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Propagation of modeling uncertainty in stochastic heat-transfer simulation using a chain of deterministic models

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Abstract

When using a chain of numerical models in a stochastic simulation, the distribution of the observed output depends on both the input parameter uncertainty and the errors of the individual models in the chain. In this work, the propagation of model uncertainty is studied in a simple one-dimensional heat transfer system. The errors in temperature are found to depend on the heat flux coupling scenario and on the type of the input parameter distributions. The radiation heat flow boundary condition limits the error propagation by compensating the gas temperature errors through enhanced heat losses. Model biases were found to be detrimental to the accuracy of the predicted probabilities of exceeding safety criteria. Finally, corrections to the predicted distribution moments are proposed and tested, showing that the error contributions can be effectively eliminated from the observed distributions if the properties of the individual models are well known.

1 Introduction

Uncertainty quantification method can be deterministic or stochastic [1]. In the deterministic method, output uncertainty is expressed as a function of input uncertainties using Taylor expansion method, Hermite polynomials or basis functions [2, 3]. Some examples of such method are

generalized Polynomial Chaos (gPC), Gaussian closure, Equivalent linearization and the use of meta-models [4, 5]. Deterministic methods are popular for linear problems and in cases in which uncertainties are relatively small. Similarly, some examples of stochastic methods are Monte-Carlo (MC), Latin Hypercube Sampling (LHS) and Fourier Amplitude Sensitivity Test (FAST) [6–8]. Stochastic methods involve a large number of model realizations with randomly sampled inputs. Despite the high computational cost, they are widely used in various fields of science and engineering.

For a chain of non-linear models involving multi-physics problems, deterministic uncertainty quantification becomes mathematically challenging. There is also a problem with the stochastic method as the stochastically calculated output uncertainty can be due to both input parameter uncertainty and modeling uncertainty [9]. If the effect of the input parameter uncertainty on output uncertainty is far higher than the effect of the modeling uncertainty, one can simply ignore modeling uncertainty and conclude that the stochastically calculated output uncertainty to be the desired output uncertainty. However, there are uncertainty quantification cases in which the effect of modeling uncertainty on the output uncertainty cannot be ignored. Stochastic methods can then be used to investigate the relative importance of the parameter and model uncertainties [10, 11]. The main aim of this work is to develop a method using which one can eliminate the effect of the modeling uncertainty from the stochastically simulated output uncertainty.

An example case is the design of buildings parts against fire load. It involves prediction of compartment wall temperatures for various possible fire scenarios [12–14]. The wall temperature is predicted using two numerical models; the gas model that estimates the gas temperature near the wall and the Finite Element Method (FEM) based wall model that simulates heat-diffusion in the wall [15]. Inputs for the gas model are the location and size of the fire, ambient temperature, and the material and dimensional properties of the compartment. Similarly, inputs for the wall model are the heat-flux predicted by the gas model, ambient temperature, and the material and dimensional properties of the wall. Uncertainties in the input parameters of both models propagate to the wall temperature prediction. In addition to the input parameter uncertainty, modeling errors associated with each model also propagate along the chain and may affect the wall temperature

prediction.

In fire safety engineering, it is common to model the model uncertainty using two components; systematic error (bias) and random error. Such parameters can be determined for each model and each quantity through the validation process. McGrattan and Toman present the model uncertainty parameters associated with the gas model output quantities and a technique using which one can estimate the likelihood of the output exceeding an assumed threshold value [16, 17]. Similar method has not been proposed for the stochastic simulations where the input and model uncertainties combine. In this work, we calculate the model uncertainty parameters for a chain of numerical models and demonstrate a method to correct the result of the stochastic simulations. We present MC simulation using one of the deterministic models, i.e., wall model, for a range of different boundary conditions, error-cases, and input parameter distributions. The model uncertainty parameters of the chain are estimated by comparing the result of the MC simulations obtained under different error conditions. The stochastically calculated output distribution is corrected using a simple deterministic relationship and the parameters representing model uncertainty.

2 Methods

2.1 Heat Transfer in Solids

The heat transfer along the thickness x of a solid wall exposed to a heat source can be predicted using a one-dimensional parabolic heat diffusion equation,

$$\rho c_p \frac{\partial T(x, t)}{\partial t} = \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x}(x, t) \right), \quad (1)$$

where k is thermal conductivity, ρ is density and c_p is specific heat capacity. The initial condition is $T(x, 0) \equiv T_0 = 20^\circ\text{C}$. Denoting the hot- and cold-side temperatures at time t as $T_1 \equiv T(0, t)$ and $T_2 \equiv T(l, t)$, the boundary conditions of Eq. 1 consist of a linear convective and a nonlinear

radiative heat-fluxes,

$$\begin{aligned} -k \frac{\partial T}{\partial x} \Big|_{x=0} &= -h(T_g - T_1) - e\sigma(T_g^4 - T_2^4), \\ -k \frac{\partial T}{\partial x} \Big|_{x=l} &= -h(T_2 - T_\infty) - e\sigma(T_2^4 - T_\infty^4), \end{aligned} \quad (2)$$

where h is heat transfer coefficient, e is thermal emissivity and σ is the Stefan-Boltzmann constant. T_g and T_∞ are the hot-side and cold-side ambient temperature respectively.

A numerical solution is obtained using a discretized system, shown in Figure 1. The FEM solution for the temperatures obtained using 1st order shape function and explicit Euler time scheme is [18]

$$\mathbf{T}_{1..N,j} = [\mathbf{C}]^{-1} [(\mathbf{C} - \Delta t \mathbf{K}) \mathbf{T}_{1..N,j-1} - \Delta t \mathbf{F}], \quad (3)$$

where $\mathbf{K}$, $\mathbf{C}$ and $\mathbf{F}$ are conductance matrix, capacitance matrix and force vector given as

$$\frac{k}{\Delta x} \begin{pmatrix} 1 & -1 & \cdots & 0 & 0 \\ -1 & 2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 2 & -1 \\ 0 & 0 & \cdots & -1 & 1 \end{pmatrix}, \quad \frac{\rho c_p \Delta x}{6} \begin{pmatrix} 2 & 1 & \cdots & 0 & 0 \\ 1 & 4 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 4 & 1 \\ 0 & 0 & \cdots & 1 & 2 \end{pmatrix}, \quad \text{and} \quad \begin{pmatrix} F_{1,j} \\ 0 \\ \vdots \\ 0 \\ F_{2,j} \end{pmatrix}$$

respectively, where $F_{1,j} = -h(T_g - T_{1,j-1}) - e\sigma(T_g^4 - T_{1,j-1}^4)$ and $F_{2,j} = -h(T_{N,j-1} - T_\infty) - e\sigma(T_{N,j-1}^4 - T_\infty^4)$.

Figure 1: Heat transfer model, α represents the wall material parameters.

2.2 Uncertainty Propagation

The modelling chain consists of the gas (G) and wall (W) models. The gas model output T_g serves as an input to the wall model, from which we observe as an output the cold-side temperature

$$T_2 = f_W(T_g, \alpha) \quad (4)$$

where α is a vector of wall model inputs, e.g. material properties. Figure 2 illustrates the connections between model inputs, output T_2 and the model errors.

Figure 2: The modeling chain.

We assume that for each model, the relation between simulated and true output can be explained using two different model error parameters; bias factor and random error. The bias factor is a measure of a multiplicative factor by which the observed output is away from the true value. On average, it is the ratio of observed and true output. The random error represents an additive error that could be observed as a fluctuation of the model output around its true value. Both bias and random error are defined and evaluated using the temperature increase above the initial temperature $(T - T_0)$, but this detail is omitted in the following to simplify the notation. In the presence of gas temperature bias factor δ_G and random error ϵ_G , the observed gas temperature is

$$T_g\{G\} = \delta_G T_g\{\} + \epsilon_G, \quad (5)$$

where $T_g\{\}$ denotes the true gas temperature. Similarly, assuming errors δ_W and ϵ_W in the wall

model but no models in the gas phase, would yield observed wall model outputs

$$T_2\{W\} = \delta_W f_W(T_g\{\}, \alpha) + \epsilon_W = \delta_W T_2\{\} + \epsilon_W. \quad (6)$$

Finally, having errors in both models, we would observe

$$T_2\{G,W\} = \delta_W f_W(T_g\{G\}, \alpha) + \epsilon_W. \quad (7)$$

Alltogether, we have four possible combinations of model errors propagating into the wall temperature (Table 1).

Table 1: Observed outputs for different combinations of modeling errors.

		$T_2 = f_W(T_g, \alpha)$	
		No error	Error
T_g	No error	$T_2\{\}$	$T_2\{W\}$
	Error	$T_2\{G\}$	$T_2\{G,W\}$

According to the definition, T_g and ϵ_G are independent, and the first order approximation for the observed mean and variance of the error-containing $T_g\{G\}$ becomes

$$\mu_{T_G}\{G\} = \delta_G \mu_{T_G}\{\}, \quad \sigma_{T_G}^2\{G\} = \delta_G^2 \sigma_{T_G}^2\{\} + \sigma_{\epsilon,G}^2 \quad (8)$$

where $\mu_{T_G}\{\}$ and $\sigma_{T_G}^2\{\}$ are the mean and variance of the error-free gas temperatures, and $\sigma_{\epsilon,G}^2$ represents the variance of ϵ_G . For the two models in a chain, we can write

$$\mu_{T_2}\{W\} = \delta_W \mu_{T_2}\{\}, \quad \sigma_{T_2}^2\{W\} = \delta_W^2 \sigma_{T_2}^2\{\} + \sigma_{\epsilon,W}^2 \quad (9)$$

and

$$\mu_{T_2}\{G,W\} = \delta_W \mu_{T_2}\{G\}, \quad \sigma_{T_2}^2\{G,W\} = \delta_W^2 \sigma_{T_2}^2\{G\} + \sigma_{\epsilon,W}^2 \quad (10)$$

Combining Eq 9 and 10, we can solve for the mean and variance of the true, error-free wall

temperatures

$$\mu_{T_2}\{\} = \frac{\mu_{T_2}\{G\}\mu_{T_2}\{W\}}{\mu_{T_2}\{G,W\}}, \quad \sigma_{T_2}^2\{\} = \sigma_{T_2}^2\{G\} - \frac{\mu_{T_2}^2\{G\}}{\mu_{T_2}^2\{G,W\}} [\sigma_{T_2}^2\{G,W\} - \sigma_{T_2}^2\{W\}]. \quad (11)$$

2.3 Stochastic analysis

The stochastic analysis was carried out using the modeling chain (Figure 2) with assumed distributions of the gas model output T_g and the wall model inputs α . Both T_g and α are inputs for f_W but the modeling errors were introduced only into T_g .

Three types of distributions were used for the uncertain parameters: Normal, Uniform, and a single value (equal to the mean of the Normal). These are commonly used in the context of the fire risk analyses. Table 2 lists the uncertain parameters. Columns 2-3 show the first and second moments of normal distribution and columns 4-5 indicate the bounds the uniform distributions.

Table 2: Parameters of the distributions presenting stochastic uncertainties.

Input parameters	Mean, μ	Standard deviation, σ	Lower value	Upper value	Unit
Gas Temperature, T_g	1200	100	780	1580	$[^{\circ}C]$
Thermal conductivity, k	1	0.1	0.6	1.4	$[Wm^{-1}K^{-1}]$
Specific Heat Capacity, c_p	2200	200	1400	2900	$[JKg^{-1}K^{-1}]$
Density, ρ	900	90	530	1260	$[Kgm^{-3}]$
Heat transfer coefficient, h	9	0.7	6	12	$[Wm^{-2}k^{-1}]$
Emissivity, e	0.7	0.07	0.4	1	$[Wm^{-2}k^{-1}]$

Altogether 18 different combinations of the heat flux boundary conditions and T_g and α parameter distribution types were studied, as listed in Table 3. To investigate the effect of the dominating mode of heat flux on the uncertainty propagation, separate sets of simulations were carried out for the cases where the heat flux is purely convective, purely radiative, or includes both modes. For each heat flux combination, two different gas temperature distributions and three different wall parameter distributions were considered.

For each of the 18 cases, we generated $N = 10000$ input variable samples using LHS. In this method, the cumulative density function (CDF) corresponding to each variable is divided into N partitions. One sample is randomly chosen from each partition, resulting in N samples for each variable. Then out of N samples corresponding to each variable, one sample is randomly chosen

to form one set of input variable sample. This is repeated N times to form N sets of input variable samples. Using LHS, one can obtain the converged solutions with comparatively small sample size, hence is a computationally cost-effective method to carry out the Monte Carlo simulations. [7].

Table 3: Study cases.

S.N.	Symbol	Heat-flux	Gas temperature	Wall parameters
1	conv.u.s	convective	uniform	single
2	conv.u.u	convective	uniform	uniform
3	conv.u.n	convective	uniform	normal
4	conv.n.s	convective	normal	single
5	conv.n.u	convective	normal	uniform
6	conv.n.n	convective	normal	normal
7	rad.u.s	radiative	uniform	single
8	rad.u.u	radiative	uniform	uniform
9	rad.u.n	radiative	uniform	normal
10	rad.n.s	radiative	normal	single
11	rad.n.u	radiative	normal	uniform
12	rad.n.n	radiative	normal	normal
13	both.u.s	both	uniform	single
14	both.u.u	both	uniform	uniform
15	both.u.n	both	uniform	normal
16	both.n.s	both	normal	single
17	both.n.u	both	normal	uniform
18	both.n.n	both	normal	normal

For each of the 18 cases, four different uncertainty scenarios were investigated (Table 1). The 1D FEM model, $T_2 = f_W(T_G, \alpha)$ was assumed to be an ideal model that produces an error-free output. Model errors were then imposed on T_g and T_2 as bias and random error. For both models, the relative bias factor was $\delta = 1.1$ and the random errors were sampled from a normal distribution with a relative second central moment $\tilde{\sigma}_\epsilon \equiv \sigma_\epsilon/\mu = 0.1$ respectively. Appendix 1 shows the histogram plots of the absolute random errors.

3 Results

3.1 Wall temperature

Figure 3 shows the simulated hot-side and cold-side wall temperatures that were obtained with the mean values of the input parameters. With the constant gas temperature T_g , the hot-side surface temperature, T_1 quickly rises and gradually approaches T_g . The cold-side surface temperature, T_2 rises much slower due to the thermal inertia and low conductivity of the wall. The black dashed line indicates a steady-state cold-side surface temperature, T_τ .

Figure 3: Wall hot-side (red line) and cold-side (black line) surface temperatures.

After 10000 LHS iterations, the statistics of the wall temperature T_2 were collected. As an illustrative example, Figure 4 visualizes the effects of the imposed model errors on T_2 at 180 min of simulation in case n.o 18 of Table 3. Plots corresponding to remaining 17 cases are presented in Appendix 2. $T_2\{\}$ represents the output in which no model errors were present. $T_2\{G,W\}$, $T_2\{G\}$ and $T_2\{W\}$ represent the outputs with model errors in both models, only in the Gas model and only in the Wall model, respectively.

Figure 4 indicates that the presence of the gas model error primarily shifts the T_2 distribution towards the higher temperatures, while the presence of the wall model error, in addition to shifting, widens the distribution. The wall model error directly influences the output distribution, while the gas model error first needs to propagate through the heat-diffusion model. Similar trends can be seen for the remaining 17 cases.

Figure 4: Distribution of cold-side surface temperatures obtained under different model error scenarios.

3.2 Temperature uncertainty

The propagated errors are evaluated by comparing the observed quantities, $T_2\{*\}$, and the true quantity, $T_2\{\}$. Since the mean of random error is zero, the propagated bias factor, δ is simply the ratio of observed mean, $\mu_{T_2}\{*\}$ and the error-free mean, $\mu_{T_2}\{\}$.

$$\delta\{*\} = \frac{\mu_{T_2}\{*\}}{\mu_{T_2}\{\}} \quad (12)$$

The second central moment of the propagated random errors, σ_ϵ can be calculated using equation 6, and the relative second central moment of the random errors, $\tilde{\sigma}_\epsilon$ is simply the ratio of σ_ϵ and the observed mean, $\mu_{T_2}\{*\}$.

$$\sigma_\epsilon\{*\} = \left[\frac{1}{N-1} \sum_{i=1}^N \epsilon_i^2 \right]^{\frac{1}{2}} = \left[\frac{1}{N-1} \sum_{i=1}^N (T_{2,i}\{*\} - \delta T_{2,i}\{\})^2 \right]^{\frac{1}{2}}, \quad \tilde{\sigma}_\epsilon\{*\} = \frac{\sigma_\epsilon}{\mu_{T_2}\{*\}} \quad (13)$$

where N is the sample size.

Figure 5 presents the propagated modeling-errors for T_2 at 180 min in terms of δ (marker) and $\tilde{\sigma}_\epsilon$ (error bars). Symbol α indicates the distribution of the wall model parameters, and T_g indicates the type of distribution chosen for the gas temperature.

Figure 5: Propagated model-error associated with the prediction of cold-side surface temperature.

Figure 5 shows that the propagated model-errors are independent of the type of input parameter distribution. They are, however, sensitive to the type of the heat flux. In case of the convective heat flux, the propagated bias factor $\delta\{G,W\}$ is simply a product of individual biases, but with radiation, the bias values are lower, as the nonlinear heat loss compensates for the increased surface temperature. The $\tilde{\sigma}_\epsilon$'s are slightly less than 0.1, due to the positive biases included in the mean values. The second moments for $T_2\{G\}$ are smaller than for $T_2\{W\}$, indicating that the random errors in the gas temperature are partially suppressed in the chain.

Figure 6: Model-error associated with the time to reach $T_{2,cr} = T_\infty + 0.9(T_\tau - T_\infty)$.

In fire safety analyses, it is common to compare the predicted temperatures against some critical temperature, and to investigate the time for reaching that threshold. To find out how the propagated errors in T_2 affect the timing results, we defined a critical temperature

$$T_{2,cr} = T_\infty + 0.9(T_\tau - T_\infty), \quad (14)$$

where T_τ is the steady state temperature corresponding to the mean values of the input parameters. The values of T_τ were 427 °C, 400 °C, and 328 °C for heat flux cases "conv", "rad" and "both", respectively.

Figure 6 presents the errors in the predicted times to reach $T_{2,\text{cr}}$. As the temperatures $T_2\{*\}$ were biased upwards, the time predictions have $\delta < 1$, i.e., the critical temperatures are achieved earlier than in the reality. The different uncertainty distribution types are found to produce quite different biases in time.

To investigate the sensitivity of the propagated errors on the assumed mean gas temperature T_g , we calculated the uncertainty parameters for gas temperatures between 200 °C and 1200 °C. We imposed σ_ϵ as $0.1(T_g - T_\infty)$ for the T_g , and $0.1(T_\tau - T_\infty)$ for the wall model. Figure 7 shows the resulting uncertainty of T_2 for the three different uncertainty scenarios. The model error $\delta \pm \sigma_\epsilon\{W\}$ remains as the imposed value at all gas temperatures, but $\delta \pm \sigma_\epsilon\{G\}$ and $\delta \pm \sigma_\epsilon\{G,W\}$ are reduced when T_g increases, i.e. when the radiative heat transfer becomes effective.

Figure 7: Propagated model-error in cold-side temperature prediction, for different values of gas temperature.

3.3 Uncertainty of reliability estimates

Reliability analysis is a common part of any design optimization or risk assessment process. As a measure of reliability, we use here the stochastically estimated probability of T_2 exceeding a critical temperature $P(T_2 > T_{2,\text{cr}})$, where $T_{2,\text{cr}}$ is defined in Eq. 14. The simulations were here carried out for a 24 h time period.

The propagation of the model uncertainties to the estimated probabilities are reported as

Figure 8: The effect of model-error on reliability estimate; The ratio of probability inferred from the observed distribution to the one inferred from the error-free distribution.

ratios of the observed and error-free probabilities $P\{*\}/P\{\}$ in Figure 8. The results show that the deviation from the actual probability depends on both the type of the boundary heat flux and the type of the input parameter distribution. Highest errors are found in the absence of the radiative component because the steady-state cold-side temperatures were comparatively higher in these cases. Up to 20 % overestimation is obtained from only 10% errors in a single model. Wider input distributions seem to enhance the propagation of model errors into the probability observations, as can be seen from the single-value, uniform and normally distributed wall inputs.

The use of the numerical models for engineering requires balancing the computational costs and desired level of accuracy. Making stochastic analyses possible may require simplifications in the deterministic models, and the information about uncertainty propagation can then be used for defining the performance requirements. To learn how the total error of our two-model chain depends on the magnitude of the individual model uncertainty, we varied δ and $\tilde{\sigma}_\epsilon$ of one model keeping the δ and $\tilde{\sigma}_\epsilon$ of the other model constant at 1.1 and 0.1 respectively. The upper plots in Figure 9 shows $\tilde{\sigma}_{T_2}\{G,W\}$ as a function of random error $\tilde{\sigma}_\epsilon$ for the different values of δ . The curves on the upper left plot are almost overlapping, except for the small $\tilde{\sigma}_{\epsilon,W}$. According to the results, we can see that, for example, in order to obtain $\tilde{\sigma}_{T_2}\{G,W\} < 0.1$, one must ensure that $\tilde{\sigma}_{\epsilon,W}$ is less than about 0.08. Additionally, due to the presence of the gas model uncertainty, reducing $\tilde{\sigma}_{\epsilon,W}$ below about 0.02 does not bring any additional benefit. The upper right plot shows that the gas model bias significantly influences the output uncertainty. With non-linear boundary conditions of the heat transfer problems, the total error of the two-model chain is not a linear function of the two components.

Figure 9: Uncertainty in the cold-side temperature at 180 min (Upper) and probability to cross 312°C (lower) with respect to the modeling uncertainty.

The lower plots in Figure 9 show how the probability values depend on the model uncertainty parameters. The critical temperature in this case was $T_2 > 312^\circ\text{C}$ and the simulation time was 20 hours. 312°C was the error-free, steady-state T_2 corresponding to the mean of the input parameters, i.e. $P(T_2 > 312^\circ\text{C})\{\} = 0.5$. The results show that the bias of both models is detrimental to the accuracy of the probability estimation. With unbiased wall model, $P\{G,W\}$ is between 0.7 and 1.0, being highest at $\sigma_{\epsilon,W} = 0$. Hence, more accurate wall model would yield less accurate probability estimates. Increasing the wall model uncertainty would apparently improve the probability predictions, but in general this cannot be seen as a means to compensate for the gas model uncertainty.

3.4 Estimation of error-free distribution

Error-free output, $T_2\{\}$ can be estimated from the observed output, $T_2\{*\}$ using Eq 11 and the propagated modeling-error values, i.e., δ and $\tilde{\sigma}_\epsilon$. The corrected mean and standard deviation of

the observed output are

$$\mu_2\{\text{Corrected}\} = \frac{\mu_2\{*\}}{\delta}, \quad \sigma_2\{\text{Corrected}\} = \frac{1}{\delta} \left(\sigma_2^2\{*\} - \tilde{\sigma}_\epsilon^2 \mu_2^2\{*\} \right)^{\frac{1}{2}}. \quad (15)$$

Figure 10 compares the error-free distribution of $T_2\{\}$, and the observed distribution of $T_2\{G,W\}$ in the study case n:o 18 (see Table 3). Assuming that the corrected distribution is normally distributed with a mean $\mu_2\{\text{Corrected}\}$ and standard deviation $\sigma_2\{\text{Corrected}\}$, we can draw an estimate of the corrected distribution, shown as a continuous line. Similar plots are presented in Appendix 2 for the remaining 53 cases.

Figure 10: Estimation of error-free cold-side temperature from the simulated one.

In the study case 18, the difference between $\mu_2\{\}$ and $\mu_2\{\text{Corrected}\}$ is in the order of 10^{-12} , and the difference between $\sigma_2\{\}$ and $\sigma_2\{\text{Corrected}\}$ is in the order of 10^{-3} . This shows that, in case of normally distributed inputs, the method is remarkably effective in correcting the first moment of the stochastically simulated output distribution. The accuracy of the correction method in all study cases is shown in the upper row of Figure 11. The observed and corrected standard deviations of T_2 are divided by the corresponding error-free $\sigma_2\{\}$. The circular markers show the errors in the observed distribution shapes, and the square markers show the corrected ratios. The results in Figure 11 show that the ratio of the second central moment is improved from above 1.2 to nearly 1 in all study cases.

The second row of Figure 11 shows a similar comparison for the 95 percent fractiles. The corrected fractile values were calculated using $\mu_2\{\text{Corrected}\}$ and $\sigma_2\{\text{Corrected}\}$. The observed 95 percent fractile values were 10 to 30 % higher than the true fractile. After the correction, the

fractile estimates are within few percent from the true, error-free fractile.

Figure 11: Upper: Ratios of standard deviations in cold-side temperature, $\sigma_2\{*\}/\sigma_2\{\}$ and $\sigma_2\{\text{Corrected}\}/\sigma_2\{\}$. Lower: Similar ratios for 95 percent fractiles.

4 Discussion

The results of the present study illustrate how the model errors are combined with the stochastic (parameter) uncertainties in the simulation of fire barrier performance. The results are specific to the studied scenarios, but some general trends can be found as well. Most importantly, we learned that the chain of gas and wall model temperature solvers tends to damp the errors in the first model when analyzing the cold-side wall temperature. In particular, the radiative heat losses compensate for the too high gas temperatures.

The type of the boundary heat-flux was found to affect the error in T_2 but not in time to reach a critical temperature (see Figure 6). It is important to notice that in our analysis, the critical temperature was defined as a fraction of the steady-state temperature, not as an absolute value which would be the case in real applications. The critical temperatures were thus different for each heat flux type. Different results would have been obtained, had the critical temperature been defined differently.

Finally, we explored a method that can be used for improving the accuracy of the simulated output distribution by introducing corrections to the observed bias and standard deviation, using the known metric δ and σ_ϵ ,

$$\mu_{\text{true}} = \frac{\mu_{\text{observed}}}{\delta}, \quad \sigma_{\text{true}}^2 = \frac{1}{\delta^2} (\sigma_{\text{observed}}^2 - \sigma_\epsilon^2). \quad (16)$$

For a chain of two models, Eq 11 provides an alternative expression that would not require δ and σ_ϵ . However, in many applications it is not practical to calculate $\mu_{T_2}\{\text{W}\}$, $\mu_{T_2}\{\text{G}\}$, $\sigma_{T_2}^2\{\text{W}\}$ and $\sigma_{T_2}^2\{\text{G}\}$. The Eq 11 is useful when it is difficult to carry out the simulation with refined mesh in both models. In such case, the solutions equivalent to the refined mesh, $\mu_{T_2}\{\}$ and $\sigma_{T_2}^2\{\}$ can be obtained from three different not-refined cases: $\mu_{T_2}\{\text{G}\}$ and $\sigma_{T_2}^2\{\text{G}\}$ when only the wall model is finely meshed, $\mu_{T_2}\{\text{W}\}$ and $\sigma_{T_2}^2\{\text{W}\}$ when only the gas model is finely meshed, and $\mu_{T_2}\{\text{G,W}\}$ and $\sigma_{T_2}^2\{\text{G,W}\}$ when both models are coarsely meshed.

In Section 2.3, we assumed that the wall model was error-free. The real simulations will always contain errors, as do the experimental measurements and specifications that are used to quantify the model error parameters. The motivation behind adopting the stochastic method is the difficulty in achieving a deterministic expression for the output uncertainty. The deterministic uncertainty quantification approach would require derivatives of Eq. 4 and a numerical integration of the uncertainty expression over time. Finding the derivatives of a complex numerical system would be extremely difficult.

5 Conclusions

The aim of this work was to study modeling uncertainty propagation in a chain of two numerical models and develop a method to improve the accuracy of stochastically simulated output uncertainty. To achieve this, we studied the relationship between error-free output, simulated output, and the assumed modeling errors. Assuming a one-dimensional heat transfer problem, motivated by the fire safety analyses, we carried out a stochastic analysis to estimate propagated model-error in the cold-side temperature. The stochastic analysis was performed for different study cases

in order to investigate the effect of the boundary conditions and the types of input parameter distributions.

According to the simulation results, the model error propagation in a chain of two models depends upon the type of boundary that links them. In the absence of a non-linear component, propagated bias is simply a multiple of individual model biases. In the presence of a non-linear radiation component, the model error propagation is affected by the boundary heat losses. Interestingly, the super-linear radiation heat fluxes seem to reduce the magnitude of the errors propagating from the gas phase model to the wall model.

Errors in the predicted time and reliability estimates were found to depend on the width of the distributions representing the input parameter uncertainty. Both the predicted times and the estimated probabilities were further away from the actual values when the input parameters were taken from uniform distributions instead of normal distribution. The reliability estimates were strongly affected by the model biases and wall model random errors.

Finally, we showed that the accuracy of the simulated output distribution can be improved by estimating the true moments of the distribution using the proposed deterministic relationship. Although the results were promising in here, the method may be difficult to use in real applications where individual model uncertainties cannot be eliminated. More simple approach was proposed for such situations.

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Appendices

1. Random error in T_g and T_2 .

2. Cold-side temperature, T_2 at 180 min: The observed, $T_2\{*\}$ and the error-free distributions, $T_2\{\}$ for different study cases.

(a) Error type: $\delta_W = 1.1$, $\tilde{\sigma}_{\epsilon,W} = 0.1$, and $\delta_G = 1$, $\tilde{\sigma}_{\epsilon,G} = 0$.

(b) Error type: $\delta_W = 1$, $\tilde{\sigma}_{\epsilon,W} = 0$, and $\delta_G = 1.1$, $\tilde{\sigma}_{\epsilon,G} = 0.1$.

(c) Error type: $\delta_W = 1.1$, $\tilde{\sigma}_{\epsilon,W} = 0.1$, and $\delta_G = 1.1$, $\tilde{\sigma}_{\epsilon,G} = 0.1$.