid accumulation method	40, 41, 42
Chowdhury et al., (2010)	Lava rock	0.7	2560	-	Waste water	-	-	D = 50	-	-	-	43
Eldyasti et al., (2010)	Lava rock	0.7	2560	-	Landfill leachate	-	-			-	-	44
Natarajan et al., (2011)	Blue stone	0.337 0.55	2850	-	Water	1000	0.001	D=94 H=240	D=304 H=610	20- 180 mm.s ⁻¹	Pressure gradient method; Solid	5, 45, 46

	Sand Silica gel Ion exchang e resins	0.3- 0.463 0.55 0.463 - 0.655	270 0 165 0 132 5					0			accumulation method	
Shilapuram et al., (2011)	Glass beads	1.36	246 8	131- 208	Water and glycer ol solutio n	100 0- 111 0	0.0009 03- 0.0032 2	D=80 H=220 0	-	90- 420	Pressure gradient method; Solid accumulation method	47, 48, 49,50
Razzak et al., (2012a)	Glass beads	0.5	250 0	-	Water	100 0	0.001	D=76. 2 H=597 0	D=200 H=5050	56- 350	Electrical resistance tomography; Solid accumulation method	26, 51
Sang and Zhu, (2012)	Plastic beads	0.58- 1.19	133 0- 152 0	25- 64	Water	100 0	0.001	D=76. 2 H=540 0	-	50- 450	Fibre optic; Solid accumulation method	52, 53
Prince et al., (2012)	Ion exchang e resin	0.35- 0.6	-	-	Soy protein solutio n	-	-	D=40 H=422 0	D=100 H=3790	-	-	54
Razzak, (2012b)	Glass beads Lava rocks	0.5- 1.25 0.5- 0.92	250 0 221 0		Water	100 0	0.001	D=76. 2 H=597 0	D=200 H=5050	56- 350	Electrical resistance tomography; Solid accumulation method	1, 6, 26, 55, 56

Shilapuram and Sai, (2012)	Glass beads	1.36	246 8	131- 208	Water and glycerol solution	100 0- 111 0	0.0009 03- 0.0032 2	D=80 H=220 0	-	90- 420	Pressure gradient method; Solid accumulation method	5, 31, 57, 58
Nirmala and Muruganandam, (2013)	Sodium alginate beads	4	101 0	33	Water	100 0	0.001	D=50 H=100 0	-	33- 55	Pressure gradient method; Solid accumulation method	59, 60
Lau et al., (2013)	Polymeric resin	0.32	108 0	4.5	Protein solution	-	-	D=38 H=300 0	D=120 H=2500	5- 11.3	Pressure gradient method; Solid accumulation method	61, 62
Dadashi et al., (2014a)	Glass beads	0.508	249 0	59	Water	100 0	0.001	D=76 H=300 0	-	120- 290	Pressure gradient method; Solid accumulation method	1, 6, 63
Gnanasundaram et al., (2014)	Glass beads Sand	2 0.5	246 0 240 0	70- 291. 9	Water and glycerol solution	100 0- 103 6	0.001- 0.0013 6	D=100 H=240 0	-	56- 421	Pressure gradient method; Solid accumulation method	3, 6, 33, 64
Natrajan et al., (2014)	Blue stone Sand Silica gel Ion exchange resin	0.337 0.3- 0.55 0.55 0.46- 0.655	285 0 270 0 165 0 132 5	-	Water	100 0	0.001	D=94 H=240 0	-	30- 150	Pressure gradient method; Solid accumulation method	65

Lim et al., (2014)	Glass beads	0.5- 3	250 0	-	Water	100 0	0.001	D=102 H=350 0	-	100- 415	Pressure gradient method; Solid accumulation method	6, 45, 66, 67
Nirmala and Muruganan dam, (2015)	Polymeri c resin Sand	0.5	140 0 240 0	30.6 4-70	Water and glycer ol solutio n	100 0- 103 6	0.001- 0.0013 6	D=100 H=240 0	-	25- 130	Pressure gradient method; Solid accumulation method	3, 33, 68

Table 2: Summary of previous work (some important observations for Table 1)

Sr. No.	Important observations
1	Radial non-uniform distribution of voidage and liquid velocity in circulating fluidization regime with higher voidage at center and less at wall has been observed.
2	The radial non-uniformity of the voidage increases with an increase of superficial liquid velocity and solid circulation rate.
3	The uniform axial flow structure has been observed.
4	Transition criteria into circulating fluidization regime have been proposed with a comprehensive regime map.
5	The empirical correlations based on homogeneous fluidization are not applicable in circulating fluidization regime due to radial non-uniformity in flow structure.
6	The solid holdup decreases with an increase in superficial liquid velocity for a given solid circulation rate and reverse is true for a given superficial liquid velocity.
7	The solid holdup decreases with an axial position for a given superficial liquid velocity in the riser column.
8	The solid phase velocity reaches somewhat constant value with increasing superficial liquid velocity.
9	Circulating fluidization regime is subdivided into two regimes viz., (i) initial circulating regime and (ii) fully developed circulating regime.
10	Light particles show an axial similarity in the flow structure, but heavy particles present non-uniformity in the initial zone of the circulating fluidization regime.
11	Transition from conventional fluidization regime to circulating regime occurs at particle terminal settling velocity.
12	Transport regime is homogenous like conventional fluidization regime.
13	A model has been developed for ion exchange system based on ion exchange kinetics and plug flow assumptions for the down comer and the riser section.
14	The dynamic seal between the riser and downcomer column has been achieved by maintaining the solids in the solid return pipe in packed moving bed regime.
15	The pressure drop across the valve (placed between the riser and the storage vessel) should be equal to pressure heads difference between the riser and storage vessel at the bottom.
16	The model has been proposed to estimate pressure drop across the valve. The maximum solid circulation rate and stable operating range are strongly influenced by the total solid inventory and unit geometry.
17	The time averaged particles velocity exhibit a clear pattern although the instantaneous solid particles velocity field is highly turbulent and chaotic.
18	The liquid phase presumes plug flow while solid particles are severely backmixed in the riser column.
19	The extend of solid particle backmixing depends on the solids loading (increases with increased loading) and on the liquid flow rate (increases with increased flow rate).
20	The Computational Fluid Dynamics simulation has provided a quantitative and qualitative micro-scale picture of the riser and overall flow pattern. The technique

	based on the use of radioactive isotope for estimating the solid velocity and voidage has been proposed.
21	The higher solid circulation rate can be achieved by increasing solid inventory for a given superficial liquid velocity.
22	The critical transition velocity decreases with an increase in solid inventory.
23	The onset velocity has been proposed based on time emptying method in a batch operated fluidized bed.
24	The onset velocity increases with particle density and size.
25	The onset velocity is lower for solid particles having wider size distribution and/or irregular shapes than for particles with narrow size distribution and/or regular shapes.
26	The radial non-uniformity of solid holdup decreases with an increase in superficial liquid velocity.
27	Particle density significantly influences the flow structure. Heavier particles lead to a steeper radial profile of solids holdup and a more visible acceleration region.
28	Under optimized condition, the system indicated a high protein removal efficiency of upto 98% and a overall protein recovery of upto 84%.
29	The primary inlet location has been optimized to obtain steadily increase in solid circulation rate with liquid velocity.
30	Diameter of solid return pipe has significant effect on solid circulation rate (increases with an increase in diameter) than length.
31	The viscosity of the liquid phase reduces the non-uniform distribution of the solid particles in the radial direction.
32	The continuous enzymatic polymerization of phenol has been carried out using SLCFB and 54% conversion of phenol has been achieved.
33	Solid circulation rate and solid holdup increases with an increase in auxiliary liquid velocity.
34	Under constant set of primary and auxiliary liquid velocity, the solid circulation rate and solid holdup increase with an increase in solid inventory.
35	Operating window for stable operation of multistage solid liquid fluidized bed has been proposed.
36	Empirical correlations have been proposed to estimate pressure drop voidage and average bed height during cross flow fluidization apparently prevalent on the stage.
37	The maximum pressure drop occurs in the column due to the hydrostatic head.
38	An impact of carbon to nitrogen ratio on nutrient removal in solid liquid circulating fluidized bed bioreactor has been investigated with 82% nitrogen removal at COD/N ratio of 6:1.
39	The empirical correlation has been proposed to determine local voidage in the riser column
40	A novel multi-staged radially cross-flow fluidized bed ion-exchanger has been to remove dissolved solutes from aqueous stream.
41	Approximately 40% improvement in mass transfer rate in comparison to that in conventional staged-column has been demonstrated.
42	The empirical correlation has been developed to determine mass transfer coefficient based on their experimental data.
43	The removal of approximately 90% organic, 80% nitrogen, and 70% phosphorous

	at loading rates of 4.12 kg COD/m ³ .d, 0.26 kg N/m ³ .d, and 0.052 kg P/m ³ .d and empty bed contact time of 1.5 hours.
44	The COD, nitrogen, and phosphorus removal efficiencies of 85%, 80%, and 70%, respectively at a low carbon-to-nitrogen ratio of 3:1 and nutrients loading rates of 2.15 kg COD/(m ³ d), 0.70 kg N/(m ³ d), and 0.014 kg P/(m ³ d) have been achieved as compared with 60–77% COD and 70–79% nitrogen removal efficiencies achieved by upflow anaerobic sludge blanket (UASB) and moving bed bioreactor (MBBR), respectively.
45	The slip velocity increases with an increase in the solid velocity, size, and density and decreases with an increase in solid holdups.
46	Drift flux model closely predicts slip velocity.
47	The critical transitional liquid velocity to circulating fluidization bed regime decreases with increase in liquid viscosity.
48	The onset average solids holdup increases with increase in auxiliary liquid velocity and solids inventory.
49	The axial and average solids holdup decreases with increasing primary liquid velocity and increases with increasing either auxiliary liquid velocity or solids inventory.
50	The empirical correlation has been proposed to estimate average solid holdup in the riser column with due consideration of viscosity of liquid medium.
51	The solid holdup profiles are successfully modeled by ANN model
52	An excess superficial liquid velocity ($V_L - V_{s\infty}$) is an appropriate parameter to evaluate the effect of particle properties on hydrodynamic characteristics.
53	$V_L - V_{s\infty} = V_s$ (solid particle velocity) when $\varepsilon_s < 0.1$.
54	An effectiveness of SLCFB has been demonstrated for recovery of soy protein with 50% recovery
55	ANFIS model successfully predict solid holdups in axial and radial location in a riser.
56	The solid holdups are higher in the case of glass beads as compared to irregular shaped lava rock particles for a given superficial liquid velocity.
57	At any axial position, axial solid holdup decreases with an increase in the primary liquid velocity and increases with an increase in auxiliary liquid velocity.
58	Solid holdup is a strong function of location of primary liquid inlet. Solid holdup is higher when primary inlet is located at no dynamics seal position as compared to with dynamic seal.
59	The SLCFB has been used for biosorption of cadmium using immobilized micro-organism and 64% removal has been achieved.
60	Solid circulation rate increases with an increase in superficial liquid velocity upto certain value and then decreases
61	A general purpose, extensible, and dynamic theoretical compartmental model based upon a tanks-in-series framework incorporating the equilibrium and hydrodynamics of liquids and solid particles has been developed for continuous protein recovery in SLCFB systems.
62	Protein recovery increases with an increase in solid circulation rate and feed concentration while it decreases with an increase in superficial liquid velocity.

63	Dispersed k- ϵ turbulent model satisfactorily predicts flow characteristics of riser column.
64	The empirical correlation has been proposed to estimate average solid hold up which shows that average solid hold up varies linearly with viscosity of the liquid phase.
65	The empirical new similarity parameter $[(V_{La}/V_L)^{0.3}]$ has been proposed which shows similar hydrodynamic behavior under circulating fluidization regime for a given $[(V_{La}/V_L)^{0.3}]$.
66	The overall particle rising velocity in the riser increases with increasing superficial liquid velocity or solid circulation rate but decreases with increasing particle size.
67	The empirical correlation has been proposed in terms of diameter of particle, superficial liquid velocity, and solid circulation rate to determine solid hold up in the riser column.
68	The average solid hold up increases with an increase in liquid viscosity as the critical transitional velocity decreases with an increase in liquid viscosity.

3.1 Transition from conventional to circulating regime

For a given system and geometrical parameters, solid liquid system observes four fluidization regimes *viz.* fixed bed, expanded bed, circulating, and transport regimes with a change in superficial liquid velocity. The system falls under fixed bed regime when superficial liquid velocity is less than minimum fluidization velocity (V_{mf}). When the superficial liquid velocity increases beyond minimum fluidization velocity, a fixed bed gets transformed into conventional fluidization regime. The conventional fluidization regime prevails until superficial liquid velocity is less than the terminal settling velocity of solid particles. The entrainment of solid particles starts when superficial liquid velocity becomes equal to terminal settling velocity of the solid particle. The entrained solid particles are circulated back to a column to keep voidage at desired value. This operating state is defined as solid-liquid circulating fluidization regime in which the fluidized bed is maintained by continuous particle circulation and a proper pressure balance between the riser and the particle return system.

The reports are available which deal with determination of a transition velocity or the onset velocity to the circulating fluidization. A transition from conventional fluidization regime to circulating fluidization regime occurs when superficial liquid velocity becomes equal to terminal settling velocity of the solid particle under consideration (Zheng et al. 1999, Zheng and Zhu 2001). Liang et al. (1996) have shown that the radial distribution of voidage becomes non-uniform when transition from conventional regime to circulating regime occurs. A superficial velocity at which voidage profile becomes non-uniform is probably termed as transition velocity. The method, however, lacks in accuracy since it is difficult to demarcate voidage nonuniformity as a function of superficial liquid velocity with precision. In their subsequent investigation (Liang et al. 1997), they have determined transition velocity by plotting the particle circulation rate against the superficial liquid velocity. A superficial liquid velocity at which the particle circulation rate approaches 0 is termed as transition velocity. It has been shown that transition velocity is a function of operating conditions and the equipment configuration, which leads to an uncertainty while demarcating the boundary between conventional fluidization and the circulating fluidization. The value of transition velocity obtained was found to be higher than the terminal settling velocity of the particle. On the contrary, in their previous work (Liang et al. 1993), they have reported normalized transition velocity (ratio of transition velocity and terminal

settling velocity of particle) at about 0.6, suggesting that the value transition velocity is lower than the terminal settling velocity of the particle under consideration. This seems to be unreasonable.

Zheng et al. (1999) have observed that the onset velocity is a strong function of particle density. For lighter solid particles ($\rho_s = 1100 \text{ kg m}^{-3}$), the transition from conventional fluidization regime to circulating fluidization regime occurs for a given superficial liquid velocity. On the contrary, the transition occurs for heavy particles ($\rho_s = 7000 \text{ kg m}^{-3}$) over a wide range of superficial liquid velocity. Similar results have been obtained by Sang and Zhu (2012). Zheng and Zhu (2001) have obtained transition velocity by measuring the time required to empty solid particles in a batch-operated SLFB as a function of liquid velocity. In contrast to the observation of Liang et al. (1997), they have shown that transition velocity is independent of solid inventory and the solid feeding system, while it is dependent only on physical properties of liquid and solid. Zheng et al. (2002) have proposed a method for demarcating the boundary between convention fluidization and circulating fluidization regimes based on measurement of pressure drop gradient along the riser length as a function of superficial liquid velocity. The superficial liquid velocity corresponding to a peak of the static pressure gradient profile at the uppermost section of the riser is defined as an onset velocity of circulating fluidization regime where solid particles are about to entrain out of the riser column. Vatanakul et al. (2005) and Shilapuram et al. (2008, 2011) have adopted a similar procedure to demarcate transition from conventional zone to circulating zone.

Shilapuram et al. (2009) have determined the onset velocity to circulating fluidization regime by three experimental methods wherein primary and/or auxiliary liquid velocity is varied to vary total liquid velocity in the riser section. In the first method, primary liquid velocity is varied and auxiliary liquid velocity is set to fixed value, while the reverse is true for the second method. In the third method, the primary liquid velocity is introduced until the solid in riser is about to entrain, and then the auxiliary liquid velocity is set to a constant value and the primary liquid is increased until the transported regime is reached. It has been seen that the onset velocity obtained by the first and second methods is fairly the same and is less than by the third method for a given solid inventory. In contrast to the finding of Liang et al. (1997) that transition liquid velocity is independent of the auxiliary liquid velocity, it has been demonstrated that transition occurs at different combinations of primary and auxiliary liquid velocities in the same method of operation and is different by different methods of operation. In a report by Shilapuram et al. (2011), the effect of liquid viscosity on onset velocity to circulating regime has been studied by using solid circulation as the method of determination of the onset velocity. The onset velocity decreases with an increase in liquid viscosity. This is noticeably due to the decrease in the value of terminal settling velocity of particle as viscosity enhances shear stress acting on the particle. This enhancement results in increase of drag force acting on the particles, and therefore the circulating regime starts at lower velocity with an increase in liquid viscosity. A similar observation has been made by Nirmala et al. (2015) while studying the effect of liquid viscosity on transition from conventional fluidization regime to circulating fluidization regime.

It can be concluded that there are mainly three methods available in the literature for regime transition identification *viz.* (i) solid circulation method, (ii) bed emptying method, and (iii) pressure gradient method. The bed emptying method seems to be precise among the other methods to determine the onset velocity of SLCFB.

3.2 Flow regime map

Zhu et al. (2000) have modified the flow regime map initially proposed by Liang et al. (1997) to illustrate the flow regimes in solid-liquid fluidization system. Figure 5 shows a regime map where the solid-liquid circulating fluidization regime is identified as a new operating regime in a plot of dimensionless superficial liquid velocity against dimensionless particle diameter. The dimensionless superficial velocity (V_L^*) is defined as follows:

$$V_L^* = V_L \left(\frac{\rho_L^2}{\mu_L g \Delta \rho} \right)^{1/3} \quad (1)$$

and dimensionless diameter (d_p^*) is defined as follows:

$$d_p^* = d_p \left(\frac{\rho_L g \Delta \rho}{\mu_L^2} \right)^{1/3} \quad (2)$$

The flow regime map proposed by Liang et al. (1997) shows that the onset velocity to circulating fluidization is less than the terminal settling velocity of the particle. On the contrary, it has been shown by various investigators that the onset velocity is always greater than the terminal settling velocity (Zheng et al. 1999, Zheng and Zhu 2001, Shilapuram et al. 2008, 2009, Chavan et al. 2009).

The regime map is divided into four regimes *viz.* fixed bed, conventional, circulating, and transport fluidization regimes. For a given system and geometrical parameters, the bed is initially in the fixed bed regime when V_L is less than V_{mf} . When V_L crosses V_{mf} , the operation of bed gets transferred into conventional fluidization regime. The circulating fluidization regime begins when V_L is beyond the onset velocity. With a further increase in V_L , the operating stage leaves the circulating regime and enters the transport regime. The shaded area gives the stable operation regime for circulating fluidized bed. When superficial liquid velocity is higher than transition velocity, it becomes essential to recirculate solid particles in order to maintain the desired solid holdup. For a given solid holdup, a higher solid particle circulation rate would require higher liquid velocity. This means that the stable operation of circulating fluidization regime is dependent not only on liquid velocity but also on solid particle circulation rate.

It is clear from Figure 5 that if V_L^* is increased beyond onset velocity for a given d_p^* , a point will eventually be reached where the circulating regime enters into the transport regime. This transition velocity is shown by the dotted line in Figure 5, illustrating an upper limit for stable operation of SLCFB. It is expected that transition from circulating regime to transport regime varies with the solid particle circulation rate. A high solid particle circulation rate can delay the transition since a higher solid particle circulation rate increases the solid particle holdup which in turn increases the radial flow non-uniformity. In order to impart radial uniformity in the flow structure (so as to enter the transport regime), the higher liquid velocity would be required.

Figure 5: Flow regime map for liquid-solid circulating fluidized bed.

3.3 Axial flow structure

Unlike conventional solid-liquid fluidization, Liang et al. (1996) have observed non-uniformity in the radial flow structure in solid-liquid circulating fluidization. However, a uniform axial flow structure has been found. The experiments were conducted using glass beads as a solid phase alone, and no attempt has been made to illustrate the effect of particle property on axial flow structure. In the subsequent publication, Liang et al. (1997) have found the same results with respect to axial flow structure wherein they have used glass bead and silica gel particles for fluidization. The work of Zheng et al. (1999) has shown that the distinct hydrodynamic behavior exists in two zones, the initial circulating fluidization zone and the fully developed circulating fluidization zone, for low-density particles (plastic beads, $\rho_s = 1100 \text{ kg m}^{-3}$; glass beads, $\rho_s = 2490 \text{ kg m}^{-3}$) and high-density particles (steel shot, $\rho_s = 7000 \text{ kg m}^{-3}$). In the initial zone, the axial profiles for lighter particles are uniform along the length of the riser. This is in agreement with what has been reported by Liang et al. (1997). However, the behavior of the steel shots deviates from that of lighter particles in the initial zone. The axial distribution of solid holdup was found to be nonuniform. This is noticeably due to the fact that the solid flow to the riser is not fully developed for steel shots. The axial flow structure, however, was found to be uniform for lighter as well as heavier particles in fully developed circulating fluidization zone. With increasing particle density, the decrease of solid holdup with liquid velocity becomes slow, and

the circulating fluidization develops more gradually from the initial circulating fluidization zone to the developed zone. For lighter particles, an increase of solid circulation rate and the decrease of solid holdup with increasing liquid velocity are more rapid. Therefore, the transition from the initial circulating fluidization zone to the developed circulating fluidization zone becomes sharper with a reduction in a particle density for both cases. The article by Shilapuram and Sai (2012) essentially demonstrates that the location of the liquid distributor affects the axial solid holdup in the circulating fluidization regime. At any axial position higher solid holdup can be obtained when there is no dynamic seal as compared to when there is a dynamic seal. Further, at any axial position, axial solid holdup decreases with an increase in the primary liquid velocity and increases with an increase in the auxiliary liquid velocity. It has also been shown that axial solid holdup fairly remains uniform with a change in liquid viscosity. However, a distinct observation has been made that the solid holdup increases with an increase in liquid viscosity and decreases with a further increase in the liquid viscosity. Numerous experimental investigations, reported in literature, conform to uniformity in axial flow structure in fully developed circulating fluidization regime (Kuramoto et al. 1998, Zheng and Zhu 2000b, Roy and Dudukovic 2001, Zheng et al. 2002, Zheng 2004, Shilapuram et al. 2008, Natarajan et al. 2008, Chavan et al. 2009, Gnanasundaram et al. 2014).

3.4 Radial flow structure

Flow structure in SLCFB differs greatly when compared with SLFB wherein uniformity in radial and axial flow structures has been well accepted. Although uniform axial flow structure exists in SLCFB, non-uniform radial flow structure has been observed. The extent of radial nonuniformity, however, varies depending upon the operating conditions and the physical properties of the solid and liquid phases. Literature essentially reports the effect of superficial liquid velocity, primary and auxiliary liquid velocity, solid circulation rate, and particle density and size on flow structure in SLCFB.

Liang et al. (1996, 1997) and Liang and Zhu (1997a) have studied the effect of superficial liquid velocity and solid circulation rate on radial distribution of bed voidage and liquid velocity in the riser column using glass beads and silica gel beads as a solid phase. The radial non-uniformity has been observed with respect to liquid velocity and solid holdup. An increase in superficial liquid velocity increases the non-uniform radial distribution of liquid velocity. Further, it has been seen that the solid holdup increases with an increase in solid circulation rate, and the radial non-uniformity of the solid holdup increases slightly with an increase in solid circulation rate. The radial solid holdup profiles, however, remain the same along the length of the column, indicating uniform flow structure along the axial direction. Similar results have been reported in gas-solid-liquid circulating fluidization by Vatanakul et al. (2005) using ultrasonic technique to investigate phase holdups in multiphase flows. The non-uniformity in radial distribution was observed with respect to solid holdup and gas holdup. The local solid holdup increases from the center to the wall of the column, while gas holdup follows the opposite trend. Roy et al. (1997) and Roy and Dudukovic (2001) have measured particle velocity and solid holdup using noninvasive flow monitoring methods *viz.* computer automated radioactive particle tracking and computer tomography in 150 mm i.d. and 2.10 m long column using glass beads (diameter 2.5 mm). It has been seen that the solid particles ascend at the center of the column and descend at the wall as a function of superficial liquid velocity. An increase in solid circulation rate increases the average solid holdup for a given superficial liquid velocity, whereas an increase in superficial liquid velocity decreases average solid holdup for a given solid circulation

rate. The particle velocity roughly follows cubic function of radial location, the highest velocity being at the center of the column and the velocity being negative near the wall. Since the solid holdup is higher at the wall as compared to the center of the column, the annulus of negative velocities at the wall makes downward mass flow of particles in the column. The downflow at fixed superficial liquid velocity increases with solid to liquid flow ratio and increases with superficial liquid velocity at fixed solid to liquid flow ratio. Zheng et al. (2002) have reported the radial flow structure with respect to solid holdup and liquid velocity at seven radial positions using glass and plastic beads as a solid phase. A non-uniformity increases with an increase in superficial liquid velocity till circulation regime persists. However, a transition from circulating regime to transport regime tends to decrease non-uniformity with respect to both solid holdup and liquid velocity. Further, it has been observed that the radial non-uniformity of solid holdup and the cross-sectional average solid holdup increases with an increase in solid circulation rate. It has also been reported that the particle density affects the radial non-uniformity of solid holdup and liquid velocity. The heavier particles lead to steeper radial profiles of solid holdup as compared to lighter particles. These results agree with an earlier report by Zhang et al. (1999). Zheng and Zhu (2000b) have studied micro-structural aspects of circulating regime. The hardware configuration and solid phase conform to their earlier work (Zheng et al. 1999). A radial non-uniformity exists with dilute particle concentration at the axis and a denser particle concentration at the wall region. The particles at a wall region were found to be in vigorous movement state as compared to particles at axis, identifying radially segregated flow structure. The effect of particle density on flow segregation has been enumerated. The heavier particles tend to offer intensified flow segregation as compared to lighter particles.

The effect of liquid viscosity on radial distribution of solid holdup has been investigated by Zheng (2004). The flow structure was found to be axially uniform but radially non-uniform. The solid holdup decreases with an increase in superficial liquid velocity and increases with an increase in viscosity. The viscosity of fluidizing liquid tends to reduce non-uniformity of particle distribution in the radial direction. Similar results have been obtained by Gnanasundaram et al. (2014) and Shilapuram and Sai (2012). Chavan et al. (2009) have measured local particle velocity and solid holdup in the riser column using UVP and γ -ray tomography, respectively. The flow characteristics have been investigated using polymeric resins in the size range of 0.35–0.725 mm. The following empirical correlation for estimation of local radial voidage has been proposed:

$$\varepsilon_L = \varepsilon_L - \left[0.95 + 0.75 \left(\frac{r}{R} \right)^{2.65} \right] \quad (3)$$

where ε_L is local voidage, while ε_L is an average voidage over cross-sectional area. It has been observed that solid holdup decreases with an increase in liquid velocity and decrease in solid circulation rate. Further, the experimental results follow a similar trend obtained by Liang et al. (1996) and Zheng et al. (2002), confirming that solid holdup is minimum at the center and maximum at the wall of the column, while the particle velocity is maximum at the center and minimum at the wall of the column. Sang and Zhu (2012) have investigated the effect of density and size of particle on radial distribution of solid holdup in the range of 1520–7000 kg m⁻³ and 0.51–1.19 mm, respectively, for a given superficial solid velocity. It has been observed that for five types of particles, solid holdup is non-uniform across the radial position: lower at the center and higher at the wall of the column. At all the radial locations, local solid holdup increases with an increase in particle density for a given superficial liquid velocity. A similar trend has been observed with respect to size of particles at a given superficial liquid velocity.

3.5 Pressure drop

SLCFB is operated in a continuous mode with respect to a solid phase so that both steps of utilization (catalytic reaction, adsorption, etc.) and regeneration would be carried out simultaneously. One needs two liquid streams of fairly opposite chemical nature to accomplish these steps. A proper pressure balance between the riser and main columns avoids intermixing of two liquid streams and ensures stable operation of SLCFB. Further, an analysis of pressure balance in whole circulation loop would be helpful to elucidate the origin of possible flow stability and to evaluate the effect of operating conditions on the steady state hydrodynamics.

3.5.1 Pressure drop in the riser (ΔP_R)

The pressure drop in the riser is due to (i) liquid and solid static heads, (ii) solid acceleration, (iii) friction between liquid and solid, and (iv) wall friction due solid and liquid. The pressure drop due to solid acceleration, friction between liquid and solid, and wall friction can be neglected in circulating system (Basu and Cheng 2000, Zheng and Zhu 2000a, Kim et al. 2002). The pressure drop due to liquid and solid static head can be written as follows:

$$\Delta P_R = \rho_s (1 - \varepsilon_L) + \rho_L \varepsilon_L H_R g \quad (4)$$

The mass flow rate of solid particles leaving the riser section can be estimated by the following expression:

$$m_s = (1 - \varepsilon_L) \rho_s A_R V_s \quad (5)$$

V_s is the true velocity of solid particle. As the voidage is very high, V_s can be approximated by the following equation:

$$V_s = V_L - V_{s\infty} \quad (6)$$

where V_L is superficial liquid velocity and $V_{s\infty}$ is particle terminal settling velocity.

3.5.2 Pressure drop in the downcomer (ΔP_D)

The downcomer is composed of various subcomponents *viz.* solid-liquid separator, solid return pipe, storage vessel, and solid feed pipe. The pressure drop in each component can be computed.

3.5.3 Pressure drop in solid-liquid separator (ΔP_{SL})

In solid-liquid separator, the solid particles conveyed through riser section spontaneously get settled due to high ratio of separator diameter to riser diameter. The liquid flows from the top of the separator. The clear liquid height and the solid bed height contribute to pressure drop in the solid-liquid separator

$$\Delta P_{SL} = H_{SL} \rho_L g + [\rho_s (1 - \varepsilon_L) + \rho_L \varepsilon_L] H_R g \quad (7)$$

3.5.4 Pressure drop in solid return pipe (ΔP_{SR})

The solid particles from the solid-liquid separator move to the solid storage section via the solid return pipe. The solid particles move down slowly as a moving packed bed flow, and hence the pressure drop can be obtained using the Ergun equation:

$$\Delta P_{SR} = \frac{150 \mu_L V_L H_{SR} (1 - \varepsilon_L)^2}{d_p^2 \varepsilon_L^3} + \frac{1.75 \rho_L V_L^2 H_{SR} (1 - \varepsilon_L)}{d_p \varepsilon_L^3} \quad (8)$$

3.5.5 Pressure drop in solid storage vessel (ΔP_{ss})

The solid particles move from the solid storage vessel to the riser section via the solid feed pipe in the fixed bed flow. The pressure drop in the solid feed pipe can be estimated using Eq. (8). The pressure drop in the solid storage vessel is given by the following equation:

$$\Delta P_{ss} = H_{ss} \rho_L g + [\rho_s (1 - \varepsilon_L) + \rho_L \varepsilon_L] H_R g \quad (9)$$

3.5.6 Pressure drop across the valve

The solid particles flow to the riser section via the solid flow control device, mainly a non-mechanical valve. The non-mechanical valve does not contain any moving parts, and solid flow is controlled by auxiliary liquid flow rate and the geometry of the pipe. Zheng and Zhu (2000a) have proposed the following semi-empirical equation to estimate pressure drop across the valve:

$$\Delta P_v = \frac{G_s^{1.51} g D}{0.25 V_{La}} \quad (10)$$

where G_s is the flux of the solid-liquid mixture through the valve ($\text{kg m}^{-2} \text{s}^{-1}$), D is the diameter of the feeding pipe (m), and V_{La} is the auxiliary liquid flow rate (m s^{-1}). It has been shown that SLCFB operates at stable conditions when pressure head difference between the bottom of the storage vessel (P_{st}) and the riser (P_r) balances pressure loss across the non-mechanical solid control valve (ΔP_v). The values of pressure drop (ΔP_v) are compared with the values of pressure drop ($P_{st} - P_r$), estimated experimentally for given operating conditions. A good agreement has been seen with a deviation of $\pm 10\%$. Although Eq. (10) predicts pressure drop within an acceptable limit, it has been developed using only a single solid phase with density 2490 kg m^{-3} and particle diameter of $508 \mu\text{m}$. The effect of particle density and size and fluidizing medium density and viscosity has not been considered for Eq. (10). Further, the effect of geometrical parameters (length and diameter of feeding pipe, in particular) has also not been considered. Therefore, it is necessary to develop a unified correlation considering a wide range of system, geometric, and operating parameters. The aforementioned section indicates that unlike conventional SLFB, SLCFBs are fairly cumbersome to operate under stable condition. This is due to (i) inclusion of solid circulation as a separate operating variable, and its net flux is a strong function of the operating flow rates in the riser and downcomer sections, and (ii) solid and liquid flows in co-current and countercurrent manner in riser and downcomer sections, respectively, and therefore pressure drop and hydrostatic head follows different functions in both sections. A judicious pressure balance has to be maintained in the two sections for a desirable stable solid flow from downcomer to riser at the bottom and riser to downcomer at the top. Thus, for a given SLCFB system, there is a region of operating flow rates in the two sections that will mark the flow rates required for stable SLCFB operation. This is called the operating window. The operating window can be estimated from pressure balances if only the dispersion densities are known in both sections for a given set of operating conditions (Bi and Zhu 1993, Cheng and Basu 1999, Basu and Cheng 2000, Zheng and Zhu 2000a, Kim et al. 2002, Han et al. 2003, Gaikwad et al. 2008).

3.6 Drag coefficient

The drag coefficient for a particle is defined as the ratio of the resistance force per unit projected area in a plane perpendicular to the direction of motion of fluid (R_f) to the kinetic energy of the fluid per unit volume. The drag coefficient can be mathematically expressed as follows:

$$C_D = \frac{R_f}{\frac{1}{2} \rho_L V_L^2} \quad (11)$$

The resistance force per unit projected area can be estimated as follows:

$$R_f = \frac{\pi/6 d_p^3 (\rho_s - \rho_L) g}{\pi/4 d_p^2} = \frac{2}{3} d_p (\rho_s - \rho_L) g \quad (12)$$

If the particle is settling in the gravitational field at its terminal settling velocity ($V_{S\infty}$), then combining Eqs. (11) and (12) gives

$$C_{D\infty} = \frac{4}{3} \frac{\rho_s - \rho_L}{\rho_L} \frac{d_p g}{V_{S\infty}^2} \quad (13)$$

Equation (13) defines drag coefficient for a particle based on its terminal settling velocity. In the fluidization system, however, flow field around the particle gets affected due to the presence of other particles, and hence drag force acting on a single particle needs to be modified to account for the presence of the surrounding solid particles. In a fluidized bed, the forces acting on a solid particle are in dynamic equilibrium. These forces are gravitational, buoyancy, and fluid drag. The gravitational force acting on a particle is proportional to the density of a particle and well defined by Newton's second law. However, buoyancy and drag forces are not being defined as precisely as gravitational force. In fact, drag force can be evaluated if the buoyancy force is defined properly. The two basic approaches are available in the literature to define buoyancy force. Foscolo et al. (1983) and Foscolo and Gibilaro (1984) have defined the buoyancy force based on suspension density rather than fluid density. Similar definition of buoyancy force has been used by Gibilaro et al. (1984, 1987a,b), Richardson and Meikie (1961), Barnea and Mizrahi (1973), and Rietema (1982). However, Epstein (1984), Joshi (1983), Clift et al. (1987), Fan et al. (1987), and Jean and Fan (1992) state that the buoyancy force is the net force arising from the pressure distribution over the surface of the particle with the fluid at rest, and the drag force represents the additional force exerted on the particle by a flowing fluid. In general, a drag coefficient can be defined by analogy with Eq. (13) as follows:

$$C_{Dx} = \frac{4}{3} \frac{\rho_s - \rho_x}{\rho_L} \frac{d_p g}{V_{Lx}^2} \quad (14)$$

where ρ_x is either the fluid density (ρ_L) or the suspension density (ρ_C), and V_{Lx} is either superficial liquid velocity or the relative velocity. Combining Eqs. (13) and (14) and putting $\rho_x = \rho_L$ and $V_{Lx} = V_L$, we get the following expression:

$$\frac{C_{Dx}}{C_{D\infty}} = \left(\frac{V_{S\infty}}{V_L} \right)^2 \quad (15)$$

The dependence of drag coefficient on the voidage can be expressed as follows using Richardson and Zaki (1954) equation:

$$\frac{C_{Dx}}{C_{D\infty}} = \epsilon_L^{-2n} \quad (16)$$

where n is a Richardson and Zaki parameter. The value of drag coefficient thus can be obtained if the value of n is estimated.

There are plenty of semi-empirical and empirical correlations available in the literature to estimate drag coefficient in homogeneous fluidization. The widely used drag coefficient

formulas are by Chen (2009), Almedeij (2008), Brown and Lawler (2003), Grbavcic and Vukovic (1991), Turton and Levenspiel (1986), Khan and Richardson (1987), Flemmer and Banks (1986), and Concha and Barrientos (1982). Homogeneous fluidization presumes that bed voidage remains constant across the cross section of the column for a given system and operating parameters. Therefore, drag coefficient estimated using Eq. (16) gives a constant value across the cross section of the column. The situation pertaining to SLCFB, however, is dissimilar when compared to conventional SLFB. The non-uniformity has been found with respect to voidage, superficial liquid velocity, and solid particle velocity in the riser section of SLCFB. Therefore, the equation applied to estimate drag coefficient in conventional fluidization cannot be applied to circulating fluidization. It can be seen that the value of drag coefficient varies along the radial direction due to the non-uniform distribution of the flow structure in the riser section. In the higher bed voidage region (center region of column), there is a lower drag coefficient, and in the lower bed voidage region, there is a higher drag coefficient. The drag coefficient values obtained from Eq. (16) are higher than the experimental values due to radial flow segregation (Shilapuram and Sai 2012). It is imperative that new modeling work is needed to account for non-uniformity in the flow structure. For the sake of simplicity, Eq. (16) can be used to determine the local drag coefficient, and the average value can be considered for estimation of pressure drop in the column. The average drag coefficient over the cross-sectional area can be estimated using local drag coefficient values obtained from Eq. (16) as follows:

$$\overline{C_D} = \frac{1}{A} \int_0^R 2\pi r C_{Dx} dr \quad (17)$$

Alternatively, average drag coefficient can be estimated if the average value of voidage is known over the cross section:

$$\frac{\overline{C_D}}{C_{D\infty}} = \varepsilon_L^{-2n} \quad (18)$$

The average voidage over cross-sectional area can be estimated if local voidage is known:

$$\overline{\varepsilon_L} = \frac{1}{A} \int_0^R 2\pi r \varepsilon_L dr \quad (19)$$

4 Mixing studies

An accurate knowledge of mixing behavior of liquid and solid phases is essential to model, design, and optimize the liquid fluidization system. Accordingly, the mixing behavior of individual phases has been extensively studied in conventional SLFB since 1960. However, scanty information is available in the literature on liquid phase mixing in SLCFBs. A summary of previous work on liquid phase mixing in SLCFB is given in Tables 3 and 4. The previous studies on hydrodynamics have shown that the flow characteristics in SLCFB systems are distinct from those in conventional SLFBs. The non-uniform radial distribution of liquid velocity, particle velocity, and solid holdup has been observed in SLCFBs. The empirical models developed for conventional SLFBs cannot be applied to SLCFBs. The mixing behavior strongly depends on hydrodynamic characteristics of the system which are governed by geometrical (column diameter, column verticality, type of distributor, presence of internals, and end effects), system (physical properties of solid and liquid phases), and operating (superficial liquid velocity and solid circulation rate) parameters. Therefore, the study of liquid mixing is necessary for further understanding of the flow behavior in SLCFBs.

Table 3: Summary of previous work on mixing studies

Investigator	Liquid Phase	Solid Phase	System Parameters		Geometric Parameters		Operating Parameters		Input Method*/ Fluidization system [#]
			Liquid Phase properties Density, (kg.m ⁻³), Viscosity, (kg.m ⁻¹ .s ⁻¹)	Solid Phase properties Density, (kg.m ⁻³), Diameter, (mm)	Column diameter (mm)	Column Height (m)	Superficial liquid velocity (mm.s ⁻¹)	Solid circulation rate (kg.m ⁻² .s ⁻¹)	
Zheng, (2001)	Water	Glass beads	$\rho_L = 1000$ $\mu_L = 0.001$	$\rho_S = 2590$ $dp = 0.508$	76.0	3.0	130- 200	0.84-10.0	2/A
Chen et al., (2001)	Water	Glass beads	$\rho_L = 1000$ $\mu_L = 0.001$	$\rho_S = 2490$ $dp = 0.4$	140	3	40- 145	5- 25	2/A
Cho et al., (2005)	Water, Aqueous solution of carboxymethylcellulose	Glass beads	$\rho_L = 1000- 1003$ $\mu_L = 0.00096-0.038$	$\rho_S = 2500$ $dp = 1- 3$	102	3.5	150- 450	-	2/A
Singh et al., (2008)	Water	Glass beads	$\rho_L = 1000$ $\mu_L = 0.001$	$\rho_S = 2500$ $dp = 0.5-0.92$	100	0.5	8.5- 10.6	23- 45 g.min ⁻¹	1/B
Verma et al., (2010)	Water	Glass beads	$\rho_L = 1000$ $\mu_L = 0.001$	$\rho_S = 2500$	100	0.5	3.4- 5.5	24- 35 g.min ⁻¹	1/B
Kalaga et al., (2012)	Water	Resin, glass beads	$\rho_L = 1000$ $\mu_L = 0.001$	$\rho_S = 1100- 2500$ $dp = 0.1-0.72$	50- 100	1.7- 2	10.62-29.72	-	2/C

* Input method: 1-Step, 2- Pulse; # Fluidization system: A- SLCFB, B- multistage SLFB, C- multistage SLCFB

Table 4: Summary of empirical correlations available to estimate liquid phase dispersion coefficient

Investigator	Correlation Proposed	Range of Applicability	Remark
Chen et al., (2001)	$Pe_a = 0.05(1 - \varepsilon_L)^{-0.41}(\text{Re})^{0.44}$ $Pe_r = 0.044(1 - \varepsilon_L)^{-0.287}(\text{Re})^{0.64}$	<p>Re = 16- 58 Pe = 7- 13</p>	The dispersed plug flow model has been used to develop the correlation for axial and radial Peclet numbers. The axial and radial Peclet numbers increases with an increase in superficial liquid velocity and solid hold up. The transverse mixing in SLCFB is high as compared to conventional SLFB.
Cho et al., (2005)	$Pe_r = \frac{d_p V_L}{D_r} = 8.47 \left(\frac{d_p}{D} \right)^{1.303} \left(\frac{d_p V_L \rho_s}{\mu_L} \right)^{-0.056} \left(\frac{G_s}{V_L \rho_s} \right)^{-0.311}$	<p>Re = 19.48- 121.9 Pe = 0.059- 0.49</p>	The radial dispersion increases with an increase in particle size and solid circulation rate. However, reverse is true with respect to superficial liquid velocity and liquid viscosity.
Kalaga et al., (2012)	$\ln \left(\frac{D_L}{d_p V_L} \right) = 5.609 - 0.179 \ln(\text{Re}) - 0.199 \ln(Ga) - 0.568 \ln \left(\frac{\rho_L}{\rho_s - \rho_L} \right) + 1.339 \ln \left(\frac{V_L}{V_L - V_{mf}} \right)$ $\ln \left(\frac{D_L}{d_p V_L} \right) = 5.734 - 0.167 \ln(\text{Re}) - 0.262 \ln(Ga) - 0.706 \ln \left(\frac{\rho_L}{\rho_s - \rho_L} \right) + 0.704 \ln \left(\frac{V_L}{V_L - V_{mf}} \right)$ $\ln \left(\frac{D_L}{d_p V_L} \right) = 9.067 - 1.592 \ln(\text{Re}) - 0.198 \ln(Ga) - 0.462 \ln \left(\frac{\rho_L}{\rho_s - \rho_L} \right) - 1.470 \ln \left(\frac{V_L}{V_L - V_{mf}} \right)$	<p>Re = 0.34- 9.23 Pe = 0.0038- 0.051</p>	The dispersed plug flow model has been used to estimate dispersion coefficient. The dispersion coefficient increases with an increase in superficial liquid velocity, particle size, and particle density.

4.1 Physical properties of liquid and solid phases

The effect of particle density, particle size, liquid viscosity, and liquid density on longitudinal liquid phase dispersion has been well investigated in conventional SLFB by various researchers. Cairns and Prausnitz (1960) and Chung and Wen (1968) have found that liquid phase axial dispersion coefficient increases with an increase in particle density and particle size. Similar observation has been reported by Asif et al. (1992) and Kalaga et al. (2012). Mehta and Shemilt (1976) have reported that axial dispersion coefficient decreases with an increase in liquid viscosity. Similar result has been reported by Tan and Krishnaswamy (1989) while investigating effect of liquid density on axial dispersion coefficient.

Unlike conventional SLFB, very few reports are available in the literature that deal with the effect of physical properties of solid and liquid phases on the mixing characteristics of SLCFB. These reports are mainly by Cho et al. (2005), Singh et al. (2008), and Kalaga et al. (2012). Cho et al. (2005) have studied the effect of solid particle size and liquid viscosity on radial dispersion coefficient. As the particle size increases, the terminal settling velocity also increases, which necessitates higher relative liquid velocity to maintain a given bed voidage for solid particles with a higher terminal velocity. Consequently, the power consumption per unit volume increases, which results in higher intensity of turbulence, which in turn leads to higher extent of mixing. On the other hand, the terminal settling velocity of solid particle decreases with an increase in liquid viscosity. Therefore, radial liquid phase dispersion coefficient decreases with increasing liquid viscosity. In contrast to the findings of Cho et al. (2005), interestingly, the effect of particle size on liquid phase mixing has been observed to be insignificant by Singh et al. (2008) while investigating liquid phase mixing characteristics of multistage SLFB. This may be due to the use of a narrow size range of the solid particles. Kalaga et al. (2012) have studied the effect of particle size and particle density in the riser column. It has been observed that axial dispersion coefficient decreases with an increase in solid particle size and density in the riser column. This is due to the fact that the solid particle size and density modifies the profiles of axial mixing. For a given superficial liquid velocity, the number of solid particles having higher Galileo number circulating from riser section to multistage section is less compared to that of the solid particles having low Galileo number which means a decrease in solid holdup. As the solid holdup decreases, the number of solid particles decreases, which in turn decreases solid particle interaction resulting in decrease in liquid phase mixing.

4.2 Superficial liquid velocity

The effect of superficial liquid velocity on liquid phase axial mixing in conventional SLFB has been well studied in the past. Most of these studies report that liquid phase axial dispersion coefficient is a strong function of superficial liquid velocity. A maximum value of axial dispersion coefficient appears at superficial liquid velocity corresponding to bed voidage of about 0.7 (Cairns and Prausnitz 1960, Mehta and Shemilt 1976, Kim and Kim 1983, Tan and Krishnaswamy 1989). On the contrary, it has been observed that axial dispersion coefficient monotonically increases with an increase in superficial liquid velocity in SLCFB (Kalaga et al. 2012).

Zheng (2001) has measured axial liquid phase dispersions at a given radial position in the riser column using a dual conductivity probe. The local axial dispersion coefficients increase with increasing superficial liquid velocity. For a given superficial liquid velocity, the local axial dispersion coefficients are greater at the axis than near the wall region of the riser. This non-uniformity of axial liquid is believed to be caused by the radial non-uniform distribution of liquid

velocity and bed voidage in the riser column. Chen et al. (2001) have investigated the effect of superficial liquid velocity on axial and radial mixing behaviors of SLCFB using the electrolyte tracer technique. It has been found that the axial and radial mixing of the liquid phase depends on the superficial liquid velocity and solid holdup. With an increase in solid holdup, axial and radial Peclet numbers decrease, whereas the opposite effect is observed when the superficial liquid velocity is increased. The liquid mixing behavior of conventional SLFB has also been compared with SLCFB. The values of radial Peclet number obtained are comparable, indicating that radial mixing characteristics in both cases are similar. However, the reduced axial back mixing has been observed as compared to conventional SLFB. Cho et al. (2005) have investigated radial liquid phase mixing in the riser column using viscous liquid medium. The infinite space model has been employed to obtain the radial dispersion coefficient from the radial concentration profiles of the tracer. It has been observed that the radial dispersion coefficient decreases considerably with increasing superficial liquid velocity. This is due to the fact that the increase of superficial liquid velocity leads to the decrease in solid holdup for a given solid circulation rate. Thus, the contact efficiency of liquid phase with solid particles decreases at higher superficial liquid velocity as compared to that at lower superficial liquid velocity.

Verma et al. (2010) have carried out residence time distribution (RTD) experiments in multistage SLFB using glass beads as solid phase, water as fluidizing medium, and sodium hydroxide solution as tracer. The twodimensional axial plug flow dispersion model has been used to ascertain the extent of liquid phase mixing. The axial liquid phase dispersion coefficient increases with an increase in superficial liquid velocity within flooding and loading states of multistage column. Similar observation has been reported by them in their previous study (Singh et al. 2008) while investigating the effect of superficial liquid velocity on axial liquid phase dispersion coefficient in multistage SLFB. Kalaga et al. (2012) have investigated the mixing characteristics of the liquid phase in the riser and multistage SLCFB by using the pulse response technique. Computational fluid dynamic (CFD) modeling and the dispersed plug flow model are successfully used to describe the liquid phase mixing. The experimental findings show that the liquid phase axial dispersion coefficient increases with an increase in the liquid velocity in the riser column. The higher liquid velocity leads to non-uniform radial distribution of the liquid velocity which in turn leads to an increase of mixing of the liquid phase, consistent with the literature (Chen et al. 2001, Zheng 2001). In multistage SLFB, it has been observed that axial dispersion coefficient increases with an increase in superficial liquid velocity. The values of axial dispersion coefficient, however, are considerably lower as compared to the values obtained in the conventional SLFB. The presence of the sieve trays tends to render the velocity profile more uniform and physically reduces the backmixing in the liquid phase by eliminating any cross flow across the sieve plate. This in turn reduces the axial liquid mixing in multistage SLFB.

4.3 Solid circulation rate

The system (physical properties of solid and liquid phases) and operating (superficial liquid velocity) parameters strongly affect the mixing characteristics in SLFB. For given geometrical parameters, physical properties of liquid and solid phases, and superficial liquid velocity completely describe liquid phase mixing characteristics in SLFB. It would be worthwhile to mention that superficial liquid velocity is only the operating parameter in SLFB. However, solid circulation rate is also one of the operating parameters which need to be considered to elucidate mixing characteristics in SLCFB.

Zheng (2001) has measured axial liquid dispersion coefficient at radial positions in the riser column of SLCFB. The experimental results reveal that the axial dispersion coefficient is affected not only by the system and operating parameters but also by radial position. The local axial dispersion coefficient increases with an increase in solid circulation rate for a given superficial liquid velocity. The axial dispersion coefficients determined are greater at the axis than near the wall region of the riser. The radial distributions of liquid velocity and solid holdup show an opposite trend from the center to the wall of the riser, with lower liquid velocity but higher solid holdup appearing at the wall. Although axial dispersion coefficient is proportional to liquid velocity and solid holdup, the contribution of liquid velocity to the local axial dispersion coefficient seems to overcome the counterbalance of solid holdup and to dominate the radial profile of the liquid dispersion coefficients. This could be reasonably accepted since solid holdup alters within a narrow range across the radial positions in the riser column. Chen et al. (2001) have employed pulse response technique to investigate liquid phase mixing characteristics in the riser column of the SLCFB. The axial and radial dispersion coefficients have been determined using two-dimensional diffusion model. It has been seen that axial dispersion coefficient increases with an increase in solid circulation rate for a given superficial liquid velocity. This is due to the fact that an increase in solid circulation rate tends to increase the solid holdup which in turn tends to favor radial non-uniformity with respect to liquid velocity, solid holdup, and solid particle velocity in the riser column. The radial non-uniformity of local liquid velocity accompanied by the rise in solid holdup tends to enhance the axial backmixing of the liquid phase in the riser column. A similar trend has been observed with respect to radial dispersion coefficient. It has been found that the radial dispersion coefficients are about one order of magnitude less than the axial dispersion coefficients.

Cho et al. (2005) have measured radial dispersion coefficient in the riser column. The experimental results indicate that radial dispersion coefficient increases with an increase in solid circulation rate. An increase in solid circulation rate tends to increase solid holdup in the riser for a given operating condition, which consequently results in the increase of hindrance effects on the fluid element flowing upward continuously. The hindered fluid elements tend to spread radially rather than rise quickly; thus, the radial dispersion of the liquid phase increases. Similar observations have been seen by Singh et al. (2008) and Verma et al. (2010) while investigating the effect of solid circulation rate on axial liquid dispersion coefficient in multistage SLFB.

4.4 Empirical correlations

The mixing characteristics of SLFB have been extensively studied and adequately presented in the form of empirical correlations since 1960. However, very few attempts have been accomplished to propose empirical correlations to predict liquid phase dispersion in SLCFB. These attempts are mainly by Chen et al. (2001), Cho et al. (2005), and Kalaga et al. (2012). Chen et al. (2001) have measured axial dispersion coefficient in the riser column of the SLCFB using the electrolyte tracer technique. It has been observed that radial non-uniformity with respect to liquid velocity and voidage affects the axial and radial mixing characteristics in the riser column. Based on the experimental findings, the axial and radial Peclet numbers have been correlated with voidage and Reynolds number.

$$Pe_a = 0.05(1 - \varepsilon_L)^{-0.41}(Re)^{0.44} \quad (20)$$

$$Pe_r = 0.044(1 - \varepsilon_L)^{-0.287}(Re)^{0.64} \quad (21)$$

It has been shown that axial and radial mixing of the liquid phase depends on the superficial liquid velocity and solid holdup. With an increase in solid holdup, the axial and radial Peclet

numbers decrease, indicating that the axial and radial liquid phase mixing becomes more intensive, whereas the opposite effect has been observed when the superficial liquid velocity is increased. Cho et al. (2005) have investigated radial liquid dispersion in the riser column of the SLCFB. The effect of liquid velocity, solid circulation rate, particle size, and liquid viscosity on the liquid radial dispersion coefficient has been demonstrated using the infinite space model. The liquid radial dispersion coefficient decreases with increasing liquid velocity or viscosity, but it increases as the solid circulation rate or particle size increases. The following empirical correlation has been in terms of dimensionless groups:

$$Pe_r = \frac{d_p V_L}{D_r} = 8.47 \left(\frac{d_p}{D} \right)^{1.303} \left(\frac{d_p V_L \rho_s}{\mu_L} \right)^{-0.056} \left(\frac{G_s}{V_L \rho_s} \right)^{-0.311} \quad (22)$$

The predicted data fit well to the experimental with a correlation coefficient of 0.94. Equation (22) covers the range of variables $0.0592 \leq Pe_r \leq 0.492$, $0.0098 \leq d_p/D \leq 0.0294$, $19.48 \leq Re \leq 121.9$, and $0.0018 \leq G_s/V_L \rho_s \leq 0.0133$.

Kalaga et al. (2012) have employed the pulse response technique to investigate the axial mixing characteristics of the liquid phase in SLFB and multistage SLCFB. The polymeric resin ($\rho_s = 1100 \text{ kg m}^{-3}$) and glass beads ($\rho_s = 2500 \text{ kg m}^{-3}$) in the size range of 0.365–0.725 mm and 0.1–0.2 mm have been used, respectively, as a solid phase, while water was used as liquid phase. Based on Rayleigh's method of dimensional analysis, they have proposed the following empirical correlations for (i) conventional SLFB, (ii) multistage SLCFB, and (iii) the riser column of multistage SLCFB, respectively:

$$\ln \left(\frac{D_L}{d_p V_L} \right) = 5.609 - 0.179 \ln(Re) - 0.199 \ln(Ga) - 0.568 \ln \left(\frac{\rho_L}{\rho_s - \rho_L} \right) + 1.339 \ln \left(\frac{V_L}{V_L - V_{mf}} \right) \quad (23)$$

$$\ln \left(\frac{D_L}{d_p V_L} \right) = 5.734 - 0.167 \ln(Re) - 0.2621 \ln(Ga) - 0.706 \ln \left(\frac{\rho_L}{\rho_s - \rho_L} \right) + 0.704 \ln \left(\frac{V_L}{V_L - V_{mf}} \right) \quad (24)$$

$$\ln \left(\frac{D_L}{d_p V_L} \right) = 9.067 - 1.592 \ln(Re) - 0.198 \ln(Ga) - 0.462 \ln \left(\frac{\rho_L}{\rho_s - \rho_L} \right) - 1.470 \ln \left(\frac{V_L}{V_L - V_{mf}} \right) \quad (25)$$

The standard deviation of Eqs. (23)–(25) with experimental data has been found to be 25%, 10%, and 10%, respectively. Equations (23)–(25) demand estimation of minimum fluidization velocity prior to their use. Although there are correlations available in the literature to predict minimum fluidization velocity, they are accompanied by a certain amount of inaccuracy. Therefore, Eqs. (23)–(25) would cause corresponding deterioration in the prediction of dispersion coefficients.

5 Mass transfer characteristics

Separation processes using fixed and expanded beds of ion exchange resins (and other stationary phases) are commonly employed in the industry to recover dissolved solute from a variety of mixtures owing to the good regeneration ability of solid phase. It is cumbersome to use conventional physical separation methods such as fractional distillation, liquid-liquid extraction, solvent-aided separation, crystallization, and membrane separation to reduce the impurity to the new standards. Chromatographic separation offers an edge over the conventional methods owing to its ability to achieve higher levels of purity. The chromatographic separations carried out using classical fluidized bed, however, possess several limitations. These limitations are mentioned in Section 1 coherently. SLCFBs can expediently be used for chromatographic separation since the loading and regeneration steps can be simultaneously implemented with the

amount of stationary phase that can be theoretically equivalent to a breakthrough quantity. This feature offers substantially lower capital cost compared to the conventional fixed bed chromatography where the quantity of stationary phase is several times higher than that needed for a breakthrough. Additionally, SLCFB also provides some obvious advantages such as large liquid-solid contact efficiency, higher throughput and enhanced mass transfer due to higher velocity, reduced back mixing of phases, and uniform flow pattern.

These instant eye-catching features of SLCFBs have enabled researchers to apply them for the processes wherein mass transfer between liquid and solid phases plays a vital role. In the last two decades, SLCFBs are moderately employed for the recovery of protein from fermentation broth wherein mass transfer between solid and liquid phase determines the overall rate of production. SLCFB system was used as an integrated reactor and regenerator system wherein two different operations (adsorption and desorption) were carried out simultaneously in two separate columns (downcomer and riser, respectively) in continuous mode with continuous circulation of solid phase between the two columns (Lan et al. 2000, Gaikwad et al. 2008, Mazumder et al. 2009a). Lan et al. (2000) have developed a model for continuous protein recovery in SLCFB ion-exchange system assuming

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and diffusion mass transfer from the liquid phase to the adsorbent surface and diffusion through the pore of the ion exchange resins as rate-limiting steps. In these studies, the MTCs have been estimated by using empirical correlations developed based on homogeneous fluidization concept.

The downcomer column is operated in conventional fluidization regime (superficial liquid velocity is less than terminal settling velocity of solid particle) with continuous inlet and outlet of solid particles. Therefore, it may be assumed that solid holdup is uniform radially and axially in the column. However, the flow structure in riser column is distinct as compared to the downcomer column. The hydrodynamic studies in the riser column depict that solid holdup is noticeably non-uniform in radial direction. Nevertheless, MTC in the riser was estimated based on the postulate that homogeneous fluidization prevails in the riser column (Lan et al. 2000, Mazumder et al. 2009a). This is due to the lack of information on mass transfer characteristics of SLCFB. No mass transfer study has been conducted so far in the riser column of SLCFB to the best of the authors' knowledge. Therefore, it is essential to undertake systematic investigation to understand mass transfer characteristics of the riser column and downcomer column as well. The present part of this article reviews the reports pertaining to mass transfer studies available in the literature on SLCFB. Some pioneering mass transfer studies conducted in conventional SLFB have also been reviewed since the flow structure in the downcomer column is similar to that of conventional SLFB.

5.1 Physical properties of liquid and solid phases

Literature shows that the effect of physical properties of liquid and solid phases on mass transfer in SLFB is moderately encountered using dimensionless number *viz.* Reynolds number, Galileo number, and density number. The reports are available wherein the effect of particle size and shape has been studied separately by many investigators (Evans and Gerald 1953, Damronglerd et al. 1975, Tournie et al. 1979, Rahman and Streat 1981a,b, Ballesteros et al. 1982, Shen et al. 1985, Livingston and Noble 1993). However, the conclusions arrived at are not consistent with respect to the exponent of particle diameter. The exponent of particle diameter is reported in the range of -0.031 to -2.87 . Most of the researchers have reported the value around -0.5 (Damronglerd et al. 1975, Rahman and Streat 1981a,b, Livingston and Noble 1993). Ballesteros et al. (1982) have also investigated the influence of the shape of solid particles on the mass transfer rate and shown that MTC is proportional to sphericity with exponent equal to 1.35. Recently, Kalaga et al. (2014) have measured solid-liquid MTC in both conventional SLFB and solid-liquid multistage fluidized bed by using the system of dissolution of benzoic acid in water. The solid particle size was varied in the range of 0.16–0.89 mm and the voidage in the range of 0.4–0.9. It has been observed that MTC is proportional to $d_p^{-0.291}$ and $d_p^{-0.167}$ in conventional SLFB and solid-liquid multistage fluidized bed, respectively.

5.2 Superficial liquid velocity and bed voidage

Two approaches are commonly adopted in the literature to describe mass transfer in fixed and fluidized beds. The first approach uses dimensionless analysis method wherein Sherwood number is correlated with dimensionless groups like Reynolds number, Galileo number, particle density number, and Schmidt number. In the second approach, the Chilton-Colburn mass transfer factor is correlated with Reynolds number. The effect of superficial liquid velocity on the mass transfer has been illustrated in terms of Reynolds number.

Few reports are also available in the literature wherein MTC is directly related to superficial liquid velocity (Evans and Gerald 1953, Laguerie and Angelino 1975, Shen et al.

1985, Kikuchi et al. 1988, Seguin et al. 1996, Boskovic- Vragolovic et al. 2005). Evans and Gerald (1953) have reported that MTC increases with an increase in superficial mass velocity raised to 0.33 power. Shen et al. (1985) have reported the exponent value as 0.27, which is quite similar to the results obtained by Laguerie and Angelino (1975), who have found the exponent of superficial liquid velocity equal to 0.217. Kikuchi et al. (1988) have shown that MTC is proportional to superficial velocity raised to 0.196 power, while Seguin et al. (1996) have found the value equal to 0.33. It would be worthwhile to mention that few investigators (Damronglerd et al. 1975, Tournie et al. 1979, Ballesteros et al. 1982, Livingston and Noble 1993) have observed that MTC is independent of superficial liquid velocity. The argument of authors is that total pressure drop is independent of superficial liquid velocity above minimum fluidization velocity. However, it is known that the value of drag coefficient is dependent on superficial liquid velocity. In fact, the value of drag coefficient varies as voidage raised to power -4.8 (Richardson and Zaki 1954, Joshi 1983, Reddy et al. 2010).

The effect of bed voidage on mass transfer factor has been observed and considered by many previous investigators (Sengupta and Thodos 1962, Wilson and Geankoplis 1966, Couderc et al. 1972, Damronglerd et al. 1975, Laguerie and Angelino 1975, Dwivedi and Upadhyay 1977, Rahman and Streat 1981a, 1985, Shen et al. 1985, Wildhagen et al. 1985, Seguin et al. 1996). Sengupta and Thodos (1962) and Wilson and Geankoplis (1966) have shown that mass transfer factor is inversely proportional to bed voidage. Similar observation has been made by Dwivedi and Upadhyay (1977), Rahman and Streat (1981a, 1985), Wildhagen et al. (1985), and Seguin et al. (1996). Couderc et al. (1972) have reported that mass transfer factor is inversely proportional to the voidage raised to power two. Damronglerd et al. (1975) have shown that mass transfer factor is proportional to the bed voidage raised to power -1.20 and -2.40 when bed voidage is less than 0.815 and greater than 0.815 , respectively. Laguerie and Angelino (1975) have reported the bed voidage exponent value equal to -1.8 . Shen et al. (1985) have shown that mass transfer factor is proportional to bed voidage raised to power -1.42 . Another approach is also available in the literature wherein bed voidage has been replaced by Galileo number, particle Reynolds number ($d_p L V S^\infty / \mu L$), and often a density number (Damronglerd et al. 1975, Tournie et al. 1979, Ballesteros et al. 1982, Livingston and Noble 1993). This exercise has been performed in order to increase the predictive ability of empirical equation by reducing the error imposed due to bed voidage determination, in particular. An analysis of experimental data of Damronglerd et al. (1975), Tournie et al. (1979), Ballesteros et al. (1982), and Livingston and Noble (1993) shows that MTC does not depend either on superficial liquid velocity or on particle diameter in SLFB. This means that physical properties of solid and liquid entirely determine the effect of superficial liquid velocity and particle diameter on mass transfer.

5.3 Empirical correlations

Numerous experimental correlations have been proposed to ascertain mass transfer characteristics of solid-liquid system using dimensionless analysis. Three different equations have been commonly used in the following form, respectively:

$$Sh = f(Re, \varepsilon_L, Sc) \quad (26)$$

$$Sh = f(Re, Ga, Mv, Sc) \quad (27)$$

$$j_D = f(Re) \quad (28)$$

The dimensionless groups, especially Re and j_D , have been modified by incorporating friction factor term and/or bed voidage in empirical correlations to obtain satisfactory fit with the experimental data (McCune and Wilhelm 1949, Gamson 1951, Evans and Gerald 1953, Fan et al.

1960, Sengupta and Thodos 1962, Upadhyay and Tripathi 1975, Dwivedi and Upadhyay 1977, Tournie et al. 1979, Rahman and Streat 1981a, Shen et al. 1985, Boskovic-Vragolovic et al. 2005).

There are few investigations reported in the literature wherein empirical correlations have been used to determine MTC as a function of the physical properties of solid and liquid phases (Damronglerd et al. 1975, Tournie et al. 1979, Ballesteros et al. 1982, Livingston and Noble 1993). Damronglerd et al. (1975) have replaced bed voidage term by $Re/Ga^{1/2}$ using Richardson-Zaki equation of bed expansion. Tournie et al. (1979) have demonstrated that the MTC is independent of superficial liquid velocity for a given Galileo, density, and Schmidt numbers. Further, it has also been shown that the MTC is a weak function of particle size. Ballesteros et al. (1982) have modified the correlation proposed by Tournie et al. (1979) to incorporate the shape factor. It has been observed that the MTC is proportional to shape factor raised to 1.35 power. Livingston and Noble (1993) have proposed a correlation wherein MTC is related to particle Reynolds number, indicating that mass transfer rate is independent of superficial liquid velocity and dependent on physical properties of liquid and solid phases.

Verma et al. (2009) have studied mass transfer in radial cross flow multistage SLFB. They have modified the mass transfer correlation available in the literature in terms of dimensionless quantity, Sherwood number, by introducing a new term called slip ratio:

$$Sh = 12.758 Re^{0.9948} \varepsilon_L^{2.8855} Ga^{-0.476} \left(\frac{V_S}{V_L} \right)^{0.3218} \quad (29)$$

The equation is essentially developed for countercurrent flow of solid particle and liquid in multistage SLFB. No attempt has been made to correlate data available in the literature using Eq. (29). Kalaga et al. (2014) have critically analyzed the literature data pertaining to mass transfer in SLFB and multistage SLFB and proposed the following correlation to predict the MTC:

$$\left(\frac{k_{SL}}{V_L} \right) = 0.672 (Re)^{0.643} (Sc)^{-0.906} (Fr)^{0.649} (Mv)^{-0.216} \left(\frac{g \mu_L}{\rho_L V_L^3} \right)^{0.894} \quad (30)$$

The proposed empirical correlation satisfactorily correlates the mass transfer data reported in the literature with standard deviation of 30% and mean deviation of 21%.

The foregoing discussion indicates that adequate attempts have been made in the literature to propose mass transfer correlations in SLFB. A summary is given in Tables 5 and 6. However, only two reports are available in the literature which deal with mass transfer in multistage SLFB (Verma et al. 2009 and Kalaga et al. 2014). Further, empirical correlations developed, so far, in SLFB are not consistent, and any one correlation is capable of fitting only a subset of a huge pool of data. Therefore, it is essential to understand apparent discrepancies among the different correlations.

Table 5: Summary of previous work on mass transfer studies

Authors	Solid phase			Liquid phase			Hardware		ϵ_L	Range		
	Material	dp (mm)	ϕ	Material	ρ_L (kg. m ⁻³)	μ_L kg.m ⁻¹ . s ⁻¹	D (m)	H (m)		Re	Sh or j _D	Sc
McCune and Wilhem, (1949)	2- naphthol	3.1-6.3	-	Water	1000	0.001	0.102	-	0.5-0.95	14-1755	j _D = 0.021-0.58	1200-1500
Gamson, (1951)	β - naphthol	3.05-6.35	0.87	Water	1000	0.001	-	-	0.49-0.94	7-7000	j _D = 0.08-0.40	1-2000
Evans and Gerald, (1953)	Benzoic acid	0.5-2	0.5-0.65	Water	1000	0.001	0.054	0.5	0.47-0.91	1-100	-	970
Fan et al., (1960)	Benzoic acid	0.7-2.4	1	Water	1000	0.001	0.051	0.61	0.65-0.90	5-130	Sh= 21-116	1020-1540
Rowe and Claxton, (1965)	Benzoic acid	38.1	1	Water	1000	0.001	-	-	0.26-0.47	200-1230	Sh= 149-1455	1310-1670
Couderc et al., (1972)	Benzoic acid	4.9-6.1	1	Water	1000	0.001	0.192	0.6	0.50-0.70	112-288	j _D = 0.001-0.002	970
Damronglerd et al., (1975)	Benzoic acid	6.4-8.2	1	Water	1000	0.001	0.094	2	0.70-0.95	590-2120	j _D =0.01-0.05; Sh= 37-45	1-1.06
Upadhyay and Tripathi, (1975)	Benzoic acid	9- 12	0.5-0.81	Water	1000	0.001	0.045	-	0.50-0.90	0.0405-11610	j _D = 0.035-4.18	572-1350
Vanadu-rongwan et al., (1976)	-	-	1	Water	1000	0.001	-	-	0.53-0.96	180-1320	-	300-2000
Dwivedi and Upadhyay, (1977)	Benzoic acid, β -naphthol, MgSO ₄	0.023-0.127	-	Water	1000	0.001	-	-	0.40-0.95	0.01-1000	-	0.3-1755

Koloini et al., (1977)	Ion exchange resins	0.27-0.55	1	Dilute NaOH solution	1000	0.001	0.035	0.2	0.5-0.92	0.22-6.4	Sh= 2.02-21.4	0.052
Tournie et al., (1979)	Benzoic acid, β -naphthol, citric acid	4.6-8.2	-	Water	1000	0.001	-	-	0.40-0.95	16-1320	-	305-1595
Rahman and Streat, (1981a)	Ion exchange resins	0.4-1	1	Dilute NaOH solution	997	0.0008	0.051	-	0.55-0.85	2-25	-	336-451
Bellesteros et al., (1982)	Benzoic acid	-	0.7-1	Water	1000	0.001	0.05	1	0.8-1	-	-	-
Shen et al., (1985)	Benzoic acid	0.3-0.8	0.58-0.63	Water	1000	0.001	-	-	0.54-0.65	0.644-7.312	$j_D=0.28-0.84$, Sh= 5.69-23.44	997.6-1315.7
Livingston and Noble, (1993)	Ion exchange resin	0.4-0.9	1	NaOH solution	1000	0.0008-0.0063	0.051	-	0.61-0.81	0.25-22.2	Sh= 5.9- 29.8	368-2896
Boskovic-Vragolovic et al., (2005)	Benzoic acid	0.021	1	Water	977	0.0009	0.04	-	0.41-0.46	-	-	969
Kalaga et al., (2014)	Benzoic acid	0.02-0.89	-	Water	1000	0.001	0.1	1.6	0.4-0.95	0.008-11	$j_D= 0.16-2.1$, Sh= 0.1- 26	967

Table 6: Summary of correlations available to estimate mass transfer coefficient

Investigator	Correlation	Reynolds number	Remark
Evans and Gerald (1953)	$j_D \frac{(1-\varepsilon_L)d_p^{2.6}}{f'} = 2.3 \times 10^{-10} (Re)^{1.33}$	$Re = \frac{d_p V_L \rho_L}{\mu_L}$	The mass transfer coefficient has been determined using dissolution of non- spherical particles. The mass transfer coefficient is a sole function of liquid flow rate and particle size for a given liquid and solid system.
Fan et al., (1960)	$Sh = 2 + 1.483[(1-\varepsilon_L)Re]^{1/2} Sc^{1/3}$	$Re = \frac{d_p V_L \rho_L}{\mu_L}$	A new solid liquid contacting technique, known as semi-fluidization, has been employed to propose mass transfer correlation for fixed, semi-fluidized and fluidized beds. The correlation has been obtained by plotting mass transfer factor against modified Reynolds number.
Sengupta and Thodos (1962)	$\varepsilon_L j_D = 0.0108 + \frac{0.929}{Re^{0.58} - 0.483}$	$Re = \frac{d_p V_L \rho_L}{\mu_L}$	A single unified correlation, independent of the type of bed, has been proposed by plotting $\varepsilon_L j_D$ against Reynolds number.
Rowe and Clexton, (1965)	$Sh = \frac{2}{[1 - (1-\varepsilon_L)]^{1/3}} + \frac{0.7}{\varepsilon_L} Re^n Sc^{1/3}$ $\frac{(2-3n)}{(3n-1)} = 4.65 Re^{-0.28}$	$Re = \frac{d_p V_L \rho_L}{\mu_L}$	The mass transfer coefficient has been estimated using surface renewal theory for binary SLFB. The correlation has been proposed for low Reynolds number region, in particular.
Couderc et al., (1972)	$Sh = \frac{5.4 \times 10^{-2}}{\varepsilon_L^2} Re Sc^{1/3}$	$Re = \frac{d_p V_L \rho_L}{\mu_L}$	The mass transfer coefficient is a sole function of liquid flow rate and particle size for a given physical properties of liquid and solid phases. It has been put forth that discrepancies in available mass transfer data is may be due to either equipment or procedure.
Damronglerd et al., (1975)	$Sh = 0.301 Re^{0.038} Ga^{0.305} Sc^{1/3}$ $Sh = 0.138 Re^{-0.086} Ga^{0.412} Sc^{1/3}$	$Re = \frac{d_p V_L \rho_L}{\mu_L}$	The dependence of mass transfer coefficient on voidage has been eliminated using Reynolds number and Galileo number. The correlation has been proposed using dimensionless method of analysis. The mass transfer coefficient is independent of liquid velocity and solely depends on physical properties of liquid and solid phases.
Upadhyay and Tripathi (1975)	$j_D = 3.8155(Re'')^{0.7313} \quad Re'' < 20$ $j_D = 1.6218(Re'')^{-0.4447} \quad Re'' > 20$	$Re'' = \frac{d_p G}{\mu_L (1-\varepsilon_L)}$	The mass transfer coefficient is independent of superficial liquid velocity, particle shape, and column diameter. The mass transfer coefficient increases with decrease in particle

			size. Further, for a given Reynolds number, mass transfer rate is higher in fixed bed as compared to fluidized bed.
Vanadurongwan et al., (1976)	$Sh = 0.215 Re^{0.011} Ga^{0.309} Mv^{0.303} Sc^{0.436} \epsilon > 0.815$ $Sh = 0.115 Re^{-0.076} Ga^{0.39} Mv^{0.503} Sc^{0.436} \epsilon > 0.815$	$Re = \frac{d_p V_L \rho_L}{\mu_L}$	The mass transfer coefficient is independent of superficial liquid velocity. The Sherwood number is proportional to the product of density number and Schmidt number.
Koloini et al., (1977)	$\frac{Sh \epsilon_L}{Sc^{1/3}} = 0.245 \left[\frac{\epsilon_L^{1-2m}}{(1-\epsilon_L)^{1/3}} \right] \left[\frac{1}{(1-\epsilon_L)^{1/3}} - 1 \right] Re Sc^{1/3}$ <p>$m = 1$ for $Re > 1$ and $m = 0.5$ for $Re < 1$</p>	$Re = \frac{d_p V_L \rho_L}{\mu_L}$	The dependence of Sherwood number on voidage has been investigated using modified Rowe's theory
Dwivedi and Upadhyay, 1977	$\epsilon_L j_D = 1.1068 Re^{-7.2} \quad \text{for } Re < 10$ $\epsilon_L j_D = 0.4548 Re^{-0.4069} \quad \text{for } Re > 10$	$Re = \frac{d_p V_L \rho_L}{\mu_L}$	The mass transfer data is arbitrary divided into two parts based on Reynolds number to propose unified correlations. The mass transfer coefficient is inversely proportional to voidage.
Tournie et al., (1979)	$Sh = 0.245 Ga^{0.323} Mv^{0.3} Sc^{0.4}$	$Re = \frac{d_p V_L \rho_L}{\mu_L}$	The mass transfer coefficient is independent of superficial liquid velocity and sphere diameter. The discrepancies in available mass transfer data is may be due to either equipment or procedure.
Rahman and Streat, (1981a)	$Sh = 0.75 Re_\infty^{1/2} Sc^{1/3}$	$Re_\infty = \frac{d_p V_{S\infty} \rho_L}{\mu_L}$	The mass transfer coefficient increases with an increase in superficial liquid velocity and is independent of initial bed height.
Ballesteros et al., (1982)	$Sh = 0.245 \psi^{1.35} Ga^{0.323} Mv^{0.3} Sc^{0.4}$	$Re = \frac{d_p V_L \rho_L}{\mu_L}$	The effect of particle shape on mass transfer has been enumerated in particular using dissolution of non- spherical particles. For a given particle shape, mass transfer coefficient is independent of superficial liquid velocity and particle size. The shape of particle does not vary during dissolution.
Shen et al., (1985)	$j_D = 0.5060 Re^{-0.4527} \epsilon^{-1.4165}$ $j_D = 1.4891 Re^{-0.4506}$	$Re = \frac{d_p V_L \rho_L}{\mu_L}$ $Re'' = \frac{d_p V_L \rho_L}{\mu_L (1 - \epsilon_L)}$	The mass transfer coefficient is independent of particle size and increases with an increase in superficial liquid velocity. It has been put forth that satisfactory correlations can be obtained if both the fixed and fluidized beds data have been treated separately.

Kikuchi et al., (1988)	$Sh = 2 + 0.52 \left(\frac{E_s^{1/3} d_p^{4/3}}{v} \right)^{0.59} Sc^{1/3}$	$Re' = \frac{d_p V_L \rho_L}{\varepsilon_L \mu_L}$	The correlation has been proposed for single particles, fixed beds, and fluidized beds using Kolmogoroff theory. A new parameter (E_s), energy dissipation rate per unit mass of liquid flowing around a particle, has been used to estimate mass transfer coefficient.
Livingstone and Noble (1993)	$Sh = 0.65 Re_\infty^{-0.468} Sc^{1/3}$	$Re_\infty = \frac{d_p V_{s\infty} \rho_L}{\mu_L}$	The correlation has been developed for low Reynolds number ($0.1 < Re < 10$). The mass transfer coefficient is independent of superficial liquid velocity.
Seguin et al., (1996)	$Sh = 1.615 \left[\frac{4\varepsilon_L}{A_{vd}(1-\varepsilon_L)\tau h_p} \right]^{1/3} Re_p^{1/3} Sc^{1/3}$	$Re_p = \frac{4V_L \tau \rho_L}{\mu_L A_{vd}(1-\varepsilon_L)}$	The correlation has been developed to estimate mass transfer coefficient in fixed bed of various shape of solid particles at low Reynolds number.
Boskovic-Vragolovic et al., (2005)	$j_D \varepsilon_L = 0.64 \left(\frac{Re}{1-\varepsilon_L} \right)^{-0.4}$	$Re = \frac{d_p V_L \rho_L}{\mu_L}$	The mass transfer correlation has been developed using binary fluidized bed. An enhancement in mass transfer has been observed in the presence of inert material. The mass transfer coefficient decreases with an increase in superficial liquid velocity and depends on particle size in fluidized bed.
Verma et al., (2009)	$Sh = 12.758 Re^{0.9948} \varepsilon_L^{2.8855} Ga^{-0.476} \left(\frac{V_s}{V_L} \right)^{0.3218}$	$Re = \frac{d_p V_L \rho_L}{\mu_L}$	A dimensionless analysis method has been employed to propose the correlation for multistage SLFB using their own experimental data. Apart from conventional dimensional numbers, a new term, slip ratio, has been introduced in the correlation.
Kalaga et al., (2014)	$\left(\frac{k_{SL}}{V_L} \right) = 0.672 (Re)^{0.643} (Sc)^{-0.906} (Fr)^{0.649} (Mv)^{-0.216} \left(\frac{g \mu_L}{\rho_L V_L^3} \right)^{0.894}$	$Re = \left(\frac{d_p V_L \rho_L}{\mu_L} \right)$	An increment of ~15% has been found in multistage SLFB as compared to SLFB. For both SLFB and multistage SLFB, mass transfer coefficient increases with an increase in superficial liquid velocity and also in the presence of inert particles. Further, mass transfer coefficient increases with an increase in hole density of distributor and an increase in pressure drop.

5.4 Relationship between mass transfer coefficient and pressure drop

In SLFB, the pressure gradient can be related to drag force acting on a single solid particle as follows:

$$\frac{\Delta P}{L} = \frac{F_d(1 - \varepsilon_L)}{\frac{\pi}{6}d_p^3} \quad (31)$$

The drag force on an individual solid particle in a solid-liquid fluidized bed is given as follows:

$$F_d = C_D \left(\frac{\pi}{4}d_p^2 \right) \left(\frac{1}{2}\rho_L V_L^2 \right) \quad (32)$$

Substituting Eq. (32) in Eq. (31), we get following expression:

$$\frac{\Delta P}{L} = \frac{3}{4}C_D \frac{1}{d_p} (\rho_L V_L^2) (1 - \varepsilon_L) \quad (33)$$

Pandit and Joshi (1998) have obtained the following expression for pressure gradient in solid-liquid fluidized for an entire regime:

$$\frac{\Delta P}{L} = \left(\frac{18}{\text{Re}} + 0.33 \right) \frac{1}{d_p} (\rho_L V_L^2) (1 - \varepsilon_L) \varepsilon_L^{-4.8} \quad (34)$$

Comparing Eqs. (33) and (34), we get the following expression for drag coefficient:

$$C_D = \frac{4}{3} \left(\frac{18}{\text{Re}} + 0.33 \right) \varepsilon_L^{-4.8} \quad (35)$$

A convenient dimensionless parameter for relating the MTC to the physical properties of the liquid system is the mass transfer factor (j_D). This parameter is defined as follows:

$$j_D = \frac{Sh}{\text{Re} Sc^{1/3}} \quad (36)$$

In order to develop the relationship between pressure drop and j_D , the j_D can be correlated with drag coefficient (C_D). In the present attempt, the j_D has been modified and correlated with C_D by the following expression to propose the unified correlation:

$$j_D' = AC_D^c \quad (37)$$

where j_D' is a modified mass transfer factor, defined as follows:

$$j_D' = \frac{Sh}{\text{Re}^{0.71} Sc^{1/3}} \quad (38)$$

The correlation constants (A and c) have been obtained by least squares regression by using the experimental data set of previous investigators (McCune and Wilhelm 1949, Evans and Gerald 1953, Couderc et al. 1972, Damronglerd et al. 1975, Upadhyay and Tripathi 1975, Koloini et al. 1977, Rahman and Streat 1981a, Shen et al. 1985, Livingston and Noble 1993, Yang and Renken 1998, Kalaga et al. 2014). The following expression has been obtained using 480 data points:

$$j_D' = 0.33C_D^{0.20} \quad (39)$$

Figure 6 shows that values of j_D' predicted by Eq. (39) match satisfactorily with experimental values with a standard deviation of 30%.

Figure 6: Parity plot: ♦ McCune and Wilhelm (1949); ■ Evans and Gerald (1953); – Couderc et al. (1972); × Upadhyay and Tripathi (1975); ● Damronglerd et al. (1975); ◇ Koloini et al. (1977); □ Rahman and Streat (1981a); + Shen et al. (1985); Δ Livingston and Noble (1993); * Yang and Renken (1998); ▲ Kalaga et al. (2014).

6 Heat transfer characteristics

The application of SLCFB reactor in chemical, petrochemical, and pharmaceutical industries has been considerably increased in recent years. SLCFB reactor is a potential candidate, especially where the chemical processes involve solid phase catalyst. In petrochemical industry, for example, new solid catalysts have been developed in order to replace the corrosive HF and $AlCl_3$ catalysts in the production of the linear alkyl benzene, an intermediate for detergent. SLCFB reactor can be effectively used to accommodate catalytic alkylation reaction and regeneration of spent catalyst simultaneously on a continuous mode. However, catalytic alkylation reaction and regeneration of solid catalyst demand control temperature environment in order to maximize the production rate. Therefore, accurate information about the heat transfer characteristics is crucial wherein a desired temperature is to be maintained in the reactor to maximize the throughput. The heat transfer characteristics of SLCFB are distinct when compared to conventional SLFB and are dependent on physical properties of solid and liquid phases, superficial liquid velocity, and solid circulation rate. Tables 7–9 provide a summary of previous work on heat transfer characteristics of SLCFB.

Table 7: Summary of previous work on heat transfer characteristics of SLCFB

Investigator	Solid Phase			Liquid Phase			Geometrical Parameters		Liquid velocity and SCR	Important observations
	Material	d_p , mm	ρ_s , kg.m ⁻³	Material	ρ_L , kg.m ⁻³	μ_L , kg.m ⁻¹ .s ⁻¹	Riser, mm	Downcomer, mm		
Kuramoto et al., (1999)	Glass beads	0.18	2580	Water	1000	0.001	D=49 H=4000	-	0.11-0.32 m.s ⁻¹ 4.44-153.90 kg.m ⁻² .s ⁻¹	1
Shin et al., (2005)	Glass beads	1-3	2500	CMC-water	1000-1003	0.00096-0.038	D=102 H=3000	-	0.16-0.44 m.s ⁻¹ 2-8 kg.m ⁻² .s ⁻¹	2, 3, 4, 5
Hashizume and Kimura, (2008)	Glass beads and Ceramic	2.10-4.95	2530 3700	Water	1000	0.001	D= 12-24 H= 2000	-	0.22-1.85 m.s ⁻¹	2, 4, 6, 7
Hashizume et al., (2011)	Glass beads Ceramic	2.19-4.95 2.1-4.02	2530 3700	Water	1000	0.001	D= 12-36 H= 2000	-	0.22-1.85 m.s ⁻¹	6, 8, 9
Jin et al., (2013)	Glass beads	0.5-3	2500	Aqueous ethanol solutions	998-1000	0.00094-0.00096	D=102 H=2500	-	0.2-0.3 m.s ⁻¹ 2.5- 8 kg.m ⁻² .s ⁻¹	2, 4, 10, 11

Table 8: Summary of previous work (some important observations for Table 7)

Sr. No.	Important observations
1	The non- uniform flow structure with respect to solid hold up and liquid velocity leads to complex convective motions of solid- liquid flow. This non- uniformity results in the promotion of heat transfer rate in aggregative fluidization.
2	HTC increases with an increase in solid circulation rate for a given superficial liquid velocity.
3	HTC decreases slightly with an increase of velocity of the viscous liquid medium at the relatively higher range of superficial liquid velocity.
4	HTC increases with an increase in solid particle size for a given system and operating parameters.
5	HTC decreases gradually with an increase in liquid viscosity for a given system and operating parameters.
6	HTC of SLCFB is higher than that of liquid single phase flow. It increases gradually with an increase in liquid velocity and approaches that of a liquid single phase flow.
7	The effect of wall on HTC is insignificant within the experimental range ($d_p/D \leq 0.26$).
8	Empirical correlation to predict HTC has been proposed. The predicted values match well with experimental values with an accuracy $\pm 15\%$.
9	The relationship between frictional pressure drop and heat transfer coefficient has been proposed.
10	HTC increases with an increase in superficial liquid velocity even in the higher range with a given solid circulation rate.
11	HTC increases gradually with an increase in the surface tension of liquid phase for a given system and operating conditions.

Table 9: Summary of empirical correlations available to estimate HTC

Investigator	Correlation	Operating range		j factor and Reynolds number	Remark
		Liquid velocity, m.s ⁻¹	SCR, kg.m ⁻² .s ⁻¹		
Shin et al., (2005)	$j_H = 0.0549 \text{Re}^{-0.224}$	0.16-0.44 m.s ⁻¹	2-8 kg.m ⁻² .s ⁻¹	$j_H = \frac{\varepsilon_L h}{\rho_L C_P \mu_L} \text{Pr}^{2/3}$ $\text{Re}'' = \frac{d_p V_L \rho_L}{\mu_L (1 - \varepsilon_L)}$	The proposed correlation is valid for Reynolds number range of 16.44-9027 with a correlation coefficient of 0.96.
Hashizume et al., (2011)	$\text{Nu} = (\text{Nu}_{\text{EHR}}^5 + \text{Nu}_{\text{LSP}}^5)^{1/5}$ $\text{Nu}_{\text{EHR}} = 0.06 \text{Re}_S^{0.75} \text{Pr}^{0.4} \left(\frac{D}{d_p} \right)$ $\text{Nu}_{\text{LSP}} = 0.023 \text{Re}^{0.8} \text{Pr}^{0.4}$	0.22-1.85 m.s ⁻¹	-	$\text{Re}_S = \frac{V_R d_p \rho_L}{\mu_L}$ $\text{Re} = \frac{d_p V_L \rho_L}{\mu_L}$	The predicted values of HTC match well with experimental values with an accuracy of ±15% within Reynolds number (Res) range of 440- 1835.
Jin et al., (2013)	$h = 4147 V_L^{0.07} G_S^{0.10} d_p^{0.09} \sigma_L^{0.08}$	0.2-0.3 m.s ⁻¹	2.5- 8 kg.m ⁻² .s ⁻¹	$\text{Re} = \frac{d_p V_L \rho_L}{\mu_L}$	The proposed correlation is valid in the following range of operating variables: $0.2 \leq V_L \leq 0.3 \text{ m.s}^{-1}$; $2.5 \leq G_S \leq 8.0 \text{ kg.m}^{-2}.\text{s}^{-1}$; $0.5 \leq d_p \leq 3.0 \times 10^{-3} \text{ m}$; and $55.73 \leq \sigma_L \leq 72.75 \times 10^{-3} \text{ N.m}^{-1}$ with a correlation coefficient of 0.954.

6.1 Physical properties of solid and liquid phases

Voluminous reports are available in the literature on heat transfer characteristics of conventional SLFB. However, there has been little information available on heat transfer in SLCFBs. The effect of the size/shape and density of solid particle and the density, viscosity, and surface tension of liquid has been investigated in the literature. Shin et al. (2005) have investigated the effect of particle size and viscosity on HTC in the riser column of SLCFB. The heat transfer resistance between the immersed heater and the bed has been estimated based on the two resistances in series model as in the case of conventional SLFB (Muroyama et al. 1986, Kang and Kim 1988, Kang et al. 1991, Kim and Kang 1997). It was also reported that HTC increases with an increase in particle size due to the fact that the larger particles can have a larger inertial force to move and generate the turbulence in the viscous liquid medium. However, HTC was found to decrease gradually with increasing liquid viscosity. This can be attributed to the effective mobility of the solid particles in the bed with an increase in liquid viscosity. With slower movement of the particles, there is decrease of turbulence in the bed with a higher-viscosity liquid medium which in turn results in a decrease of HTC. According to this study, the thickness of liquid thin film around the heater surface which has a governing effect for the determination of overall HTC was not found to increase progressively with an increase in liquid viscosity due to the enhanced adhesion force of the liquid medium to the heater surface. In other terms, it advocates the decrease in HTC with increasing viscosity of liquid medium. Similar effect of liquid viscosity on heat transfer has also been observed in conventional SLFB (Kang et al. 1985, Kim et al. 1986, Kim and Kang 1997). Hashizume and Kimura (2008) have investigated the effect of solid particle size and density on HTC using water as a fluidizing medium. It has been seen that HTC increases with an increase in solid particle size and density. In their subsequent publications (Hashizume et al. 2009, 2011), similar observations have been reported with respect to effect of solid particle size and density on HTC. Jin et al. (2013) have studied the effect of liquid surface tension on HTC in SLCFB using ethanol-water mixtures as a liquid phase. It has been observed that HTC increases slightly with an increase in the surface tension of liquid phase. It is implicit that the thermal conductivity of liquid phase decreases slightly with increasing content of ethanol in the continuous liquid phase. In addition, an increase in liquid surface tension leads to increase in the cohesive force between liquid and solid particles at the interface, which promotes the drag force acting on the particles fluidized in the continuous liquid medium. This results in the increase in solid holdup that in turn increases the turbulence in the riser. Therefore, the value of HTC increases with an increase in the surface tension of liquid phase.

6.2 Superficial liquid velocity

In conventional SLFB, the HTC increases with an increase in superficial liquid velocity and attains maximum value at superficial liquid velocity corresponding to bed voidage of about 0.7. HTC decreases significantly when superficial liquid velocity is increased beyond a threshold value (superficial liquid velocity corresponding to bed voidage of about 0.7). This is mainly attributed to a significant decrease of solid holdup at higher superficial liquid velocity (Kang et al. 1985, Kim and Kang 1997). However, in SLCFB, the effect of superficial liquid velocity on HTC is distinct when compared with conventional SLFB. Kuramoto et al. (1999) have measured the fluctuations of bed voidage and HTC to analyze the particulate and aggregative fluidization in the riser of SLCFB as a function of superficial liquid velocity. It has been observed that a higher heat transfer rate can be obtained in the aggregative fluidization regime compared with

that in the particulate fluidization regime. The non-uniformity of solid holdup and interstitial liquid velocity was found to be significant in aggregative fluidization than that in particulate fluidization, leading to the complex convective motions of solid-liquid flow. This non-uniformity was observed to result in the promotion of the heat transfer rate in the bed. Shin et al. (2005) have shown that HTC decreases slightly with an increase in superficial liquid velocity in the riser column. It has been well recognized that the solid holdup tends to decrease with an increase in superficial liquid velocity, leading to decrease in turbulence arising from fluidized solid particles. Simultaneously, an increase in superficial liquid velocity tends to increase the liquid turbulence. Because of these competing effects, the HTC value decreases slightly with an increase in superficial liquid velocity.

Hashizume and Kimura (2008) have experimentally shown that HTC measured in the riser column of SLCFB is usually higher than that of single-phase liquid flow when operated with low superficial liquid velocities. HTC in the riser column gradually increases with an increase in superficial liquid velocity and approaches toward the HTC of a single-phase liquid flow. The region where the HTC is higher than that of the single-phase liquid flow has been termed as “the heat transfer enhanced region or enhanced heat transfer region (EHR)”. With further increment of superficial liquid velocity, the HTC coincides with the HTC for a single-phase liquid flow. This region is termed as liquid single-phase heat transfer region (LSP). In their subsequent publication (Hashizume et al. 2009, 2011), they have investigated an analogy between pressure drop and heat transfer in SLCFB. The empirical correlation has been proposed to relate modified j factor and the friction factor. The HTC predicted from the correlation agrees well with their own experimental data and previous experimental data available on heat transfer in SLCFB with an accuracy of $\pm 15\%$. Jin et al. (2013) have investigated the effect of superficial liquid velocity on HTC in the riser column. It was noticed that HTC increases with increasing superficial liquid velocity even at high superficial liquid velocity range. According to this study, the thermal boundary layer around the heater immersed in the riser constitutes the major resistance for heat transfer from the heater surface to the riser content. This thermal boundary layer resistance was found to decrease with an increase in superficial liquid velocity. This is ascribed that the momentum of fluidized solid particles increases with increasing liquid velocity which makes solid particles highly susceptible to attack the thermal boundary layer. This phenomenon prevails even at higher range of superficial liquid velocity if suitable solid circulation rate is maintained.

6.3 Solid circulation rate

Shin et al. (2005) have investigated the effect of solid circulation rate on the heat transfer based on the two resistances-in-series model between the immersed heater and the bed proper in the riser column. The immersed heater-to-bed HTC was observed to increase with an increasing solids circulation rate since solid holdup increases with an increase in solid circulation rate in the riser column for liquid film which constitutes the major resistance to heat transfer was found to decrease gradually with the increasing rate of solids circulation because of the increase of contacting frequency between the fluidized particles and the heater surface by increasing the solids holdup in the bed. Jin et al. (2013) have studied heat transfer characteristics between the immersed heater and the bed content in the riser column of SLCFB. They found that HTC increases with an increase in solid circulation rate due to the higher potential of solid particles to contact with heater surface and promote turbulence near the heater surface. Further, they have also observed that HTC is directly proportional to the solid circulation rate raised to 0.10 power.

6.4 Empirical correlations

Shin et al. (2005) have estimated overall HTC based on two resistances in series model.

$$\frac{1}{h} = \frac{1}{h_h} + \frac{1}{h_b} \quad (40)$$

where h , h_h , and h_b are the overall HTC, HTC in the region adjacent to the heater surface, and that in the bed proper, respectively. They have correlated h_h in terms of the modified Colburn j factor ($j_{H,h}$) containing superficial liquid velocity and bed voidage as a function of modified Reynolds number:

$$j_{H,h} = 0.0632 \text{Re}^{-0.184} \quad (41)$$

Equation (41) fits well to their experimental data with a correlation coefficient of 0.98. The values of the modified Colburn j factor, based on the overall HTC (j_H), have also been well correlated as a function of the modified Reynolds number:

$$j_H = 0.0549 \text{Re}^{-0.224} \quad (42)$$

Equation (42) is applicable in the Reynolds number range of 16.44–9027 with correlation coefficient of 0.96.

Hashizume et al. (2011) have proposed the following empirical equation to determine HTC in the riser column:

$$Nu = (Nu^5_{EHR} + Nu^5_{LSP})^{1/5} \quad (43)$$

They have demarcated heat transfer in the riser column by defining two regions: EHR and LSP. The HTC gradually increases and approaches toward the HTC a single-phase liquid flow. This region where the HTC is larger than that of the single-phase liquid flow has been termed as EHR. With further increment of liquid velocity, the HTC coincides with the HTC for a single-phase liquid flow. This region is termed as LSP. The HTC in EHR region is estimated by the following correlation:

$$Nu_{EHR} = 0.06 \text{Re}_s^{0.75} \text{Pr}^{0.4} \left(\frac{D}{d_p} \right) \quad (44)$$

where Re_s is particle Reynolds number, defined using slip velocity. The HTC in LSP region is obtained using Dittus- Boelter equation:

$$Nu_{LSP} = 0.023 \text{Re}^{0.8} \text{Pr}^{0.4} \quad (45)$$

On the basis of the experimental data, Eq. (43) predicts HTC with an accuracy of $\pm 15\%$. Jin et al. (2013) have proposed the following empirical correlation to predict HTC in the riser column of the SLCFB in terms of superficial liquid velocity, solid circulation rate, particle diameter, and surface tension of liquid phase:

$$h = 4147 V_L^{0.07} G_S^{0.10} d_p^{0.09} \sigma_L^{0.08} \quad (46)$$

Equation (46) fits to experimental data with a correlation coefficient of 0.95. Equation (46) is valid in the following range of operating variables: $0.2 \leq V_L \leq 0.3 \text{ m s}^{-1}$, $2.5 \leq G_S \leq 8.0 \text{ kg m}^{-2} \text{ s}^{-1}$, $0.5 \leq d_p \leq 3.0 \times 10^{-3} \text{ m}$, and $55.73 \leq \sigma_L \leq 72.75 \times 10^{-3} \text{ N m}^{-1}$. In terms of dimensionless numbers, Eq. (46) reads as follows:

$$Nu = 1.58 \text{Pr}^{-0.15} \text{Re}^{0.02} We^{-2.99} \quad (47)$$

7 Modeling

SLCFBs are gaining extensive attraction in diverse fields of biotechnology, food processing, wastewater treatment, and petrochemical and metallurgical processing (Zhu et al. 2000, Atta et

al. 2009). This is because of their several instant eye-catching advantages such as efficient liquid-solid contact, favorable mass and heat transfer, reduced back-mixing of phases, and integrated reactor and regenerator design. It has been well recognized that conventional SLFBs expand in a homogeneous manner with a spatially and uniformly distributed concentration of solid particles in both axial and radial directions (Richardson and Zaki 1954, Chavan et al. 2009). However, previous hydrodynamic studies conducted on SLCFBs show that flow structure of SLCFB is notably different as compared with SLFB. The radial non-uniformity with respect to solid holdup, liquid velocity, and solid particle velocity has been clearly observed in SLCFBs (Liang et al. 1996, 1997, Roy et al. 1997, 2014, Zheng et al. 1999, 2002, Zhu et al. 2000, Chavan et al. 2009). Therefore, conventional modeling approaches based on homogeneity assumption fail to describe the complicated flow phenomenon prevailing in SLCFBs. Consequently, design and scale-up of SLCFBs remain a great challenge in their development.

The solids holdup and solids circulation rate are the imperative hydrodynamic variables that govern the overall performance of the system, wherever SLCFB is used as a contacting device or as a simultaneous reactor and regenerator. As detailed in Section 3, these hydrodynamic variables are strongly affected by the dimensions of the SLCFB, operating variables such as primary liquid velocity, auxiliary liquid velocity, physical properties of solid and liquid phases, method of operations, and solids inventory in the downcomer. In literature, both the experimental and the theoretical approaches have been reported wherein the effect of the aforementioned variables on the solid holdup and solid circulation rate has been studied. As expected, both the experimental and the modeling approaches have certain merits and demerits. Preferably, the experimental studies are acknowledged over the theoretical approaches. However, in most of the cases, the experimental studies are costly and time consuming. Therefore, the theoretical modeling approaches are receiving growing attention to study hydrodynamic aspects of SLCFBs.

7.1 Computational fluid dynamic models

CFD modeling and simulation approach mainly comprise Eulerian-Lagrangian (E-L) and Eulerian-Eulerian (E-E) methods to simulate two-phase flow. In the E-L approach, the liquid phase is considered as a continuous phase, and Navier-Stokes equations are solved for the liquid phase. The solid phase is treated as a discrete phase, and each solid particle is tracked by solving the Lagrangian force balance equation. The solid-phase flow and concentration distribution can be obtained by averaging the movement parameters of a large number of particles tracked. The main drawback of this approach is that the particle particle interaction is neglected. Therefore, this method is usually not used for dense two-phase flows. In the E-E approach, each phase is treated as an interpenetrating continuum, and the concept of phase volume fraction is used. The governing equations based on the conservation of the mass, momentum, and energy are solved for both liquid and solid phases based on the kinetic theory of granular flow (Sinclair and Jackson 1989, Gidaspow 1994, Enwald et al. 1996, Gidaspow and Bacelos 2017). In the case of two-phase flow in SLCFB where the number of solid particles is huge, the E-E method is attractive and practical.

Roy and Dudukovic (2001) have adopted the Eulerian two-fluid approach wherein the liquid phase is not only treated as a continuous phase, but the entire ensemble of solid particles is treated as a pseudo-continuous phase, admitting the existence of two interpenetrating continua. The momentum balance equations for solid phase and liquid phase have been solved based on the kinetic theory model of granular solids and the standard k - ϵ model, respectively. The

momentum exchange between the liquid and solid phases (solid-liquid drag) was modeled using the correlation of Wen and Yu (1966). Their computed time-averaged solids axial velocity profile when compared with the time-averaged velocity profiles obtained by computer automated particle tracking showed very good agreement. The satisfactory comparison of the time averaged solids radial holdup was also presented against the axially averaged, time averaged solids holdup profile measured by γ -ray computed tomography. Cheng and Zhu (2005) have simulated the experimental results of previous investigators (Roy et al. 1997, Zheng et al. 2002), demonstrating the effect of liquid velocity and solid circulation rate on radial solid holdup, using Eulerian two fluid approach. With the successful development of a robust model by validating previous experimental data, they have made an attempt to predict the effect of physical properties of fluids/solids on the two-phase flow structures. They have investigated three different systems (case I: water-glass beads; case II: water-steel shots; and case III: air-glass beads) to compare radial flow structures. For solid-liquid systems, it was observed that there is hardly any difference in uniformity of the radial flow profile for different solid particles. However, quite logically, solids velocities are less in magnitude for heavier particles (steel shots) as compared to lighter particles (glass beads). For a solid-gas system, it was observed that there is a significant radial non-uniformity in flow structure. Further, they have investigated the influence of bed dimension on the radial flow structure. It was seen that the computed radial distribution of liquid velocity and particle velocity becomes uniform with the increase of bed diameter. Higher negative velocities for liquid and solid phases were observed near the walls in larger diameter columns, which in turn signify severe backmixing in the axial direction for larger diameter columns. Doroodchi et al. (2005) have applied the E-E method to study the effect of the inclined plates on the expansion behavior of SLFB. The viscous stress of the solid particles has been neglected in the simulation. The CFD model has successfully predicated the general trends in the experimental data. Lettieri et al. (2006) have also used E-E approach based on granular kinetic theory to simulate SLFB of lead shot in slugging mode to describe the solid pressure and solid stress tensor. The CFD results have shown a good agreement with the experimental data at low superficial liquid velocities. However, it has been seen that the CFD modeling fails at higher superficial liquid velocities.

Cheng and Zhu (2008) have concentrated on the CFD simulations to investigate hydrodynamics characteristics in SLCFBs with different bed dimensions and solid liquid systems to establish dynamic similarity between SLCFBs in order to surpass the scale-up concern. Accordingly, they have applied the similitude method to establish the dynamic similarity among SLCFBs by varying physical properties of solid and liquid phases, operating conditions, and bed dimensions to match scaling sets of dimensionless numbers. The hydrodynamic behavior has been simulated by a validated CFD model (Cheng and Zhu 2005) and compared in terms of the axial and radial flow structures characterized by the solid fraction, solid particle and liquid velocities, and solid circulation rate. The results have demonstrated that only the full set scaling parameters obtained from similitude method, i.e., five dimensionless groups, together with fixed bed geometry, solid particle sphericity, solid particle size distribution, and solid particle collision properties, can ensure the similarity of hydrodynamics in the fully developed region of different SLCFBs, except for the turbulent kinetic energy of the liquid phase. Reducing the number of dimensionless groups leads to less desirable matching. Scale-up based on the viscous limit dimensionless group set causes larger deviations compared with the other scaling sets which implies that the fluid inertia should not be neglected in modeling and scaling of SLCFBs. With the limitation on experimental validation in practical uses, it has been suggested to use the

combination of a reliable CFD model with the proper similitude scale-up for more promising and better reactor design, scale-up, and operation.

Razzak et al. (2008) have carried out ample parametric numerical studies to investigate operation of SLCFB under a wide range of operating parameters which mainly include the solids circulation rate and primary and auxiliary liquid velocities. Thereafter, numerical modeling has been carried to predict the behavior of different particles with different densities upon fluidization in SLCFB, which resolves the problem of experimentation with a wide spectrum of new particles that might have a wide variety of applications in the SLCFB. The simulations were carried out using glass beads of three different diameters: 0.2, 0.5, and 1 mm. The cross-sectional average solids holdup was found to be almost identical for all three different size glass beads (0.2, 0.5, and 1 mm). The radial parabolic profile trend of dimensionless solid holdup for different size glass beads was also similar in all cases. However, larger particles tend to have a higher degree of non-uniformity in the radial profiles of the solids holdup. Solids and liquid velocities of the smaller particles (0.2 mm) were much higher than the velocities of larger particles (0.5 and 1 mm) in the central region because the weight of the particle increases with the increase in size. It is apparent that the heavier particles have less capability of flowing. The particle-particle interactions and drag forces are also high for larger particles.

Guodong et al. (2013) have pointed out that the formation of particle clusters in SLCFBs significantly affects macroscopic hydrodynamic behavior of the system. Accordingly, they have proposed a multiscale interfacial drag coefficient (MSD) to determine effects of particle clusters on the meso-scale structure by taking momentum and energy balance of dense phase, dilute phase, and inter-phase into account. Based on the transportation and suspension energy-minimization method, the MSD model was combined with the E-E two-fluid model to simulate the heterogeneous behaviors of SLCFB. By means of the MSD model, the core-annulus flow is observed in the riser. The liquid-solid flow regime was significantly affected by the downflow of particles in the form of clusters near the walls in the riser. The calculated concentration of particles inside the riser compared reasonably well with the available experimental data obtained by Razzak et al. (2009). Dadashi et al. (2014a) have proposed axisymmetric CFD model to provide qualitative and quantitative pictures of the turbulent two-phase flows in the riser column. Three different types of $k-\epsilon$ multiphase turbulent models have been examined, and it has been found that the dispersed $k-\epsilon$ turbulence model is more efficient than the others because of the lower computational time and higher accuracy. The predicted liquid velocity profiles and solid holdup profiles were found to be in good agreement with experimental data. It was found that the non-uniformity of liquid velocity distribution in the riser is higher than that in a conventional SLFB. Additionally, it was shown that the increase in the liquid velocity decreases the average cross-sectional solid holdup under the same solid circulation rate in the riser. Further, it has been shown that dispersed $k-\epsilon$ can be applied effectively to predict RTD of solid and liquid phases. Similar observation has been reported by Kalaga et al. (2012) while investigating RTD of liquid phase in SLCFB using CFD approach. In their successive investigation (Dadashi et al. 2014b), they have presented a CFD model to simulate the protein extraction process in the SLCFB ion exchange system wherein both adsorption and desorption processes were taken into account. The simulation of the desorption process in the riser was based on the E-E approach, incorporating the kinetic theory of granular flow, while the adsorption process in the downcomer was formulated by a onedimensional mathematical model using the adsorption kinetics correlations developed before (Mazumder et al. 2009a). The numerical results were validated favorably with reported experimental data. It was found that both the rate of protein production and the total

system efficiency decrease with the increase in the superficial liquid velocity in the riser. In contrast, with the increase in the feed flow rate and the feed protein concentration, the rate of protein production increases, but the overall system efficiency decreases.

Recently, Dadashi et al. (2015) have made a first attempt to develop a sophisticated CFD model to simulate the countercurrent solid-liquid flow in downcomer of SLCFB. The model was based on E-E approach incorporating the kinetic theory of granular flow. The proposed model was used to examine the effect of the operating condition on the hydrodynamic aspects and to obtain RTD of solid particles using a pulse technique. The numerical results were validated favorably with the reported experimental data (Lan et al. 2000). It was found that the bed expansion of the solid particles in the downcomer is directly affected by superficial liquid velocity and the solid circulation rate. The solid holdup in the dense zone of the downcomer decreases with the increase in either superficial liquid velocity or solid circulation rate. It was also found that the radial distribution of solid holdup is very uniform. Additionally, it was demonstrated that the increase in superficial liquid velocity decreases the solid dispersion in the downcomer. Han et al. (2015) have proposed a new approach to obtain the onset velocity of circulating fluidization in SLCFBs by estimating the particle RTD using CFD-discrete element method (DEM) simulation. The CFD-DEM simulation, as an E-L approach, was used to simulate the solid-liquid two-phase flow wherein DEM simulation directly tracks the motion of each solid particle. The predicted values of onset velocities were found in good agreement with experimental values (Zheng and Zhu 2001) with 0.6–2.1% deviation. It was observed that the onset velocity gets strongly affected by density and size of particles and gets weakly affected by riser height and diameter. A power-law function was proposed to correlate the mean particle residence time with the superficial liquid velocity. Further, it has also been seen that the mean residence time and hence the onset velocity gets insignificantly affected by solid particle collision parameters, which affects the solid particles RTD shape.

In summary, the E-E model has been presented to provide a good estimation of the behavior of different types of particles in the flow system when coupled with dimensionless analysis without conducting real-life experiments which can substantially reduce the experimental effort to generate the large amount of data that is essential for scale-up and commercial applications.

7.2 Computer intelligence based approach

Computer intelligence based modeling and simulation approaches are being extensively employed when the system has nonlinear response for a specific input as exemplified while investigating hydrodynamic aspects in SLCFB. These approaches are widely used to determine the input-output relationships based on experimental experiences and to provide suitable models for complex nonlinear processes. Recently, computational intelligence models such as artificial neural network (ANN), adaptive neuro-fuzzy inference system (ANFIS), and Abductive Network are explored in the hydrodynamic study of SLCFBs.

Razzak et al. (2012a) have investigated ANN approach to study the radial and axial solid holdup distributions in SLCFBs. The results show that the ANN model is capable of predicting the radial distribution of the solids holdup at a given height of the riser column and operating conditions for different combinations of primary liquid velocity, auxiliary liquid velocity, and superficial solids velocity. The capability of the developed model was investigated by comparing the model output and experimental data obtained from a pilot scale SLCFB reactor using glass beads (0.50 mm) as solid phase with water as liquid phase. The ANN model successfully

predicted the radial non-uniformity (higher close to the wall and lower in the central zone) of phase holdup that was observed in the pilot scale SLCFB experiments. The non-uniform solids holdup generally decreases with increasing superficial velocity of water. When compared, the model predicted output and trend of radial flow structure for solids holdup are in good agreement with the experiments. The mean absolute percentage error was found to be around 6% with the correlation coefficient value of 0.992. In a later study, Razzak et al. (2012b) has applied ANFIS approach to investigate the effect of solid particle size and shape on solid holdup distribution in SLCFBs under the different sets of auxiliary and primary liquid velocities and superficial solid velocities. The spherical glass beads (0.5 and 1.25 mm) and irregular shape lava rocks (0.5 and 0.92 mm) were used as a solid phase with water as a liquid phase. The competency of the developed model was examined by comparing the model predicted data sets with experimental data sets of solid holdup. It was observed that the solid holdup is higher for glass beads as compared to lava rocks due to higher drag force imposed on the spherical shape particles for a given superficial liquid velocity. The solid holdup decreased with increasing liquid velocity at all axial locations. The non-uniformity of solid holdup in central region decreased axially as the flow is well developed at higher levels as revealed by both the experimental observations and model predictions.

Razzak (2013) has applied Abductive Network modeling approach to enhance prediction accuracy over ANN and ANFIS modeling techniques. The model was validated for solid holdup distribution using their previous pilot scale SLCFB experimental data (Zheng et al. 2002). It was observed that model predicted values of solid holdup match well with experimental values with the mean absolute percentage error of around 4.67% and the correlation coefficient of 0.992. In a later study, Razzak et al. (2014) have applied support vector machine modeling approach to predict the solid holdup distribution in SLCFB. The model was validated by using their previous experimental data wherein glass beads (0.5 and 1.25 mm) were used as a solid phase (Razzak et al. 2009). For both sizes of solid particles, the mean absolute error was found to be less than 5% with the values of correlation coefficient equal to 0.998 and 0.994 for 0.5 mm and 1.25 mm solid particles, respectively. Very recently, Palkar and Shilapuram (2017) have applied ANN approach to investigate the effect of liquid viscosity, solid inventory in downcomer, auxiliary liquid velocity, total liquid velocity (sum of primary and auxiliary liquid velocity) on average solid holdup in the riser column, and solid circulation rate. The results showed that an average solid holdup increases with increasing liquid viscosity. However, the change in average solid holdup fairly becomes independent of liquid viscosity after liquid viscosity of about 1.5 cP. The average solid holdup increases with increasing solid inventory and auxiliary liquid velocity for given operating conditions. However, for a given liquid viscosity and solid inventory, the average solid holdup decreases with increasing total liquid velocity. The effect of liquid viscosity, solid inventory, and auxiliary liquid velocity on solid circulation rate has also been investigated. The solid circulation rate increases with increasing liquid viscosity up to a threshold value and then decreases with further increase in liquid viscosity. The solid circulation increases with increasing auxiliary liquid velocity and solid inventory in the downcomer. The ANN simulation shows good agreement between predicted and experimental values of average solid holdup and solid circulation rate with correlation coefficient of 0.97.

7.3 Drift flux models

Natarajan et al. (2008) have modified the drift flux model which is extensively used to describe the flow behavior of gas-solid systems and have applied it to the experimental data of SLCFB for

the first time. The model takes into account the effect of non-uniform flow and concentration distribution across the column as well as the effect of the local relative velocity between the phases by using distribution parameter (C_0) and weighted average drift velocity (u_{sj}), respectively. The model was applied to their experimental data and the data reported in literature covering a range of particle Reynolds number from 0.60 to 49 and liquid flow rates from 0.04 to 0.4 m s^{-1} . It was found that the C_0 ranges from 0.78 to 0.99, indicating that the solids concentration is minimum at the riser core and maximum at the wall of the riser. For fully established profiles, the value of C_0 remains constant. The weighted average drift velocity was found to depend on the terminal rise velocity of the particle and insignificantly on auxiliary liquid flow rate that controls solids input into the riser. The values of solid holdup predicted by using drift flux model match well with experimental values reported in the literature with a maximum root mean square deviation of 9.85%. In their subsequent study (Natarajan et al. 2011), they have investigated the effect of auxiliary liquid velocity, total liquid velocity, particle properties (size and density), and solid holdup on slip velocity with a prime objective of developing empirical correlation for slip velocity in SLCFB. The results show that slip velocity increases with an increase in the solid velocity, size and density, and decreases with increasing solid holdup. The variation in slip velocity was found to be marginal with increase in the liquid velocity. The experimental values of slip velocities were compared to the values predicted by using the proposed empirical correlation. The proposed empirical correlation predicts the slip velocity with root mean square deviation of less than 11%.

Besides the aforementioned modeling and simulation approaches, there are few reports in the literature wherein the efficacy of SLCFB has been demonstrated for continuous protein recovery. The model essentially considers hydrodynamics, adsorption-desorption kinetics, and solid-liquid mass transfer to delineate the overall performance of SLCFB. Lan et al. (2000) have developed the first model on the continuous protein recovery in the SLCFB wherein the surface reaction is assumed as a rate determining step. Gaikwad et al. (2008) have developed a model considering the film mass transfer as the limiting step. Their model, however, considers protein adsorption in downcomer, and desorption process was not considered. Mazumder et al. (2009a) have developed a comprehensive model on continuous adsorption and desorption of protein in two separate columns of the SLCFB considering detailed hydrodynamics of the system. The model was subsequently validated using reported experimental data (Lan et al. 2000, 2002a). They have also reported the trends of the adsorption and the desorption behavior as a function of operating parameters. They have observed that the rate of protein production and an overall recovery increases with an increase in solid circulation rate and the height of the downcomer dense phase, while both of them decrease with the increase in superficial liquid velocity in the riser. In their successive study, Mazumder et al. (2009b) have developed the model for multi-objective optimization of the SLCFB ion exchange system for continuous protein recovery at the operation stage. They have solved a number of two- and three-objective functions optimization problems using their model (Mazumder et al. 2009a) to find out optimal operating conditions for the existing SLCFB ion exchange system for continuous protein recovery. The performance of SLCFB was optimized by maximizing the production and recovery of protein and minimizing the amount of resin required for the experimental results (Lan et al. 2000, 2002a).

8 Proposed model of SLCFB

8.1 Setup - Figure 7 shows the experimental setup of the proposed SLCFB. It mainly consists of two sections (each 100 mm i.d. and ~700 mm length): (i) loading section and (ii) regenerating section wherein solid and liquid phases are contacted countercurrently. Each section comprises several stages (each 100 mm i.d. and 100mm length) assembled together with flange joints. A SS made mesh with openings smaller than the solid particle size is fitted onto an aluminum ring sandwiched between every pair of adjoining flanges. The solid particles move across on the stage to the next stage through a downspout, as the liquid flows upward through mesh openings. There are two types of downspouts arranged alternatively on successive stages: one is fitted at the center of the stage as the central downspout and other fitted around the outer periphery of the stage as the circumferential downspout.

Figure 7: Schematic of proposed model: 1, loading section; 2, regenerating section; 3, solid return pipe; 4, solid transport line; 5, feed tank; 6, SS stage with SS mesh; 7, downcomer; 8, overflow; V, valve; D, diffuser/solid distributor.

A feed tank (300 mm i.d. and 500 mm length) is used to charge regenerated solid particles from the top to the loading section through solid transport line wherein solid transport is controlled by valve V1. The solid particles are kept in expanded state by the liquid charged from the bottom of feed tank via calming section of 150 mm. The feed tank is provided with top outlet for the discharge of the liquid via mesh to avoid loss of fine solid particles. The solid particles are charged via diffuser at the top of the loading section provided with two liquid inlets at the bottom: primary inlet and auxiliary inlet. The primary inlet is given through a SS conical distributor, made up of SS plate having 70 mm diameter and 30 holes of 1 mm diameter. This plate is covered with the mesh of size smaller than the particle size. The auxiliary stream is introduced at the bottom of the loading section through the mesh of size smaller than particle size. The solid particles are transported to regenerating section via SS pipe of 10 mm i.d. and 150 mm length via a control valve V2. The primary and auxiliary inlet arrangement is provided in the regenerating section similar to that of loading section. The solid particles coming out of the regenerating section are conveyed to the feed tank via solid transport line.

8.2 Operation

The solid transport lines, connecting loading and regenerating sections and regenerating and collector sections, are closed initially using valve V2 and valve V3, respectively. The primary and auxiliary liquid streams are started in loading and regenerating sections, and subsequently the fresh solid particles are charged using valve V1 at the top of the loading section via diffuser in order to distribute solid particles uniformly. Initially, in the loading section, as the flow rate of liquid stream is gradually increased, the solid bed on the stage expands up to the weir height and subsequently flows to the next lower stage since solid particles continue to pour from the adjacent upper stage through downspout, creating a difference between bed depths from the center to the periphery of the stage or vice versa, depending upon the type of the downspout in that stage (center or circumferential). Thus, the state of fluidization on every stage is cross-current, although the overall flow of the solid and liquid phases is in the countercurrent direction. A similar flow pattern is achieved in the regenerating section when the valve V2 is gradually opened and solid particles are allowed to enter into the section for a given liquid flow rate in the section. When solid particles reach the bottom of the regenerating section, valve V3 is gradually opened to transport solid particles to the top feed tank for continuous operation.

8.2.1 Dynamic seal between two sections

In the proposed system, there are two sections (loading and regenerating) in which two distinct unit operations (for example, adsorption and desorption) take place simultaneously, and they are further interconnected by the solid transport line through which solid particles flow from one section to another. In such a system, the dynamic seal between the two sections is of critical importance since two liquid streams of opposite properties are involved in the two sections and a little intermixing of streams may result in the failure of the whole column. The dynamic seal could be achieved by maintaining particle plug in the solid transport lines, meaning allowing solid particles to flow in fixed bed mode.

The instant eye-catching benefits of proposed SLCFB are (i) a reduction in operational time as compared to the time required in batch-wise conventional SLFB and existing SLCFB, leading to reduction in the operational cost, (ii) a reduction in the pressure drop and enhancement in the rate of mass transfer (thus, reduction in number of stages) due to the modified stage configuration wherein solid particles follow radial cross flow which results into uniform

fluidization and RTD of solid particles, and (iii) a reduction of impurity (recalcitrant solute) up to picobillion level from feed solution with higher throughput. Therefore, the proposed SLCFB will be highly beneficial in various sectors like petrochemical, bioelectronics, pharmaceutical, biotechnology, and medical sciences where solid-liquid contact plays an important role in determining the overall performance of the system.

9 Conclusions and recommendations for future work

(i) It has long been recognized that solid-liquid fluidization is homogeneous in nature. This homogeneous behavior forms the basis of theoretical background of solid-liquid fluidization. The flow structure of SLCFB is, however, distinct as compared to conventional SLFB. The radial non-uniformity has been observed with respect to voidage, superficial liquid velocity, and solid particle velocity in the riser section of SLCFB. Therefore, the empirical relationships developed for homogeneous fluidization (velocity-voidage relationship, drag-voidage relationship, etc.) are not applicable to elucidate the flow structure of SLCFB. The new modeling work is needed to account for the radial non-uniformity in SLCFB.

(ii) The major fraction of literature pertaining to hydrodynamic aspects of SLCFB is dedicated to the riser section. The flow structure of the riser section has been studied to a desirable extent. However, very little information is available on the main section of the SLCFB. Although the main section operates in conventional fluidization regime, an additional operating variable (solid circulation rate) affects the flow characteristics considerably. Kishore and Verma (2006), Singh et al. (2008), and Verma et al. (2009) are the few studies wherein hydrodynamic characteristics of multistage SLFB have been investigated. In these studies, however, solid particles are fed from the top using a conveyor belt and therefore neglecting the risk of maintaining a dynamic seal between the two columns. Other investigations (Lan et al. 2000, 2002a,b, Mazumder et al. 2009a,b, 2010) are inclined towards effective utilization of SLCFB, keeping hydrodynamic study in the main section aside. Overall, there is a need to study both the riser and main column simultaneously to standardize SLCFB.

(iii) The literature reports the methods to determine transition velocity (velocity at which conventional fluidized bed transforms to circulating fluidized bed). However, no theoretical/empirical equation is available in the literature to predict transition velocity to the best of authors' knowledge. Further, no method is reported to demarcate transition from circulating fluidization regime to transport regime.

(iv) The riser column of the existing SLCFBs operates in circulating fluidization regime and therefore poses certain limitations like high liquid and solid phases mixing, low residence time, and need of proper dynamic seal. Therefore, an investigation shall be directed to the direction wherein both the steps of utilization (catalytic reaction, adsorption, etc) and regeneration can be carried out simultaneously at higher throughputs in conventional fluidization regime.

(v) Voluminous reports are available in the literature which deal with liquid phase mixing in conventional SLFB. However, very little work is available on mixing characteristic of SLCFB. Further, it is essential that mixing characteristics should be studied when both the sections riser and main column are under steady state condition.

(vi) The empirical correlation developed in the literature commonly contains special system properties *viz.* minimum fluidization velocity, voidage, and particle terminal settling velocity. Although there are correlative approaches available for estimating these properties for a given system, they are subject to a certain amount of inaccuracy. Therefore, the empirical correlation

may be developed based on physical properties of solid and liquid phases and operating parameters.

(vii) The empirical correlations proposed to estimate MTC in conventional SLFB are used to elucidate mass transfer in downcomer column of SLCFBs. However, mass transfer characteristics of riser column are scantily studied. Therefore, it is necessary to undertake systematic investigation in riser column with respect to mass transfer. Further, whatsoever studies conducted using SLCFB separately considers riser and main column sections. Thus, these studies overlook the interaction between the riser section and the main section.

(viii) The two resistances in series model satisfactorily presents the heat transfer aspects of the SLCFBs. The empirical correlations developed satisfactorily predict overall HTC with an accuracy of $\pm 15\%$.

(ix) The CFD, computer intelligence, and drift flux models predict non-uniform solid holdup profile in the riser section with a sufficient degree of confidence. However, very few reports are available in the literature that deals with modeling of the downcomer column of SLCFB. Therefore, systematic investigation of modeling of the downcomer column should be undertaken. Furthermore, the mixing time measurements in the SLFBs and/or SLCFBs have not been studied either experimentally or numerically. Therefore, it is necessary to (a) perform the mixing time measurement studies in the liquid fluidized bed and (b) validate the CFD codes, using the experimental results, for prediction of mixing time.

(x) The novel SLCFB proposed overcomes the limitations of existing SLCFBs and offers several advantages like reduction in operational time as compared to the time required in batch-wise conventional SLFB and existing SLCFB, leading to reduction in the operational cost and a reduction in the pressure drop and enhancement in the rate of mass transfer (thus, reduction in number of stages) due to the modified stage configuration wherein solid particles follow radial cross flow which results in uniform fluidization and RTD of solid particles.

Nomenclature

A_R	= cross sectional area of the riser column, (m^2)
A_{vd}	= dynamic specific surface area of porous medium, (m^{-1})
$\overline{C_D}$	= cross sectional average drag coefficient, (-)
C_D	= drag coefficient, (-)
$C_{D\infty}$	= drag coefficient defined when solid particle settling at terminal velocity, (-)
C_{Dx}	= drag coefficient defined by equation 14, (-)
D	= column diameter, (m)
D_L	= liquid dispersion coefficient, ($m^2.s^{-1}$)
d_p	= diameter of solid particle, (m)
d_p^*	= dimensionless solid particle diameter defined by equation 2, (-)
D_r	= radial liquid dispersion coefficient, ($m^2.s^{-1}$)
E_s	= Energy dissipation rate defined by Kikuchi et al., (1988), ($m^2.s^{-3}$)
f'	= modified friction factor defined by Evans and Gerald, (1953), (-)
F_d	= drag force acting on a single solid particle, (N)
Fr	= Froude number, (-)
g	= gravitational acceleration, ($m.s^{-2}$)
G_a	= Galileo number, (-)
G_s	= flux of solid- liquid mixture, ($kg.m^{-2}.s^{-1}$)
h	= overall heat transfer coefficient, ($W.m^{-2}.K^{-1}$)

h_b	= heat transfer coefficient in bed proper, ($\text{W.m}^{-2}.\text{K}^{-1}$)
h_h	= heat transfer coefficient in the region adjacent to the heater surface, ($\text{W.m}^{-2}.\text{K}^{-1}$)
h_p	= thickness of layer of particles, (m)
H_R	= height of solid-liquid mixture, (m)
H_{SL}	= height of clear liquid in solid-liquid separator, (m)
H_{SR}	= length of solid return pipe, (m)
H_{SS}	= height of clear liquid in solid storage vessel, (m)
j_D	= mass transfer factor, (-)
j_D'	= modified mass transfer factor defined by equation 38, (-)
j_H	= heat transfer factor based on overall HTC defined by equation 42, (-)
$j_{H,h}$	= heat transfer factor in heater surface region, defined by equation 41, (-)
k_{SL}	= solid-liquid mass transfer coefficient, (m.s^{-1})
m	= arbitrary constant defined by Koloini et al., (1977), (-)
m_s	= mass flow rate of solid particles (kg.s^{-1})
M_v	= density number, (-)
n	= Richardson – Zaki parameter, (-)
Nu	= Nusselt number, (-)
Pe	= Peclet number, (-)
Pe_a	= axial Peclet number, (-)
Pe_r	= radial Peclet number, (-)
Pr	= Prandtl number, (-)
$\Delta P/L$	= pressure gradient, (N.m^{-3})
ΔP_R	= pressure drop across riser, (Pa)
ΔP_{SL}	= pressure drop across solid-liquid separator, (Pa)
ΔP_{SR}	= pressure drop across solid return pipe, (Pa)
ΔP_{SS}	= pressure drop across solid storage vessel, (Pa)
ΔP_v	= pressure drop across valve, (Pa)
r	= any radial distance, (m)
R	= radius of the riser column, (m)
Re''	= modified Reynolds number defined by Upadhyay and Tripathi, (1975), (-)
Re'	= Reynolds number based on interstitial liquid velocity, (-)
Re	= Reynolds number based on superficial liquid velocity, (-)
Re_∞	= Reynolds number based on terminal settling velocity, (-)
Re_{mf}	= Reynolds number based on minimum fluidization velocity, (-)
Re_p	= Reynolds number defined by Seguin et al., (1996), (-)
Re_s	= Reynolds number based on slip velocity, (-)
R_f	= resistance force per unit projected area, (N.m^{-2})
Sc	= Schmidt number, (-)
Sh	= Sherwood number, (-)
V_L	= superficial liquid velocity, (m.s^{-1})
V_L^*	= dimensionless superficial liquid velocity defined by equation 1, (-)
V_{La}	= auxiliary superficial liquid velocity, (m.s^{-1})
V_{Lc}	= transition velocity (conventional fluidization to circulating fluidization regime), (m.s^{-1})
V_{Lt}	= transition velocity (circulating fluidization to transport regime), (m.s^{-1})
V_{Lx}	= Superficial liquid velocity defined by equation 14, (m.s^{-1})
V_{mf}	= minimum superficial liquid fluidization velocity, (m.s^{-1})

V_R = slip velocity, (m.s^{-1})
 V_S = superficial solid velocity, (m.s^{-1})
 $V_{S\infty}$ = Terminal settling velocity of particle, (m.s^{-1})
 We = Weber number, (-)

Greek letters

ρ_C = suspension density, (kg.m^{-3})
 ρ_L = liquid density, (kg.m^{-3})
 ρ_S = solid density, (kg.m^{-3})
 ν = kinematic viscosity of liquid ($\text{m}^2.\text{s}^{-1}$)
 $\Delta\rho$ = density difference between solid phase and liquid phase, (kg.m^{-3})
 ρ_x = liquid or suspension density defined by equation 14, (kg.m^{-3})
 τ = bed tortuosity, (-)
 ψ = shape factor, (-)
 μ_L = viscosity of liquid, ($\text{kg.m}^{-1}.\text{s}^{-1}$)
 ε_L = voidage of the bed, (-)
 $\overline{\varepsilon_L}$ = average voidage of the bed, (-)
 σ_L = liquid surface tension, (N.m^{-1})

Subscripts

EHR = heat transfer enhanced region
 LSP = liquid single phase heat transfer region

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