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Near Field Active Imaging Using Sparse Arrays

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Abstract—Sensor arrays designed for far field operation may experience performance degradation when imaging near field objects. Specifically, sparse active arrays utilizing the additional degrees of freedom provided by the sum co-array are susceptible to these effects, as the co-array depends on both the range and direction of scatterers close to the array. Consequently, a uniform far field sum co-array may become non-uniform in the near field. As a result, co-array processing algorithms, such as image addition, are subject to undesired grating lobes in the presence of near field scatterers. This paper proposes an extension to image addition for mitigating such undesired distortions. The method compensates for near field effects by computing spatially varying transmit and receive element weights. These weights minimize the discrepancy between the desired and achieved near field point spread function, while using as few image addition components as possible. Given a desired point spread function and a set of calibration measurements of the near field array steering vectors, a regularized convex optimization problem is then solved for each pixel of the image.

I. INTRODUCTION

Sparse sensor arrays can achieve comparable performance to conventional filled arrays, despite using only a fraction of the number of elements and RF front ends. For example, a sparse array of N elements can match the far field *point spread function* (PSF) of a uniformly spaced array with $\mathcal{O}(N^2)$ elements [1], or resolve $\mathcal{O}(N^2)$ sources in direction finding applications [2], [3]. This is enabled by a virtual array called the *co-array* [4], which is constructed using vector sums or differences of element positions. For instance, the *sum co-array* arises in active sensing applications, including far field coherent imaging and radar [1], [5]. In the near field however, the underlying plane wave assumption in co-array processing is no longer valid. Hence, the co-array becomes spatially varying. This *spatially varying co-array* depends on both the focus range and look direction of the array [6]–[8]. Consequently, near field scatterers distort the PSF, when applying beamforming weights designed for far field operation. This distortion can be compensated for by adjusting the weights in the near field.

A straightforward approach to near field compensation is to correct for the wavefront curvature by multiplying the far field array weights by appropriate phase and amplitude factors [9], [10]. These factors are determined by the array geometry and assumed scatterer positions, which need to be either known or estimated a priori. This restriction may be overcome by decomposing the desired beampattern into spherical harmonics, and computing the near field element weights based on a finite harmonical series approximation [11]–[13]. Other works formulate (often convex) optimization problems, which minimize the discrepancy between the desired and realized beampatterns, while satisfying certain design constraints imposed on the

pattern’s mainlobes, sidelobes or nulls [14]–[17]. Most of the aforementioned works address the near field compensation problem from the perspective of passive (receive-only) arrays, whereas active (transceiving) arrays are of interest in this work. Furthermore, these works do not jointly consider near field compensation and co-array processing techniques, such as *image addition* [1]. Image addition synthesizes a desired PSF by adding together component images acquired using multiple sets of elements weights. These weights may be obtained from the SVD of the co-array weight matrix [18]. The rank of this matrix determines the number of component images, Q . In case of a phased array, Q determines the number of transmissions, or time, required for image acquisition [19].

This paper develops a novel near field compensation method in the context of the sum co-array model and the image addition method. The main contributions of the paper are twofold. Firstly, we demonstrate that sparse arrays can be sensitive to near field effects, as their co-arrays contain fewer redundancies than uniform arrays. Secondly, we formulate a low-rank matrix retrieval problem for optimizing the near field element weights. This formulation utilizes calibration measurements of the near field array steering vectors to approximate a desired beampattern, while imposing a matrix rank constraint to keep the number of component images low. Since the optimization problem is non-convex and generally difficult to solve exactly, we propose two convex relaxations to it. We evaluate the performance of the proposed approaches in simulation, where we consider imaging a scene of coherent near field scatterers. The suggested near field compensation methods show improved image quality over the uncompensated case, even when only few noisy narrowband calibration measurements are available.

The paper is organized as follows. Section II reviews the signal model. Section III describes how near field scatterers distort an array’s PSF, and motivates the need for mitigating such effects. Section IV formulates the near field compensation problem, and section V presents the proposed solutions. Section VI demonstrates the performance of these solutions in numerical simulations, and section VII concludes the paper.

II. SIGNAL MODEL

Let an active linear array with N_t transmitters and N_r receivers be located on the x' -axis at $\mathcal{D}_t = \{d_{t,m} \in \mathbb{R}\}_{m=1}^{N_t}$ and $\mathcal{D}_r = \{d_{r,n} \in \mathbb{R}\}_{n=1}^{N_r}$. The array is used to image a collection of K coherent point reflectors. Fig. 1 illustrates the setup, showing a single transmitter (Tx), receiver (Rx), and scatterer. For simplicity, we derive our signal model under the assumption that each transmitter is operated sequentially within the coherence time of the scene. In the absence of noise

and interference, the resulting signal model is equivalent to the phased array case, which is of interest here. For clarity of presentation, we do not consider extensions to 2D or 3D arrays in this paper.

Fig. 1. Imaging in the plane with a linear array. Transmit element m and receive element n are located at $d_{t,m}$ and $d_{r,n}$. Scatterer k with reflectivity γ_k has polar coordinates (r_k, φ_k) , or alternatively (r_k, v_k) , where $v_k = \sin(\varphi_k)$. The distance from the transmitter to the receiver, via the k th target is $l_{nmk} = l_{mk}(r_k, v_k) + l_{nk}(r_k, v_k)$. Similarly, the distance via the pixel in focus at (r, u) is $l_{nm} = l_m(r, u) + l_n(r, u)$, where $u = \sin(\varphi)$.

A. Received signal

Let $p(t) = s(t)e^{j2\pi ct/\lambda_c}$ denote the transmitted pulse, where $s(t) \in \mathbb{C}$ is the complex amplitude, λ_c the carrier wavelength, and c the propagation speed of the wave. The pulse is transmitted from the m th transmitter, and scattered off the k th scatterer with reflectivity $\gamma_k \in \mathbb{C}$. The backscattered signal is then received at the n th receiver after delay $\tau_{nmk} = l_{nmk}/c$, where $l_{nmk} = l_{mk} + l_{nk}$ denotes the distance from the m th Tx element to the n th Rx element via the k th scatterer. In the absence of clutter and noise, the Rx signal x_{nm} is a superposition of the attenuated scatterer reflections:

$$x_{nm}(t) = \sum_{k=1}^K \frac{g_{nmk}}{l_{nmk}} \gamma_k s(t - \tau_{nmk}) e^{j2\pi c(t - \tau_{nmk})/\lambda_c}. \quad (1)$$

Here $g_{nmk} = g_{r,nk}g_{t,mk} \in \mathbb{C}$ is the two-way *active element pattern* in the direction of scatterer k , where $g_{r,nk}$ and $g_{t,mk}$ are the respective Rx and Tx active element patterns. The active element pattern accounts for element gain, and any non-idealities of real world sensor arrays, such as mutual coupling [20]. In (1), g_{nmk} and γ_k are assumed to be frequency independent, and higher order reflections are ignored.

B. Signal after beamforming, image addition, and scaling

The array is focused by delaying the received signals in (1) by $\tau_{nm}(r, u) = l_{nm}(r, u)/c$, where r is the range to the pixel in focus, and $u = \sin(\varphi)$ the direction of the pixel. Furthermore, $l_{nm} = l_n + l_m$ denotes the Tx-to-pixel-to-Rx distance, with $l_i(r, u) = \sqrt{r^2 + d_i^2 - 2rd_i u}$. Signals x_{nm} are then multiplied by Tx/Rx image addition weights $w_{t,mq}, w_{r,nq} \in \mathbb{C}$, scaled by $2r$ to compensate for free-space propagation loss, and summed, yielding

$$y(t, r, u) = \sum_{q=1}^Q \sum_{n=1}^{N_r} \sum_{m=1}^{N_t} w_{r,nq} w_{t,mq} 2r x_{nm}(t + \tau_{nm}). \quad (2)$$

Here Q is the number of image addition components, which determines the number of transmitted pulses when Tx elements are operated coherently, as in a phased array.

C. Reflectivity estimate after matched filtering

Matched filtering of (2) with the transmitted pulse $p(t)$ produces an estimate of the reflectivity at (r, u) . The output of the matched filter sampled at $\tau = 0$ is $\hat{\gamma}(r, u) = \int_{-\infty}^{\infty} p^*(t - \tau)y(t, r, u)dt|_{\tau=0}$, which evaluates to

$$\hat{\gamma}(r, u) = \sum_{q=1}^Q \sum_{n=1}^{N_r} \sum_{m=1}^{N_t} w_{r,nq} w_{t,mq} \sum_{k=1}^K G_{nmk} \gamma_k R_s e^{j2\pi c \Delta \tau_{nmk} / \lambda_c}, \quad (3)$$

where $R_s(\Delta \tau_{nmk}) = \int_{-\infty}^{\infty} s^*(t - \tau)s(t + \Delta \tau_{nmk})dt|_{\tau=0}$ is the autocorrelation of the transmitted baseband signal s , and $(\cdot)^*$ denotes complex conjugation. Furthermore, $\Delta \tau_{nmk} = (l_{nm}(r, u) - l_{nm}(r_k, v_k))/c$ is the difference between the focusing delay of the pixel and the round-trip delay to the k th scatterer in direction $v_k = \sin(\varphi_k)$. The scaled two-way active element pattern in that same direction is denoted by $G_{nmk} = g_{nmk} \cdot 2r/l_{nmk}$.

D. Point spread function and co-array weight matrix

Consider a single scatterer with unit reflectivity ($\gamma_k = 1$) in direction $v \in [-1, 1]$ and in range focus ($r_k = r$). In this case, (3) yields the spatially varying PSF, which can compactly be expressed as:

$$\psi(r, u, v) = \text{Tr}(\mathbf{A}_{tr}^T \mathbf{W}), \quad (4)$$

with $\mathbf{A}_{tr}, \mathbf{W} \in \mathbb{C}^{N_r \times N_t}$. The *two-way steering matrix* in (4) is

$$\mathbf{A}_{tr}(r, u, v) = \mathbf{G} \circ \mathbf{R}_s \circ e^{j2\pi c \Delta \tau / \lambda}, \quad (5)$$

where $\circ$ denotes the Hadamard product, and $\mathbf{R}_s = R_s(\Delta \tau)$. Function R_s and the exponential function in (5) are applied element-wise to delay matrix $\Delta \tau$, whose (n, m) th entry is $\Delta \tau_{nm} = (l_{nm}(r, u) - l_{nm}(r, v))/c$. Similarly, the (n, m) th entry of the gain matrix $\mathbf{G}$ is $G_{nm} = g_{nm}(r, v) \cdot 2r/l_{nm}(r, v)$. The *co-array weight matrix* $\mathbf{W} \in \mathbb{C}^{N_r \times N_t}$ is given by the outer product of Q receive and transmit weight vectors $\mathbf{w}_{r,q} = [w_{r,1q}, \dots, w_{r,N_r q}]^T \in \mathbb{C}^{N_r}$ and $\mathbf{w}_{t,q} = [w_{t,1q}, \dots, w_{t,N_t q}]^T \in \mathbb{C}^{N_t}$:

$$\mathbf{W} = \sum_{q=1}^Q \mathbf{w}_{r,q} \mathbf{w}_{t,q}^T. \quad (6)$$

The weight vectors in (6) may be obtained through the SVD of $\mathbf{W}$, in which case $Q = \text{rank}(\mathbf{W}) \leq \min(N_r, N_t)$ [18]. Matrix $\mathbf{W}$ is in turn determined by the desired point spread function, and the sum co-array [18]. The support of the sum co-array consists of the sums of all Tx-Rx element pair positions, which in this case is the set $\{d_{t,m} + d_{r,n}\}_{m,n=1}^N$.

E. Sampling the two-way steering matrix

Subsequent sections assume that the two-way steering matrix $\mathbf{A}_{tr}$ in (5) is known for (r, u, v) tuples of interest. Sampling the angular domain of the scatterer in a set of directions $\{v_i\}_{i=1}^V$ yields a set of matrices $\{\mathbf{A}_{tr}(r, u, v_i)\}_{i=1}^V = \{\mathbf{A}_{tr,i}\}_{i=1}^V$ for each (r, u) pair. Vectorizing and collecting these matrices into a matrix $\mathbf{A} = [\text{vec}(\mathbf{A}_{tr,1}), \dots, \text{vec}(\mathbf{A}_{tr,V})]^T \in \mathbb{C}^{V \times N_r N_t}$, allows the PSF in (4) to be written as:

$$\psi = \text{Avec}(\mathbf{W}). \quad (7)$$

In practice, analytical expressions for the terms in (5) may be unavailable, and the two-way steering matrix $\mathbf{A}_{\text{tr}}(r, u, v)$ must be obtained through measurements or simulations. This implies that V calibration matrices of size $N_r \times N_t$ should be collected for each pixel $\{(r_i, u_i)\}_{i=1}^P$, leading to a total of VP matrices, where P is the number of pixels. Fortunately, this number may be substantially reduced by noting that (5) can also be written as

$$\mathbf{A}_{\text{tr}}(r, u, v) = \tilde{\mathbf{A}}_{\text{tr}}(r, v) \circ \tilde{\mathbf{A}}_{\text{tr}}(r, u) \circ \mathbf{R}_s(r, u, v), \quad (8)$$

where the (n, m) th entries of $\tilde{\mathbf{A}}_{\text{tr}}(r, v)$ and $\tilde{\mathbf{A}}_{\text{tr}}(r, u)$ in (8) are $\tilde{A}_{\text{tr},nm}(r, v) = \exp(-j2\pi l_{nm}(r, v)/\lambda_c) g_{nm}(r, v)/l_{nm}(r, v)$, respectively $\tilde{A}_{\text{tr},nm}(r, u) = l_{nm}(r, u) \exp(j2\pi l_{nm}(r, u)/\lambda_c)$. When the transmitted signal is narrowband, function R_s is constant, and (8) becomes separable in v and u . In this case, only $\tilde{\mathbf{A}}_{\text{tr}}(r, v)$ needs to be measured, since $\tilde{\mathbf{A}}_{\text{tr}}(r, u)$ can be precomputed for any pixel. Matrix entry $\tilde{A}_{\text{tr},nm}(r, v)$ is obtained by measuring the response of Rx element n to a narrowband transmission from Tx element m , with a point scatterer present at (r, v) . If the range is sampled at a finite set of $J \ll P$ bins $\{r_i\}_{i=1}^J$, a total of VJ matrices of size $N_r \times N_t$ need to be collected.

III. ARRAY SENSITIVITY TO NEAR FIELD EFFECTS

As a rule of thumb, the transition between near and far field at wavelength λ occurs when $r \propto L^2/\lambda$, where L is the array aperture [6], [9]. The difference between the near and far field beampatterns depends on the employed element weighting [21], [22], look direction [6], and array configuration, specifically when using co-array processing [23].

The effect of the near field on the array beampattern may be understood as spatial aliasing [24], since in the near field, the plane wave assumption no longer holds, and scatterers cease to be spatially narrowband. An alternative interpretation may be given in terms of the co-array, which varies spatially for near field scatterers. Consider the path difference $c\Delta\tau_{nm} = l_{nm}(r, u) - l_{nm}(r, v)$ between the pixel and a single scatterer in range focus at angle v . Approximating $c\Delta\tau_{nm}$ by the two leading-order terms of its Taylor expansion yields [6]

$$c\Delta\tau_{nm} \approx \left(d_{r,n} + d_{t,m} + (d_{r,n}^2 + d_{t,m}^2) \frac{v+u}{2r} \right) (v-u). \quad (9)$$

For finite r , the term $(d_{r,n}^2 + d_{t,m}^2)(v+u)/(2r)$ in (9) can be interpreted as a perturbation to the sum co-array $d_{r,n} + d_{t,m}$, such that it is not necessarily uniform anymore. A sum co-array with few redundancies will be more susceptible to these perturbations, as the elements of the spatially varying co-array become non-uniformly distributed.

For example, consider the two arrays with co-located transceivers in Fig. 2. The filled *Uniform Linear Array* (ULA) has $N_r = N_t = 23$ sensors, whereas the sparse *Minimum-Redundancy Array* (MRA) [25], [26] has 11. The MRA minimizes the number of elements in the array subject to the sum co-array being a ULA. Both the ULA and MRA have the same sum co-array, therefore achieving the same far field PSF. When the inter-element spacing is half a wavelength, the physical aperture of both arrays is $L = 11\lambda_c$. Fig. 3 shows the spatially varying co-array of the MRA and ULA, given a single boresight scatterer ($v = 0$) at view angle $u = \sin(\pi/4)$ and a distance of 11 baseline lengths ($r = 121\lambda_c$). The spatially

varying co-array of the MRA is visibly less uniform than that of the ULA, because the ULA has more unique elements in its spatially varying co-array. Fig. 4 shows the PSF of the ULA and MRA, assuming ideal omnidirectional elements ($g_{nm} = 1$) and a narrowband transmitted waveform ($R_s = 1$). The ULA requires $Q = 2$, and the MRA $Q = 6$ component images to achieve the desired far field Dolph-Chebyshev beampattern [27] given by the solid red line. The dashed blue lines are the near field PSFs using the far field weights. The finite range does not affect the ULA, whereas the MRA displays clear grating lobes. These grating lobes may be mitigated by reducing the field of view, increasing the imaging range, or by adding elements to the array, as suggested by (9). Alternatively, we may compute new element weights that approximately achieve the desired PSF in the near field, as discussed next.

Fig. 2. MRA and ULA with aperture $L = 11\lambda_c$. Both arrays have the same sum co-array, which is a 45 element virtual ULA with aperture $22\lambda_c$.

IV. CO-ARRAY WEIGHT OPTIMIZATION PROBLEM

The co-array weight optimization problem may be expressed as follows. For a given (r, u) , we seek matrix $\mathbf{W} \in \mathbb{C}^{N_r \times N_t}$ that minimizes the discrepancy between the true PSF $\psi(r, u, \mathbf{W}) \in \mathbb{C}^V$ in (7) and a desired PSF $\psi_d \in \mathbb{C}^V$. Furthermore, we constrain the rank of $\mathbf{W}$ to the desired number of component images Q . Choosing the ℓ_2 -norm as the loss function yields the non-convex optimization problem:

$$\begin{aligned} & \underset{\mathbf{W} \in \mathbb{C}^{N_r \times N_t}}{\text{minimize}} \quad \|\psi_d - \text{Avec}(\mathbf{W})\|_2 \\ & \text{subject to} \quad \text{rank}(\mathbf{W}) = Q, \end{aligned} \quad (\text{P1})$$

which is a *low-rank matrix recovery* problem [28]. Note that $\text{rank}(\mathbf{W})$ may be minimized when $\text{Avec}(\mathbf{W}) = \psi_d$, which was the case considered in our previous work [19]. However, $\text{Avec}(\mathbf{W}) = \psi_d$ cannot be guaranteed in (P1), since knowledge of the spatially varying co-array is not assumed. Nevertheless, (P1) can be solved offline and in parallel for each (r, u) of interest, provided that the non-convexity of the problem is addressed first.

V. PROPOSED SOLUTIONS

This section proposes two convex relaxations to (P1). The first solution modifies the rank constrained problem to a Tikhonov regularized least squares (LS) problem. The resulting semi closed-form expression is fast to evaluate, but typically results in a high number of component images Q , which may unnecessarily increase the image acquisition time. The second solution relaxes the rank constraint to its closest convex surrogate, the nuclear norm [29]. This semidefinite program (SDP) allows a trade-off between Q and approximation error, at the expense of a higher computational cost. Note that non-convex approaches have also been proposed for solving (P1). For example, alternating minimization solves a biconvex problem, which essentially involves iteratively solving two LS problems is succession [28], [30]. Unfortunately, such algorithms have no guarantees of convergence to global optima.

Fig. 3. Spatially varying co-array of ULA and MRA for single boresight scatterer at finite range ($r = 121\lambda_c$, $u = \sin(\pi/4)$, $v = 0$). Due to having fewer physical elements than the ULA, the MRA also has a less uniform spatially varying co-array (especially around the midpoint). Note that both arrays have the same uniform (far field) sum co-array.

Fig. 4. PSF of ULA (top) and MRA (bottom) in the far field (solid red line) and near field (dashed blue line) using far field weights (Dolph-Chebyshev sum co-array weighting with -40 dB side lobes). The MRA suffers from grating lobes in the near field, since its spatially varying co-array is non-uniform.

A. Least squares approximation

Squaring the objective function in (P1), and replacing the rank constraint by a Frobenius norm regularizer $\beta\|\mathbf{W}\|_F^2$, $\beta > 0$ yields the convex unconstrained optimization problem:

$$\underset{\mathbf{W} \in \mathbb{C}^{N_r \times N_t}}{\text{minimize}} \quad \|\psi_d - \text{Avec}(\mathbf{W})\|_2^2 + \beta\|\mathbf{W}\|_F^2. \quad (\text{P2})$$

Problem (P2) admits a unique, semi closed-form solution given by the diagonally loaded LS, or ridge regression solution:

$$\text{vec}(\mathbf{W}) = (\mathbf{A}^H \mathbf{A} + \beta \mathbf{I})^{-1} \mathbf{A}^H \psi_d. \quad (10)$$

Matrix $\mathbf{A}^H \mathbf{A} \in \mathbb{C}^{N_r N_t \times N_r N_t}$ may be ill-conditioned, especially if $V < N_r N_t$. In practice, the diagonal loading parameter β has to be tuned manually so that it is as small as possible, yet does not cause numerical instability or overfit the solution to noisy measurements. Evaluating (10) requires $\mathcal{O}(N_r^3 N_t^3)$ arithmetic operations, assuming $V \gg N_r N_t$.

B. Nuclear norm relaxation

Replacing the rank operator in (P1) with the nuclear norm, the tightest convex relaxation [29], leads to the convex unconstrained optimization problem

$$\underset{\mathbf{W} \in \mathbb{C}^{N_r \times N_t}}{\text{minimize}} \quad \|\psi_d - \text{Avec}(\mathbf{W})\|_2 + \alpha\|\mathbf{W}\|_*, \quad (\text{P3})$$

where $\alpha > 0$ is again a regularization parameter. Selecting the value of α is not a trivial problem, and one often has to resort to heuristics, such as trial-and-error, to find a value that works satisfactorily. Problem (P3) may be expressed as an SDP with an $(N_r + N_t) \times (N_r + N_t)$ PSD matrix of auxiliary variables [29]. This SDP can be solved by interior-point methods using roughly $\mathcal{O}(N_r^2 N_t^2 (N_r + N_t)^{2.5})$ arithmetic operations, assuming $V \propto N_r N_t$ [31]. We use the Matlab [32] toolbox CVX [33] with SDPT3 [34] to solve (P3) in our subsequent simulations. However, faster algorithms are also readily available for problems closely related to (P3). Examples include both proximal [28], and greedy methods [35]. These algorithms have the complexity of SVD, i.e., $\mathcal{O}(N_r^3 N_t^3)$, provided that the number of iterations is kept low.

VI. NUMERICAL EXPERIMENTS

In this section, we numerically demonstrate the performance of the proposed solutions (P2) and (P3) using the arrays in Fig. 2. First however, we define the *cumulative energy*: $p(Q) = (\sum_{q=1}^Q \sigma_q) / (\sum_{q=1}^{\min(N_r, N_t)} \sigma_q)$, where $\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_Q \geq \dots \geq \sigma_{\min(N_r, N_t)}$ are the singular values of matrix $\mathbf{W}$ sorted in descending order. We require that the Q largest singular values of $\mathbf{W}$ explain at least 99.99% of its variance, i.e., $p(Q) \geq 0.9999$. Second, since Q may vary from pixel to pixel, we also compute its average value, $\bar{Q} = \sum_{i=1}^P Q(r_i, u_i) / P$, where P is the number of pixels.

A. Near field compensated PSF

Let us revisit the narrowband PSF of the MRA at range $r = 121\lambda_c$ considered in section III. We assume perfect knowledge of $\mathbf{A}_u(r, u, v)$ in $V = 100$ directions $v \in \{\sin(\varphi_i)\}_{i=1}^{100}$. Angle φ_i is uniformly sampled between $-\pi/2$ and $\pi/2$, which requires collecting 100 calibration matrices of size 11×11 .

Fig. 5(a) shows the near field PSF using the LS solution (10). Grating lobes are visibly suppressed compared to Fig. 4, although the average number of component images has increased from $\bar{Q}=6$ to 11. Fig. 5(b) shows the PSF obtained by solving the nuclear norm regularized problem (P3). Fig. 5(b) is practically indistinguishable from Fig. 5(a), despite requiring 19% fewer component images on average, as $\bar{Q}=9$.

Fig. 5. Near field PSF of MRA using weights solving the (a) LS problem in (P2), and (b) nuclear norm regularized problem in (P3). Grating lobes in Fig. 4 are suppressed at the expense of an increase in the average number of component images: (a) $\bar{Q}=11$, (b) $\bar{Q}=9$, compared to $\bar{Q}=6$ in Fig. 4.

B. Imaging with noisy near field calibration measurements

Consider three unit reflectivity scatterers at coordinates $(r, \varphi) \in \{(109\lambda_c, -\pi/4), (121\lambda_c, 0), (133\lambda_c, \pi/4)\}$ with respect to the center point of the array. The transmitted waveform is a square pulse lasting 10 carrier frequency cycles. Such a signal has a relative 3 dB bandwidth of approximately 9%. The two-way steering matrix is sampled using a narrowband signal of wavelength λ_c , such that only $\hat{\mathbf{A}}_{tr}(r, v)$ in (8) is measured. The measurements are contaminated by complex additive white Gaussian noise at the receiver, where the SNR of a single sensor is 30 dB. The imaging process itself is assumed noiseless, as in (3), in order to highlight the effects of calibration errors. The calibration grid is sampled uniformly at $V=100$ angles, and a single range $r = 121\lambda_c$. The range of the imaging grid is uniformly sampled at 100 points between $96\lambda_c$ and $146\lambda_c$, and the look direction u at 200 points between $-\pi/2$ and $\pi/2$. Optimization problems (P2) and (P3) are thus solved for $P=200 \times 100$ pixels.

Fig. 6(a) and 6(b) show the images produced by the ULA and MRA using far field weights. As expected, the MRA suffers from grating lobes, which do not affect the ULA. Fig. 6(c) and 6(d) show the same scene imaged by the MRA using weights obtained by solving (P2), respectively (P3). Near field compensation effectively suppresses grating lobes

over the whole imaging range, despite using noisy narrowband calibration measurements from a single range. The nuclear norm formulation again produces a solution with a lower average number of component images than LS ($\bar{Q} = 9$ in Fig. 6(d) compared to $\bar{Q}=11$ in Fig. 6(c)).

VII. CONCLUSION

This paper considered active near field imaging of coherent scatterers using sparse linear phased arrays. We demonstrated that sparse arrays using image addition are susceptible to grating lobes in the near field, due to the non-uniformity of the spatially varying (sum) co-array. In order to address this issue, we proposed a novel near field compensation method applicable to sum co-array processing using image addition. The method adapts the array weights to the imaging range and direction by trying to match a desired PSF with support on the sum co-array. A constraint was also added for keeping the number of component image additions low, such that the acquisition time of the image is minimized. Our problem formulation takes the form of a rank-constrained non-convex optimization problem, which uses samples of the array's near field response. We proposed two convex solutions to this problem: a Tikhonov regularized LS approximation, and an SDP relaxation using the nuclear norm. These solutions were evaluated in simulations, which demonstrated that the proposed approaches mitigate near field effects, even when calibration measurements are noisy and only available at a limited number of ranges.

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Fig. 6. Images of three coherent point reflectors at a distance of 11 array baselines. When far field weights are used, the (b) MRA is severely affected by grating lobes in comparison to the (a) ULA. Both (c) LS and (d) nuclear norm based near field compensation effectively suppresses these grating lobes.