Gallo, Pasquale; Remes, Heikki; Romanoff, Jani

Influence of crack tip plasticity on the slope of fatigue curves for laser stake-welded T-joints loaded under tension and bending

Published in:
INTERNATIONAL JOURNAL OF FATIGUE

DOI:
10.1016/j.ijfatigue.2017.02.025

Published: 01/06/2017

Please cite the original version:
Influence of crack tip plasticity on the slope of fatigue curves for laser stake-welded T-joints loaded under tension and bending

Pasquale Gallo*, Heikki Remes*, Jani Romanoff*

*Aalto University, Department of Mechanical Engineering, Marine Technology, Puumiehenkuja 5A, 02150 Espoo, Finland

*Corresponding author: pasquale.gallo@aalto.fi

Abstract

The paper investigates the crack tip plasticity of laser stake-welded T-joints when loaded in tension and bending. Irwin’s first-order plastic zone size is kept equal for the selected load levels, and Finite Element analyses are carried out to investigate the stress redistribution in the plastic zone varying the applied loads. The results allow us to explain why the fatigue slope $k$ is different for bending and tension. Moreover, a new method is proposed that permits the number of cycles to failure under bending to be derived directly from the tension fatigue curve, employing an effective $J$-integral. The results prove that the method is able to give an estimation of the number of cycles to failure under bending with a very good approximation and acceptable errors.

Keywords: crack tip plasticity; laser stake-welds; fatigue assessment; slope of fatigue curve; fatigue strength
1. Introduction

Weight reduction is a crucial design challenge for the transport industry. Lightweight materials and structures are important not only for the automotive and aerospace industries, but also in shipbuilding. For large vessels, a reduction in weight leads to increased efficiency in terms of the cost reduction per ton of goods transported. To achieve the weight reduction, one approach has been to replace thick monolithic plates by sandwich panels made from thin plates. This substitution introduces new engineering challenges: current methods to estimate fatigue life have been developed for thick plates and are not suited to these more complex geometries assembled from thin plates [1–6].

Sandwich panels, especially those obtained through a laser welding process, offer numerous advantages, e.g. a high stiffness-to-weight ratio, heat and noise insulation, and high energy absorption under impulsive loads [7]. In detail, the sandwich panel consists of laser stake-welded T-joints connecting thin plates, as shown in Fig. 1. When the laser penetrates the face plate thickness, $t_f$, the metal in both the face plate and web plate is melted, creating the joint. As the source of the heat is highly localized, the thickness of the web plate, $t_w$, is not fully melted, and a so-called crack-like notch [8] is created on each side; see Fig. 1b. Even if it is small, a gap within the face and the plate is present. It should also be mentioned that in practical applications the resulting welded zone is not centred with respect to the web plate, and as a consequence the two cracks have different lengths.

![Diagram of sandwich panel](image)
Fig. 1. Illustration of a) web-core steel sandwich panel, b) geometrical details of the laser stake welded T-joint, and c) schematic geometry of the T-joint.

In view of the application of these structures in load-carrying situations, investigation of the fatigue behaviour of the welded joints is of primary interest in order to understand the influence of different parameters on the fatigue strength, e.g. the quality of the weld [9] and the complex loading conditions [10,11]. Nevertheless, experimentally verified design procedures for laser stake-welded T-joints are rare, as well as not being included in the actual standards.

Sonsino et al. [4] showed that the International Institute of Welding (IIW) design recommendation [12] applied to thin welded structures results in an overestimation of fatigue strength at high load levels and a conservative estimation at low load levels, in many cases regardless of the fatigue assessment approach applied. This is mainly due to the fact that the assumed slopes of the design S-N curves have been defined in the past for thick and stiff structures, with a thickness higher than 5 mm. To overcome this inconsistency for welded thin structures, the slopes k=5 for normal and k=7 for shear stresses were suggested [4]. Fricke et al. [13] focused on the fatigue strength evaluation of laser stake-welded T-joints subjected to combined axial and shear loads. T-joints and cruciform specimens with a web thickness of 5 and 8 mm were considered. The fatigue tests with axial, shear, and multiaxial loads showed very different slope exponents of the S-N curves, varying between 3 and 22. On the basis of their results, the authors provided several suggestions for the fatigue assessment of those components, following the IIW recommendations [12] and Eurocode [14].

A few studies have tried to explain why the slope of the fatigue curve changes with the loading conditions. The main contributions on the topic are from Frank and collaborators [15–18]. In particular, Ref. [16] presents a study on the fatigue slope of laser stake-welded T-joints for different loading modes. The test results were presented using a J-integral approach and showed that the slope of the fatigue curve depends upon two parameters: the web plate thickness and the type of loading (i.e. tension or bending). These authors also used the Finite Element method to study the highly localized stress distribution around the crack tip [17]. They showed that a relationship exists between the stress gradient at the weld notch and several parameters such as the thickness, weld geometry, and loading condition (tension and bending).

It is clear, from the brief literature review reported above, that the fatigue curve slope is strongly influenced by a different stress distribution close to the crack tip, which can be caused by variation in the loading mode (tension and bending), as well as in the thickness. If, for laser stake-welded T-joints under axial loading, the results and the fatigue slope can often be compared approximately to conventionally welded specimens, this is not true when the same components are subjected to
bending. The key to analyzing the phenomenon under consideration is to find a suitable general condition that enables a comparison within different local stress gradients generated by different loading modes and/or thicknesses. The importance of the stress gradient in fatigue assessment when dealing with notch components in general is clear. The gradient does indeed play an important role in the fatigue analyses, because the mechanisms which cause fatigue damage take place within a finite volume of material, rather than at a single point [19].

The present paper concentrates on the influence of crack tip plasticity on the slope of the fatigue curve for laser stake-welded T-joints, and proposes a new fatigue model that permits the bending number of cycles to failure under bending to be derived by introducing an effective J-integral, $\sqrt{J_{\text{eff}}}$, in the tension fatigue curve. The idea is that in order to compare the bending and tension loading type, the condition of the same first-order plastic zone as defined by Irwin [20] must be imposed. Therefore, an introduction to Irwin’s approach to the estimation of the crack tip plastic zone is given. A new method based on the results obtained is later proposed for the fatigue assessment of laser stake-welded T-joints if the fatigue data for tension or bending is known; if one is known, the other can be computed, assuming small-scale yielding, from the fatigue limit to the high number of cycles region. $\sqrt{J_{\text{eff}}}$ is obtained through a correction factor derived from the analysis conducted on Irwin’s first-order plastic radius. The method is then validated against experiments [17].

2. Crack tip plasticity under different loading conditions

2.1 Crack tip plastic zone: the Irwin approach

Linear elastic stress analysis of cracks proves the stresses tend to infinity at the crack tip. However, in real materials and applications, the stresses at the crack tip must be finite because of localized plasticity at the crack tip, which affects the distribution of the stresses. The crack then becomes a so-called blunt crack with a plastic zone surrounding the tip (see Fig. 2).
In 1968, Irwin [20] proposed a method for the quantification of the size of the crack tip yielding zone. In detail, assuming a mode I singular field in the crack plane, along the bisector the stresses in the $y$-direction (as well in the $x$-direction) can be defined by the well-known relationship:

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}}$$

(1)

Equation (1) holds near the crack tip, in the singularity zone, and expresses the stress as a function of the mode I stress intensity factor and distance from the crack tip $r$. As a first approximation, the plastic zone ahead of the crack tip is obtained by combining a yield criterion and Eq. (1). Assuming, for example, plane stress conditions, the limit within plastic and elastic behaviour occurs when the stress reaches the yielding stress of the material, i.e. when $\sigma_{yy} = \sigma_{YS}$. With a simple substitution of the latter condition in Eq. (1), and solving for the variable $r$, one obtains a so-called first-order estimation [21] of the plastic zone size $r_y$ (see Fig. 2a):

$$r_y = \frac{1}{2\pi} \left( \frac{K_I}{\sigma_{YS}} \right)^2$$

(2)

Inside the plastic zone, the stresses can be considered constant and equal to the yielding value, i.e. elastic perfectly plastic behaviour of the material is assumed. However, because of the plastic yielding at the notch tip, the cross-hatched region depicted in Fig. 2a, represented by the force $F_1$, cannot be carried directly in the plastic zone, because the stresses cannot exceed the yield limit. In order to satisfy the equilibrium conditions, the force has to be carried through by the material beyond $r_y$. For this reason, stress redistribution occurs, increasing the plastic zone by $\Delta r_y$ (Fig. 2b).
By a simple force balance within the forces $F_1$ and $F_2$ depicted in Fig. 2b, one can obtain the so-called second-order estimation of the plastic zone size $r_p$ [21]:

$$r_p = \frac{1}{\pi} \left( \frac{K_I}{\sigma_{YS}} \right)^2,$$  \hspace{1cm} (3)

which is twice as large as $r_y$.

The same considerations can be drawn for plane strain conditions, in which the size of the first- and second-order plastic zones has the same formulation as in Eqs. (2)-(3) but is smaller by a factor of 3. This reduction is due to the triaxial stress state, which implies that the limit within plastic and elastic behaviour occurs when $\sigma_{yy} = \sqrt{3}\sigma_{YS}$ [20–22]. The equations under plane strain conditions are reported below for the sake of clarity:

$$r_y = \frac{1}{6\pi} \left( \frac{K_I}{\sigma_{YS}} \right)^2$$ \hspace{1cm} (4)

$$r_p = \frac{1}{3\pi} \left( \frac{K_I}{\sigma_{YS}} \right)^2,$$ \hspace{1cm} (5)

where Eq. (4) represents the size of the first-order plastic zone and Eq. (5) the size of the second-order plastic zone.

The plastic zone size equations given following the work by Irwin [20], valid under small-scale yielding, should be considered to be rough estimates only. On the other hand, the results obtained with the Irwin approach [20] are very simple from both the theoretical and practical points of view, and are a good method to analyze and compare the crack tip plasticity of different cases. This is especially true when, as in the present paper, the final aim is not a precise estimation of the size of the plastic zone, but the changes in that. Moreover, despite its simplicity, it permits the evaluation of loading quantities related to the size of the plastic zone, such as the force $F_1$.

### 2.2 Definition of the representative crack driving force ratio

In order to analyze and compare the different effects of tension and bending loads on the stress redistribution at the crack tip, it is necessary to define a suitable comparison criterion that is more general than the nominal stress or linear elastic local approach.

Let us consider two hypothetical specimens, with the same geometry and mechanical properties and thus the same yield strength $\sigma_{YS}$, under mode I loading. In general, the imposition of the same plastic zone size as defined by Irwin [20] for both specimens implies that the stress intensity factors
and thus the stress distribution have the same value. As shown by Williams [23,24] and later by Sih et al. [25,26], the stress around a crack tip (as well as the stress intensity factor) in an infinite plate as a result of the bending loads is different from the tension case, despite the fact that it possesses the characteristic inverse square-root singularity in terms of distance from the crack tip [23,24]. However, Irwin still provides useful tools for comparing the tension and bending loading condition if the first-order plastic radius is considered instead:

- it provides an “effective zone”, defined by the first-order plastic radius \( r_y \) (see Fig. 2);
- it permits the stress distribution over the effective zone to be quantified easily.

Going back to the original problem, the same first-order plastic radius and thus the same effective zone is assumed as a comparison condition between tension and bending loads. In other words, an equivalent portion of material over which the stress redistribution and fatigue phenomena take place is defined within tension and bending. The force \( F_1 \), depicted in Fig. 2 is then considered to be the crack driving force acting on the plastic (fatigue) zone, and can be used to quantify the difference between the tension and bending. This idea is very close to the concept of the force method presented by Raju in [27]. Other authors have shown both in the past and in recent applications the potential of the crack driving force concept when dealing with different problems such as creep [28–30] and elastic-plastic stress distribution [31–34]. The crack driving force is theoretically defined by the following integral both for tension and bending:

\[
F_1 = \int_0^{r_y} \sigma_{yy} \, dr - \sigma_{ys} \cdot r_y
\]

where \( \sigma_{yy} \) is the generic elastic stress distribution. The ratio within the force \( F_1 \) of the bending and the tension, assuming the same \( r_y \), is defined as the representative crack driving force ratio \( F_R \):

\[
F_R = \frac{F_1 \text{ bending}}{F_1 \text{ tension}}
\]

The analytical calculation of \( F_R \) is possible but difficult since an analytically correct value of the plastic radius as a function of the applied load is needed in the evaluation of \( F_1 \). This leads to a set of equations to be solved numerically [28,33]. Additional comments are given in the discussion.

In the present paper, the crack driving force ratio is instead easily derived numerically with the aid of finite element analysis, and used as a correction parameter to define an effective J-integral \( \sqrt{J_{eff}} \). This is shown in detail in the next section.

3. Estimation model for the number of cycles to failure of bending load
The fatigue curves of tension and bending are assumed to be linked to each other. Indeed, at two million cycles, both the tension and bending fatigue curves present the same J-Integral value [17,35], i.e. 0.37 kJ$^{0.5}$/m; in the medium and high cycles range, however, the two loading cases differ, since the bending presents a higher fatigue curve slope; see Fig. 3. Small variations in the slopes for the same loading conditions refer to different thicknesses. See Ref. [17] for more details.

![Fatigue curves](image)

**Fig. 3.** Fatigue resistance curves for laser stake-welded T-joints loaded in tension and bending. Data reproduced from [17].

For these reasons, the present authors assume that the bending curve can be derived from the tension curve, and that the slope effect is taken into account through the crack driving force ratio $F_R$ presented previously. On the basis of these assumptions, the new procedure for the fatigue assessment of the bending case is developed below.

The tension and bending fatigue curves, in terms of the square root of the J-Integral, are defined by the following equations [17,18] according to the common Wöhler form:

$$
\left( \sqrt{J} \right)^{k_T} \cdot N_{f,T} = C_T \text{ for tension} 
$$

$$
\left( \sqrt{J} \right)^{k_B} \cdot N_{f,B} = C_B \text{ for bending,}
$$

where $\sqrt{J}$ a generic load, $k$ is the slope of the curve, $N_f$ is the number of cycles to failure and $C$ is a constant. In order to take into account the slope effect and the relationship between the tension and
bending curves, the latter is expressed as a function of the tension curve parameters \((k_T, C_T)\) through an *effective J*-integral defined as follows:

\[
\sqrt{J}_{\text{eff}} = \sqrt{J} \cdot F_R ,
\]

(10)

where \(F_R\) is the crack driving force ratio defined in the Section 2.2. As a consequence the bending fatigue curve Eq. (9) assumes the following form:

\[
\left(\sqrt{J}_{\text{eff}}\right)^{b_T} \cdot N_{f,B} = C_T
\]

(11)

The meaning of Eq. (11) is very important: if the effective J-integral is employed in the fatigue curve of the tension load, the number of cycles to failure under a bending load is easily derived. On the basis of the crack driving force ratio concept \(F_R\), the following procedure for the fatigue assessment of laser stake-welded T-joints under a bending load is proposed:

A. once the desired \(\sqrt{J}\) is selected, evaluation through finite element analysis of the corresponding force \(F_1\) tension and first-order plastic radius \(r_y\);

B. evaluation of the equivalent force \(F_1\) bending for the bending case, assuming the same first-order plastic radius evaluated in the previous step, through finite element analysis (the external load is determined by trial and error until the target \(r_y\) is obtained);

C. evaluation of \(F_R\) as defined by Eq. (7);

D. evaluation of the effective \(\sqrt{J}_{\text{eff}}\) according to Eq. (10);

E. the new effective J-Integral is then used as the input parameter in the fatigue curve equation of the tension load and the number of cycles to failure of the bending is obtained; Eq. (11).

A simplified diagram of the procedure is proposed in Fig. 4.

---

**Fig. 4.** Simplified diagram of the proposed procedure for the evaluation of the number of cycles to failure of laser stake-welded T-joints under a bending load.
By repeating the steps from A to E and varying $\sqrt{J}$, the fatigue curve in the case of a bending load can easily be derived and experimental testing avoided. The proposed method only requires the experimental fatigue curve under tension loading to be available or obtained experimentally considering the geometry of interest. Moreover, the quantities to be evaluated in the procedure do not explicitly need the definition of the externally applied loads since the comparison is carried out only on local parameters.

The relationship within the experimental fatigue curves can also be derived for comparison with $F_R$ if an experimental effective J-Integral is defined as follows:

$$\sqrt{J}_{eff,exp} = \sqrt{J} \cdot H,$$

(12)

where H is the experimental correction parameter. The bending fatigue curve again assumes the form of Eq. (11). With this assumption, the number of cycles to failure evaluated through Eq. (9) and (11) must be equivalent for any J-Integral, that is:

$$\frac{C_B}{(\sqrt{J})^{k_B}} = \frac{C_T}{(\sqrt{J}_{eff,exp})^{k_T}}$$

(13)

Substitution of Eq. (12) in Eq. (13) gives:

$$\frac{C_B}{(\sqrt{J})^{k_B}} = \frac{C_T}{(\sqrt{J})^{k_T} H^{k_T}}$$

(14)

At this point, by simple mathematical manipulations, from Eq. (14) the experimental correction factor H is obtained:

$$H = \left(\frac{C_T}{C_B}\right)^{k_T} \cdot (\sqrt{J})^{k_B - 1}$$

(15)

The parameter H is a function of the fatigue constants, fatigue slopes, and J-Integral. The factor expresses the ratio within $\sqrt{J}$ of the tension and of the bending fatigue curve when the same number of cycles to failure is considered. At this point, equivalence within H and the crack driving force ratio $F_R$ is implicitly assumed and expected from the results:

$$\sqrt{J}_{eff} = \sqrt{J} \cdot F_R = \sqrt{J} \cdot H,$$

(16)

but unlike the correction factor H, $F_R$ is obtained numerically, and knowledge of the experimental bending fatigue curve is not needed. It should be noted that as shown in detail experimentally by [17,35], run-out specimens are obtained for very high numbers of cycles. Therefore the fatigue
curve can be approximated with a straight line over two million cycles, and the parameter H assumes the value of 1. In the light of the above, the final set of equations to be verified is:

\[
F_R \simeq H = \left( \frac{C_T}{C_B} \right)^{\frac{1}{k_T}} \cdot \left( \sqrt{\mathcal{J}} \right)^{k_T - 1} \quad \text{when } \sqrt{\mathcal{J}} > \sqrt{\mathcal{J}_A}
\]

\[
F_R \simeq H = 1 \quad \text{when } \sqrt{\mathcal{J}} \leq \sqrt{\mathcal{J}_A}
\]

where \(\sqrt{\mathcal{J}_A}\) is the J-integral at two million cycles.

4. Case study

4.1 Geometry and mechanical properties

A case study re-adapted from Ref. [17] was selected. A thick specimen was chosen, in order to avoid any influence of the low thickness, and some simplifications were introduced to isolate any other factor. With respect to a real T-joint, the main simplifications were the symmetry along the web plate axis (the cracks have the same length) and the null web-face gap, which, based on Ref. [17], was very small, being on average only 9 \(\mu\)m. In addition, when the contact between the crack surfaces is not modelled, as explained in the next section, 4.2, the modelling of the gap is unnecessary. Other authors also found that there was a negligible influence of the gap when dealing with similar joints [13,36].

In detail, a symmetric web-face T-joint as shown in Fig. 5 was adopted, with the following geometric parameters: thickness of the face plate \(t_f\) and web plate \(t_w\) equal to 8 mm, length of the face plate \(l_f\) equal to 100 mm, and length of the web plate \(l_w\) equal to 60 mm. A crack of length \(a = 2.5\) mm was assumed on each side, which returns an un-cracked ligament \(2b = 3\) mm, while the face-web gap was neglected. The crack length considered here is a medium value close to the common lengths found by [17] for this kind of geometry.

The mechanical properties are reported in Table 1, together with the geometric parameters, for the sake of clarity.
Fig. 5. Case study: laser stake-welded T-joint geometry under a) tension and b) bending load.

Table 1. Case study mechanical properties and geometry parameters

<table>
<thead>
<tr>
<th>σYS</th>
<th>σUTS</th>
<th>E</th>
<th>lw</th>
<th>lw</th>
<th>tf</th>
<th>lf</th>
<th>a</th>
<th>2b</th>
</tr>
</thead>
<tbody>
<tr>
<td>235</td>
<td>400</td>
<td>206</td>
<td>8</td>
<td>60</td>
<td>8</td>
<td>100</td>
<td>2.5</td>
<td>3</td>
</tr>
</tbody>
</table>

4.2 FE analysis

The geometry presented in Fig. 5 and described in the previous section was modelled by means of the ANSYS® APDL15.0 finite element software package and several numerical simulations under linear elastic conditions were carried out. The 2D 8-node element type PLANE183 was employed with unit thickness, while plain strain condition was assumed.

For the tension case, only half of the joint was modelled, and symmetry was imposed along the web plate axis. A uniform nominal stress was applied along the web plate, as shown in Fig. 5a, while all the degrees of freedom were constrained along the face plate. For the bending load, the moment was generated by applying a force P, as depicted in Fig. 5b. Because of the asymmetry of the loading condition, the whole joint was modelled, and constraints were applied along the face plate. In addition to the simplifications of the geometry listed in the previous section, 4.1, the contact within the web and the face along the crack surfaces was neglected. This assumption does not affect the results of the tension loading, while may have an influence when the bending is being studied. Anyway, the contact plays an important role only when the load levels are very high, with a sudden increase in the stiffness, as reported in Ref. [37]. This effect is also reduced when a gap is present. Moreover, in [17], it is shown that the contact on the compressive side does not have a strong influence on the stress gradient, which relates the linear-elastic stress distribution ahead of the crack tip and the slope of the fatigue line. For these reasons, in a first analysis the simplification introduced here becomes reasonable.

A very accurate mesh, i.e. with elements of a size 2.5E-05 mm close to the crack tip, was realized, and sensitivity analysis was carried out on the stress singularity, the stress intensity factor, and the first-order plastic radius. The concentration “keypoint” option [38] was employed to model the crack tip properly.

The following procedure was followed when performing the analysis: first, a very low value of the external load was applied for the tension case, in order to obtain a low starting value of the \( r_y/b \) ratio. Then the numerical stress distribution ahead of the crack tip was stored. Later, following the Irwin formulation, the first-order plastic radius was determined by considering the distance from the crack tip at which the yield stress \( \sigma_{YS} \) was detected, directly from the numerical stress distribution.
Moving on to consider the bending case, the external load was gradually varied until the same first-order plastic radius as that obtained for the tension case was detected. Then the bending stress distribution ahead of the crack tip was stored. At this point, the force $F_1$ could easily be determined by a trapezoidal method algorithm for both tension and bending according to Eq. (6). The J-integral was also evaluated through the FE analysis at the same time for each load case.

Several FE simulations were carried out for the geometry under consideration and five load levels have been selected and presented here for the sake of brevity. The load levels are classified as low and high as a function of the $r_y/b$ ratio, and are listed in Table 2 together with the corresponding numerical J-integral. These are in agreement with the theoretical elastic J-integrals $J_{th}$ [39,40]. The values of $J_{th}$ are evaluated using the mechanical properties and the stress intensity factors listed in Table 1 and Table 3, respectively. The nominal stress $\sigma_{nom}$ for tension and the applied load $P$ for bending shown in Figure 5 are also reported in the same table for the sake of clarity.

Table 2. Load levels and corresponding numerical J-integral, theoretical J-integrals $J_{th}$, and applied nominal loads; $\sigma_{nom}$ refers to the gross area (see Fig. 5), $b=1.5$ mm.

<table>
<thead>
<tr>
<th>Load Level</th>
<th>$r_y$ [mm]</th>
<th>$r_y/b$</th>
<th>$J_{tension}$ [kJ/m²]</th>
<th>$J_{bending}$ [kJ/m²]</th>
<th>$J_{th_tension}$ [kJ/m²]</th>
<th>$J_{th_bending}$ [kJ/m²]</th>
<th>$\sigma_{nom}$ [MPa]</th>
<th>$P$ [N/mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.010</td>
<td>0.007</td>
<td>0.0159</td>
<td>0.0163</td>
<td>0.0159</td>
<td>16.875</td>
<td>1.65</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.075</td>
<td>0.050</td>
<td>0.1133</td>
<td>0.1269</td>
<td>0.1131</td>
<td>45</td>
<td>4.60</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.152</td>
<td>0.101</td>
<td>0.2141</td>
<td>0.2733</td>
<td>0.2138</td>
<td>61.875</td>
<td>6.75</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.231</td>
<td>0.154</td>
<td>0.3146</td>
<td>0.4711</td>
<td>0.3149</td>
<td>65.125</td>
<td>8.75</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.306</td>
<td>0.204</td>
<td>0.4053</td>
<td>0.6996</td>
<td>0.3976</td>
<td>85.125</td>
<td>10.80</td>
<td></td>
</tr>
</tbody>
</table>

5. Results

5.1 Crack driving force ratio $F_R$

The results obtained in terms of $F_1$ as a function of the normalized first-order plastic radius are depicted in Fig. 6 and summarized in Table 3. The $K_I$ values and the $K_{II}/K_I$ bending ratio are also reported in the same table in order to detect any mode II contribution. For a very low value of the $r_y/b$ ratio, the tension and bending perfectly match each other, being $F_1=2.42$ N/mm for the tension and $F_1=2.45$ N/mm for the bending. In other words, there is no difference in terms of the local stress gradient and re-distribution between bending and tension. With the $r_y/b$ ratio increasing, the two loading conditions start to differ. The difference is shown in detail in Table 3, where the crack driving force ratio $F_R$ becomes 1.39 when $r_y/b = 0.2$, being $F_1=65.12$ N/mm for the tension and $F_1=90.30$ N/mm for the bending. The same trend is found for the stress intensity factors. The effect of mode II in the bending case can be neglected since the stress intensity factor ratio $K_{II}/K_I$ equals 0.
for all of the load levels. This is easily achievable by modelling a web plate long enough to generate high stresses at relatively small values of the applied external force $P$.

![Graph showing the evolution of the force $F_1$ as a function of the normalized first-order plastic radius; $b=1.5$ mm is half un-cracked ligament.]

Table 3. Force $F_1$ for the selected load levels, $F_R$ ratio (bending/tension), stress intensity factors, $K_{II}/K_I$ ratio of the bending load configuration, $b=1.5$ mm

<table>
<thead>
<tr>
<th>Load Level</th>
<th>$r_y/b$</th>
<th>$F_1$ tension [N/mm]</th>
<th>$F_1$ bending [N/mm]</th>
<th>$F_R$ ratio (bend./tens.)</th>
<th>$K_I$ tension [MPa mm$^{0.5}$]</th>
<th>$K_I$ bending [MPa mm$^{0.5}$]</th>
<th>$K_{II}/K_I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.007</td>
<td>2.42</td>
<td>2.45</td>
<td>1.01</td>
<td>60</td>
<td>60</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.050</td>
<td>17.50</td>
<td>18.72</td>
<td>1.07</td>
<td>160</td>
<td>160</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0.101</td>
<td>33.53</td>
<td>39.25</td>
<td>1.17</td>
<td>220</td>
<td>247</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0.154</td>
<td>49.41</td>
<td>63.31</td>
<td>1.28</td>
<td>267</td>
<td>321</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0.204</td>
<td>65.12</td>
<td>90.30</td>
<td>1.39</td>
<td>300</td>
<td>388</td>
<td>0</td>
</tr>
</tbody>
</table>

The difference between the bending and tension crack driving forces is synthesized by the crack driving force ratio $F_R = F_1$ bending/$F_1$ tension, and it is interesting to investigate the evolution of this parameter with respect to several variables such as $r_y/b$ and the number of tension cycles to failure $N_f$. The number of cycles to failure is evaluated on the basis of the J-integral values shown in Table 2 and fatigue curves proposed in Fig. 3. To cover the fatigue strength variation resulting from thickness for the same loading condition (see Fig. 3 and [17]), a simplified unified fatigue curve for
tension is assumed: an average value of the fatigue slope \( k \), which represents the geometry under consideration better, is assumed to equal 4.2, while a unique fatigue strength at two million cycles equal to 0.37 kJ\(^{0.5}/m\) is considered as reported in [17].

By simple substitutions in Eq. (18), for each numerical \( \sqrt{J} \) associated with the tension load levels, the corresponding number of cycles is easily evaluated:

\[
\left( \sqrt{J} \right)_A^k \cdot N_{f,A} = \left( \sqrt{J} \right)^k \cdot N_f,
\]

where

- \( \sqrt{J}_A \) is the fatigue strength at two million cycles equal to 0.37 kJ\(^{0.5}/m\) [17];
- \( N_{f,A} \) is the number of cycles to failure at the fatigue strength equal to two million cycles;
- \( \sqrt{J} \) is a generic tension load level;
- \( N_f \) is the number of cycles to failure corresponding to the selected \( \sqrt{J} \);
- \( k \) is the fatigue slope, assumed here to equal 4.2 for tension.

The results are summarized in Table 4 and depicted in semi-log scale in Figure 7. For a high number of cycles, the ratio tends to assume the value of 1, while when \( N_f \) approaches low values, the ratio increases. The crack driving force ratio perfectly shows what happens when the number of cycles to failure varies, underlining that the bending and the tension are not equivalent.

Table 4: Evolution of \( F_R \) as a function of \( r_y/b \) and number of cycles to failure of the tension loading case. *simplified curve: \( k = 4.2 \), fatigue strength at two million cycles=0.37 kJ\(^{0.5}/m\).

<table>
<thead>
<tr>
<th>Load Level</th>
<th>( r_y/b )</th>
<th>( \sqrt{J} ) tension [kJ^{0.5}/m] )</th>
<th>( N_f ) (Tension)*</th>
<th>( F_R ) ratio (bend./tens.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.007</td>
<td>0.1263</td>
<td>1.83E+08</td>
<td>1.01</td>
</tr>
<tr>
<td>2</td>
<td>0.050</td>
<td>0.3365</td>
<td>2.98E+06</td>
<td>1.07</td>
</tr>
<tr>
<td>3</td>
<td>0.101</td>
<td>0.4627</td>
<td>7.82E+05</td>
<td>1.17</td>
</tr>
<tr>
<td>4</td>
<td>0.154</td>
<td>0.5609</td>
<td>3.48E+05</td>
<td>1.28</td>
</tr>
<tr>
<td>5</td>
<td>0.204</td>
<td>0.6366</td>
<td>2.05E+05</td>
<td>1.39</td>
</tr>
</tbody>
</table>
Fig. 7. Crack driving force ratio against number of cycles to failure of the tension load case.

5.2 Evaluation of the proposed model

To evaluate the proposed model, first, the equivalence within the experimental correction factor $H$ and the crack driving force ratio $F_R$ expressed by Eq. (17) was verified; later, the proposed procedure was applied to experimental data taken from the literature [17].

The parameter $H$ was derived for several arbitrary $\sqrt{J}$ through Eq. (15), and plotted as a function of the tension number of cycles to failure in Fig. 8 on a log-log scale, together with the force ratio $F_R$ that was obtained (bending/tension). Only the load levels within the $10^4$ and $2\cdot10^7$ cycles were considered. Two million cycles are indicated by a vertical line. An average value of the fatigue slopes $k=4.2$ for tension and 7.0 for bending that better represent the geometry under consideration are assumed, while a unique fatigue strength at two million cycles equal to 0.37 kJ$^{0.5}$/m is considered [17]. The fatigue constants ratio $C_T/C_B$ based on the experimental curves is equal to 16.187. All the parameters needed for the estimation of $H$ are reported in the figure for the sake of clarity. Figure 8 shows that the trend of the force ratio $F_R$ was found to be equal to the trend of the correction factor $H$ within medium and high cycles range. This result is very important, since the equivalence within the two parameters is proved and the effective J-Integral can be evaluated through $F_R$. 
Fig. 8. Trends of the correction factor $H$ and crack driving force $F_R$ ratio as a function of the tension number of cycles to failure; subscript $B$ refers to the bending curve, $T$ to tension.

The proposed approach was tested with consideration being given to the available tension load levels since for these values the exact $F_R$ was known. The estimated number of cycles to failure for bending evaluated according to Eq. (11) was then compared with the expected experimental value [17]. The results are reported in Table 5. Load levels 1 and 2 present a $J$-Integral value lower than the fatigue limit. In these cases, a number of cycles to failure greater than two million is reported, and $F_R$ is assumed to be equal to 1, according to our hypothesis. The results clearly prove that the method is able to give an estimation of the number of cycles to failure under bending with a very good approximation. The errors, if compared to the common scatter band of fatigue data, are absolutely acceptable. It must be noted that anyway, the number of cycles to failure is always slightly overestimated. In detail, the error varies from -3% to 16%.

Table 5. Comparison between the estimated and expected number of cycles to failure for bending load. *simplified curve: $k=4.2$, fatigue strength at two million cycles=0.37 kJ$^{0.5}$/m. ** simplified curve: $k=7$, fatigue strength at two million cycles=0.37 kJ$^{0.5}$/m [17].

<table>
<thead>
<tr>
<th>Load Level (tension)</th>
<th>$F_R$ ratio (bend./tens.)</th>
<th>$\sqrt{J}$ [kJ$^{0.5}$/m]</th>
<th>$\sqrt{J}_{eff}$ [kJ$^{0.5}$/m]</th>
<th>$N_f$ bending Estimated*</th>
<th>$N_f$ bending Experimental**</th>
<th>$\Delta%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0.1263</td>
<td>0.1263</td>
<td>&gt;2.00E+06</td>
<td>&gt;2.00E+06</td>
<td>-</td>
</tr>
</tbody>
</table>
6. Discussion

The results show that for high external load levels the two load cases cannot any longer be considered equivalent; see Fig. 6. The bending first-order plastic zone, in which the fatigue phenomena take place, is subjected to a bigger crack driving force $F_1$ and so to a more critical stress field. When the results are considered in terms of the J-integral, the same trends can be found. It can be asserted that, if an interpretation from the fatigue viewpoint is given, the bigger bending crack driving force $F_1$ generates a higher level of damage in the representative fatigue volume, which results in a lower number of cycles. This is true despite the fact that the same crack is considered for bending and tension. The physical meaning of this statement is clearly shown in Fig. 9. The low load values situation and corresponding stress distribution is labelled as number 1, the high load values situation as number 2. In case 1, the tension and bending return the same stress redistribution; for case 2 (high load levels) the crack driving force $F_1$ resulting from bending ($F_{1,b}$ and the area depicted in red) is higher than the tension case ($F_{1,t}$ and the area depicted in black). From the fatigue point of view, for a chosen number of cycles to failure, this phenomenon is taken into account by a lower value of $\sqrt{J}$ for the bending needed to generate the same amount of damage.
Fig. 9. High load level (2) and low load level (1) crack driving force, and matching with fatigue curves; the subscript \( b \) stands for bending, \( t \) for tension.

The slope issue, as briefly summarized in the introduction, has been found by different authors who provided recommendations but did not address the phenomenological aspect. Frank et al. [17,18] mainly focused on laser stake-welded T-joints and loading conditions, but other results can be found when considering thin and thick joints [4]. The interpretation given here can be extended to those cases in which different slopes have been found for different loading conditions, such as in Sonsino et al. [4] or Fricke et al. [13]. In [17], where an analysis of the slope of laser stake-welded T-joints is presented, an interpretation of the slopes in terms of the relative stress gradient was also given. Despite the fact that the analysis did not address the plasticity occurring at the crack tip, the relative stress gradient defined by the author also showed a relationship between the slopes of the fatigue curves as a function of the loading conditions, including the case when mode II was considered. These results are in agreement with the conclusion drawn in the present paper, where a deeper and more detailed analysis gave a more detailed explanation which takes the local plasticity into consideration. Further analysis based on the size of the first-order plastic zone considering the thickness variation may lead to the same conclusions but giving a physical explanation of the phenomenon.

The parameter \( F_R \) proved to estimate very well the ratio within \( \sqrt{J} \) for bending and tension at the same number of cycles to failure. For this reason, it resulted in being equal to the parameter \( H \) (see Fig. 8), which mathematically expressed the relationships between the experimental fatigue curves, and can be employed as a correction factor for the evaluation of the effective J-integral. Following the proposed procedure, the fatigue assessment of a laser stake-welded T-joint can be carried out only on the basis of the geometry of interest and the corresponding tension fatigue curve. The results are in good agreement with the experimental ones (see Table 5). Even if the fatigue testing under a bending load is not needed any more, finite element analysis is still necessary for the
evaluation of $F_R$. It is worth noting that all the analyses are carried out in linear elastic condition, and that few analyses are needed if the relationship between $F_R$ and the $r_y/b$ ratio is derived as follows. Figure 10 plots the crack driving force ratio against the $r_y/b$ ratio, showing that the two parameters are linearly dependent, and in agreement with comments provided by Morais [41]. The load levels present a good alignment in general and the dependence can be mathematically expressed by the following equation, which was derived without load level 1 being considered:

$$F_R = 2.155 \frac{r_y}{b} + 0.9587$$  \hspace{1cm} (19)

![Figure 10. Crack driving force evolution against $r_y/b$; b=1.5 mm.](image)

If the steps from A to E of the procedure proposed in Section 3.1 and Fig. 4 are repeated for limited numbers of $\sqrt{J}$, Eq. (19) is easily obtained for the geometry of interest. At this point, the $F_R$ can be evaluated for any given $r_y$ (and so loading condition) directly from step A, avoiding any additional analysis of the bending configuration (step B). This expedient drastically reduces the amount of analysis that is needed.

From the theoretical point of view, some considerations regarding the crack driving force can be drawn. In the present paper, the forces $F_1$ and crack driving force ratio were evaluated numerically, with the stress distribution obtained through FE analysis. But Equation (6) can be further developed analytically. For example, under a tension load, the integral is solved once stress distribution is assumed, e.g. the Irwin solution, leading to Eq. (20):
Equation (20) can be further developed when a yielding criterion is selected. It should be noted that the crack driving force is a function of the first-order plastic radius and field parameter stress intensity factor. The latter would be different when a double-edged crack plate was subjected to tension and to bending, while \( r_y \) is assumed constant end equal within the two loading conditions. For these reasons, the analytical calculation of \( F_R \) is possible but difficult, and numerical methods are needed. In fact, while for a double-edged crack plate several stress intensity factor expressions as a function of the externally applied load are given [42–45], to the best of the authors’ knowledge this is not true when the first-order plastic radius is considered.

Another important consideration is the possibility through Eq. (20) of involving the nominal load. In fact, the stress intensity factor is also defined by the following formulation:

\[
K_I = \sigma_{\text{nom}} Y \sqrt{\pi a},
\]

where \( \sigma_{\text{nom}} \) is the applied nominal load, \( Y \) is the shape factor, and \( a \) is the crack length. Once the nominal load of tension and bending is defined, the force \( F_1 \) is linked to the applied nominal load:

\[
F_1 = \frac{\sigma_{\text{nom}} Y \sqrt{\pi a}}{\sqrt{2\pi}} \cdot 2\sqrt{r_y} - \sigma_{YS} \cdot r_y
\]

It must be underlined that in general a plastic zone correction factor is needed when the plastic zone size is theoretically developed. Generalizing:

\[
F_1 = \left( \frac{\sigma_{\text{nom}} Y \sqrt{\pi a}}{\sqrt{2\pi}} \right)^2 \cdot g, \text{ where}
\]

\[
g = f(\sigma_{YS}; C_{r_y})
\]

\( C_{r_y} \) is the plastic zone correction factor, e.g. [34], that would be different for bending and tension, as well as the shape factor \( Y \). The development of these parameters is currently challenging, and it is left for future work. Potentially, once the parameters that are needed are known, the forces \( F_1 \) and thus the force ratio \( F_R \) are evaluated analytically as a function of the applied nominal load, without the employment of FE analysis or any other heavy computation. In addition, Eq. (20), should be further developed considering an alternative formulation [46,47] for the plastic radius to the one provided by Irwin, which, as is well known, does not give an exact solution.

The approach based on the crack driving force ratio has the potential to be extended to several cyclic problems, dealing with different geometries, from notches to cracks, and to several loading conditions. In fact, with the aim of superposition principles, different loading modes can be taken into account, noted that the geometry factor for axial and bending is different and the stress
intensity factors of different modes cannot be added together. Several examples and further developments of the crack driving force are available in the literature [28–30,32–34] dealing with different applications and notch configurations, as well as multiaxial loading. On the basis of those developments, the procedure proposed here can be extended to different notch geometries from cracks, such as blunt U-notches [34], blunt V-notches [28], and sharp V-notches [29]. On the other hand, to the best of the authors’ knowledge, a numerical method is still necessary in the evaluation of the size of the plastic zone once stress distribution equations and a yielding criterion are combined.

7. Conclusions
In this paper, the influence of the loading condition on the fatigue behavior of laser stake-welded T-joints was investigated, and a new model for the fatigue life estimation of the joint under bending was presented.

Finite Element analyses were carried out on a stake-weld T-joint loaded under tension and bending and the first-order plastic zone size was kept equal for selected load levels. For small plastic zones (i.e. low values of \( r_y/b \)), bending and tension loads are completely equivalent and return the unified fatigue strength of Fig. 9. For large plastic zones (i.e. high values of \( r_y/b \)) the bending presents a higher crack driving force called \( F_1 \). The difference is quantified through a crack driving force ratio, \( F_R \), which increases as the load does (Fig. 7). The obtained \( F_R \), permits to take into account the more critical condition of the bending load type, and it has been employed as a correction factor for \( \sqrt{J} \) to define an effective J-integral \( \sqrt{J_{eff}} \). The effective J-integral is later employed in the tension fatigue curve to obtain the number of cycles to failure of the bending corresponding to \( \sqrt{J} \). The results are in agreement with the experimental tests (see Table 5). Future work should explore the accuracy of the proposed method for different loading modes and notch geometries. The method also has good potential to be completely defined analytically, providing the correct analytical formulation for the first-order plastic radius and the formulation of the stress intensity factor as a function of the nominal loads in tension and bending.

References


[38] Mandenci E, Guven I. The finite element method and applications in engineering using ANSYS. Springer; 2006.


