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Fast synthetic X-mode Doppler reflectometry diagnostics for the full-f global gyrokinetic modeling of the FT-2 tokamak

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The fast linear (Born approximation) version of the X-mode Doppler reflectometry (DR) synthetic diagnostics is developed in the framework of the ELMFIRE global gyrokinetic modeling of the FT-2 tokamak ohmic discharge. The DR signal frequency spectra and the dependence of their frequency shift and shape on the probing antenna position are computed and shown to be similar to those measured in the high magnetic field side probing DR experiment at the FT-2 tokamak. The fluctuation poloidal velocities are determined using the DR experiment and synthetic diagnostics and shown to be within 15%. However, the computed and measured dependences of the DR signal power on the antenna position appear to be different presumably due to underestimation of the small-scale trapped electron mode turbulence component in the measurement region by the code.

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I. INTRODUCTION

The anomalous energy transport remains the main unresolved enigma on the way to the realisation of magnetic confinement fusion using the tokamak concept. According to the present day understanding,^{1–4} the anomalous transport is determined by the drift-wave turbulence and is strongly dependent on the nonlinear interaction of micro-turbulence excited due to specific instabilities caused by the plasma inhomogeneity with large-scale mean $E \times B$ flows and meso-scale zonal flows. That interaction, controlling the anomalous transport, has been in the focus of theoretical and experimental research in magnetically confined plasmas for a long time. The massively parallelized particle-in-cell simulations of the gyrokinetic (GK) distribution function and the electric field^{5,6} provide an efficient theoretical tool for studying the nonlinear turbulent plasma dynamics, which, however, needs a validation and comprehensive benchmarking against the experimental data.^{7–9} In the recent papers,^{10,11} such simulations of the electron and ion distribution functions from the first principles were performed for the small research tokamak FT-2 with a limiter configuration. The complex interplay of different turbulence components was studied in the simulations, and the successful quantitative comparison with the measurements carried out mostly at the low magnetic field side of the machine, where the turbulence is excited, was presented.

In the present paper, the results of the global GK particle-in-cell simulations are quantitatively compared with the X-mode Doppler reflectometry experimental data obtained at the high magnetic field side of the FT-2 tokamak and presented in part in Refs. 11 and 12. These data, obtained with standard Doppler reflectometry utilizing variable antenna positions, characterize the tokamak plasma turbulent dynamics at different scales: micro-scales [density fluctuations of trapped electron mode (TEM)] and macro-scales (the velocity of global plasma flows). All the cases of agreement and

disagreement of comprehensive comparison of synthetic and real experimental diagnostics are discussed in detail.

Doppler reflectometry (DR) is the microwave diagnostic widely used nowadays in toroidal fusion experiments to get information on the anomalous transport phenomena—namely, on plasma rotation (the mean value and its oscillations)¹³ and the turbulence poloidal wavenumber spectrum.¹⁴ Unfortunately, in the case of X-mode DR, the probing wave turning point, providing the enhanced contribution to the DR signal in the slab or cylinder plasma model, cannot be determined unambiguously due to the 2D propagation effects essential in a tokamak, which complicates the experimental data interpretation and its comparison with the GK theory predictions. Unlike the cylinder geometry, in the 2D case, the radial coordinate of the ray trajectory turning point is dependent not only on the value of the poloidal wavenumber, but also on the poloidal coordinate of the ray starting point at the boundary. In the 2D case, it is not given by explicit expression; therefore, to determine it, one needs to integrate the differential equation for the ray trajectory. Development of a synthetic diagnostics, allowing the DR signal computation based on the results of GK modeling, could be helpful in this case both for the interpretation of the experimental results and for the code benchmarking.^{10,11,15–21} The linear version of the DR synthetic diagnostics, based on the computation of the probing microwave spatial distribution in the background plasma and evaluation of the DR signal in the Born approximation, was developed and used in Refs. 10, 11, and 15 for the O-mode and in Refs. 16, 20, and 21 for both O- and X-mode DR measurements at the low field side (LFS) of the torus. The approach of Refs. 10, 11, and 15 utilized the reciprocity theorem of electrodynamics integrating contributions from all the plasma volume to the DR signal under assumption of cylindrical plasma neglecting the discharge toroidal curvature, poloidal magnetic field component, and Shafranov shift. Under these simplifying assumptions

making the problem effectively one-dimensional (cylindrical), the electric field of the probing wave supposedly propagating perpendicular to the external magnetic field was found by solving a set of ordinary differential equations for different poloidal harmonics. In Refs. 16, 20, and 21, only the input of the maximum of the probing wave diagram close to the its reflection region, was taken into account when calculating the DR signal spectrum, as a convolution of the diagram and the turbulence spectrum provided by the gyrokinetic code. The non-linear version of the DR synthetic diagnostics utilizing the full-wave computation of the probing microwave in the plasma with the density profile provided by the GK code, but neglecting the poloidal magnetic field, was recently developed and applied for comparison of computed and measured Doppler backscattering signals dependence on the incidence angle of the O- and X-mode probing at the LFS.^{17–19}

In this paper, the linear version of the X-mode high field side (HFS) DR synthetic diagnostics is developed in the framework of the global gyrokinetic ELMFIRE modeling of the ohmic FT-2 tokamak discharge similar to that used in Refs. 10 and 11. The X-mode DR signal is computed within the framework of the theory linear in the density fluctuation amplitude (Born approximation) using the reciprocity theorem technique and utilizing the probing wave field pattern provided by the full-wave computation that takes into account the 2D plasma inhomogeneity effects and the poloidal magnetic field component. Therefore, by solving the Maxwell equations, we do not assume the probing wave propagation to be strictly perpendicular to the external magnetic field and take into account the variation of the probing wave field along the magnetic field. The dependencies of the DR signal frequency spectrum amplitude, frequency shift, and shape on the probing antenna position are computed and compared with those measured in the experiment at the HFS of the FT-2 tokamak, where the decay of the TEM turbulence takes place according to the GK code predictions. Strong disagreement obtained in the case of the spectrum amplitude dependence, as well as a close agreement demonstrated in the latter cases are discussed. The fluctuation poloidal velocities are determined using the DR experiment and synthetic diagnostics and shown to be within 15%.

II. THE EXPERIMENTAL APPROACH

The experiment was performed at the FT-2 tokamak (major radius $R = 55$ cm, limiter radius $a = 7.9$ cm) in the hydrogen ohmic discharge (with plasma current $I_p = 19$ kA, central density $n_e(0) = 4 \times 10^{13} \text{ cm}^{-3}$ and electron temperature $T_e = 370$ eV). The duration of the discharge is 40 ms whereas the flat-top stage lasts for 10 ms [Fig. 1(a)]. The discharge is similar to that utilized for successful comprehensive benchmarking of the ELMFIRE GK code in Refs. 10 and 11, however, the toroidal magnetic field at the discharge axis is slightly smaller ($B_t(0) = 1.7$ T instead of 2.1 T). The majority of the experimental figures in this work are obtained using data from the flat-top phase of multiple highly reproducible discharges similar to that shown in Fig. 1(a). The ASTRA code fits of the actual electron density (measured with a microwave interferometer and laser Thomson scattering) and temperature (provided by laser Thomson scattering) profiles for this discharge used in the gyrokinetic modelling, as well as the ion temperature profile (determined with a neutral particle analyser and impurity spectroscopy) are shown in Fig. 1(b) together with the corresponding error bars. The vertically movable (by ± 2 cm) X-mode double antenna set (shown in scale in Fig. 2) installed at HFS near the equatorial plane allowed plasma probing at variable incidence angle with frequencies in the range $f_i = (50\text{--}75)$ GHz. The distance from the limiter position to the antenna opening is 0.2 cm, whereas the microwave beam diameter there is 2 cm. In the past this antenna, described in more detail in Ref. 22, was used for the ETG mode studies using the enhanced scattering in the upper hybrid resonance.^{23,24} In the present experiment, it was used in standard DR measurements utilizing the quadrature detection scheme. The DR spectrum was obtained via averaging over 155 random DR power spectra obtained by using Fourier transform of successive $12.8 \mu\text{s}$ samples of the DR signal.

The backscattering spectra $p_s(f)$ measured by the X-mode DR diagnostics at different vertical antenna displacements are shown in Figs. 3(a) and 3(b). There the frequency f is equal to the difference of the scattered and incident wave frequencies: $f = f_s - f_i$. One can see that with the increase in the antenna vertical displacement (and, correspondingly, of the probing wave incidence

FIG. 1. (a) The temporal evolution of the loop voltage, plasma current, toroidal magnetic field, and chord integrated density. (b) The plasma parameter profiles. Density—solid curve; electron temperature—squares; and ion temperature—triangles.

FIG. 2. The DR experiment geometry (the antennae, the X-mode cut-off, and the upper hybrid resonance positions) and the probing wave poloidal electric field distribution computed for $y_a = 1.5$ cm.

angle), the backscattering power decreases, while the spectra frequency shift and width increase.

III. THE GK COMPUTATION APPROACH AND THE SYNTHETIC DIAGNOSTICS

A. The GK computation approach

The simulations are performed with the global electrostatic particle-in-cell code ELMFIRE⁵ to model the evolution of the full distribution function of drift kinetic electrons, gyrokinetic protons, and O^{6+} ions, which dominate in the FT-2 discharges according to spectroscopic measurements.²⁵ The simulation geometry has a circular cross-section and covers the radial range of $r/a = [0.25-1]$. The time step is set to $\delta t = 30$ ns and spatial grid to $120 \times 150 \times 8$ cells in radial (r), poloidal (θ), and toroidal (φ) directions, which limits the largest resolvable density fluctuation wavenumber at $r = 5.5$ cm to $k_{\theta \max} = 14 \text{ cm}^{-1}$ corresponding to $k_{\theta \max} \rho_s = 0.8$. The maximum resolution is limited by the memory requirements of the global full-f code, and thus, the complete grid convergence has not been proven. However, as discussed in Ref. 26, the

radial and poloidal resolutions have been set such that the magnetic shear should be accurately modeled for all the poloidal modes.

Approximately, 2400 electrons and ions are simulated per grid cell on average. According to the particle number convergence studies presented in Ref. 26 for FT-2-like plasma parameters, this should be enough to guarantee a noise level of approximately 1%. Momentum and energy conserving binary collision operator²⁷ are applied for collisions between all the particle species and allow the inclusion of neoclassical physics in the simulations.

The particles are initialized according to a prescribed temperature and density profiles taken close to the experimentally measured ones at the low field side of the tokamak, where the trapped electron mode (TEM) typical for FT-2²⁸ is driven [see Fig. 1(b)]. The profiles are allowed to develop self-consistently in time, while the turbulence develops, and heat sources and sinks (namely, Ohmic heating and radiation losses) are applied. The Ohmic heating is induced by the spatially homogeneous loop voltage,^{26,29} while electrons are cooled according to the impurity radiation loss distribution measured by the bolometry diagnostics. The initial oxygen O^{6+} impurity density is chosen in order to provide the experimental value of $Z_{\text{eff}} = 2.2$, as determined from ASTRA modeling and experimental loop voltage measurements, with equal temperatures for the impurities and main ions.

Neumann and Dirichlet boundary conditions are used at the inner and outer boundary of the simulation geometry for the electrostatic potential. Particles passing through the inner boundary are simply reflected back into the simulation domain, whereas particles passing through the outer boundary are returned to the simulation domain as an electron-ion pairs (according to a probability distribution proportional to the measured radial profile of neutral hydrogen density) with temperature equal to the wall temperature. The total number of particles in the simulations is thus conserved.

After the initialization, the turbulence emerges in the simulation near the middle of the simulation volume at $r/a = 0.63$ in the first 30 μs and reaches a saturated state. That initial growth period of turbulence is neglected in the analysis, but the energy and particle fluxes remain variable in the whole plasma even in the saturated phase, after the initial growth, as it is seen in Fig. 4. The strong oscillations of

FIG. 3. The measured DR signal spectra for antennae shifts (a) below the equatorial plane and (b) above it.

FIG. 4. The temporal variation of GK energy fluxes. Solid line—for ions; and dashed—for electrons.

energy fluxes are related to the geodesic acoustic mode (GAM)³⁰ excited in the FT-2 plasma.³¹

Convergence studies for numerical parameters of corresponding discharges are presented in Ref. 26. The quality of a flux-driven full-f simulation is given by the balance of sources and sinks. In this case, the sources and sinks are not able to completely match the turbulent transport in the core and vice versa in the edge plasma. This causes the temperature profiles to change slightly from the input values, while the density profile, which is the main turbulent drive for the trapped electron mode here, remains close to the original. Possible reasons for the partial mismatch are underestimated prediction of transport from the small scales and uncertainty in the sources and sinks that depend on the ASTRA analysis. The normalized root mean square (rms) density fluctuation level is approximately 3.78% at $r = 5$ cm and increases to a maximum of 13% on the edge of the simulation volume at $r = 8$ cm.

B. The fast synthetic diagnostics

The fast synthetic X-mode DR diagnostics developed for this code is based on the reciprocity theorem of electrodynamics³² relating the high-frequency current in the plasma volume and the signal radiated by it. This approach, providing directly the scattering signal received by an antenna without computation of the scattered field in the plasma volume, is equivalent to the Born approximation treating the density fluctuations as a small perturbation. In the case of microwave backscattering, the reciprocity theorem results in the following expression for the signal $A_{sw}(t)$:

$$A_{sw}(t) = \frac{\sqrt{P}}{4} \int_V \frac{\tilde{n}(\vec{r}, t)}{n_e} [\hat{\sigma}(\vec{r}, \omega) \vec{E}_a(\vec{r}, \omega)] \vec{E}_a^+(\vec{r}, \omega) d\vec{r}, \quad (1)$$

where P is the probing wave power; the DR signal power is given by $|A_{sw}(t)|^2$; $\frac{\tilde{n}(\vec{r}, t)}{n_e}$ is the relative density fluctuation distribution provided by the GK computation; $\hat{\sigma}(\vec{r}, \omega)$ is the high-frequency conductivity tensor of the unperturbed

plasma: $\sigma_{xx} = \sigma_{yy} = \frac{i\omega}{4\pi} \frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2}$; $\sigma_{xy} = -\sigma_{yx} = \frac{\omega_{ce}}{4\pi} \frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2}$; and $\sigma_{zz} = \frac{i\omega}{4\pi} \frac{\omega_{pe}^2}{\omega^2}$, the z-axis of the Cartesian coordinate system is directed along the magnetic field calculated accounting for its poloidal component; $\sqrt{P} \vec{E}_a(\vec{r}, \omega)$ stands for the probing wave electric field distribution, and $\vec{E}_a^+(\vec{r}, \omega)$ stands for the wave electric field distribution produced by the probing antenna at the unit power and the opposite sign of the plasma magnetic field. The integration in Equation (1) should be, and was, performed over the whole discharge poloidal cross-section. The electric field distribution was determined by solving the Maxwell equations in the realistic toroidal geometry and for the density profile shown in Fig. 1(b). The poloidal magnetic field component and its inhomogeneity, provided by the ASTRA transport code were taken into account. The equations describing the probing wave propagation in two-dimensionally inhomogeneous plasma

$$\vec{\nabla} \times [\vec{\nabla} \times \vec{E}_a] = \frac{\omega^2}{c^2} \left(1 + \frac{4\pi i}{\omega} \hat{\sigma} \right) \vec{E}_a \quad (2)$$

were solved numerically by the full-wave code WaveTOP2D³³ initially developed for the ICR frequency range, modernized here and applied in the microwave frequency domain. The boundary conditions at the antenna were modeling the experimentally measured Gaussian wave beam focusing on air at the distance of 14 cm from the antenna where its diameter was equal to 1.5 cm in the poloidal direction. The infinite size of the beam in the toroidal direction (toroidal mode $n = 0$) was assumed. The reflection from the tokamak metallic wall was suppressed by introducing an artificial absorbing layer in the wall vicinity. The corresponding distribution of the poloidal component of the microwave electric field in the presence of plasma is shown in Fig. 2 for the incident wave frequency $f_i = 70$ GHz and the vertical shift of the antenna $y_a = +1.5$ cm. The DR diagnostics weighting function $w(\vec{r}, \omega) = \frac{\vec{E}_a^+(\vec{r}, \omega)}{n_e(\vec{r})} [\hat{\sigma}(\vec{r}, \omega) \vec{E}_a(\vec{r}, \omega)]$ is plotted in Fig. 5. As it

FIG. 5. Spatial structure of the weighting function W for $y_a = 1.5$ cm and probing wave ray trajectories for $y_a = 0.5$ cm (solid), $y_a = 1.0$ cm (dashed), $y_a = 1.5$ cm (dash dot), and $y_a = 2.0$ cm (short dash).

is seen, the weighting function absolute value is maximal at the minor radius $r \approx 5.8$ cm in a relatively narrow (0.4 cm in the minor radius direction) region tangential to the X-mode cut-off, and shifted vertically from the equatorial plane by 2 ± 1 cm. In the case of statistically homogeneous turbulence, this region provides the dominant contribution to the DR signal. With decreasing antenna vertical displacement, the vertical shift becomes smaller so that at the plasma probing along the density gradient (at $y_a = 0$ cm) it is 0 ± 1 cm; simultaneously, the main maximum of the weighting function penetrates slightly (0.3 cm) deeper in the minor radius direction. Thus, the expected radial position of the region providing maximal contribution to the synthetic DR signal changes from $r = 5.6$ cm at $y_a = 0$ cm up to $r = 5.9$ cm at $y_a = 2$ cm. The typical poloidal wavelength of the fluctuations contributing to the DR signal is determined by the poloidal period of the weighting function oscillations in the zone of their maximal amplitude. In the case shown in Fig. 5, it is equal to 0.8 cm. The consequence of turbulent density fluctuations computed during 0.12 ms in the eight toroidal sectors corresponding to the grid cells were used to obtain the DR signal lasting 0.96 ms. This realization was divided into 0.012 ms samples in which the random Fourier spectra were calculated and used later to obtain the average DR spectrum. The radial distribution of density fluctuations was taken exactly the same as provided by the GK code, whereas they were supposed constant in the toroidal direction inside the toroidal sector.

It should be stressed, however, that the approach based on the linearized reciprocity theorem neglects the perturbations of the probing wave propagation caused by the density fluctuations: the unperturbed wave field is substituted in (1). In this approach, nonlinear effects, in particular multiple small-angle scattering, are not taken into account; however, the single small-angle scattering important at small DR incidence angles is accounted for. In this sense, as it was already

mentioned, it is equivalent to the so-called Born approximation linear in the fluctuation amplitude. The main merit of this approach is the possibility of fast computations, whereas the main drawback is related to the neglect of the nonlinear effects, which should be not that strong a limitation in the case of a small machine. The Born approximation criteria for the backscattering in the cut-off vicinity given in Ref. 34 in the case of X-mode takes the form $\frac{2\bar{n}(\vec{r},t)}{n_e} \left(\frac{\omega L}{c}\right)^{2/3} \ll 1$, where L is the distance from the plasma boundary to the cut-off. This criteria leads to condition $\frac{\bar{n}(\vec{r},t)}{n_e} < 0.06$ which holds in the measurement region, where, according to the GK computations $\frac{\bar{n}(\vec{r},t)}{n_e} \approx 0.04$.

IV. COMPARISON OF THE DOPPLER BACKSCATTERING SIGNALS

A. DR frequency spectra form, shift, and width

All the DR spectra, both obtained in the experiment and provided by the synthetic diagnostics were normalized to unity and then compared in Fig. 6 for negative and positive vertical antenna shifts. The absolute comparison of the experimental and synthetic diagnostics results is not possible here (like in many DR experiments—see, for example, Ref. 21) because the measurements are not calibrated absolutely and the synthetic DR signal computations are two-dimensional. In spite of that, due to the two-dimensional character of the drift-wave turbulence strongly elongated along the magnetic field possessing small pitch angle (1.7°), the computation utilizing only the central ($n = 0$) component of the toroidal antenna diagram is capable of describing the main features of the DR signal. In particular, within this approach, the comparison of the synthetic and experimental DR spectra and the DR signal dependence on the antenna vertical displacement is meaningful after normalization even without absolute calibration because these characteristics are

FIG. 6. Comparison of normalized DR spectra for different antenna vertical displacement. circles—experiment; and triangles—synthetic DR.

FIG. 7. (a) Dependence of the DR signal frequency shift on the antenna vertical displacement y_a for $f_i = 70$ GHz. Circles—experiment; and triangles—synthetic DR. (b) Dependence of the DR signal frequency width (rms) on the antenna vertical displacement y_a for $f_i = 70$ GHz. Circles—experiment; and triangles—synthetic DR.

determined only by the turbulence wavenumber spectrum perpendicular to the magnetic field and by the antenna poloidal diagram. The latter was measured and then used in computations.

As it is seen in Fig. 6, the synthetic spectra fit reasonably well the experimental ones for all the antenna positions. Not only are the shifts of both spectra close in all the cases, but their shapes (with the exception of the case $y_a = -10$ mm) also look alike. This impression is confirmed more rigorously in Figs. 7(a) and 7(b), where the mean frequency shift $f_D = \int f \cdot p_s(f) df / \int p_s(f) df$ and the mean frequency width $\Delta f = [\int (f - f_D)^2 \cdot p_s(f) df / \int p_s(f) df]^{1/2}$ of the experimental and synthetic spectra obtained as the first and the second moment of the spectrum, accordingly, are plotted as functions of the antenna vertical displacement. For both spectral characteristics, the data provided by the experiment and GK modeling agree within 20%, even at $y_a = -1$ cm, where the form of the spectrum is different, as it is seen in Fig. 6. For the majority of points, the difference of the measured and computed Doppler frequency shift is substantially smaller than 20%. The meaningful difference in the Doppler frequency shift is observed only at the largest antenna displacement, $y_a = \pm 2.0$ cm. This difference could be presumably attributed to the overestimation by the code of fluctuation

phase velocity at the largest resolvable wavenumber, according to the Nyquist or Kotelnikov theorem, determined by the grid cell poloidal size ($\kappa_\theta = 14 \text{ cm}^{-1}$), which by chance is close to the value corresponding to the largest antenna shift ($\kappa_\theta = 11 \text{ cm}^{-1}$). The agreement of the experimental and synthetic data demonstrated in Figs. 6 and 7 is not specific for the chosen probing frequency $f_i = 70$ GHz, as it is shown in Fig. 8, where the results of the comparison performed for the probing frequency $f_i = 67$ GHz are presented. (In the experimental day, when the probing at frequency for $f_i = 67$ GHz was performed, the discharge was shifted vertically by 0.5 cm that allowed measurements in the vertical antenna displacement interval $[-1.5 \text{ cm}, 2.5 \text{ cm}]$ with respect to the discharge equatorial plane.)

The obtained agreement between the measured and computed DR spectra form, shift, and width, which are determined by the plasma flows, provides arguments in favor of correct modeling of plasma poloidal rotation dynamics by the GK code.

B. DR signal dependence on the probing wave poloidal wavenumber

We also compared the dependences of DR signal intensity $P_\Sigma = \int p_s(f_s - f_i) df_s$ on the vertical antenna displacement,

FIG. 8. (a) Dependence of the DR signal frequency shift on the antenna vertical displacement from the discharge equatorial plane y_a for $f_i = 67$ GHz. Circles—experiment; and triangles—synthetic DR. (b) Dependence of the DR signal frequency width (rms) on the antenna vertical displacement from the discharge equatorial plane y_a for $f_i = 67$ GHz. Circles—experiment; and triangles—synthetic DR.

FIG. 9. The dependence of the backscattering power on the fluctuation poloidal wavenumber. (a) Linear scale: circles—experiment; triangles—synthetic DR; solid and dashed lines—Gaussian approximations; and squares—extrapolation of the GK turbulence spectrum computed in the backscattering region. (b) Double logarithmic scale: circles—experiment; triangles—synthetic DR; and squares—the GK turbulence spectrum computed over the HFS part of the torus at $r = 5.5$ cm.

which are presented in Fig. 9 in terms of the poloidal wavenumber spectrum of the turbulence. (Both measured and computed dependences were normalized to unity at $\kappa_\theta = 0$ ($y_a = 0.0$ cm) and then compared.)

The relation between the antenna displacement and the approximate value of the probing wave poloidal wavenumber (providing the largest contribution to the DR signal) is shown in Fig. 10. This wavenumber is obtained using both the analysis of the weighting function periodic structure (see Fig. 5 and comments there) and the X-mode ray-tracing.

In the ray-tracing computations, we assume concentric circular flux surfaces and use the orthogonal toroidal coordinate system (ρ, θ, φ) , where ρ (varying from 0 to 1) is the dimensionless radial coordinate normalized by the tokamak minor radius a_0 , and θ and φ are the poloidal and toroidal angles, respectively. We also assume that plasma parameters are independent of the toroidal co-ordinate φ , and set the toroidal wavenumber equal to zero. Then, the ray tracing equations describing a ray trajectory in the poloidal cross-section have a very simple form

FIG. 10. The relation between the antenna vertical displacement and the probing wave poloidal wavenumber. The circles are provided by ray tracing; the crosses—by weighting function.

$$\begin{aligned} \frac{d\rho}{ds} &= \frac{\partial D}{\partial k_1}; & \frac{dk_1}{ds} &= -\frac{\partial D}{\partial \rho} \\ \frac{d\theta}{ds} &= \frac{\partial D}{\partial k_2}; & \frac{dk_2}{ds} &= -\frac{\partial D}{\partial \theta}, \end{aligned}$$

where s is an independent variable (e.g., normalized trajectory length), D is the left part of the dispersion relation $D(\rho, \theta, k_1, k_2) = 0$ (we choose the solution corresponding to X-mode); k_1, k_2 are associated with k_ρ and k_θ — radial and poloidal components of wave vector $\mathbf{k}$ — by the relations $k_1 = k_\rho$ and $k_2 = \rho k_\theta$. Ray trajectories corresponding to different antenna positions and launched from the centrum of its beam are shown in Fig. 5. The value of the probing wave poloidal wavenumber corresponding to the measurement was taken at the minimal radial coordinate of the trajectory.

Using this relation, we converted the dependences of the measured and synthetic DR signal on the antenna displacement into the dependence of the DR signal intensity on the fluctuation poloidal wavenumber (assumed to be equal to twice the probing wave poloidal wavenumber in the turning point $\kappa_\theta = -2k_\theta$) plotted in Fig. 9. The extrapolation of the density fluctuation poloidal wavenumber spectrum computed in the DR measurement area provided by ELMFIRE is also shown there.

Both the measured and the synthetic dependences of P_Σ on κ_θ in Fig. 9(a), which are related to the poloidal wavenumber spectrum of density fluctuations, as well as the locally computed poloidal wavenumber spectrum of the fluctuations itself, could be well approximated by the Gaussian dependences $P_\Sigma \sim \exp[-(\kappa_\theta l_{c\theta})^2/4]$. The parameter $l_{c\theta}$, possessing the meaning of the poloidal correlation length, is given by $l_{c\theta} = 0.4$ cm in the case of computed fluctuations, whereas in the experiment its value is substantially smaller: $l_{c\theta} = 0.25$ cm. This difference is a consequence of a steeper decay of the synthetic DR signal with growing antenna vertical displacement leading to a significant (by a factor of 10) excess of the measured P_Σ over the synthetic one for backscattering off the fluctuations possessing the poloidal wavenumber $\kappa_\theta = 11 \text{ cm}^{-1}$. There are several effects possibly contributing to this excess at larger fluctuation wavenumbers

(and antenna displacements, $y_a = \pm 2.0$ cm). The main effect is the underestimation of the computed turbulence poloidal wavenumber spectrum in the small-scale domain at HFS due to the grid-scale limitations. [As it was mentioned above the largest resolvable wavenumbers ($\kappa_\theta = 14 \text{ cm}^{-1}$), by chance is close to the value corresponding to the largest antenna shift ($\kappa_\theta = 11 \text{ cm}^{-1}$).] Another important effect is the non-linear multiple scattering providing contribution to the Doppler backscattering signal. In the case of the steep wavenumber spectrum, it can become larger than the standard linear Bragg backscattering taken into account in this paper.³⁵ Finally, the finite width and different form of the actual and computational Gaussian antenna diagram at high angles at the level of -20 dB can contribute to the effect as well, as they take part in the convolution over the fluctuation poloidal wavenumber that produces the DR signal.

The measured and synthetic dependencies of P_Σ on κ_θ plotted in the double logarithmic scale are presented in Fig. 9(b). In the case of experimental dependence, the linear approximation results in the power index -0.9 ; however, the scattering of points is quite large compared to the Gaussian approximation demonstrated in Fig. 9(a). Moreover, in the case of synthetic spectra, the approximation with one or two linear dependences is questionable. Generally speaking, the linear approximation of the measured and synthetic dependencies of P_Σ on κ_θ plotted in the double logarithmic scale is worse than the Gaussian one shown in Fig. 9(a)—probably because our measurements and computations are carried out at the HFS ($\theta \approx 150^\circ$) where the TEM turbulence decay takes place. The turbulence in computations is clearly inhomogeneous in poloidal direction. The density fluctuations are substantially stronger at the LFS thus demonstrating the ballooning effect. This effect is presumably associated with the higher density of trapped electrons at the low field side providing drive to the instability and making the drift wave unstable. In the case of the density fluctuation poloidal wavenumber spectrum computed at $r = 5.5$ cm over all the HFS region ($90^\circ < \theta < 270^\circ$) including the top of the torus, where the turbulence level is maximal, the linear approximation results in two power indexes -0.5 and -2.2 for two parts of the knee-like spectrum, as it is shown in Fig. 9(b).

C. Fluctuation poloidal velocity

The data presented in Figs. 7 and 10 allow us to plot the dependence of the fluctuation poloidal velocity on the measurement poloidal position (and/or on the fluctuation poloidal wavenumber which was varied simultaneously with position) shown in Fig. 11. It should be mentioned that the values of fluctuation poloidal velocity provided by the DR experiment and by synthetic diagnostics agree within the 20% accuracy for majority of points excluding those corresponding to the maximal poloidal wavenumbers, where the modeling is less reliable. Performing averaging of the fluctuation poloidal velocity obtained at antenna vertical displacements, where measurements are more reliable, we obtain an estimation of the mean measured fluctuation poloidal velocity in the high field side equatorial plane region $v_\theta = \pi f_D / k_\theta \approx 1.96 \pm 0.1 \text{ km/s}$, whereas in synthetic DR

FIG. 11. Dependence of the fluctuation poloidal velocity on the poloidal position of the measurement point. Circles—experiment; and triangles—synthetic DR.

computation, $v_\theta = \pi f_D / k_\theta \approx 2.25 \pm 0.03 \text{ km/s}$. The determined synthetic and experimental velocities are within 15%, thus demonstrating a correct reproduction of electric field behavior in the FT-2 tokamak by the ELMFIRE GK code. However, the value provided by the synthetic DR differs from the poloidal component of the plasma $E \times B$ velocity given by the code locally in the vicinity of the probing wave turning point (at $r = 5.5$ cm). As a result of averaging over the turbulent period of computation and all toroidal sectors, we get $cE_r / B_\phi = 2.1 \text{ km/s}$. This difference could be attributed to the contribution to the fluctuation velocity from the value of the drift wave phase velocity in the plasma reference frame, as well as to the radial and poloidal³⁶ inhomogeneity of plasma poloidal rotation. The calculation of the most unstable drift-wave phase velocity at $r = 5.5$ cm performed with the linear version of the GENE code³⁷ in the local flux-tube geometry without collisions, impurities, and electromagnetic effects resulted in the value $v_{ph} = 0.49 \text{ km/s}$. The values of the fluctuation velocity extracted from Fig. 8 and corresponding to the 67 GHz probing wave turning point $r = 6.3$ cm are $v_\theta = \pi f_D / k_\theta \approx 2.45 \pm 0.1 \text{ km/s}$ in the experiment and $v_\theta = \pi f_D / k_\theta \approx 2.6 \pm 0.03 \text{ km/s}$ for synthetic DR diagnostics. The contribution of the plasma velocity parallel to the magnetic field ($v_\parallel = 4.0 \text{ km/s}$) is small there ($|\delta v_\theta| < 0.1 \text{ km/s}$) due to a very small magnetic field pitch angle in FT-2.

V. CONCLUSIONS

To summarise, we can conclude that the fast linear version of the X-mode high field side (HFS) DR synthetic diagnostics based upon the reciprocity theorem is developed in the framework of the global gyrokinetic ELMFIRE modeling of the FT-2 tokamak discharge. The ELMFIRE code benchmarking against the X-mode Doppler reflectometry experimental data, performed for the HFS of the FT-2 tokamak with the help of the DR synthetic diagnostics, has demonstrated a good agreement between the measured and computed DR frequency spectra. For all antennae positions and probing frequencies used for comparison both the spectra

frequency shift and width, and in many cases, the spectra shape, were similar, thus demonstrating a correct reproduction of the electric field behavior in the FT-2 tokamak by the ELMFIRE GK code. The mean fluctuation velocities determined by the DR measurements and the synthetic diagnostics appeared to be close. However, the fluctuations velocity provided by the synthetic diagnostics was slightly different from the plasma $E \times B$ velocity computed by the GK code—most likely due to the contribution from the fluctuation phase velocity, determined in the plasma reference system, and to the radial and poloidal inhomogeneity of plasma poloidal rotation.

Dependences of the measured and synthetic DR signal power on the antenna vertical displacement are well approximated by Gaussian curves. However, the estimated «poloidal correlation lengths» corresponding to these dependences are different by a factor of 1.7. Presumably, this difference could be attributed to the fact that GK computations underestimate the small-scale turbulence level in its decay region at the high field side of the torus.

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