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Heat switch and thermoelectric effects based on Cooper-pair splitting and elastic cotunneling

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In this paper, we demonstrate that the hybrid normal-superconducting-normal (NSN) structure has potential for a multifunctional thermal device which could serve for heat flux control and cooling of microstructures. By adopting the scattering matrix approach, we theoretically investigate thermal and electrical effects emerging in such structures due to the Cooper pair splitting (CPS) and elastic cotunneling phenomena. We show that a finite superconductor can, in principle, mediate heat flow between normal leads, and we further clarify special cases when this seems contradictory to the second law of thermodynamics. Among other things, we demonstrate that the CPS phenomenon can appear even in the simple case of a ballistic NSN structure.

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I. INTRODUCTION

Superconductors are typically regarded as thermal insulators because, at temperatures much less than the superconducting energy gap, $\Theta \ll \Delta$ ($\Theta$ is expressed in energy units), their thermal conductivity is exponentially small. Nevertheless, this is not necessarily true in the case of a finite normal-superconducting-normal (NSN) structure.

Consider a ballistic NSN contact at low temperatures. If the length of the superconducting region ($L$) is significantly larger than the superconductor coherence length ($\xi$), the quantum transport is completely defined by Andreev reflection (AR) [1] [see Fig. 1(a)]. Due to the fact that the subgap transport is fully determined by the Cooper pairs and the total energy of the electrons in a pair is zero (counted from the chemical potential), heat does not propagate through the superconductor. In the case of a finite superconducting region, however, the elastic cotunneling (EC) process appears [Fig. 1(b)], which gives rise to electrical and thermal currents through the superconductor.

Besides AR and EC, yet another process occurs in the presence of electron-to-electron scattering on the NS border. In this CPS process [2,3] [Fig. 1(c)], two electrons from the opposite normal regions combine to form a Cooper pair. Alternatively, one can say that an incident electron from one side is transmitted as a hole to the other side. This phenomenon, also referred to as crossed Andreev reflection (CAR) [4,5], allows for the subgap energy flow and changes the thermal properties of a hybrid NSN structure. This CPS process attracts particular attention since it potentially provides an efficient way for generating entangled electron pairs in solid-state systems [2,6,7]; for example, by employing pair selection via well-defined energy levels in quantum dots [3]. Until now, CPS has been demonstrated experimentally in systems involving a superconductor connected to ferromagnetic leads [8], to bulk normal metal leads [9,10], to carbon nanotubes [11–13], to InAs nanowire [14], and to graphene quantum dots in Coulomb blockade regime [15,17].

Thermoelectric effects in mesoscopic systems have been extensively studied in quantum dots [18–22], Andreev interferometers [23,24], atomic point contacts [25–27], and, lately, in nanowire heat engines [28]. Among other things, considerable attention has been given to the manifestations of thermoelectricity in the superconducting systems [29,30]. For instance, it has been predicted [31] that thermoelectricity may be witnessed in ferromagnet-superconductor-based CPS devices. To date, a growing number of papers have also examined thermoelectricity in bulk nonmagnetic hybrid NSN structures by means of quasiclassical techniques based on Eilenberger and Usadel equations, see, e.g., Refs. [32–34]. In particular, Cao et al. suggested [7] that the CPS may occur in the sole presence of the temperature difference between the normal leads with no bias voltages applied. In the present paper, building on the scattering matrix approach, we investigate thermal and thermoelectric effects arising from CPS and EC in NSN structures going beyond quasiclassics. We explicitly show that the superconductor can, in principle, mediate heat flux. We also clarify certain cases where the CPS and EC processes seem to be in contradiction with the second law of thermodynamics. Intriguingly, we demonstrate that CPS can occur even in the trivial case of a ballistic NSN structure. We then consider how the CPS and EC effects can be utilized in heat transport control. Finally, we discuss a possible experimental setting which would facilitate detection of the considered effects.

II. NSN THERMAL PROPERTIES

Let us start by considering thermal properties of the NSN structure at low temperatures, $\Theta \ll \Delta$. Assume that electrons in the normal parts are noninteracting. In this case, the left-to-right heat current in the left (right) normal region $I^{L(R)}_Q$
from Eq. (1) we obtain a non-negative left-to-right heat current:

\[
I_{Q}^{L} = I_{Q}^{R} = \frac{2}{\hbar} \int d\varepsilon \, \frac{\partial f_{ee}}{\partial \varepsilon} \delta \Theta \left[ 1 - R_{eL}^{LL} - R_{eL}^{LR} \right] = \frac{\delta \Theta}{2\hbar \Theta_{0}^{2}} \int d\varepsilon \, \varepsilon^{2} \frac{T_{LR}^{LR} + T_{LR}^{RL}}{\cosh^{2} \frac{\varepsilon}{2\Theta_{1}}} > 0.
\]  

Note that since the number of quasiparticles in the superconductor is exponentially small (\(n_{eL}^{LL} \sim e^{−\Delta/(\Theta_{0})}\)), they do not contribute to the thermal current. For \(L > \xi\), we have \(T_{LR}^{LR} = T_{LR}^{RL} \equiv 0\), and the thermal current vanishes as expected. In other situations, the thermal current may occur due to the CPS and EC processes.

Equation (2) remains valid even if the temperature in the superconducting region is higher than in the normal leads, \(\Theta_{S} > \Theta_{L} > \Theta_{R}\). This may seem contradictory to the second law of thermodynamics, since apparently the heat flows from the colder left normal region to the warmer superconducting region. The subject of quantum thermodynamics and quantum extension of the second law has attracted much attention lately. It has been discovered that under certain circumstances, the law in its classical sense can be violated [36–39]. However, in the present case, the transfer of a particle from the warmer left reservoir to the colder right one is still associated with the overall increase in entropy, \(\Delta S = -\varepsilon \delta \Theta + \varepsilon \delta \Theta > 0\), in which sense it does not violate the second law. At the same time, the entropy change is nonlocal, and this effect may be considered nontrivial as it cannot be found in normal metal structures. We may also note that such nonlocality disappears at temperatures large compared with \(\Delta\), when the transport is no longer determined by the Cooper pairs.

III. NS SCATTERING BEYOND ANDREEV APPROXIMATION

In this section, we discuss the situation which demonstrates that the CPS process can be observable even in a very simple system—a finite, fully ballistic NSN structure. To begin, it is important to recognize the conditions necessary for the CPS process to occur in the first place.

Consider a structure for which the length of the superconductor far exceeds the coherence length, i.e., the size of a Cooper pair [see Fig. 1(a)]. In such a case, the CPS becomes highly improbable. Instead, the paired electrons may split only in accordance with a local AR process, which forces both resulting electrons into the same normal lead. A different NSN configuration with a finite superconducting region, as shown in Fig. 1(b), does not necessarily constitute a CPS device either. Nevertheless, now, as it appears from the boundary conditions, the CPS may occur if there is a nonzero probability of the electron-to-electron scattering on the NS interface [see Fig. 1(c)]. Dzhikaev [40] showed that such specular reflections can, in fact, take place, if the incident electrons move nearly parallel to the interface; several basic effects emerging from this phenomenon have been studied in Refs. [41–43]. Here we shall explicitly show that the particles “sliding” along the interface can give rise to the CPS process. Namely, we demonstrate that the CPS is possible with a small effective chemical potential \(\mu_{\perp} = \mu - \hbar^{2}k_{F}^{2}/(2m)\), where \(k_{F}\)
For the transmitted wave, we have
\[ \alpha \] electron’s (hole’s) wave vector is defined by \[ \bar{\mu} \] NSN hybrid structure. The perpendicular component of the reflected hole move nearly perpendicular to the NS boundary dreev approximation, in which an incident electron and a and thus can be controlled. Our results go beyond the An-
\[ \mu \] is the wave vector’s component parallel to the interface. The solutions of the Bogoliubov–de Gennes equations \[ \epsilon \] superconducting region, the wave function is given by
\[ \epsilon \] should be noted that in a certain range of \[ \epsilon \] TLR and \[ T ee \] transmission probabilities and \( T LR \) exceeds \( T ee \) [see Fig. 2(c)]; in other words, CPS process is stronger than EC. Yet, for the effect to appear, \( L \) should be comparable with a rather small effective coherence length \( \xi \). 

\[ L \sim \frac{1}{\xi} \sim \xi. \]

In the case of aluminum superconductor, \( L \) should be \( \sim 10 \) nm. We should emphasize, however, that the result is obtained for a one-dimensional structure, and may be invalid for other geometries (see discussion in Ref. [47]). For instance, as was pointed out in Ref. [3], in the case of a three-dimensional junction, the CPS effect is suppressed if \( L \) is large compared to \( k_0^{-1} \), which, for metals, is typically \( \sim \AA \).

IV. HEAT SWITCH

In this section, we discuss the possibility to utilize NSN structures in the control of heat transport. Let us consider Eq. (2), which indicates that the thermal conductivity of the structure is directly dependent on the transmission probabilities \( T ee \) and \( T ch \). Therefore, if one can control these values, the structure may be operated as a heat switch [48], i.e., a device that switches on demand between the thermal conductor and thermal insulator modes.

As we have seen in the previous section, the electron-to-hole transmission can take place even in ballistic NSN structures and, furthermore, can to some extent be controlled. In reality, however, this approach may be unsuitable for the practical needs. To this end, we devise our heat switch using an NXSXN structure that utilizes scatterers (X) with the energy-dependent transmission probability, e.g., quantum dots [50]. The transparency function \( T^X (\epsilon) \) of an individual scatterer is characterized by its resonance position \( \epsilon_X \), resonance analytically, but the corresponding expressions become too cumbersome (see Appendix A). Therefore, we shall base our analysis on the numerically evaluated plots.

The dependence of \( T ch \) and \( T ee \) on the dimensionless parameter \( Lk_0 \) is shown in Figs. 2(a) and 2(b), where we choose \( \mu \) to be \( \{0.01\Delta, 0.02\Delta, 0.03\Delta\} \) and \( \epsilon = 0 \). One can see that the maximum value of \( T ch \) is close to 0.25, which makes the effect quite significant. Moreover, it should be noted that in a certain range of \( L \), \( T ch \) exceeds \( T ee \) [Fig. 2(c)]; in other words, CPS process is stronger than EC. Yet, for the effect to appear, \( L \) should be comparable with a rather small effective coherence length \( \xi \).

\[ L \sim \frac{1}{\xi} \sim \xi. \]

In the case of aluminum superconductor, \( L \) should be \( \sim 10 \) nm. We should emphasize, however, that the result is obtained for a one-dimensional structure, and may be invalid for other geometries (see discussion in Ref. [47]). For instance, as was pointed out in Ref. [3], in the case of a three-dimensional junction, the CPS effect is suppressed if \( L \) is large compared to \( k_0^{-1} \), which, for metals, is typically \( \sim \AA \).

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the X barriers are perfectly transparent at its resonance energy, whereas particles with other energies are completely reflected. The positions of the resonances can be adjusted by the external gate voltages. (a) The resonance settings realizing ideal transmission. Symmetric configuration (X). Each barrier is perfectly transparent at its resonance energy, whereas particles with other energies are completely reflected. The positions of the resonances were given in the caption. From the expressions for the transmission probabilities (see Appendix B), it follows that in the case of the symmetric resonance configuration [see Fig. 3(a)], the maximal electron-to-hole transmission probability, \( T_{X}^{LR} \), can reach unity (alternatively, the resonances may be positioned on the same level; in this case, CPS would be replaced by EC and \( T_{LR}^{LR} \) double barriers for \( pL = \{n\pi, 0.15\pi + n\pi, 0.5\pi + n\pi\} \) \((n \text{ is integer})\). \( E_F \gg \Delta, L = \xi_0 = k_F\Delta/(2E_F) \) and \( \Theta = 0.1\Delta; \) the parameters of the X barriers are \( \Gamma_X = 0.03\Delta, T_{X}^{LR} = 1 \).

The physics of the device we propose is based on the scattering matrices outlined in Ref. [6]. From the expressions for the transmission probabilities (see Appendix B), it follows that in the case of the symmetric resonance configuration [see Fig. 3(a)], the maximal electron-to-hole transmission probability, \( T_{X}^{LR} \), can reach unity (alternatively, the resonances may be positioned on the same level; in this case, CPS would be replaced by EC and \( T_{LR}^{LR} = 1 \)). Conversely, certain settings may completely block the transmission; an example is depicted in Fig. 3(b).

A fuller understanding of the thermal properties of NXSN structures can be achieved by considering the dependence of the heat conductance, \( G_Q = I_Q^2/\delta\Theta \), on the positions of the left and right resonances. The situation where the Fermi energy \( E_F \) is much larger than \( \Delta \) is shown in Figs. 3(c)–3(e) (the parameters are given in the caption) plotted for \( pL = \{n\pi, 0.15\pi + n\pi, 0.5\pi + n\pi\} \), where \( n \) is integer (if \( E_F \gg \Delta \), \( pL \) can be regarded independent from \( \varepsilon \)). One can see how the variation of the quantum dot gate potentials can drastically change the thermal conductivity of the system. Further in this paper, we demonstrate that it is also possible to configure the structure in such a way that it would essentially become an electrical insulator, but would still conduct thermal current.

V. EC AND CPS COOLING

In this section, we discuss a CPS-based cooling device involving voltage bias. The working principle of such a device is not distinctive to structures with superconductors; however, the presence of a superconducting electrode in some situations allows for better efficiency. We start by discussing the NXN design [51] and then proceed with the NXSXN version.

A. NXN scheme

Let us consider two normal leads connected via a quantum dot with a narrow resonance. Suppose also that the leads are biased at negative constant voltage \( V_B \), i.e., \( \mu_L = \mu_R + eV_B \), and the resonance of the dot is positioned slightly above the chemical potential of the left lead, \( \delta\varepsilon = \varepsilon_0 - eV_B \ll eV_B \) [see Fig. 4(a)]. According to Eq. (1), in which we put
that the normal leads have equal chemical potentials which are
on CPS process. Suppose the voltage is applied in such a way
Supposing that the resonance is narrow, i.e.,
the NXN one.
which CPS is replaced by EC. The scheme is essentially reduced to
quasiparticles in the normal leads. (c) The NXSXN configuration for
scheme, the cooling is based on the concentration difference of the
electron-to-hole transmission probability. Similar to the previous
resonance configuration of the barriers corresponds to the perfect
around the resonance of the X scatterer. (b) CPS refrigerator: The
lead occurs due to the electron concentration difference at energies
T LR

We notice that the current is positive and hence the left region
is cooling when the left term in the brackets is greater than
the right one, i.e., when the number of electrons with energies
close to δε0 (counted from μL) in the left terminal is higher
than in the right one. The heat current may appear even
opposed to the temperature gradient δθ (see Fig. 5). Note that
in its physical sense, this process is similar to Peltier effect.

B. NXSXN scheme

The NXSXN cooling device, depicted in Fig. 4(b), is based
on CPS process. Suppose the voltage is applied in such a way
that the normal leads have equal chemical potentials which are

heat currents are given by

Supposing that the resonance is narrow, i.e., \( \Gamma_X \ll \Theta_{L,R} \), the
relation for the heat extraction from the left region can be
rewritten as

We notice that the current is positive and hence the left region
is cooling when the left term in the brackets is greater than
the right one, i.e., when the number of electrons with energies
close to \( \delta \varepsilon_0 \) (counted from \( \mu_L \)) in the left terminal is higher
than in the right one. The heat current may appear even
opposed to the temperature gradient \( \delta \theta \) (see Fig. 5). Note that
in its physical sense, this process is similar to Peltier effect.

FIG. 4. (a) NXN refrigerator: The heat extraction from the left
lead occurs due to the electron concentration difference at energies
around the resonance of the X scatterer. (b) CPS refrigerator: The
resonance configuration of the barriers corresponds to the perfect
electron-to-hole transmission probability. Similar to the previous
scheme, the cooling is based on the concentration difference of the
quasiparticles in the normal leads. (c) The NXSXN configuration for
which CPS is replaced by EC. The scheme is essentially reduced to
the NXN one.

\[
I^L_Q = I^L_Q \equiv 0 \quad \text{and} \quad T^L_{\text{LR}}(RL) = T^X, \quad \text{the left-to-right}
heat currents are given by
\]

\[
I^L_Q = \frac{2}{h} \int \epsilon \left( \epsilon - \mu_{LR} \right) T^X \left[ f_L - f_R \right]. \quad (8)
\]

FIG. 5. Heat currents \( I^N_Q \) (blue; \( T^X_\text{res} = 1 \)) and \( I^{NSN}_Q \) (red, solid, and dashed lines correspond to, respectively, \( T^L_{\text{LR}} \) and \( T^L_{\text{LR}}(\text{res}) = 0.5 \) (nonideal CPS)) as functions of the bias voltage \( V_B \).
The dashed vertical line indicates where the solid lines are separated
the most. The positiveness of the heat current directed from the
left colder region to the right warmer one (\( \Theta_R > \Theta_L \)) means that
the heat flows against the temperature gradient. The parameters are
\( \delta \varepsilon = 0.05 \Delta, \Theta_L = 0.1 \Delta, \Theta_R = 0.2 \Delta, \Gamma_X = \Gamma_{XXS} = 0.003 \Delta. \)

higher than that of the superconductor, \( \mu_L = \mu_R = \mu_S + eV_B \). We
consider a symmetrical resonance configuration when the
left resonance is situated at \( \varepsilon_L = \varepsilon_0 \) above the superconduc-
tor’s chemical potential and the right one lies below it at
\( \varepsilon_R = -\varepsilon_0 \) (the energies are counted from \( \mu_S \)). Using Eq. (1),
one can find the left-to-right heat current in the left normal
lead in the case where \( T^L_{\text{LR}} \) has a small resonance half-width
\( \Gamma_{XXS} \ll \Theta_{L,R} \):

\[
I^{NSN}_Q \equiv I^L_Q = \frac{2}{h} \int \left[ \frac{1}{e^{(\varepsilon - \varepsilon_0)/\Theta_L} + 1} \right] \text{d} \varepsilon = \frac{2 \pi T^Y_{\text{LR}}(\text{res})}{h} \Gamma_{XXS} \delta \varepsilon \left( \frac{1}{e^{(\varepsilon - \varepsilon_0)/\Theta_L} + 1} - \frac{1}{e^{(\varepsilon + eV_B)/\Theta_R} + 1} \right). \quad (9)
\]

\[
I^L_Q = \frac{2}{h} \int \epsilon \left( \epsilon - \mu_{LR} \right) T^X \left[ f_L - f_R \right]. \quad (8)
\]

\[
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\]

It can be seen that the heat extraction from the left region takes
place when the number of electrons with energies \( \varepsilon_0 \) in the left
lead surpasses the number of holes at energies \( -\varepsilon_0 \) in the right
lead. Thus, the operating principle of both cooling schemes is
essentially the same.

The efficiency coefficient of the NXSXN refrigerator, ex-
tracting heat \( I^L_Q \) from the colder left reservoir using input
electrical power \( W \), is given by

\[
\eta = \frac{I^L_Q}{W}. \quad (11)
\]

Here \( W \) can be expressed in terms of the bias voltage and
the electrical current in the left lead \( I^L_Q \), which will be given below:
\( W = I^L_Q V_B = 2 I^L_Q V_B \). For \( \Gamma_{XXS} \ll \Theta_{L,R} \) we have

\[
\eta = \frac{I^L_Q}{W} \quad (11)
\]
\( \eta = (\epsilon_0 - eV_B)/(2eV_B) \). In the limit \( \epsilon_0 \to eV_B/2\), the efficiency assumes the Carnot value, \( \eta_C = \frac{\pi}{2\sigma_L} \).

### C. Advantage of CPS process

A comparison of Eqs. (9) and (10) shows that, provided \( \Gamma_x = \Gamma_{\text{NSN}} \) and \( T_{\text{NSN}}^{LR} = T_{\text{NSN}}^{RR} \) at the same voltage bias and with fixed \( \delta e \), the CPS device has larger cooling power than the NXN system. In Fig. 5, we plot \( I_{\text{NSN}}^{(\text{res})} \) (blue solid line) and \( I_{\text{NN}}^{(\text{res})} \) (red solid line), corresponding to the unity maximal transmission probabilities as functions of the bias voltage (the parameters are given in the caption). The dashed vertical line marks the point where these heat currents differ the most. If \( eV_B \gg 0 \), the enhancement in the heat current is close to zero:

\[
\Delta I_Q = I_{\text{NSN}}^{(\text{res})} - I_{\text{NN}}^{(\text{res})} \sim \frac{2\pi}{h} T_{\text{res}} \frac{\delta e}{\hbar} e^{-(\delta e + eV_B)/\Theta} [1 - e^{-(\delta e + eV_B)/\Theta}] .
\]

In the real experiment it is, of course, possible that \( T_{\text{res}}^{LR} \ll T_{\text{res}}^{RR} \), in which case the NXN system may have an advantage at some \( V_B \). This is reflected in Fig. 5 by the red dashed line which corresponds to NXNS cooling power for \( T_{\text{res}}^{LR} = 0.5 \).

When configured as depicted in Fig. 4(c), the NNXSN scheme is essentially reduced to the NXN one. With these settings the CPS process is completely replaced by EC.

### VI. THERMOELECTRICITY AND JOULE HEATING

We proceed by addressing thermoelectric properties of the NSN structure. We may write the left-to-right electric current in NSN structure in the form similar to Eq. (1) [35]:

\[
I_e^{LR} = -\frac{2e}{h} \int \delta e \left\{ -T_{eL}^{LR}(\delta e) \right\} [1 - f_{L}(\delta e)] - T_{eR}^{LR}(\delta e)
\]

\[
\times [1 - f_{R}(\delta e)] + \left[1 - T_{eL}^{RR}(\delta e) \right] f_{L}(\delta e) - T_{eR}^{RR}(\delta e) f_{R}(\delta e) \right\} .
\]

Bearing in mind that the currents vanish in equilibrium, at zero bias voltage this formula gives the simple result:

\[
I_e^{LR} = \frac{2e}{h} \int \delta e \left[ T_{eL}^{LR} - T_{eR}^{LR} \right] (f_L - f_R).
\]

As distinct from the thermal current given by Eq. (2), the electric current in the right lead is zero if \( T_{eL}^{LR} = T_{eR}^{LR} \). This means that the NSN structure can be configured in such a way that it would conduct heat, but not electric charge. The system therefore does not satisfy the Wiedemann–Franz (WF) law [52] stating that the ratio of the thermal conductivity (\( k \)) to the electrical conductivity (\( \sigma \)) is proportional to the temperature, \( k/\sigma = L \Theta \) where the Lorenz number \( L = \frac{\pi^2}{3} \left( \frac{k_B}{e} \right)^2 \).

Previously, it has been shown that the WF law can be violated when the thermoelectric effect is significant [53]. In the case of normal metal structures, thermoelectricity is typically associated with the energy dependent transmission [54]; namely, if this dependence is weak, the effect can be expressed by the Cutler–Mott formula. Intriguingly, this is not so when we consider NS junctions. Yet, the mechanisms by which the thermoelectricity can be established may appear in the case of the finite superconductor.

Let us now suppose that the leads are biased at voltages \( V_L \) and \( V_R \) with respect to the superconductor. The heating power of the structure \( \dot{Q} \) can be written, using Eqs. (1) and (13), as

\[
\dot{Q} = I_L^R - I_L^L = I_L^L V_L - I_R^R V_R,
\]

meaning that the system obeys the Joule law. From Eq. (1), it can also be seen that the particles dissipate energy through relaxation to the local chemical potential. Consequently, in contrast with the classical picture, the heating is nonhomogeneous, as it can vary substantially from one part of the structure to another. This creates a temperature gradient which, in turn, can result in thermoelectricity.

### VII. EXPERIMENTAL CONSIDERATIONS

Let us explore experimental detectability for heat current caused by nonlocal thermal and thermoelectric effects. A promising system is a graphene-based setting with two quantum dots etched out of exfoliated graphene [16]. We aim at nonlocal thermal phenomena at moderate charge density, and consequently we may neglect the inherent peculiar properties of AR in graphene [55–57]. The advantage of graphene for Cooper pair splitting is that its electrons are quite well isolated from lattice so that a small heat input can raise the electronic temperature substantially. Furthermore, it is quite easy to pattern part of the very same graphene flake to obtain proximity-induced superconductivity [58], which can be employed for thermometry based on switching supercurrents. The switching current \( I_{SW} \) of a diffusive graphene superconductor-graphene-superconductor (SGS) junction depends strongly on temperature when the Thouless energy \( E_T \ll \Delta \), which can be reached in junctions of length \( L \sim 400 \text{ nm} \) for standard graphene devices on SiO\(_2\) [59]. However, as found in Ref. [59], nonequilibrium quasiparticles may contribute to the heat flow out of the SGS junction and increase the coupling of the graphene sheet to the environment. Consequently, we base our estimates on the experimental results on heat relaxation in graphene obtained in Ref. [59]. In fact, their device has dimensions and characteristics close to such a temperature detector that could be adopted for thermometry on a graphene Cooper pair splitter.

The sensitivity of a switching current thermometer depends on the width of the switching distribution and the steepness of \( I_{SW}^{\Theta} \). We assume 400-nm-long SGS junctions, for which \( d_{SW}^{\Theta}/d\Theta = 100 \approx 200 \text{ nA/K} \) [59]. At temperatures below 100 mK, we estimate for the single-measurement temperature resolution \( \Delta \Theta = 3 \text{ mK} \). According to Ref. [59], a heating power of 150 fW, 2.3 pW, and 9 pW will increase the temperature of the graphene thermometer and the attached graphene heat link to about 35 mK, 120 mK, and 190 mK, respectively. Using the parameter values employed in Fig. 5, the heat current amounts to 13 fW at bias \( V_B = 0.2 \Delta e \). The corresponding \( \Delta \Theta \approx 10 \text{ mK} \) at 500 mK.

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the CPS structure requires careful tracking of the inadvertent current paths in the circuit. The prospects for heat current detection, however, become much more favorable in the situation where $\Theta_L = \Theta_R = 0.2 \Delta$, $\Gamma \simeq 0.1 \Delta$ and $\delta \varepsilon = 0.2 \Delta$, for which we can no longer apply the approximation made in Eqs. (9) and (10), and the transmission probability shall be considered as a Lorentz function. In this case, the heat current increases by two orders of magnitude. According to the experimental work of Ref. [60], the heat current may also increase by inverse proximity effect in this regime with substantial coupling between N and S conductors.

VIII. SUMMARY

To sum up, we have shown that the hybrid NSN structures can have promising applications in thermal regulation; namely, we have presented the concepts of the NSN-based heat switch and refrigerator. Using scattering matrix framework, we have uncovered thermal phenomena appearing in NSN structures. Our analytic results indicate that the heat can be conducted nonlocally through a superconducting lead in the presence of the CPS and EC. Intriguingly, we have shown that the CPS process may be witnessed even in ballistic NSN structures. Moreover, we have addressed thermoelectricity and the Joule law manifestation in the NSN systems. Lastly, we have made suggestions regarding the experimental detectability of the nonlocal effects above.

Note added. While preparing this paper, we became aware of two related works [61,62], in which the case of strong Coulomb interaction on the dots is considered.

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APPENDIX A: NSN BOUNDARY CONDITIONS

The boundary conditions for the wave functions expressed in Eqs. (3)–(5) can be written as

\[ 1 + r_{ee} = e^{ia} [A + B] + e^{-ia} [C + D], \]
\[ ik_+ - ik_+ r_{ee} = e^{ia} [A (i p - q) + B (-i p + q)] + e^{-ia} [C (i p + q) + D (-i p - q)], \]
\[ e^{ia} [A e^{(ip-q)L} + B e^{(-ip+q)L}] + e^{-ia} [C e^{(ip+q)L} + D e^{(-ip-q)L}] = \tilde{t}_{ee}, \]
\[ e^{ia} [A (i p - q) e^{(ip-q)L} + B (-i p + q) e^{(-ip+q)L}] + e^{-ia} [C (i p + q) e^{(ip+q)L} + D (-i p - q) e^{(-ip-q)L}] = ik_+ \tilde{t}_{ee}, \]
\[ r_{eh} = A + B + C + D, \]
\[ ik_- r_{eh} = A (i p - q) + B (-i p + q) + C (i p + q) + D (-i p - q), \]
\[ A e^{(ip-q)L} + B e^{(-ip+q)L} + C e^{(ip+q)L} + D e^{(-ip-q)L} = \tilde{t}_{eh}, \]
\[ A (i p - q) e^{(ip-q)L} + B (-i p + q) e^{(-ip+q)L} + C (i p + q) e^{(ip+q)L} + D (-i p - q) e^{(-ip-q)L} = -ik_- \tilde{t}_{eh}, \]
\[ \tilde{t}_{ee(ch)} = e^{\pm ik_+ r_{ee(ch)}}, \]

where \( \tilde{t}_{ee(ch)} = \mu_\perp / \Delta \). In the limit \( \varepsilon \ll \mu_\perp \ll \Delta \), where \( p = k_0 / \sqrt{2} + Q (\mu_\perp), q = k_0 / \sqrt{2} + Q (\mu_\perp) \) and \( k_\perp = k_0 \sqrt{\mu_\perp + Q (\varepsilon / \sqrt{\mu_\perp})} \), the boundary conditions become

1 + \( r_{ee} = i [A + B] - i [C + D] \),
\[ \sqrt{2} \mu_\perp (1 - r_{ee}) = A (i - 1) + B (-i + 1) - C (i + 1) - D (-i - 1), \]
\[ i [A e^{i(1-Lk_0/\sqrt{2})} + B e^{(-i+1-Lk_0/\sqrt{2})}] - i [C e^{(i+1-Lk_0/\sqrt{2})} + D e^{(-i-1-Lk_0/\sqrt{2})}] = \tilde{t}_{ee}, \]
\[ A (i - 1) e^{i(1-Lk_0/\sqrt{2})} + B (-i + 1) e^{(-i+1-Lk_0/\sqrt{2})} - C (i + 1) e^{(i+1-Lk_0/\sqrt{2})} - D (-i - 1) e^{(-i-1-Lk_0/\sqrt{2})} = \sqrt{2} \mu_\perp \tilde{t}_{ee}, \]
\[ r_{eh} = A + B + C + D, \]
\[ \sqrt{2} \mu_\perp (1 - r_{eh}) = A (i - 1) + B (-i + 1) - C (i + 1) - D (-i - 1), \]
\[ i [A e^{i(1-Lk_0/\sqrt{2})} + B e^{(-i+1-Lk_0/\sqrt{2})}] - i [C e^{(i+1-Lk_0/\sqrt{2})} + D e^{(-i-1-Lk_0/\sqrt{2})}] = \tilde{t}_{eh}, \]
\[ A (i - 1) e^{i(1-Lk_0/\sqrt{2})} + B (-i + 1) e^{(-i+1-Lk_0/\sqrt{2})} - C (i + 1) e^{(i+1-Lk_0/\sqrt{2})} - D (-i - 1) e^{(-i-1-Lk_0/\sqrt{2})} = \sqrt{2} \mu_\perp \tilde{t}_{eh}, \]
\[ r_{eh} = A + B + C + D, \]
\[ i r_{eb} \sqrt{2 \mu_\perp} = A(i-1) + B(-i+1) + C(i+1) + D(-i-1), \]  

\[ A e^{i(-1)Lki/\sqrt{\tau}} + B e^{i(+1)Lki/\sqrt{\tau}} + C e^{i(+1)Lki/\sqrt{\tau}} + D e^{i(-1)Lki/\sqrt{\tau}} = \tilde{t}_{eh}, \]  

\[ A(i-1)e^{i(-1)Lki/\sqrt{\tau}} + B(-i+1)e^{i(-1)Lki/\sqrt{\tau}} + C(i+1)e^{i(+1)Lki/\sqrt{\tau}} + D(-i-1)e^{i(-1)Lki/\sqrt{\tau}} = -i \sqrt{2 \mu_\perp} \tilde{t}_{eh}. \]

From these relations, one can obtain analytic formulas for the transmission amplitudes:

\[ \tilde{t}_{ee} = - \frac{\left( (1 + i) \sqrt{\mu_\perp} e^{i(1+Lki/\sqrt{\tau})} (-\sqrt{2} \mu_\perp + (2 - 2i) \sqrt{\mu_\perp} + (2 - 2i) \sqrt{\mu_\perp} - i \sqrt{2}) e^{i(1+Lki/\sqrt{\tau})} \right) }{ (\mu_\perp - 1)^2 e^{2Lki/\sqrt{\tau}} + (\mu_\perp + 1)^2 e^{2Lki/\sqrt{\tau}} - 8 \mu_\perp e^{(1+Lki/\sqrt{\tau})}}. \]

\[ \tilde{t}_{eh} = - \frac{2(\sqrt{2} - (1 - i) \sqrt{\mu_\perp} \sqrt{\mu_\perp} e^{i(1+Lki/\sqrt{\tau})} (-\mu_\perp + (1 - i \mu_\perp) e^{2Lki/\sqrt{\tau}} + (1 + i \mu_\perp) e^{2Lki/\sqrt{\tau}} + (\mu_\perp + i) e^{(1+Lki/\sqrt{\tau})} - i) }{ (\sqrt{2} \mu_\perp + (-1-i))((\mu_\perp - 1)^2 e^{2Lki/\sqrt{\tau}} + (\mu_\perp - 1)^2 e^{2Lki/\sqrt{\tau}} - 8 \mu_\perp e^{(1+Lki/\sqrt{\tau})}}. \]


[48] To make contrast against the device “heat valve” introduced in the work of Ref. [49], we have adopted the term “heat switch” as our device, in contrast to the heat valve, does not have a finite residual thermal conductance.


