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Lasing at \(K\) Points of a Honeycomb Plasmonic Lattice

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We study lasing at the high-symmetry points of the Brillouin zone in a honeycomb plasmonic lattice. We use symmetry arguments to define singlet and doublet modes at the \(K\) points of the reciprocal space. We experimentally demonstrate lasing at the \(K\) points that is based on plasmonic lattice modes and two-dimensional feedback. By comparing polarization properties to \(T\)-matrix simulations, we identify the lasing mode as one of the singlets with an energy minimum at the \(K\) point enabling feedback. Our results offer prospects for studies of topological lasing in radiatively coupled systems.

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Feedback provided by a resonator is essential for lasing. The resonator can be a set of mirrors [1] or periodic structures enabling distributed feedback (DFB) lasing [2–6]. Most DFB lasers rely on simple one-dimensional periodic structures. More complex geometries would offer such interesting features as distributed feedback involving multiple modes, flat bands, and increased variety of degenerate high-symmetry points and possibilities of creating topological bands [7]. The symmetry of a hexagonal Bravais lattice leads to the possibility to multiply degenerate points at the first Brillouin zone edge [8]. Here we experimentally demonstrate lasing at \(K\) points of a honeycomb plasmonic lattice.

The vast majority of the work on bosons in hexagonal (including honeycomb) lattices for photonic [9–11], microwave [12,13], and atomic [14–17] systems realize essentially the tight-binding model of the lattice. That is, the lattice sites are connected only up to the (next-)nearest neighbor; in the optical systems, this is realized by site-to-site near-field coupling. Our system consists of an array of plasmonic nanoparticles that are radiatively coupled over the whole system size. This renders tight-binding models useless, and we base our theoretical description on symmetry arguments and \(T\)-matrix scattering simulations.

Plasmonic nanohole and nanoparticle arrays combined with organic and inorganic gain materials are emerging as a versatile platform for room-temperature, ultrafast lasing [18–32] and Bose-Einstein condensation [33,34]. These works, however, focus on lasing action or condensation at the \(\Gamma\) point, that is, at the center of the Brillouin zone of systems with a Bravais lattice that is rectangular or square [18–22,24–27,29], hexagonal [30,32], or one-dimensional [28] (Ref. [31] studies lasing action in the \(X\) point of a square lattice).

\(K\)-point lasing or condensation in radiatively (long-range) coupled hexagonal lattices has been studied in photonic crystal [35–37] and exciton-polariton [38] systems. In those works, however, the polarization properties of the output light were not analyzed. Here we demonstrate lasing at the \(K\) points and show that the polarization properties and real-space patterns of the laser emission contain essential information about the lasing mode. We identify the lasing mode as one of the singlets allowed by symmetry and explain why this mode is selected by the lasing action.

We fabricate cylindrical gold nanoparticles with electron-beam lithography on a glass substrate in a honeycomb lattice arrangement. The particle separation is varied between 569–583 nm. Individual nanoparticles have a nominal diameter of 100 nm and height of 50 nm. An organic dye molecule IR-792 is added on top of the array in 25 mM solution and the structure is sealed with a glass superstrate [Fig. 1(b)]. The dye molecules act as the gain material and are optically pumped with 100 fs laser pulses (750 nm central wavelength). For details, see Supplemental Material [39], which includes Refs. [40–48].

The energies of diffracted orders (DOs) of a 2D hexagonal lattice are shown with dashed lines in Fig. 1(a) for the \(\Gamma-K\) in-plane \((x-y)\) plane momentum direction. The DOs correspond to diffraction without resonant phenomena at the lattice sites, so-called empty lattice approximation. In our samples, the nanoparticles have a broad plasmonic resonance (at 1.87 eV, width \(\sim 300\) meV), which hybridizes with the DOs, leading to narrow (width \(\sim 20\) meV) dispersive modes called surface lattice resonances (SLRs) [49,50], see Fig. 1(a).

A dispersion obtained by multiple-scattering \(T\)-matrix simulation (for details, see Ref. [24] and the Supplemental Material [39]) agrees with the experiments, see the insets of Fig. 1(a). The dispersions are measured with a Fourier imaging setup used in our previous works [24,34,51] but are now extended to larger angles.
The geometry of an infinite honeycomb lattice belongs to the group \( p6m \times \sigma_h \), the wallpaper group \( p6m \) extended by the horizontal reflection \( \sigma_h \). The horizontal reflection ensures that the eigenmodes can be divided into two classes according to the electric field orientation at the mirror plane: the electric field \( E \) is either parallel (in plane \( E \)) or perpendicular to the mirror plane (perpendicular \( E \), magnetic field \( H \) in plane) [8].

A single unit cell of the reciprocal lattice of our system contains six high symmetry points [Fig. 2(d)]: one \( \Gamma \) point with \( D_6 \) point symmetry, as well as two \( K \) points with \( D_3 \) and three \( M \) points with \( D_2 \) point symmetries. The \( K \) points are mutually related by parity inversion symmetry. Whenever the distinction between the two \( K \) points is relevant, we label the other one as \( K' \). To a large extent, group theory determines the properties of the eigenmodes supported at the high-symmetry points. As the reciprocal lattice has \( D_3 \) point group symmetry around the \( K \) points, the \( K \)-point modes must constitute irreducible representations of the \( D_3 \) group. Using standard group-theoretical reduction methods [53], we can determine for instance the electric dipole polarizations of the nanoparticles in the respective modes. The irreducible representations of \( D_3 \) are either one- or two-dimensional, so the eigenmodes are, apart from accidental degeneracies, either nondegenerate (“singlets,” 1D representation) or doubly degenerate (“doublets,” 2D representation). Six dispersion branches meet at the \( K \) point (see Supplemental Material [39]), and the eigenmodes constitute two singlets and two doublets.

Figure 2(a) shows the admissible patterns of nontrivial nanoparticle dipole polarizations in the \( E \) plane case for the singlets and one doublet. Any linear combination of the depicted doublet states is possible as well. Figure 2(b) shows spatial Fourier transforms of these patterns, corresponding to the polarizations of the far-field beams escaping the array.
In real space, the magenta color in Fig. 2 means clockwise rotating electric dipole polarizations while orange means the dipoles rotate counterclockwise for all $K$ modes. For $K'$ modes, the polarization rotation directions are reversed. If the system is excited simultaneously in the $K$ and corresponding $K'$ states with the same intensities, the polarizations will, instead of rotating, oscillate in a linear direction, with the exact direction depending on the relative phase between the $K$ and $K'$ modes. This will be important in analyzing the experimental real-space images.

To characterize the lasing action, we perform angle, energy, polarization, and position resolved emission measurements. Above a critical pump threshold, the sample exhibits lasing at six specific angles with small spontaneous emission coupling to the lasing mode (small $\beta$ factor [54]) [24,26,28,29,31]. Increased temporal coherence due to lasing is evident from the line width of the emission (2 meV), which is well below the natural line width of the SLR mode at the $K$ point ($\sim$ 20 meV). The 2 meV line width is smaller than those in Refs. [18,21,22,26,31,32] (3.6–27 meV), but larger than the values 0.26–1.5 meV in Refs. [20,24,25,28,29].

In Fig. 4(a), we show the angle resolved emission of the sample. The system exhibits lasing at six specific angles that correspond to three $K$ and three $K'$ points of the lattice. We measure the polarization properties of each point by recording the emission intensities with several different linear polarizer angles. For each point, we recover a typical dipolar emission pattern; however, the direction of linear polarization is different; see Fig. 4(b). The results match excellently the calculated angular distributions of linearly polarized light having a polarization along the six $\Gamma$–$K$ directions (the red dashed lines). We find that the $A_1'$ singlet mode has corresponding polarization properties, see Fig. 2(b). The linear polarization degree $\rho_L = (I_{max} - I_{min})/(I_{max} + I_{min})$ reaches on average 0.8 for the six $K$ points.

The identification of the lasing mode as the singlet $A_1'$ can be further confirmed by analyzing the real-space images with variously oriented polarization filters at the output. While the dipole polarization directions of the nanoparticles cannot be measured directly, we can estimate them using the spatial intensity variations due to wave interference in case of different filter orientations. The intensity variations should be most clear in the case where the system lases in the $K$ and $K'$ modes simultaneously, with a fixed (modulo $\pi/3$) relative phase such that the dipoles are oriented as in Fig. 2(a). If the system lases only in one of the $K$ or $K'$ modes, or if the relative phase is random, the real-space intensity distribution should become more uniform due to time averaging (see Supplemental Material [39]).

Figure 5 shows an image of a small piece of the array for three choices of polarization filters for the lasing emission, with the predicted intensities and nanoparticle electric dipole polarizations of the singlet mode $A_1'$ for the ideal, namely zero phase-difference combination of the $K$ and $K'$ modes, as defined in Fig. 2(a) (cases with other polarization filter orientations and details of the theoretical predictions are shown in Supplemental Material [39]). The intensity maxima appear at the places where the surrounding adjacent dipoles, or their projections according to the polarization filter orientation, have the same or similar directions and therefore interfere constructively. Comparing the real-space

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**FIG. 3.** (a) Measured emission spectra of a honeycomb lattice with $P = 1.38 P_{th}$, where $P_{th} = 0.47$ mJ/cm$^2$ is the threshold pump fluence for the $K$-point lasing mode (particle distance $p = 576$ nm and diameter $d = 100$ nm). (b) The mode output power (squares) and the line width (circles) at the $K$-point angle ($35^\circ \pm 0.4^\circ$) as a function of pump fluence. Note that due to low intensity, we cannot determine the line width at pump fluences below the threshold, for below threshold emission, see Supplemental Material [39]. (c) The emission intensity as a function of angle at the $K$-point energy ($\sim$ 1.426 eV) with several pump fluences.

**FIG. 4.** Lasing mode polarization. (a) Angle resolved emission of the sample without any polarizer in detection. All six $K$ points are clearly visible. (b) Polar emission intensities at each $K$ point in the presence of a linear polarizer. The angles refer to the polarizer angles and the radii refer to the measured intensities. The red dashed lines are the calculated intensity distributions for linearly polarized light (along the $\Gamma$–$K$ directions) passing through the polarizer at the corresponding angle.

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images with dipole orientations predicted for the other
omodes ($A_1^2$ and the doublet $E'$) results in inconsistencies
(for details, see Supplemental Material [39]). This confirms
that the system indeed lases in the singlet mode $A_1^1$. The
intensity variations in the observed patterns show that the
system lases in the $K$ and $K'$ singlet $A_1^1$ modes simulta-
nously, with comparable intensities and with a fixed, or at
least strongly correlated, relative phase. The existence of
interference patterns over the whole sample, furthermore,
proves the spatial coherence of the observed lasing. Since the
$K$ point of our system corresponds to the crossing of
diffractive orders in three directions with 120° angles
between them, the feedback in the lasing action is two
dimensional, different from one dimensional DFB lasing [2]
in nanoparticle arrays [24,25,28,55]. This is reflected in the
nontrivial 2D polarization patterns.

DFB-type lasing typically occurs at a band edge or an
extremum of the dispersion because zero group velocity
enables feedback. Both the measured and simulated dis-
persions [Fig. 1(a)] show crossings of the modes at the $K$
point, without any visible gap and zero group velocity
point. Why does a mode of a certain symmetry (the $A_1^1$
singlet) lase, if the $K$ point apparently has a degeneracy of
several modes? To answer this we computed the energies of
the eigenmodes using symmetry-adapted $T$-matrix simu-
lations (for details, see Supplemental Material [39]). Figure 2(e)
shows that indeed there is a difference in the
energies of the $A_1^1$ singlet and the $E'$ doublet near the $K$
point. This band gap means that the singlet $A_1^1$ has an
energy minimum at the $K$ point, which explains why lasing
is possible in this mode. The narrower peak for $A_1^1$
compared to that for $E'$ indicates higher quality factor,
making the former mode more amenable for lasing. The $A_2^1$
singlet mode seems almost degenerate with $A_1^1$, but the
resonance is a bit weaker {slightly smaller dip in Fig. 2(e); see
Supplemental Material [39] for a larger picture}. The
energy difference between $A_1^1$ and $E'$ is only 3.2 meV,
smaller than the natural linewidth of the SLR mode around
20 meV, which explains why the gap is not visible in the
dispersions. On the other hand, the lasing emission has
2 meV linewidth, similar to the scale of the band gap.

In summary, we have observed lasing action at the $K$
and $K'$ points of a honeycomb plasmonic lattice. Both the
polarization of the six output beams and the real space
interference patterns provide distinct features that, when
combined with the group theory description, reveal the
lasing mode as the singlet $A_1^1$. Analysis of the $T$-matrix
simulation results using the group theory eigenmodes
showed that the singlet $A_1^1$ has an energy minimum at the
$K$ point, which enables the feedback necessary for
lasing. Our results demonstrate the potential of plasmonic
nanoparticle array systems for tailoring the polarization and
beam direction of laser output by the lattice geometry. The
 tunability of the beam direction (here $\sim 35°$) can be used
for bringing the beam close to the in plane direction. If
realized in a less lossy platform, this could enable on-chip
planar integration.

Our study gives a promising starting point for inves-
tigations of topological photonics and lasing [7,56–63] in
radiatively coupled systems. Plasmonic nanoparticle array
lasers offer a unique combination of easy fabrication, room
temperature operation, ultrafast speeds, long-range radi-
ative coupling, and strong coupling to emitters (the gain
medium) [26,64,65]. Radiatively coupled systems offer
topological phenomena different from tight-binding models
[66]. Arrays of magnetic nanoparticles have been realized
[67], and the magnetization of nanoparticles could be used
for opening topological gaps at the high-symmetry points
where we have shown lasing. Time reversal symmetry
breaking is one of the main mechanisms leading to
topologically nontrivial systems but topological gaps based
on magnetic materials [62,63] are extremely small at
optical frequencies [56]. The polarization and interference
analysis demonstrated here should prove helpful in iden-
tifying topological modes, and the observed stability of the
lasing action despite a narrow gap is promising concerning
topological lasing relying on small topological gaps.

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FIG. 5. Upper row: examples of real-space images for different
polarization filters (no filter, horizontal, vertical) used for
the analysis of the lasing mode. In each case, the expected positions
of the nanoparticles (small yellow-cyan circles) and the dipole
polarizations (arrows) of the singlet mode $A_1^1$ for the ideal (zero)
$K$–$K'$ relative phase are depicted, projected to the corresponding
filter direction. Lower row: theoretical prediction for the intensities
for the ideal $K$–$K'$ relative phase. The scale bar length is 1 μm. For
images over larger areas, see Supplemental Material [39].
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