Qubit Measurement by Multichannel Driving

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We theoretically propose and experimentally implement a method of measuring a qubit by driving it close to the frequency of a dispersive coupled bosonic mode. The separation of the bosonic states corresponding to different qubit states begins essentially immediately at maximum rate, leading to a speedup in the measurement protocol. Also the bosonic mode can be simultaneously driven to optimize measurement speed and fidelity. We experimentally test this measurement protocol using a superconducting qubit coupled to a resonator mode. For a certain measurement time, we observe that the conventional dispersive readout yields close to 100% higher average measurement error than our protocol. Finally, we use an additional resonator drive to leave the resonator state to vacuum if the qubit is in the ground state during the measurement protocol. This suggests that the proposed measurement technique may become useful in unconditionally resetting the resonator to a vacuum state after the measurement pulse.

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Since the birth of quantum mechanics, quantum measurements and the related collapse of the wave function has puzzled scientists [1,2], culminating in various interpretations of quantum mechanics such as that of many worlds [3]. With the recent rise of quantum technology [4–6], the quantum measurement has become one of the most important assets for practical applications. For example, measurements of single qubits are the key in reading out the results of quantum computations [7–10] and parity measurements in multiqubit systems are frequently required in quantum error correction codes such as the surface and color codes [11–15]. Furthermore, single-qubit measurement and feedback can be used to reset qubits [16–18] or even solely provide the nonlinearity needed to implement multiqubit gates [19–21].

One of the most widespread ways to measure qubits is to couple them to one or several bosonic modes, such as those of the electromagnetic field, and to measure their effect on the radiation [22]. This method is currently used, for example, in quantum processors based on superconducting circuits [23–28], quantum dots [29–31], and trapped ions [32]. Especially with the rise of circuit quantum electrodynamics [33,34], this measurement technique has become available to many different hybrid systems such as mechanical oscillators [35,36] and magnons [37].

Theoretically, the interacting system of a qubit and a bosonic mode is surprisingly well described by the Jaynes–Cummings model [38,39]. If the qubit frequency is far detuned from the mode frequency, i.e., we operate in the dispersive regime, the interaction term renders the mode frequency to depend on the qubit state. Consequently, a straightforward way to implement a nondemolition measurement on the qubit state is to drive the mode at a certain frequency close to the resonance and measure the phase shift of the output field with respect to the driving field. This kind of dispersive measurement has been extremely successful, for example, in superconducting qubits [40] with increasing accuracy and speed [24,41–43] currently culminating in 99.2% fidelity in 88 ns [26].

In the dispersive measurement, one of the key issues has been the ability to quickly populate the bosonic mode in the beginning of the measurement protocol [24] without surpassing the critical photon number, and to quickly evacuate the excitations from the mode after the measurement [17,44]. These requirements point to the need for a fast, low-quality readout mode. However, this poses a trade-off on the qubit lifetime, which to some extent can be answered using Purcell filters [24,26,45] with the cost of added circuit complexity. A simple and fast high-fidelity measurement scheme is of great interest not only to the field of superconducting qubits, but also to other quantum technology platforms utilizing bosonic modes as the measurement tool.

Inspired by our recent work [46] on quickly stabilizing resonator states by a qubit drive, we propose in this Letter a qubit measurement protocol that is based on driving the qubit close to the frequency of the bosonic mode through a nonresonant channel. Owing to the dispersive coupling, the initial vacuum state of the resonator begins to rotate selectively on the qubit state about a point controlled by the strength and phase of the qubit drive. Importantly, this rotation begins immediately after the drive pulse arrives at...
the qubit with no bandwidth limitation imposed by the resonator. We demonstrate this nondemolition readout scheme in planar superconducting qubits and observe that it leads to a significant speedup. Furthermore, we discuss how this method can be used to unconditionally reset the resonator state into vacuum after the readout without the need for feedback control. We experimentally demonstrate a related effect where the resonator is left in the vacuum state provided that the qubit was in the ground state.

Let us theoretically study a qubit coupled to a single bosonic mode such as that of an electromagnetic resonator, as shown in Fig. 1(a). Instead of using the conventional readout by populating the resonator with a coherent pulse [40,41], we measure the qubit state by driving the qubit at the resonator frequency \( \omega_r \). In addition, we can apply a compensation pulse to the resonator to eliminate cross-coupling effects with the qubit or otherwise control the resonator state. The qubit and the resonator couple to their respective driving fields with different strengths, which together with the pulse envelopes constitute the effective Rabi angular frequencies \( \Omega_q \) and \( \Omega_r \), respectively. The qubit may be first excited from the ground state \( |g\rangle \) to the excited state \( |e\rangle \) by a separate drive tone at the transition angular frequency \( \omega_q = \omega_r + \Delta \), where \( \Delta \) is the detuning.

In the frame rotating at \( \omega_r \) with respect to the bare qubit and resonator Hamiltonians, \( \hbar \omega_q \hat{\sigma}_z \hat{\sigma}_- \) and \( \hbar \omega_r \hat{a} \hat{a}^\dagger \), respectively, the system is described by the Jaynes–Cummings Hamiltonian

\[
\hat{H}/\hbar = \Delta \hat{\sigma}_z \hat{\sigma}_- + \left( g \hat{\sigma}_+ \hat{a} + \Omega_q \hat{\sigma}_+ + i \Omega_r \hat{a}^\dagger + \text{H.c.} \right),
\]

where \( g \) denotes the qubit-resonator coupling strength, \( \hat{a}^\dagger \) and \( \hat{\sigma}_\pm = |e\rangle \langle g| \) are the creation operators of the resonator mode and of the qubit, respectively. Above, we have introduced the rotating-wave approximation justified by \( g \ll \omega_q, \omega_r \).

To demonstrate the benefit of driving the qubit at the frequency of the resonator, we employ the standard dispersive approximation [47] in the regime \( g \ll |\Delta| \). This yields, up to constant energy terms, the Hamiltonian [48]

\[
\hat{H}''/\hbar \approx (\Delta + \chi) \hat{\sigma}_z \hat{\sigma}_- + \left( \Omega_q + i \Omega_r \frac{\chi}{g} \right) \hat{\sigma}_+ + \text{H.c.}\]

\[
- \chi \hat{\sigma}_z \hat{a}^\dagger \hat{a} + \left[ \left( i \Omega_r - \Omega_q \frac{\chi}{g} \right) \hat{\sigma}_z \right] \hat{a}^\dagger + \text{H.c.},
\]

where \( \chi = g^2/\Delta \) is the dispersive shift for a two-level system and \( \hat{\sigma}_z = |g\rangle \langle g| - |e\rangle \langle e| \). The term proportional to \( \hat{a}^\dagger \) is a generator of a displacement operator that depends on the state of the qubit. Thus, driving the qubit effectively realizes longitudinal coupling [56,57] for the duration of the readout, implying that the rate of state separation is not limited by the rate at which the resonator is populated.

In our work, the resonator is accurately described by a coherent state \( |\alpha\rangle \) such that \( \hat{a} |\alpha\rangle = \alpha |\alpha\rangle \), \( \alpha \in \mathbb{C} \). The drive amplitude \( \Omega_q \) may be turned on very fast, causing the amplitudes \( \alpha_{g/e} \) corresponding to the eigenstates of the qubit, \( |g\rangle \) and \( |e\rangle \), to separate in the complex plane at least with the initial speed \( 2\Omega_q \chi/\gamma \). This minimum speed is achieved with \( \Omega_r = 0 \) for an initial vacuum state in the resonator.

As the resonator becomes populated, the trajectories begin to curve due to the dispersive term \( -\chi \hat{\sigma}_z \hat{a}^\dagger \hat{a} \) in Eq. (2) and, in fact, to rotate about the point \( \alpha_{vo} \equiv -\Delta \Omega_q / \chi \). This behavior is intuitively understood in a frame displaced by \( \alpha_{vo} \). Introducing a shifted annihilation operator \( \hat{b} = \hat{a} - \alpha_{vo} \), the last line of Eq. (2) yields

\[
\hat{H}''/\hbar \approx -\chi \hat{\sigma}_z \hat{b}^\dagger \hat{b} + \left( i \Omega_r \hat{b}^\dagger + \text{H.c.} \right).
\]

The first term in Eq. (3) corresponds to a rotation of the amplitude \( \alpha \) in the complex plane about the virtual origin \( \alpha_{vo} \) with an angular frequency \( \chi \) in a direction determined by the qubit state. Thus driving the qubit at the resonator frequency \( \omega_r \) effectively shifts the origin of the resonator phase space to a point \( \alpha_{vo} \) in the rotating frame.

The term \( \Omega_q + i \Omega_r \chi/g \hat{\sigma}_+ \) in Eq. (2) shows that the drives slightly tilt the qubit Hamiltonian. The tilt of the quantization axis determines the speed at which the drives
can be turned on while maintaining adiabaticity, the lowest-order condition being approximately $\Omega_q \ll \Delta^2 / \sqrt{2}$. Since $\Omega_q \ll \Delta$, the rise time of the qubit drive pulse can be negligibly short compared with the relevant dynamics of the resonator states. Thus, the qubit-state-dependent separation dynamics of the resonator state starts to take place essentially instantly in this readout protocol.

In contrast to the multichannel readout visualized in Fig. 1(c), the usual dispersive readout relies solely on the term $-\chi \hat{a} \hat{a}^\dagger \hat{a}^\dagger$, which implies that one needs to use the resonator drive to populate the resonator for the state separation to take place, see Fig. 1(b). The characteristic timescale for the population dynamics $1 / \kappa$ is determined by the internal and external damping rates of the resonator $\kappa_i$ and $\kappa_x$, respectively, as $\kappa = \kappa_i + \kappa_x$.

In addition to the potentially faster readout, our scheme offers more control over the evolution of the states than the usual dispersive readout. For example, we may continuously drive the resonator such that either $\alpha_e$ or $\alpha_i$ end in any desired position at the end of the readout. For example, choosing $i \Omega = \Omega_q \chi / g$ in Eq. (2) causes $\alpha_i$ to remain in vacuum while $\alpha_e$ is displaced. Interestingly, we may also reset the resonator to the vacuum state unconditionally on the qubit state and without feedback control. As illustrated in Fig. 1(c), one may shift the phase of $\alpha_{oo}$ by $\pi$ after the actual measurement pulse and wait for both of the amplitudes $\alpha_i$ and $\alpha_e$ to rotate on top of each other. Subsequently, both distributions may be shifted to the vacuum state using a single pulse on the resonator.

Note that due to the finite resonator bandwidth, the resonator will slowly saturate towards a steady state. We obtain the steady states by solving the standard Lindblad master equation $\dot{\rho} = -i [\hat{H}, \rho] / \hbar + \kappa \mathcal{L}[\hat{a}]\rho / 2$, where $\mathcal{L}[\hat{a}]$ is the Lindblad superoperator and $\rho$ is the density operator of the qubit-resonator system. Forcing the states to remain coherent, the steady states $|\alpha^s_{g/e}\rangle$ are given by $\alpha^s_{g/e} = (i \Omega + \Omega_q \chi / g) / (i \kappa / 2 \pm \chi)$. Above, we have restricted our theory to the case of a two-level system. However, the scheme also works in the case of many nonequidistant levels [48] such as those of a superconducting transmon qubit [58] studied below. Here, the driving frequency needs to be slightly offset from that of the resonator and an additional resonator drive is needed to obtain essentially Eq. (2) for the transmon. Note that qubit nonlinearity is pivotal to obtain a nonvanishing dispersive shift $\chi$.

To implement our theoretical scheme we have fabricated [48] a superconducting Xmon qubit [59] shown in Fig. 2(a). It is coupled with strength $g = 2 \pi \times 130$ MHz to a coplanar waveguide resonator of frequency $\omega_r / 2 \pi = 6.02$ GHz. The resonator has internal and external loss rates $\kappa_i = 2 \pi \times 0.5$ and $\kappa_x = 2 \pi \times 1.5$ MHz, respectively. We tune the qubit to the point of optimal phase coherence [48], $\omega_q / 2 \pi = 7.86$ GHz, where it is characterized by the energy relaxation time $T_1 = 3.0$ $\mu$s. This leads to a dispersive shift $\chi = -2 \pi \times 1.6$ MHz. We mount the sample to the base temperature stage, $T = 20$ mK, of a dilution refrigerator and extract the effective qubit temperature $T_{\text{eff}} = 73$ mK from histograms of single-shot measurements [48]. For this purpose, we use a traveling-wave parametric amplifier [60] and a heterodyne detection setup to measure the two quadratures $\text{Re} \hat{a}$ and $\text{Im} \hat{a}$ of the resonator field.

Figures 2(b) and 2(c) present the experimentally measured temporal trajectories of ensemble-averaged expectation values $\alpha(t) = \langle \hat{a}(t) \rangle$ for the conventional readout and our method, respectively. The trajectories show qualitative agreement with our theory: In the conventional readout, the states move in the general direction of the drive and separate as the distance to the origin increases. In our scheme, the states move to opposite directions owing
to precession about the virtual origin lying on the negative real axis. The dominating differences between Figs. 1(b), 1(c) and 2(b), 2(c) can be explained by the higher levels of the transmon [48].

To characterize the performance of our method, we implement single-shot measurements $S$, of the observable

$$\hat{S} = \int_0^T \left[ W_{\text{Re}}(t) \text{Re} \hat{a}(t) + i W_{\text{Im}}(t) \text{Im} \hat{a}(t) \right] dt$$

by temporal integration of the readout signal. Here, the normalized weight functions are determined from the previously measured trajectories as $W_{\text{Re}}(t) \propto |\text{Re}[\alpha_g(t) - \alpha_e(t)]|$ and $W_{\text{Im}}(t) \propto |\text{Im}[\alpha_g(t) - \alpha_e(t)]|$. Thus the most weight is given to the signal when the state separation is known to be the largest. We also determine reference points $\alpha_g^{\text{ref}}$ by averaging shots conditioned on the qubit being in state $j \in \{g, e\}$. For a single measurement shot $S$, we infer that the qubit was in state $|g\rangle$ if $|S - \alpha_g^{\text{ref}}| < |S - \alpha_e^{\text{ref}}|$, see Figs. 2(b) and 2(c).

The error probability of assigning an incorrect label for the intended qubit state is calculated as $e_{\text{total}} = |p(e|g) + p(g|e)|/2$, where $p(j|k)$ is the sampled probability to assign the label $j$ to a state supposedly prepared in $|k\rangle$. To extract the error due to readout, we independently measure the state preparation errors caused by faulty gate operations, spontaneous decay, and thermal excitation. We estimate that these sources account for $e_{\text{prep}} = 2.6\%$ of the total error, mainly limited by $T_1$ decay of our sample (see Ref. [48] for details).

We benchmark the speed and fidelity of our readout scheme against the conventional method in Fig. 3, which demonstrates that driving the qubit directly, with or without the compensation tone on the resonator, yields considerably lower errors for integration times $\tau \leq 350$ ns. Thus, measuring the qubit state by direct or multichannel driving results in a noticeable speedup over driving only the resonator. For each readout scheme, the drive power is independently maximized with the condition that the third level of the transmon is negligibly excited during readout, to ensure that the readout realizes a non-demolition measurement. For the multichannel readout, the relative phase between the resonator and qubit drives $\phi_f - \phi_e$ is also optimized to achieve the fastest decrease in error. We observe that for a given integration time, the conventional readout bears close to 100% larger measurement error than the multichannel driving scheme.

Combining the two drive channels provides versatile tools for controlling the state of the resonator. In Fig. 4(a), we show that as a function of the phase $\phi_f$, the steady states draw circles in the complex plane, the centers and radii of which depend on the qubit state. This behavior is in agreement with the above result $\alpha_g^{\text{ref}} = (i\Omega_s \mp \sqrt{\Omega_s^2 - g^2})/(ik/2 \pm \chi)$. It appears possible to choose the phase of $\Omega_s$ such that the resonator state corresponding to one of the qubit states remains in the vacuum state (gray arrows), a situation inaccessible by driving only the resonator. In

FIG. 3. Average measurement error $e_{\text{total}} - e_{\text{prep}}$ as a function of integration time for the conventional readout (blue markers), qubit driving (red markers), and multichannel driving (yellow markers). Each data point shows the average and the standard deviation of 10 measurement runs consisting of $10^4$ single-shot measurements.

Fig. 4(b), we show that with the multichannel method we can indeed leave $\alpha_g$ near the origin while significantly displacing $\alpha_e$. As discussed above, a related mechanism may be utilized to unconditionally reset the resonator after

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FIG. 4. (a) Measured means of the steady states corresponding to $|g\rangle$ ($\alpha_g$) and $|e\rangle$ ($\alpha_e$) in the multichannel readout as functions of the phase $\phi_f$ of the qubit drive pulse, as indicated by the different colors of the markers. The black arrows denote phase-independent contributions to the steady state owing to the resonator drive. With a particular choice of $\phi_f$, indicated by the gray arrows, one of the resonator states returns to vacuum during the measurement. (b) Evolution of the amplitude of the coherent state (solid lines) for qubit ground (blue color) and excited (red color) states in the partial reset scheme. The phase $\phi_f$ is chosen such that the steady state for $|g\rangle$ lies at the origin.
the readout to further reduce the duration of the overall measurement protocol.

In conclusion, we have proposed and experimentally demonstrated a readout scheme for a qubit dispersive coupled to a bosonic mode. By driving both the qubit and the mode close to the mode frequency, the readout can be turned on much faster than any other relevant timescale in the system and the resonator can be unconditionally brought back to the vacuum state without the need for feedback control. Our experiments with a superconducting qubit demonstrate resonator control through the qubit. For a given readout time in our sample, we experimentally observe that the conventional readout may lead to more than 100% larger error than that of the proposed scheme.

In the future, we aim to realize the unconditional reset protocol and to optimize the sample design such that we improve on the present state-of-the-art readout [26]. Furthermore, our proposal could be implemented in a variety of systems such as qubits coupled to nanomechanical resonators [35,36]. We expect that in addition to qubit readout, an extension of our protocol may also be beneficial for resonator state control such as the creation of cat states [61].

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**Note added.**—Recently, Ref. [62] pursuing a similar readout scheme came to our attention. Our work is fully independent of this reference.


