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Secure Outsourced Principal Eigentensor Computation for Cyber-Physical-Social Systems

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Abstract—Cyber-physical-social systems (CPSS) are revolutionizing the relationships between humans, computers and things. Outsourcing computation to cloud can offer resources-constrained enterprises and consumers sustainable computing in CPSS. However, ensuring the security of data in such an outsourced environment remains a research challenge. Principal eigentensor computation has emerged as a powerful tool dealing with multidimensional cyber-physical-social systems data. In this paper, we present two novel secure principal eigentensor computation (SPEC) schemes for sustainable CPSS. To the best of our knowledge, this is the first effort to address SPEC over encrypted data in cloud without the interaction need between multiple users and cloud. More specifically, we leverage cloud server and trusted hardware component to design a collaborative cloud model. Using the model, we propose (1) a basic SPEC scheme based on homomorphic computing and (2) an efficient SPEC scheme that combines the advantages of homomorphic computing and garbled circuits, and exploits packing technology to reduce computational cost. Finally, we theoretically and empirically analyze the security and efficiency of our SPEC schemes. Findings demonstrate that the proposed schemes provide a secure and efficient way of outsourcing computation for CPSS. In addition, from the cloud user's perspective, our proposal is lightweight.

Index Terms—Cloud computing, privacy, cyber-physical-social systems, tensor, principal eigentensor computation, sustainable computing.

1 INTRODUCTION

With the rapid proliferation of intelligent sensors and devices, cyber-physical systems (CPS), the integrated systems of cyber space and physical space, have attracted tremendous attention [1]. As the extension of the CPS, cyber-physical-social systems (CPSS) are the integrated systems of cyber space, physical space, and social space [2, 3]. CPSS involve symbiotic networks of human (social networks), intelligent sensors, switches, routers, mobile phone terminals, and information networks. CPSS have the potential to provide enormous novel approaches for sustainable computing. Due to the increasing popularity of CPSS, huge amounts of data generated in CPSS form cyber-physical-social big data, which can be used to enhance the quality of sustainable computing.

Cloud computing is an emerging and promising computing tool in CPSS. Meeting the ever-growing expectations of users requires data processing and analytical capabilities beyond those of traditional computer infrastructures in CPSS. This, perhaps, explains the increasingly popularity of cloud computing due to the capability to support sustainable computing based on a pay-as-you-use business model [4]. Hence, we have witnessed a recent trend in processing cyber-physical-social big data in the cloud, despite the underlying cloud security issues, for example potential leakage of user’s network data (particularly in the aftermath of the revelations by Edward Snowden), risks of malicious insider attacks, and inference threats [5, 6]. Thus, addressing such concerns is key to widespread cloud adoption in CPSS [7].

While cloud computing can be seen as an extension of the “traditional” computing, its design is significantly more complicated due to the interconnectivity and other requirements. Unsurprisingly, designing secure cloud computing has been the subject of recent focus [7], such as ensuring security of virtual machines on the cloud server [8], and ensuring data security and enabling secure computation on the cloud server [9, 10]. However, designing efficient secure computation remains one of the most cloud research challenges. This is the gap we seek to address in this paper.

Specifically, we explore the potential of computing principal eigentensor for tensor in cloud-enabled CPSS. Tensor model has been adopted in a good deal of cyber-physical-social big data analytics research [11]. In our previous work [12], for example, we presented a tensor-based model for big data representation and demonstrated the effectiveness...
of the approach. The principal eigentensor for tensor is a natural generalization of the principal eigenvector for matrix. The principal eigentensor indicates the ultimate steady state distribution of high-order data and has various novel applications. For example, the principal eigentensor can be used in network traffic prediction based on spatio-temporal context and latent social relationships. This allows us to generate better effectiveness than principal eigenvector. As principal eigentensor computation is time-consuming when meeting big data, it is a preferred option to carry out the computations in the cloud.

To the best of our knowledge, there is no prior work in the design of secure principal eigentensor computation which can achieve protecting users’ privacy in cloud. We identify several challenges associated with the naive utilization of principal eigentensor computation. Firstly, to preserve data security, the principal eigentensor should be computed without the cloud having access to or inferring data information. Secondly, the secure principal eigentensor computation should be efficient and scalable to be practical for deployment. Thirdly, the computations (e.g. multilinear mode product, tensor-constant division, comparison of 2-norm of tensor difference and constant, and acquisition of principal eigentensor) are involved in principal eigentensor computation. However, it is not known how to design a secure protocols that will securely perform these complex computations in clouds. Finally, in practice, users generally have limited computation and communication resources (e.g. on their mobile devices), and principal eigentensor computation is time-consuming. Thus, it is not realistic to expect the users to remain online after supplying their data (e.g. loss of network connectivity) or for the data owners to carry out tasks while computing principal eigentensor. Therefore, secure principal eigentensor computation needs to be lightweight for users.

In this paper, we present two novel secure principal eigentensor computation (SPEC) schemes for protecting users’ privacy in sustainable cyber-physical-social systems. Both schemes are designed to securely and efficiently compute the principal eigentensor over encrypted data in the cloud. We regard the main contributions of this paper to be as follows:

- We design a basic SPEC scheme which leverages homomorphic computing properties, and uses the cloud and the trusted hardware component to collaboratively and securely compute the principal eigentensor. To implement the scheme, we design a set of secure building blocks including secure multilinear mode product protocol, secure tensor addition protocol, secure tensor-constant division protocol, secure comparison of 2-norm of tensor difference and constant protocol on cloud, and secure acquisition of principal eigentensor for data users.
- We then present an efficient SPEC scheme, which combines the advantages of homomorphic computing and garbled circuits, and exploits packing technology to improve the efficiency of the basic SPEC scheme in a collaborative cloud model. The improved scheme builds on some protocols involving secure packing-based multilinear mode product, secure packing-based tensor-constant division, improved secure comparison of 2-norm of tensor difference and constant, and secure packing-based acquisition of principal eigentensor. These protocols are very significant for tensor-based secure computation. To our best knowledge, this is the first work to achieve secure and efficient tensor computations over encrypted data in the cloud.

The SPEC schemes allow us to utilize the computational capabilities of the cloud, without compromising on data security. In our schemes, once the encrypted data have been outsourced to the cloud, data owners do not need to participate in any computations. In addition, the data users only need to undertake few addition computations upon receiving the un-encrypted data from the cloud and the trusted hardware component at the conclusion of the secure computing scheme. The schemes enable lightweight secure computing from the view of the data owners and the data users.

We also demonstrate the security of the SPPEC schemes under the semi-honest model. We also analyze both SPPEC schemes, in terms of accuracy, cost of users, and time in cloud server. We evaluate the utility of the SPPEC schemes using synthetic datasets. The results confirm that the SPEC schemes makes the outsourcing computation secure and efficient.

The remainder of this paper is organized as follows. In the next section, we introduce the preliminaries required in the understanding of this paper. The collaborative cloud model and the framework of secure principal eigentensor computation using the model are presented in Section 3. The basic SPEC scheme and the efficient SPEC scheme built on the collaborative cloud model are presented in Sections 4 and 5, respectively. Section 6 evaluates the security and performance of the SPPEC schemes. We review related works in Section 7, and conclude the paper in Section 8.

2 Preliminaries

2.1 Homomorphic Computing

The Paillier cryptosystem [13] is an additive homomorphic and probabilistic asymmetric encryption scheme. For any given ciphertexts $[m_1]$, $[m_2]$ and constant $c$, the Paillier cryptosystem exhibits the following properties: Homomorphic addition computing $[m_1 + m_2] = [m_1] \times [m_2] \mod n^2$; Homomorphic multiplication computing $[cm_1] = [m_1]^c \mod n^2$.

2.2 Garbled Circuits

Yao’s garbled circuit protocol [14] is one of the most effective protocols for secure cloud-based computation. The protocol is composed of two stages: circuit garbling and circuit evaluation.

In the circuit garbling stage, the circuit constructor $CC$ chooses two random values $K_i$ and $K_j$, corresponding to the bit values $a_i = 0$ and $a_i = 1$ respectively, for each wire $i$ of the circuit. Next, for each gate $g$ with wires $i, j$ and $t$, $CC$ calculates 4 values, $E((K_i, K_j)) \left( K_t^{g(a_i, a_j)} \right)$ for $a_i, a_j \in \{0, 1\}$, forming garbled table. After this, $CC$ sends the
permutated garbled table and the random values of its input bit values to the circuit evaluator CE.

In the circuit evaluation stage, CE obtains the random values of its input bits making use of the oblivious transfer protocol. Afterwards, CE calculates the entire circuit gate-by-gate to get the random values of the output bits by decrypting the garbled table. Finally, the bits of output is able to be recovered from their corresponding random values.

2.3 Principal Eigentensor

Tensors and principal eigentensor have been applied in quantities of data analyses, involving social networks and big data technologies [11].

Let $A \in \mathbb{R}^{U_1 \times T_1 \times L_1 \times U_2 \times T_2 \times L_2}$ and $X \in \mathbb{R}^{U \times T \times L}$ be a 6-th order tensor and a 3-th order tensor, respectively. If the equation $A \times_{U_1 \times T_1 \times L_1 \times U_2 \times T_2 \times L_2} X = X$, is satisfied, then tensor $X$ is known as the principal eigentensor.

The method of solving principal eigentensor is multi-linear mode product power algorithm. The inputs for the algorithm are tensors $A \in \mathbb{R}^{U_1 \times T_1 \times L_1 \times U_2 \times T_2 \times L_2}$ and $X_0 \in \mathbb{R}^{U \times T \times L}$. Suppose $x_{ult}$ is an element of the tensor $X_0$, then $x_{ult}$ should satisfy $x_{ult} \geq 0$ and the tensor $X_0$ should satisfy $\|X_0\|_1 = 1$. The output is tensor $X$. The basic idea is to make the tensor irreducible using primitivity adjustment method and to adopt the power method to obtain the principal eigentensor. The main steps of the multi-linear mode product power algorithm is outlined as follows:

1) Let $n = 1$; 2) Compute $X^{U \times T \times L} = \alpha \cdot A^{U_1 \times T_1 \times L_1 \times U_2 \times T_2 \times L_2} X^{U \times T \times L} + (1 - \alpha) \cdot E$; 3) If $\|X_n - X_{n-1}\|_2 < w$, then $X = X_n$, break; else $n = n + 1$, goto 2.

3 PROBLEM STATEMENT

This section presents the collaborative cloud model that leverages cloud servers and trusted hardware components, and utilizes homomorphic computing and garbled circuits. The framework for the SPEC is also presented.

3.1 Collaborative Cloud Model

In the collaborative cloud model, the entities are multiple data owners, a cloud server, a trusted hardware component [15] and data users. The data owners encrypt their data and send these ciphertexts to one or more cloud servers. These data users will receive a result generated by computing a function on the data owners’ data. The cloud servers and trusted hardware components collaboratively implement the secure computation of the function.

The process of securely computing function $F$ over encrypted data $[d]$ on the cloud server and whose output is the data $x_1$, is referred to as secure function computation (SFC). We define the SFC process as follows: $\text{SFC}([d]) \rightarrow x_1$. At the conclusion of the SPEC process, the result $x_1$ is known only to the data users and nothing is revealed to the cloud servers and trusted hardware components.

Let FC denote the process of calculating the function $F$ in the plain-domain without any privacy protection. Suppose data $x$ is the output of the function $F$ on data $d$ in the plain-domain. The FC process is defined as follows: $\text{FC}(d) \rightarrow x$.

Moreover, we need to ensure that the result of the SFC process equals that of the FC process. Formally, the process SFC should satisfy the following equation: $x = x_1$.

In the collaborative cloud model, we are restricted from exposing data $d$ or the intermediate results to any participants, and only the data users have access to the output result $x$. We also assume that all parties are in a semi-trusted model [16].

3.2 Framework of SPEC

To demonstrate how the collaborative cloud model can be used to implement secure computation, we use the principal eigentensor computation as an example. Fig. 1 outlines a framework for SPEC, and consists of the following three phases.

1) Initialization: The trusted hardware component uses the Paillier homomorphic cryptosystem to generate a public key $pk$ and a private key $sk$ for CPSS, and publishes $pk$ to the other users. By using the Paillier encryption with $pk$, the data $x$ from data owners is encrypted, $[x] = E_{pk}(x)$, prior to outsourcing $[x]$ to a cloud server. Encrypted tensor $[A]$ is formed by the encrypted data elements on the cloud server.

2) SPEC Using the Collaborative Cloud Model: The cloud server and the trusted hardware component collectively and securely calculate the principal eigentensor of the tensor $A$ using multilinear mode product power method, homomorphic computing properties, and garbled circuits. The cloud server obtains the encrypted principal eigentensor $[X]$.

3) Acquisition of Principal Eigentensor for the Data Users: Finally, the data users securely obtain the principal eigentensor $X$ with the assistance of the cloud server holding $[X]$ and the trusted hardware component holding $sk$.

4 A Basic SPEC Scheme

Using the collaborative cloud model, we now present a basic SPEC scheme based on homomorphic computing. We
first describe a transformational multilinear mode product power method for solving principal eigentensor. Then, for simplicity, we divide the SPEC scheme into five secure building blocks, namely: secure multilinear mode product protocol, secure tensor addition protocol, secure tensor-constant division protocol, secure comparison of 2-norm of tensor difference and constant protocol on cloud, and secure acquisition of principal eigentensor for data users. These building blocks altogether form the SPEC scheme.

4.1 A Transformational Method for Solving Principal Eigentensor

Since only integers can be encrypted by the Paillier cryptosystem, all variables in the principal eigentensor computation should be quantized to the nearest integer value. This may, however, result in error. Thus, to reduce the possibility of error, the variables are scaled using three positive integers \( \rho_1, \rho_2 \) and \( \rho_3 \) (referred to as scaling factors). At the end of our scheme, the corresponding results (used to recover the true results) can be obtained.

**Algorithm 1.** Transformational Method for Solving Principal Eigentensor

| Input: Tensors \( A \in \mathbb{R}^{U_1 \times T_1 \times L_1 \times U_2 \times T_2 \times L_2} \) and \( X_0 \in \mathbb{R}^{U \times T \times L} \). |
| Output: Principal eigentensor \( \rho_3 X \in \mathbb{R}^{U \times T \times L} \). |
| 1. Let \( n = 1 \) |
| 2. Compute \( \rho_1 \rho_2 \rho_3 X_n^{U \times T \times L} = (\rho_1 \cdot \alpha) \cdot ((\rho_2 \cdot A^1_{U_1 \times T_1 \times L_1 \times U_2 \times T_2 \times L_2} \times U_1, T_1, L_1, (\rho_3 \cdot X_{n-1}^{U \times T \times L})) + \rho_2 \cdot ((\rho_1 \cdot (1 - \alpha)) \cdot (\rho_3 \cdot E)) \) |
| 3. \( \rho_3 X_n^{U \times T \times L} = (\rho_1 \rho_2 \rho_3 X_n^{U \times T \times L}) / (\rho_1 \rho_2) \) |
| 4. if \( \| \rho_3 X_n - \rho_3 X_{n-1} \|_2^2 < \rho_3 \) then |
| 5. \( \rho_3 X = \rho_3 X_n \) |
| 6. break |
| 7. else |
| 8. \( n = n + 1 \) |
| 9. goto 2 |
| 10. end if |

In order to improve accuracy, a transformational multilinear mode product power method for solving principal eigentensor is proposed (see Algorithm 1). Using Algorithm 1, we can obtain the result of the corresponding step in multilinear mode product power algorithm (see Section 2.3) without scaling. For example, let us look at the step \( p_1 p_2 p_3 X_n^{U \times T \times L} = (p_1 \cdot \alpha) \cdot ((p_2 \cdot A_1^{U_1 \times T_1 \times L_1 \times U_2 \times T_2 \times L_2} \times U_1, T_1, L_1, (p_3 \cdot X_{n-1}^{L \times T \times L})) + p_2 \cdot ((p_1 \cdot (1 - \alpha)) \cdot (p_3 \cdot E)) \) in Algorithm 1, and the step \( X_n^{U \times T \times L} = \alpha \cdot A_1^{U \times T \times L} \times U_1, T_1, L_1, (p_3 \cdot X_{n-1}^{U \times T \times L}) + (1 - \alpha) \cdot E \) in multilinear mode product power algorithm.

From the result \( p_1 p_2 p_3 X_n^{U \times T \times L} \) of Algorithm 1, we arrive at the result \( X_n^{U \times T \times L} \) for multilinear mode product power algorithm (i.e., \( X_n^{U \times T \times L} = (p_1 p_2 p_3 X_n^{U \times T \times L}) / (p_1 p_2 p_3) \)). To avoid overflows after many iterations, step 3 is added to Algorithm 1.

From Algorithm 1, we need to resolve the following issues, namely: multilinear mode product, tensor addition, tensor-constant division, comparison of 2-norm of tensor difference and constant, and acquisition of principal eigentensor over encrypted data.

4.2 Secure Multilinear Mode Product and Secure Tensor Addition

The cloud server with input two encrypted tensors \([A], [B] \in \mathbb{R}^{U_1 \times T_1 \times L_1 \times U_2 \times T_2 \times L_2}\) and \([C] \in \mathbb{R}^{U \times T \times L}\) and the trusted hardware component with private key \(sk\) will collaboratively and securely compute the Encrypted tensor \([C]\), where \(C^{U \times T \times L} = A^{U \times T \times L} \times U_1, T_1, L_1, \mathcal{B}^{U \times T \times L} \).\(A\) and \(B\) are not known to either the cloud server or the trusted hardware component. The output \([C]\) is known only to the cloud server. During this protocol, no information about \(A\) and \(B\) will be revealed to the cloud server and the trusted hardware component. The operations in multilinear mode product are multiplication and addition. Therefore, the secure multilinear mode product protocol (SMMP) using the collaborative cloud model can be implemented by exploiting secure multiplication protocol (SM) [17] and the Paillier addition property. Formally, the protocol is defined as follows: \(SMMP([A], [B]) \rightarrow [A \times B, L] \). The secure tensor addition protocol using the collaborative cloud model considers the cloud server with two encrypted tensors \([A], [B] \in \mathbb{R}^{U \times T \times L}\) securely computes the encrypted tensor \([A + B]\) directly adopting the Paillier addition property. The entry of \([A + B]\) is \([a_{utl}] \cdot b_{utl}\). The protocol is defined as follows: \(STA([A], [B]) \rightarrow [A + B]\).

4.3 Secure Tensor-Constant Division

The secure tensor-constant division protocol using the collaborative cloud model (STCD) considers the cloud server with input encrypted tensor \([A]\) and constant \(c\), and the trusted hardware component with the private key \(sk\) and \(c\). Here, the tensor \(A\) is neither known to the cloud server nor the trusted hardware component. The goal of the STCD is to collaboratively compute the encrypted tensor \([A]/c\) known only to the cloud server. During STCD, no information about the tensor \(A\) is exposed to the cloud server and the trusted hardware component. The basic idea is based on Observation 1. The proposed protocol is described in Algorithm 2. Formally, the protocol is defined as follows: \(STCD([A], c) \rightarrow [A/c]\).

**Observation 1:** For any given integers \(a\) and \(c\), the following property holds: \(a/c = (a + rc)/c - r\), where \(r\) is a random number.

For every entry \(a_{utl}\) (\(1 \leq u \leq U, 1 \leq t \leq T, 1 \leq l \leq L\)) in the tensor \(A\), the cloud server randomly picks an integer \(r\), and randomizes \(a_{utl}\) by computing \(a'_{utl} = a_{utl} \cdot [r]\) and then sends the encrypted value \([a'_{utl}]\) to the trusted hardware component. Upon receiving the ciphertext \([a'_{utl}]\) from the cloud server, the trusted hardware component decrypts it and computes \(b_{utl} = a'_{utl}/c\) to obtain \(a_{utl}/c + r\). The trusted hardware component does not know the random number \(r\), so it cannot obtain the result \(a_{utl}/c\). Afterwards, the trusted hardware component encrypts \(b_{utl}\) and sends the ciphertext \([b_{utl}]\) to the cloud server. Finally, applying the Paillier properties, the cloud server removes the random value \(r\) from \([b_{utl}]\) to obtain the encrypted value \([a_{utl}/c]\) (i.e., \([b_{utl}] = [a_{utl}/c + r] \cdot [r]^{-1}\)). Without the private key \(sk\), the cloud server does not see the plaintext result \(a/c\).
Algorithm 2 Secure Tensor-Constant Division (STCD)

Input: Cloud Server: Encrypted tensor \([A] \in \mathbb{R}^{U \times T \times L}\) and constant \(c\). Trusted Hardware Component: Private key \(sk\) and constant \(c\).

Output: Encrypted tensor \([B]\) only to cloud server, where \(B = A/c\).

1: for \(1 \leq u \leq U, 1 \leq t \leq T, 1 \leq l \leq L\) do
2: Cloud Server:
3: choose a random integer \(r\)
4: \([rc]\) \(E_{pk}(r \cdot c)\)
5: \([a'_{ult}]\) \(utl(a_{ult})\) \([rc]\)
6: send \([a'_{ult}]\) to trusted hardware component
7: Trusted Hardware Component:
8: receive \([a'_{ult}]\)
9: \(a_{ult} = D_{sk}(a'_{ult})\)
10: \(b_{ult} = a_{ult}/c\)
11: \([b_{ult}]\) \(E_{pk}(b_{ult})\)
12: send \([b_{ult}]\) to Cloud Server
13: Cloud Server:
14: receive \([b_{ult}]\)
15: \([-r]\) \(E_{pk}(-r)\)
16: \([b_{ult}]\) \(b_{ult} = [-r]\)
17: end for

4.4 Secure Comparison of 2-Norm of Tensor Difference and Constant

The secure comparison of 2-norm of tensor difference and constant protocol using the collaborative cloud model (SCNC) assumes that the cloud server holds two encrypted tensors \([A], [B] \in \mathbb{R}^{U \times T \times L}\) and constant \(c\), and the trusted hardware component holds private key \(sk\) and \(c\). The goal is to collaboratively and securely compare 2-norm of tensor difference with constant. If \(\|A - B\|_2^F < c\), then the protocol outputs 1; otherwise, 0. Algorithm 3 shows the proposed method and the protocol is written as follows: \(SCNC([A], [B], c) \rightarrow (\|A - B\|_2^F < c)\).

Algorithm 3 Secure Comparison of 2-Norm of Tensor Difference and Constant (SCNC)

Input: Cloud Server: Two encrypted tensors \([A], [B] \in \mathbb{R}^{U \times T \times L}\), and constant \(c\). Trusted hardware component: Private key \(sk\) and constant \(c\).

Output: Comparison result \(r\), where \(r = (\|A - B\|_2^F < c)\).

1: for \(1 \leq u \leq U, 1 \leq t \leq T, 1 \leq l \leq L\) do
2: Cloud Server: \([d_{ult}]\) \([a_{ult}]\) \([b_{ult}]\) \(n\)
3: Cloud Server and Trusted Hardware Component:
4: \([d^2_{ult}] = SM([d_{ult}], [d_{ult}])\)
5: end for
6: Cloud Server: \(\sqrt{\sum_{u=1}^{U} \sum_{t=1}^{T} \sum_{l=1}^{L} (D_{ult})^2}\) \(\|D\|_2^F\)
7: Cloud Server and Trusted Hardware Component:
8: \(r = SC([\|D\|_2^F], c)\)

The encrypted difference of the tensors \(A\) and \(B\) can be calculated using the Paillier properties in the cloud server in step 2. Let the difference tensor be \(D\). Next, the problem becomes how to compare the tensor 2-norm \(\|D\|_2^F\) with the constant \(c\). The 2-norm computing \(\|D\|_2^F = \sqrt{\sum_{u=1}^{U} \sum_{t=1}^{T} \sum_{l=1}^{L} (D_{ult})^2}\) includes square root operation. However, the square root operation over encrypted data is more complex and time-consuming. Fortunately, the comparison result of \(\|D\|_2^F\) and \(c\) equals to \((\|D\|_2^F)^2\) and \(c^2\). Therefore, we now need to compare \((\|D\|_2^F)^2\) with \(c^2\) over encrypted data, which removes the square root operation. The cloud server and the trusted hardware component collaboratively compute \(\sqrt{(\|D\|_2^F)^2}\) using the SM protocol and the Paillier addition property in steps 4 and 6.

4.5 Secure Acquisition of Principal Eigentensor for Data Users

After the cloud server obtains the encrypted principal eigentensor \([X]\), we need to develop a secure and efficient protocol which enables data users to obtain the principal eigentensor \(X\). The protocol does not allow the cloud server to learn anything about the tensor \(X\). The secure acquisition of principal eigentensor protocol using the collaborative cloud model (SA) is given in Algorithm 4. Formally, the protocol is defined as follows: \(SA([X]) \rightarrow X\).

Algorithm 4 Secure Acquisition of Principal Eigentensor for Data Users (SA)

Input: Cloud Server: Encrypted tensor \([X] \in \mathbb{R}^{U \times T \times L}\).

Trusted Hardware Component: Private key \(sk\).

Output: Tensor \(X\) only to data user.

1: for \(1 \leq u \leq U, 1 \leq t \leq T, 1 \leq l \leq L\) do
2: Cloud Server:
3: choose a random integer \(r_{ult}\)
4: \([r_{ult}]\) \(E_{pk}(r_{ult})\)
5: \([x'_{ult}]\) \(utl(x_{ult})\) \([r_{ult}]\)
6: send \([x'_{ult}]\) to trusted hardware component
7: send \(r_{ult}\) to data user
8: Trusted Hardware Component:
9: receive \([x'_{ult}]\)
10: \(x'_{ult} = D_{sk}([x'_{ult}])\)
11: send \(x'_{ult}\) to data user
12: Data User:
13: receive \(x'_{ult}\)
14: receive \(r_{ult}\)
15: \(x_{ult} = x'_{ult} - r_{ult}\)
16: end for

The basic idea of the SA protocol is that for any given integer \(x\), the equation \(x = x + r - r\), where \(r\) is a random number, holds. For the entry \([x_{ult}]\) \((1 \leq u \leq U, 1 \leq t \leq T, 1 \leq l \leq L)\) in the tensor \([X]\), the protocol proceeds as follows:

The cloud server first picks a random integer \(r_{ult}\) and computes \([x'_{ult}] = [x_{ult}] [r_{ult}]\) to distort the value \(x_{ult}\), and then sends \([x'_{ult}]\) and \(r_{ult}\) to the trusted hardware component and the data users, respectively in steps 2 to 7.
5 An Efficient SPEC Scheme

This section uses the collaborative cloud model to propose an efficient SPEC scheme based on homomorphic computing and garbled circuit. In other words, the scheme takes advantage of homomorphic computing and garbled circuit, as well as employing packing technology to improve its efficiency. This efficient SPEC scheme improves the SMMP protocol, the STCD protocol, the SCNC protocol and the SA protocol of the basic SPEC scheme.

5.1 Secure Packing-Based Multilinear Mode Product

The SMMP protocol requires a number of (time-consuming) operations in the encrypted-domain. To improve the SMMP protocol’s efficiency, we propose a packing-based SMMP protocol using the collaborative cloud model described in Algorithm 5.

In order to explain the packing-based SMMP protocol, we will now outline the packing technology. Assume Alice wishes to send $K$ integers $a_1, a_2, \ldots, a_K$, whose digits are less than or equal to $w$, to Bob. Alice first adopts the following formula to pack the integers:

$$ a = \text{Pack}(a_K | \cdots | a_2 | a_1) = K \sum_{k=1}^{w} a_k(k-1), \quad (1) $$

Then, Alice encrypts the packed value $[a] = E_{pk}(a)$, and sends the encrypted value $[a]$ to Bob. Bob decrypts $[a]$ and obtains the packed value $a = D_{sk}([a])$. From the packed value $a$, Bob can easily obtain the $a_i (1 \leq i \leq K)$. The data unpacking is defined as:

$$ a_K | \cdots | a_2 | a_1 = \text{Unpack}(a). \quad (2) $$

In the remaining protocols, the number of values packed into one value is $K$, and for simplicity, we suppose $K = U \cdot T \cdot L$. To securely calculate the encrypted value $[[a]]_{\text{utlu}} = [a_{\text{utlu}}]_{\text{utlu}} b_{\text{utlu}}$ (1 ≤ $u \leq U_2, 1 \leq l \leq L_2$), the protocol proceeds as follows:

The cloud serverblind $a_{\text{utlu}}$ and $b_{\text{utlu}}$ for 1 ≤ $u \leq U, 1 \leq t \leq T, 1 \leq l \leq L$. The cloud server picks $2K$ integers $r_1, r_1', \ldots, r_{K}, r_{K}'$ randomly, and packs them into two integers $r$ and $r'$ using Equation 1 in steps 3 and 4. The cloud server encrypts the integers $a'$ and $b'$ in step 5. Then, the cloud server obtains the encrypted packed disturbed values $[a']$ and $[b']$, prior to unpacking the packed values using Equation 2, and obtain blinded values $a'_{\text{utlu}} (a'_{\text{utlu}} = a_{\text{utlu}} + r_{\text{utlu}})$ and $b'_{\text{utlu}} (b'_{\text{utlu}} = b_{\text{utlu}} + r_{\text{utlu}})$ (1 ≤ $u \leq U, 1 \leq t \leq T, 1 \leq l \leq L$) in steps 13 to 16. The trusted hardware component computes $[[a']_{\text{utlu}}]_{\text{utlu}} [b']_{\text{utlu}}$ and sends it to the cloud server for 1 ≤ $u \leq U, 1 \leq t \leq T, 1 \leq l \leq L$ in steps 17 to 20.

The cloud server then applies the property $a_{\text{utlu}} + r_{\text{utlu}} = a_{\text{utlu}} - r_{\text{utlu}} = a_{\text{utlu}} - (b_{\text{utlu}} + r_{\text{utlu}}) - a_{\text{utlu}} - b_{\text{utlu}} = a_{\text{utlu}} - b_{\text{utlu}} - r_{\text{utlu}} = a_{\text{utlu}} - b_{\text{utlu}}$, and the Pailler properties to obtain the ciphertext $[[a']_{\text{utlu}}]_{\text{utlu}}$, for 1 ≤ $u \leq U, 1 \leq t \leq T, 1 \leq l \leq L$ in steps 22 to 26.

5.2 Secure Packing-Based Tensor-Constant Division

In the STCD protocol, $UTL$ encryptions and decryptions are required. To increase efficiency, a packing-based STCD protocol using the collaborative cloud model is described in Algorithm 6. The protocol uses the packing technology to significantly reduce the time due to encryptions and decryptions.

In the packing-based STCD protocol, the cloud server initially chooses $K$ random integers $r_1, r_2, \ldots, r_K$, packs their $c$-multiple values $c \cdot r_1, c \cdot r_2, \ldots, c \cdot r_K$ applying

Algorithm 5 Secure Packing Based Multilinear Mode Product

Input: Cloud Server: Two encrypted tensors $[A] \in \mathbb{R}^{U_1 \times T_1 \times L_1 \times U_2 \times T_2 \times L_2}$ and $[B] \in \mathbb{R}^{U_1 \times T_1 \times L_1 \times U_2, \times T_2 \times L_2}$. Trusted Hardware Component: Private key $sk$.

Output: Encrypted tensor $[C]$ only to Cloud Server, where $C_{U_2 \times T_2 \times L_2} = A_{U_1 \times T_1 \times L_1} \times B_{U_1, \times T_1 \times L_1}$ and $[B] \in \mathbb{R}^{U_1 \times T_1 \times L_1}$.

1: for $1 \leq u \leq U_2, 1 \leq t \leq T_2, 1 \leq l \leq L_2$ do
2: Cloud Server:
3: choose $2K$ random integers $r_1, r_1', \ldots, r_K, r_K'$
4: $a' = \text{Pack}(r_K | \cdots | r_2 | r_1)$
5: $b' = \text{Pack}(r_K' | \cdots | r_2' | r_1')$
6: for $1 \leq u \leq U, 1 \leq t \leq T, 1 \leq l \leq L$ do
7: $k \leftarrow TL(u-1) + L(t-1) + l$
8: $[a'] \leftarrow [a'] \cdot [a_{\text{utlu}}]_{\text{utlu}}^{10^{w(k-1)}}$
9: $[b'] \leftarrow [b'] \cdot [b_{\text{utlu}}]_{\text{utlu}}^{10^{w(k-1)}}$
10: send $[a'], [b']$ to trusted hardware component
end for
11: Trusted Hardware Component:
12: receive $[a'], [b']$
13: $a' = D_{sk}([a'])$
14: $b' = D_{sk}([b'])$
15: $a_K | \cdots | a_2 | a_1 = \text{UnPack}(a')$
16: $b_K | \cdots | b_2 | b_1 = \text{UnPack}(b')$
17: for $1 \leq u \leq U, 1 \leq t \leq T, 1 \leq l \leq L$ do
18: $[a_{\text{utlu}}]_{\text{utlu}} = [a_{\text{utlu}}]_{\text{utlu}}^{10^{w(k-1)}}$
19: send $[a_{\text{utlu}}]_{\text{utlu}}$ to Cloud Server
end for
20: Cloud Server:
21: receive $UTL$ encrypted values $[a_{\text{utlu}}]_{\text{utlu}}$
22: for $1 \leq u \leq U, 1 \leq t \leq T, 1 \leq l \leq L$ do
23: $k \leftarrow TL(u-1) + L(t-1) + l$
24: $[a_{\text{utlu}}]_{\text{utlu}} = [a_{\text{utlu}}]_{\text{utlu}}^{10^{w(k-1)}}$
25: $[b_{\text{utlu}}]_{\text{utlu}} = [b_{\text{utlu}}]_{\text{utlu}}^{10^{w(k-1)}}$
26: end for
27: $[c_{\text{utlu}}]_{\text{utlu}} = \prod_{u, t, l} [a_{\text{utlu}}]_{\text{utlu}}$
28: end for
Algorithm 6 Secure Packing Based Tensor-Constant Division

Input: Cloud Server: Encrypted tensor $\mathbf{A} \in \mathbb{R}^{U \times T \times L}$ and constant $c$. Trusted Hardware Component: Private key $sk$ and constant $c$.
Output: Encrypted tensor $[\mathbf{B}]$ only to Cloud Server, where $\mathbf{B} = \mathbf{A}/c$.

1. Cloud Server:
2. choose $K$ random integers $r_1, r_2, \cdots, r_K$
3. $a' = \text{Pack}(c \cdot r_K \cdots c \cdot r_2 \cdot c \cdot r_1)$, $[a'] \leftarrow E_{pk}(a')$
4. for $1 \leq u \leq U, 1 \leq t \leq T, 1 \leq l \leq L$ do
5. $k \leftarrow TL(u-1) + L(t-1) + l$
6. $[a'] \leftarrow [a'] \cdot [a_{ult}]^{10^x(k-1)}$
7. send $[a']$ to trusted hardware component end for
8. Trusted Hardware Component:
9. receive $[a']$
10. $a' \leftarrow D_{sk}([a'])$, $a'_K | \cdots | a'_2 | a'_1 \rightarrow \text{UnPack}(a')$
11. for $1 \leq u \leq U, 1 \leq t \leq T, 1 \leq l \leq L$ do
12. $b_{ult}' \leftarrow a_{ult}/c$
13. send $[b_{ult}']$ to Cloud Server end for
14. Cloud Server:
15. receive UTL encrypted values $[b_{ult}']$
16. for $1 \leq u \leq U, 1 \leq t \leq T, 1 \leq l \leq L$ do
17. $[b_{ult}'] \leftarrow [b_{ult}'] - r_{ult}$ end for

Equation 1, and encrypts the packed value in steps 2 and 3. Then, the packed value is used to perturb the value $a_{ult}$, $(1 \leq t \leq T, 1 \leq l \leq L)$ in the encrypted-domain. The cloud server obtains the encrypted packed disturbed value $[a'] = [a_{ult} + cr_K \cdot \cdots \cdot a_{ult} + cr_2 \cdot a_{ult} + cr_1]$ and sends it to the trusted hardware component in steps 4 to 8.

The trusted hardware component then receives and decrypts $[a']$ using private key $sk$ to obtain the packed disturbed value. This is subsequently unpacked to obtain the distorted values $a_{ult} + cr_K, \cdots, a_{ult} + cr_2, a_{ult} + cr_1$ using Equation 2 in steps 10 to 11. The remaining steps of Algorithm 6 are similar to steps 10 to 16 of Algorithm 2.

5.3 Improved Secure Comparison of 2-Norm of Tensor Difference and Constant

Similarly, for the improved efficiency of the SCNC protocol, we use the packing technology to reduce both encryption and decryption times. Furthermore, we improve the SCNC protocol by replacing the underlying homomorphic encryption comparison method with the garbled circuit comparison method. The improved SCNC protocol using the collaborative cloud model is described in Algorithm 7.

To leverage the packing technology in the protocol, the cloud server first blinds $a_{ult}$ $(1 \leq u \leq U, 1 \leq t \leq T, 1 \leq l \leq L)$ with $K$ random integers to obtain the encrypted packed distorted value $[d'] = [d_{ult} + r_K \cdots d_{ult} + r_2 | d_{ult} + r_1]$ in steps 2-8. Upon receiving the encrypted value $[d']$, the trusted hardware component decrypts the ciphertext and then unpacks the result to obtain the distorted values $d_{ult} + r_1, d_{ult} + r_1, \cdots, d_{ult} + r_K$ in steps 11 and 12. The trusted hardware component encrypts the value $(d_{ult} + r_{ult})^2$ and sends the result $[(d_{ult}^2)]$ to the cloud server, for $1 \leq u \leq U, 1 \leq t \leq T, 1 \leq l \leq L$ in steps 13 to 16. After this, using the property $(d_{ult})^2 = (d_{ult} + r_{ult})^2 - 2d_{ult}r_{ult} - (r_{ult})^2$ and the Paillier properties, the cloud server computes $[(d_{ult}^2)]$ for $1 \leq u \leq U, 1 \leq t \leq T, 1 \leq l \leq L$ in steps 19 to 21.

Algorithm 7 Improved Secure Comparison of 2-Norm of Tensor Difference and Constant

Input: Cloud Server: Two encrypted tensors $[\mathbf{A}] \in \mathbb{R}^{U \times T \times L}$ and $[\mathbf{B}] \in \mathbb{R}^{U \times T \times L}$ and constant $c$. Trusted Hardware Component: Private key $sk$ and constant $c$.
Output: Comparison result $r$, where $r = \left( \frac{\|A - B\|_2}{c} \right)^{\frac{1}{2}} < c$.

1. Cloud Server:
2. choose $K$ random integers $r_1, r_2, \cdots, r_K$
3. $d' \leftarrow \text{Pack}(r_K \cdots r_2 | r_1)$, $[d'] \leftarrow E_{pk}(d')$
4. for $1 \leq u \leq U, 1 \leq t \leq T, 1 \leq l \leq L$ do
5. $k \leftarrow TL(u-1) + L(t-1) + l$
6. $[d_{ult}] \leftarrow [a_{ult}] \cdot [b_{ult}]^{10^x(k-1)}$
7. $[d'] \leftarrow [d'] \cdot [d_{ult}]^{10^x(k-1)}$
8. end for
9. send $[d']$ to Trusted Hardware Component
10. Trusted Hardware Component:
11. receive $[d']$
12. $d \leftarrow D_{sk}([d']), d_K | \cdots | d_2 | d_1' \rightarrow \text{UnPack}(x)$
13. for $1 \leq u \leq U, 1 \leq t \leq T, 1 \leq l \leq L$ do
14. $[(d'_{ult})^2] \leftarrow E_{pk}(d_{ult} \cdot d_{ult})$
15. send $[(d'_{ult})^2]$ to Cloud Server end for
16. Cloud Server:
17. receive UTL encrypted values $[(d'_{ult})^2]$
18. for $1 \leq u \leq U, 1 \leq t \leq T, 1 \leq l \leq L$ do
19. $k \leftarrow TL(u-1) + L(t-1) + l$
20. $[(d^2_{ult})] \leftarrow [(d'_{ult})^2] \cdot \left( [(d_{ult})^2]^{N-r_{ult}} \right)^2 \cdot \left( [(r_{ult})^2]^{N-1} \right)$
21. end for
22. $[(\|A - B\|_2)^2] \leftarrow \prod [(d^2_{ult})]$\text{to Cloud Server}$
23. Cloud Server and Trusted Hardware Component:
24. $r \leftarrow \text{SCGC}([(\|A - B\|_2)^2], c)$

Fig. 2. Garbled circuits based comparison method.

After step 23, the cloud server holding the ciphertext $[(\|D\|_2^2)^2]$ and the constant $c$ securely compares $\|D\|_2^2$ and $c$ with the assistance of the trusted hardware component
holding the private key $sk$. To improve efficiency, step 25 employs the secure comparison method based on garbled circuits (SCGC)—see Fig. 2. For simplicity, we assume $[a] = \left\lfloor \left(\|D\|_2^2 \right)^2 \right\rfloor$ and $b = a^2$. The SCGC method needs one addition garbled circuit and one comparison garbled circuit.

In the preparation stage, the trusted hardware component generates one garbled circuit. The circuit consists of one addition garbled circuit and one comparison garbled circuit, where the output of addition circuit serves as the input of the comparison circuit. The trusted hardware component sends the garbled table, and the random keys ($k_0$ and $k_1$) of the output wire to the cloud server.

First, the cloud server picks a random integer $r$ and encrypts it. The cloud server calculates $[a + r] = [a][r]$ to generate the ciphertext $[a + r]$ of the blinded value of $a$, and sends the ciphertext to the trusted hardware component.

The trusted hardware component receives and decrypts $[a + r]$ to obtain the blinded value $a + r$.

Next, the trusted hardware component sends the random keys corresponding to $a + r$ to the cloud server. The cloud server obliviously acquires the random keys related to $r$. Then, the cloud server decrypts the garbled table of the addition circuit to obtain the random key corresponding to $a$ using these random keys.

After this, the trusted hardware component sends the random keys related to $b$ to the cloud server. The cloud server decrypts the garbled table of the comparison circuit using these random keys corresponding to $a$ and $b$ to obtain the random key $k$ related to the comparison result.

Finally, the cloud server outputs the comparison result depending on whether $k$ equals $k_0$ or $k_1$.

5.4 Secure Packing-Based Acquisition of Principal Eigentensor for Data Users

$UTL$ encryptions and decryptions are required in the process of the SA protocol. In a similar fashion, the packing technology is employed to improve efficiency of the SA protocol. The proposed packing-based SA protocol using the collaborative cloud model is described in Algorithm 8.

$K$ random numbers in $\mathbb{Z}_n$ are selected and packed into a value $x'$ using Equation 1 in the cloud server, and $x'$ is encrypted in steps 2 and 3. Then, the ciphertext $[x'] = [x_{UTL} + r_K] \cdots [x_{112} + r_2][x_{111} + r_1]$ is generated and sent to the trusted hardware component in steps 4 to 8.

Upon receiving from the cloud server, the ciphertext $[x']$ is decrypted. And the trusted hardware component unpacks the result to obtain $x_{111} + r_1, x_{112} + r_2, x_{UTL} + r_K$ in steps 10 and 11. The rest is the same as Algorithm 4.

6 Security and Performance Evaluations

In this section, we evaluate the security and performance of the SPEC schemes. As this is the first effort to develop secure principal eigentensor computing schemes in cloud, we are not able to benchmark our schemes against another similar scheme. Hence, the performance of our SPEC schemes is evaluated under different parameter settings.

Algorithm 8 Secure Packing Based Acquisition of Principal Eigentensor for Data User

Input: Cloud Server: Encrypted tensor $[X] \in \mathbb{R}^{U \times T \times L}$. Trusted Hardware Component: Private key $sk$.

Output: Tensor $X$ only to data user.

1: Cloud Server:
2: choose $K$ random integers $r_1, r_2, \cdots, r_K$
3: $x' = Pack(r_K, \cdots, r_2, r_1)$, $[x'] \leftarrow E_{pk}(x')$
4: for $1 \leq u \leq U, 1 \leq t \leq T, 1 \leq l \leq L$ do
5: $k \leftarrow T_{UL}(u - 1) + L(t - 1) + l$
6: $[x'] \leftarrow [x'] \cdot [x_{UTL}]^{10^{(w-1)}}$
7: send $[x']$ to Trusted Hardware Component and $r_1, r_2, \cdots, r_K$ to data user
end for
8: Trusted Hardware Component:
9: receive $[x']$
10: $x' \leftarrow D_{sk}([x'])$
11: for $1 \leq u \leq U, 1 \leq t \leq T, 1 \leq l \leq L$ do
12: send $x'_{UTL}$ to data user
end for
13: Data User:
14: receive $UTL$ values $x'_{UTL}$ and $r_{UTL}$
15: for $1 \leq u \leq U, 1 \leq t \leq T, 1 \leq l \leq L$ do
16: $x' \leftarrow x'_{UTL} - r_{UTL}$
end for

6.1 Security Analysis

The security analysis of the SPEC schemes is shown under the semi-trusted model. The adversaries are the data owners, the cloud server, the trusted hardware component and the data users. The data owner can gain access to his/her own data, and not any other data owners’ data. With the exception of receiving data from the clouds at the conclusion of the (packing-based) SPA protocol, the data users do not participate in any computations of the SPEC schemes. Therefore, our SPEC schemes protect the data owners’ privacy from the data owners and the data users. The cloud server only obtains the ciphertexts of the data owners’ data, the ciphertexts of the intermediate results, and the ciphertexts of the final principal eigentensor $[X]$. The ciphertexts are generated by the Paillier cryptosystem with semantic security. Hence, our SPEC schemes are semantically secure and preserve the data owners’ privacy from the cloud server. Although the trusted hardware component holds the private key $sk$, it only obtains some blinded values or random values. Thus, our SPEC schemes also preserve the data owners’ privacy from the trusted hardware component.

6.2 Theoretical Evaluations

6.2.1 Accuracy

As previously discussed in Section 4.1, the accuracy of the (efficient) SPEC scheme is affected by the scaling factors. When the scaling factors increase, the accuracy is improved. Once the scaling factors are sufficiently large, the accuracy of the (efficient) SPEC scheme using the collaborative cloud model will be the same as that of the method in the plain-domain, and for the decrypted results of the encrypted
values in the (efficient) SPEC scheme will equal the corresponding times of the values in the plain-domain.

6.2.2 Data Owners Cost and Data Users Cost
After outsourcing the encrypted data to the cloud server, the data owners do not take part in the calculations in the (efficient) SPEC scheme. Consequently, our schemes result in significant savings, in terms of computation and communication resources, to the data owners.

From Algorithm 4 or 8, we determine that data users only need $UTL$ additions in the plain-domain using our (efficient) SPEC scheme. In contrast, the computation complexity for the data users is $O(U_{2}T_{2}L_{2}UTL)$ in each iteration if the computation is undertaken using multilinear mode product power algorithm on the data user’s end without considering the privacy concerns. Therefore, for data users, significant computation time reduction is achieved using our (efficient) SPEC scheme.

Thus, we have demonstrated that the SPEC scheme using the collaborative cloud model is lightweight scheme for data owners and data users, and is suitable for deployment on resource-constrained devices such as RFID or mobile devices.

6.2.3 Time Complexity
The time cost of the SPEC scheme using the collaborative cloud model is generated by the (packing-based) SMMP protocol, the STA protocol, the (packing-based) STCD protocol, the (improved) SCNC protocol and the (packing-based) SA protocol. The STCD protocol only needs $UTL$ multiplications over encrypted data. The time complexity of the other building block protocols is summarized in Table 1.

<table>
<thead>
<tr>
<th>Protocol</th>
<th>Encryption</th>
<th>Decryption</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMMP</td>
<td>$4U_{2}T_{2}L_{2}UTL$</td>
<td>$2U_{2}T_{2}L_{2}UTL$</td>
</tr>
<tr>
<td>Packing SMMP</td>
<td>$2U_{2}T_{2}L_{2}[(UTL/K)+UTL]$</td>
<td>$2U_{2}T_{2}L_{2}[UTL/K]$</td>
</tr>
<tr>
<td>STCD</td>
<td>$3UTL$</td>
<td>$UTL$</td>
</tr>
<tr>
<td>Packing STCD</td>
<td>$[UTL/K]+2UTL$</td>
<td>$[UTL/K]$</td>
</tr>
<tr>
<td>SCNC</td>
<td>$3UTL$</td>
<td>$UTL$</td>
</tr>
<tr>
<td>Improved SCNC</td>
<td>$[UTL/K]+2UTL$</td>
<td>$[UTL/K]$</td>
</tr>
<tr>
<td>SA</td>
<td>$UTL$</td>
<td>$UTL$</td>
</tr>
<tr>
<td>Packing SA</td>
<td>$[UTL/K]$</td>
<td>$[UTL/K]$</td>
</tr>
</tbody>
</table>

(Packing-Based) SMMP Protocol: The SMMP protocol needs to execute the SM protocol $U_{2}T_{2}L_{2}UTL$ times. The SM protocol contains 4 encryption operations and 2 decryption operations when plaintexts of two multipliers are not equal. Therefore, the SMMP protocol requires $4U_{2}T_{2}L_{2}UTL$ encryption operations and $2U_{2}T_{2}L_{2}UTL$ decryption operations. The packing-based SMMP protocol requires $2[UTL/K]$ times of packing, which generates $2[UTL/K]$ encryption operations and $2[UTL/K]$ decryption operations in steps 5 and 14 of Algorithm 5. In addition, steps 18 and 24 perform $2U_{2}T_{2}L_{2}UTL$ encryption operations. Consequently, the packing-based SMMP protocol executes $2U_{2}T_{2}L_{2}[(UTL/K)+UTL]$ encryption operations and $2U_{2}T_{2}L_{2}[UTL/K]$ decryption operations in total.

(Packing-Based) STCD Protocol: The STCD protocol needs 3 encryptions in steps 4, 11 and 15, and 1 decryption in step 9 in each iteration of Algorithm 2. So, the STCD protocol executes $3UTL$ encryptions and $UTL$ decryptions overall. In contrast, the packing-based STCD protocol contains $[UTL/K]$ encryptions and decryptions in steps 3 and 11, which are generated by the packing technology, and $2UTL$ encryptions in steps 14 and 19 of Algorithm 6. Hence, the STCD protocol requires $[UTL/K]+2UTL$ encryptions and $[UTL/K]$ decryptions.

(Improved) SCNC Protocol: Since the SM protocol contains 3 encryptions and 1 decryption when plaintexts of two multipliers are equal. Therefore, $3UTL$ encryptions and $UTL$ decryptions are required in step 4 of Algorithm 3. In Algorithm 7, due to the use of the packing technology, $[UTL/K]$ encryptions and decryptions are required. Besides, $2UTL$ encryptions are required in steps 14 and 21.

(Packing-Based) SA Protocol: Clearly, $UTL$ encryptions and decryptions are required in steps 4 and 10 of Algorithm 4; while $[UTL/K]$ encryptions and decryptions are required in steps 3 and 11 of Algorithm 8.

6.3 Experimental Evaluations
We now present the evaluations based on synthetic datasets. We implemented the basic SPEC scheme, the efficient SPEC scheme, and the conventional principal eigentensor computing without considering privacy in JAVA. The experiments were executed on computers with Intel(R) Core(TM) i5-3470 CPU, and 16GB RAM. In the experiments, the Paillier cryptosystem and garbled circuits based on FasterGC [19] were adopted. The key size of the Paillier cryptosystem $s$ was set to either 512 bits or 1024 bits long.

In order to evaluate the effectiveness of the (efficient) SPEC scheme, we use the accuracy indicator as the metric defined as: $\rho = 1 - ||X - X'||/||X'||$, where $X$ denotes the principal eigentensor computed by the (efficient) SPEC scheme, and $X'$ represents the principal eigentensor directly calculated by multilinear mode product power algorithm without considering privacy.

The accuracy of the (efficient) SPEC scheme is affected by the scaling factors $r_1$, $r_2$ and $r_3$ while that of the principal eigentensor computing scheme without any privacy consideration is not. $\alpha$ in Algorithm 1 has one or two decimal places, so hereafter we set $r_1 = 100$. $r_2$ and $r_3$ depend on the number $UTL$ of elements in tensor $X$, so we set $r_2 = \sigma(UTL)^{2}$ and $r_3 = UTU$, where the scaling factors $\sigma$ are set to $10^2$, $10^3$, $10^4$, $10^5$ and $10^6$ in the experiments.

![Accuracy of proposed (ef- SPEC scheme for varying ditional scheme for varying tensor scaling factors](image1.png)

![Time of data user using (efficient) SPEC scheme](image2.png)
Fig. 3 illustrates the accuracy of the proposed (packing-based) SPEC scheme for the varying scaling factors. From Fig. 3, it can be observed that the scaling factor plays a crucial part in improving accuracy. And accuracy is low if the scaling factor $\sigma$ is less than $10^3$. However, accuracy is almost 100% if the scaling factor is greater than $10^4$. This illustrates that the proposed (efficient) SPEC scheme ensures the principal eigentensor computing to function and demonstrates the effectiveness of our schemes.

To determine the performance gain for the data user using our proposed (efficient) SPEC scheme, we evaluated the time required in computing principal eigentensor using multilinear mode product power algorithm without any privacy consideration only by the data user (known as traditional scheme), and the time of the data user taken by computing principal eigentensor using our schemes with privacy consideration. The time required using the traditional scheme is depicted in Fig. 4 for varying tensor sizes. We find that the time rapidly increases with tensor size. The time is more than 10 minutes if the tensor size is larger than 4000. At the same time, the time of the data user using our schemes is less than 1 millisecond. The findings demonstrate the efficiency of our (efficient) scheme on the data user’s side.

![Fig. 4](image)

(a) SMMP protocol and packing-based SMMP protocol
(b) STCD protocol and packing-based STCD protocol
(c) SCNC protocol and improved SA protocol
(d) SA protocol and packing-based SA protocol

**Fig. 5. Time costs of the basic protocols and the improved protocols**

Efficient SPEC has the time costs for our basic SPEC scheme and efficient SPEC scheme.

Finally, we analyzed the computation costs of the proposed basic SPEC scheme and efficient SPEC scheme by varying the mode length product $UTL$ of the tensor. In our experiments, we supposed the scaling factor $\sigma = 10^5$. Based on the key size $s = 512$, we evaluated the computation costs for our basic SPEC scheme and efficient SPEC scheme. From the findings illustrated in Fig. 6, we observe that the time costs of the basic scheme increase from 16 minutes to 371 minutes for $UTL$ changes from 400 to 2000, while those of the efficient scheme increase from 11 minutes to 252 minutes. In other words, the efficient SPEC scheme achieves a saving of 33.72% in comparison to the basic SPEC scheme.

Several computations of our schemes can be easily parallelized and, in practice, more cloud nodes can be employed. Therefore, the time of our schemes on cloud can be significantly reduced.

### 7 Related Work

This section reviews related work on secure cloud computing.

Some studies have focused on designing secure cloud models to ensure data security during computation. The fully homomorphic cryptosystem [20] based secure cloud model, for example, uses the fully homomorphic cryptosystem to address data security issues in cloud computing. Using the model, arbitrary functions can be securely computed over encrypted data in the cloud. Yi et al. [21] proposed a secure information retrieval approach, and a new secure searching approach on streaming data [22]. Similarly, Dong et al. [23] presented a private information retrieval protocol. However, due to the prohibitive performance overhead in existing fully homomorphic cryptosystems, the model is far from being practical particularly in a real-world deployment.
Another popular line of research uses partially homomorphic cryptosystem based secure cloud model to securely process data. In such an approach, the cloud offloads majority, or all, computations of the data users in applications such as secure facial expression recognition [24], secure back-propagation neural network learning [25] over unencrypted data in the cloud, and secure query processing [26], secure large-scale similarity search [27] and secure support vector machine [17] over encrypted data in the cloud.

However, our work is significantly different from these existing work. Our work combines the advantages of the partially homomorphic cryptosystem and the garbled circuits to perform data processing tasks without compromising on data security. Furthermore, using the model, we implemented the first secure and efficient principal eigentensor computation in the cloud.

8 Conclusion

With the increasing popularity of cloud computing, how to design secure computation is critical in sustainable cloud-assisted cyber-physical-social systems (CPSS). In this paper, we presented the first basic SPEC scheme and the first efficient SPEC for sustainable cloud-assisted cyber-physical-social systems, which can leverage cloud server and trusted hardware component to securely execute all jobs in principal eigentensor computation over encrypted data without the data owners’ participation while protecting users’ privacies. We demonstrated the security and utility of the schemes (i.e. the collaborative cloud model is suitable for cyber-physical-social systems big data processing on the cloud server since most of the tensor based computation can be conducted securely and efficiently). Our schemes substantially relieve the burden of data storage and computation on the users. Due to the lightweightness of our schemes, resource-constrained users can also deploy our schemes with ease.

References


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