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Published in:
International Journal of Geomechanics

DOI:
[10.1061/\(ASCE\)GM.1943-5622.0001341](https://doi.org/10.1061/(ASCE)GM.1943-5622.0001341)

Published: 01/02/2019

Document Version
Peer reviewed version

Please cite the original version:
Tran, Q., Villard, P., & Dias, D. (2019). Discrete and continuum numerical modeling of soil arching between piles. *International Journal of Geomechanics*, 19(2), [04018195]. [https://doi.org/10.1061/\(ASCE\)GM.1943-5622.0001341](https://doi.org/10.1061/(ASCE)GM.1943-5622.0001341)

DISCRETE AND CONTINUUM NUMERICAL MODELING OF SOIL ARCHING BETWEEN PILES

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ABSTRACT

The aim of the study involves investigating the soil-arching phenomenon between piles by adopting two different numerical methods. The first method is the discrete element method that allows analysis of the phenomena at the microscopic level in which soil is represented by an assembly of particles of various shapes that interact with friction at contact points. The method naturally captures the mechanical behavior of frictional materials. The second method is the finite difference method that examines the problem at the macroscopic level. Specifically, soils are considered homogeneous materials from the macroscopic viewpoint. In the continuum approach, an appropriate constitutive model that considers the shear resistance degradation is used to capture the evolution of frictional materials as obtained by using the discrete approach. The numerical micro-parameters and macro-parameters for the two numerical models are calibrated from triaxial test simulations. A comparison between these two approaches for the case of a soft soil improved by vertical piles is conducted. The study highlights the limitations and advantages of each method with respect to the soil-arching investigation.

KEYWORDS: Discrete element method; finite difference method; soil arching; piled embankment; granular soil; softening behavior.

a	Pile cap width	G^p	Plastic shear modulus
$y_{s \max}$	Maximum vertical displacement	K_s	Sub-grade reaction of the soft soil
d_s	Shearing ratio	K_p	Sub-grade reaction of the pile
e_o	Initial void ratio	s	Pile center spacing
h_c	Soft soil thickness layer	P	Load transferred to the pile caps
h_m	Embankment height	R_f	Failure ratio constant
h_ϕ	Softening modulus	β	Calibration factor
h^*	Limited embankment height for punching failure analysis	γ	Density of the embankment layer
k_n	Normal stiffness between two clusters	γ^p	Plastic shear strains
k_t	Tangent stiffness between two clusters	ϕ	Micro-friction angle
m	Stress dependence constant	ϕ_m	Mobilized friction angle
p'	Current mean stress	ϕ_{cs}	Critical state friction angle
p_{ref}	Reference mean stress	ϕ_p	Peak friction angle
D	Micro-angularity of the cluster	ψ_m	Mobilized dilatancy angle
E	Efficacy	δe	Increment void ratio
E^e	Current Young's modulus	$\delta \gamma^p$	Increment plastic shear strains
E_{ref}^e	Reference Young's modulus	$\delta \epsilon_v^p$	Increment plastic volumetric strains
E_{oed}	Oedometric modulus	ν	Poisson's ratio
E_{def}	Deformation elasticity modulus	μ	Coulomb friction coefficient

G^e Elastic shear modulus

θ Shearing angle

G_{ref}^e Reference elastic shear modulus

INTRODUCTION

Embankments constructed on soft compressible soils lead to significant settlements. The piled embankment system (Figure 1) is an effective technique minimizes surface settlements and reduces the construction time. In order to increase the efficiency of this technique, a granular layer made of a material with high shear resistance properties is laid out over the network of piles. The soil-arching mechanism in the embankment layer plays a crucial role in the effectiveness of the system (Hewlett and Randolph 1988, Jenck et al. 2005). However, the mechanism of soil arching is strongly affected by the height of the granular layer and is sophisticated and reflected in various interpretations.

The first approach deals with the load transfer mechanism and is a frictional model introduced by Terzaghi (1943) who defined soil arching as “the stress transfer from a yielding part to a stationary part”. The stress transfer is induced by the relative displacement between the two parts along the shear band. The disadvantages of this model involve not adopting the real shear band as observed in the trapdoor laboratory test for the sake of simplicity and the lack of accurately integrating the earth pressure coefficient value (ratio between the horizontal and vertical stresses).

Figure 1. Piled embankment system

The second approach corresponds to the fixed-arch models that were introduced by Carlsson (1987) and adopted in the Scandinavian design guidelines (Rogbeck et al. 2005). The approach assumes that a fix-shaped

arch is created over the compressible soil between two piles. The fix-shaped arch is friction-angle dependent. However, the original models do not consider the role of the aforementioned mechanical properties of the embankment. These disadvantages are overcome by several other modified models (Chevalier et al. 2010) that consider the influence of the friction angle.

The last approach uses equilibrium models that consider soil arching as the rearrangement of stress distribution. Therefore, the approach assumes a stress arch in which the principal stresses are radial and tangential with a maximum earth pressure coefficient based on the plastic theory. The pressure on the compressible soft soil is calculated by considering the equilibrium condition. The aforementioned models are commonly used in several European design guidelines although each country has different assumptions in terms of the principal stress directions as well as the stress distribution in the compressible soft soils (Hewlett and Randolph 1988; Zaeske 2001; Van Eekelen et al. 2013).

The soil-arching phenomenon was also investigated by means of numerical methods. The implementation of continuum techniques using macro parameters was implemented to explore the soil-arching mechanism and include the finite difference method (FDM) (Jenck et al. 2007) in two dimensions, (Jenck et al. 2009a; Almeida et al. 2011; Nunez et al. 2013; Briançon et al. 2015; Girout et al. 2014; Szajna W. 2015; Dias et al. 2015) in three dimensions or finite element method (FEM) (Hassen et al., 2009; Okayay et al., 2010; Van der Peet 2014). However, extant studies do not consider the shear resistance degradation of the embankment materials reflected by the softening behavior. Therefore, soil arching that significantly depends on the shearing behavior is not fully captured. An alternative numerical method, namely the discrete element method (DEM), uses micro-parameters and was also applied to examine the soil-arching mechanism such as simulations of trapdoor tests (Chevalier et al. 2008; Jenck et al. 2009b; Chevalier et al. 2012; Rui et al. 2016) or piled embankments with geotextiles that are not considered in this study (Le Hello and Villard 2009) and coupling between DEM and FEM (Villard et al. 2009). The micro-behavior captures various mechanisms of granular materials (a shear resistance degradation, a critical state) although the determination of micro-parameters is not straightforward and the computation cost is significantly high for large-scale simulations.

The present study compares the continuum approach using the FDM applied to softening frictional soils and the DEM using micro-parameters to simulate piled embankments. Three types of granular material assembly densities are considered (low, medium, and dense). The macro-parameters of the discrete numerical material are defined from the micro-parameters using triaxial test simulations. The investigation of the piled embankment reveals that the degradation of the shear strength can lead to reductions in the efficacy that should be considered in the design phases. With respect to a thin embankment, the embankment is governed by a punching failure mechanism that provides a solution to determine the maximum efficacy for the piled embankment.

PILED EMBANKMENT NUMERICAL MODELS

Piled embankment model

Figure 2. Symmetric area in the numerical models

s, piles caps, an embankment layer, and soft soils. The embankment mesh in which the distance between the two piles is 3 m and the width of the pile cap is 0.6 m ($a = 0.6$ m). Four different embankment heights are considered: 5 m, 3 m, and 1 m. The area ratio ($\alpha = a^2/s^2$) is equal to 4%. Only one half of the model is simulated with a symmetric boundary condition as shown in Figure 2. The soil is modeled as elastic materials for the sake of simplicity. Figure 3 and

Figure 4 present the schematic of the discrete and continuum numerical models, respectively. The study mainly focuses on the shearing mechanisms that can occur in the embankment.

Figure 3. Discrete numerical model of the embankment

With respect to the discrete model, the numerical generation for the embankment particles is based on the radius expansion with a friction decrease process (REDF - Chareyre and Villard 2005). The method allows the generation of an assembly of particles at a fixed porosity within a rectangular box. The four numerical embankments are 1.5 m wide and 0.75 m, 1.5 m, 2.25 m, and 3 m high. The total number of clusters are 16000, 32000, 48000, and 64000 clusters, respectively. Each cluster is assembled by two rigid overlapped spheres with diameter D . The angularity (distance between the centroid of two spheres) is $0.8D$, and the ratio of distributed diameters (representative diameters of clusters) is $D_{\max}/D_{\min} = 4$. The macro-mechanical properties of the granular materials are extracted from numerical triaxial tests with a set of micromechanical parameters.

Figure 4. Continuum numerical model of the embankment

In the continuum model, the embankment layer is modeled with the cap yield model enhanced with friction hardening and softening behavior. In order to consider the stress dependence modulus with depth, the Young's modulus is initially generated as a function of the effective mean stress as follows:

$$E^e = E_{ref}^e \left(\frac{p'}{p_{ref}} \right)^m \quad (1)$$

Soft soil and Pile models

The discrete and the continuum models employed different approaches to model the soft soil layers and the piles. In the continuum approach, the piles and soft soils were modeled by an elastic constitutive model with an elasticity modulus (E_{def}) and Poisson's ratio (ν). In the discrete approach, the piles and soft soil layers were modeled using elastic springs with sub-grade reaction moduli K_p and K_s , respectively. The sub-grade reaction modulus is determined by the oedometric modulus (E_{oed}). Given the different types of models, the sub-grade reaction modulus in the discrete model (K) and elasticity modulus (E_{def}) in the continuum model are correlated by using Poisson's ratio ($\nu = 0.3$) and the thickness of the soft soil layers (h_c) as follows:

$$K = \frac{E_{oed}}{h_c} \quad (2)$$

$$E_{oed} = \frac{E_{def}}{\beta} \quad (3)$$

$$\beta = 1 - \frac{2\nu^2}{1 - \nu} \quad (4)$$

In order to determine the influence of the soft soil stiffness in the load transfer, different values of E_{oed} for the soft soil are considered (ranging from 0.05 MPa to 1 MPa). Han (2002) examined the influence of the Young modulus of piles and indicated that the pile stiffness does not affect the settlement and load transfer if the pile stiffness is 1000 times the soft soil stiffness. Therefore, $E_{oed} = 2000$ MPa is selected to eliminate the effect of the pile stiffness in the study. The height of the pile and of the subsoil layer are 1 m ($h_c = 1$ m).

MICRO- AND MACRO-PARAMETERS CALIBRATION

To compare the two numerical models, it is necessary to calibrate the macro-parameters in the continuum model to the micro-parameters in the discrete model. Given the complexity of calibrating the DEM model, the discrete soil behavior is considered as a reference for the continuum modeling. The DEM numerical sets of parameters that are retained allow the restoration of the behavior of loose (L), medium (M), and dense (D) typical granular materials. In order to obtain similar behavior between the two numerical materials, triaxial tests on different numerical samples (L, M, and D) are simulated by using micro-parameters for the DEM and macro-parameters for the FDM. Different confining pressures (10 kPa, 20 kPa, and 30 kPa) are applied to capture the stress dependency behavior of the elastic modulus.

Micromechanical parameters used in the DEM model

The behavior of a granular assembly depends on the granular skeleton shape and the type of constitutive laws defined between particles. Each granular particle interacts with other particles by normal forces and tangential forces at contact points. In the study, the normal forces (f_n) are expressed as follows:

$$f_n = k_n h_{ij} \quad (5)$$

where h_{ij} denotes the overlap of the two particles and k_n is the normal contact stiffness. The tangential forces (f_t) are determined by a linear law by using a tangential stiffness (k_t) and a Coulomb shear failure criterion with a coefficient of friction ($\mu = \tan\phi$). The tangential contact forces at the contact points are calculated as follows:

$$\frac{df_t}{d\delta u_t} = k_t \text{ and } |f_t| \leq \mu f_n \quad (6)$$

where δu_t denotes the incremental tangential displacement. The normal stiffness of the contact is defined as the stiffness of spheres K_n [N.m⁻²] with r_i and r_j corresponding to the radii of the clusters as follows:

$$k_n = K_n \frac{r_i r_j}{r_i + r_j} \quad (7)$$

Table 1. Micro-parameters in the discrete model for granular materials

Samples	Numerical Porosity	K_n (MN/m ²)	k_n/k_t	ϕ
Loose (L)	0.41	10	1	40°
Medium (M)	0.38	10	1	40°
Dense (D)	0.34	10	1	40°

The initial porosity of the granular assembly and shape of the particles play a crucial role in the mechanical behavior of materials. Increasingly realistic shape grains and the mechanical behavior of granular materials are achieved by clustering particles (Kozicki et al. 2011). Therefore, a cluster composed of two spheres with

an angularity of 0.8D was used for the simulations. The numerical sample porosities and the selected micro-parameters retained are given in Table 1.

Table 2. Interpretation of macro-parameters from the discrete model

Sample	ϕ_p (degrees)	ϕ_{cs} (degrees)	m
Loose (L)	34 °	26 °	0.36
Medium (M)	40 °	26 °	0.36
Dense (D)	46 °	26 °	0.36

Figure 5. Pressure dependency for the initial Young Modulus E of the numerical granular material ($p_{ref} = 100$ kPa)

The macro-friction angles of the granular materials in the peak and critical state are deduced from triaxial tests. Three triaxial tests at different confining pressures ($\sigma_3 = 10$ kPa, 20 kPa, and 30 kPa) are used. At different isotropic pressures, the granular materials exhibit a pressure-dependent Young's modulus. The pressure-dependent parameter 'm' used in equation (1) is computed from the slope of the line in $\log(\sigma_3/p_{ref}) - \log(E'=(\sigma_1/\epsilon_1))$ plot from the numerical triaxial tests results as shown in Figure 5. The macro-friction angles of the numerical granular materials are deduced from the Mohr–Coulomb circles obtained for three different

confining pressures. Table 2 presents the macro-friction angles at both the peak and critical state and pressure-dependent parameters from the triaxial test.

Cap yield model with friction hardening and softening used in the continuum

The embankment materials that comprise dense, medium, and loose granular materials are modeled by using the cap yield model (CYsoil) in the finite difference code (FLAC3D). The cap yield model is a strain-hardening constitutive model characterized by a frictional Mohr–Coulomb envelope and a cap-hardening law. The stress-dependent modulus law on the cap yield model yields the hyperbolic behavior as well as friction hardening as described in the UBCSAND model (Byrne, 2003). The friction hardening for the shear strains $\gamma^p(\phi_m)$ at an effective confining pressure (p') in CYsoil model is expressed as follows:

$$\gamma^p(\phi_m) = \frac{p_{ref}}{\beta G_{ref}^e} \left(\frac{p'}{p_{ref}} \right)^{1-m} \frac{\sin\phi_p}{R_f} \left[\frac{1}{1 - \frac{\sin\phi_m}{\sin\phi_p} R_f} - 1 \right] \quad \text{for } \gamma^p(0) \leq \gamma^p(\phi_m) \leq \gamma^p(\phi_p) \quad (8)$$

where β denotes the calibration factor, R_f denotes the failure ratio constant, ϕ_p denotes the peak friction angle, G_{ref}^e denotes the reference elastic Young's modulus in the reference pressure p_{ref} , and m denotes the stress dependent constant.

In the continuum approach, the increment void ratio is typically used to control the softening behavior on the friction angle. The evolution of the shear band at the microscopic scale is reflected by the softening behavior at the macroscopic scale that induces an increase in the void ratio along the shear band. In order to capture this phenomenon, a simple linear relation with a linear softening modulus (h_ϕ) (Marcher 2001) is adopted in the cap yield model. The integration for the linear softening law is solved, and the friction softening for the shear strains $\gamma^p(\phi_m)$ at an effective confining pressure (p') in the CYsoil model is expressed as follows:

$$\gamma_2^p(\phi_m) = \gamma_1^p(\phi_p) + \frac{\phi_p - \phi_m}{h_\phi(1 + e_o)\sin\psi_m} \quad \text{for } \gamma^p(\phi_p) \leq \gamma^p(\phi_m) \leq \gamma^p(\phi_{cs}) \quad (9)$$

After reaching the peak friction angle, the soil skeleton may dilate under high shear strains if the soil is dense, and a dilation strain-hardening law is used in the cap yield model incorporating the Rowe dilatancy theory.

$$\sin\psi_m = \frac{\sin\phi_m - \sin\phi_{cs}}{1 - \sin\phi_m \sin\phi_{cs}} \quad (10)$$

Figure 6. Friction hardening and softening in cap yield model

Figure 6 illustrates the capacity of the CYsoil model to reproduce the softening behavior.

Macro-parameters are obtained from the interpretation of the DEM triaxial tests and include macro-friction angle, Young's modulus, pressure-dependent parameter, porosity, and volumetric weight. Other numerical parameters, such as softening parameters, were tested to obtain a similar numerical behavior corresponding to the one obtained with the DEM model. For example, the results of the 20 kPa confining pressure for the three modeled density states (L, M, and D) are shown in Figure 7 and Figure 8 using the optimal set of parameters

selected (see Table 3). These curves exhibit similar trends to real typical granular materials tested at different density states.

The results indicate a similar stress–strain behavior. However, the maximum positive volume changes in the discrete model slightly exceed that of the continuum model (approximately 1%). The discrete technique describes the shear dilatancy given the interlocking between particles, and the continuum technique imposes the Rowe stress–dilatancy theory to capture the dilatancy phenomenon. Given the elongated particle shapes, the increase in volume in the discrete model could exceed that in the continuum model.

Table 3. Macro-parameters in the continuum model

Sample	E_{ref}^e (MN/m ²)	p_{ref} (kPa)	ν	m	ϕ_p (degree)	ϕ_{cs} (degree)	R_f	β	h_ϕ (degree)	e_o	Volumetric weight (kg/m ³)
L	100	100	0.2	0.36	34°	26°	0.9	0.12	250	0.7	1470
M	120	100	0.2	0.36	40°	26°	0.9	0.12	250	0.61	1550
D	160	100	0.2	0.36	46°	26°	0.9	0.12	250	0.51	1650

Figure 7. Stress–strain response for the Loose, Medium, and Dense granular materials

Figure 8. Volumetric response for the Loose, Medium, and Dense granular materials

PARAMETRIC STUDY AND RESULTS

The discrete approach and continuum model that uses the CY soil constitutive law provide a precise analysis of the impact of geotechnical parameters. In the study, the variation in the embankment height, the soft soil stiffness and the granular material porosity are considered to investigate the soil-arching mechanisms. In order to compare the discrete and continuum models, the medium density material (M) was first considered with embankment heights corresponding to 0.75 m, 1.5 m, 2.25 m, and 3 m and for different subsoil stiffness. The comparison focuses on the evaluation of the efficacy change, stress distribution of the soft soil, and settlements in the embankment.

The efficacy E is defined as the proportion of the embankment weight carried by the pile caps. It is expressed as the proportion of the total embankment weight that is transferred to the pile caps as follows:

$$E = \frac{P}{s^2 \gamma h_m} \quad (11)$$

where P denotes the total force carried by the pile cap, s denotes the pile center spacing, γ denotes the unit weight of embankment materials, and h_m denotes the embankment height.

The shearing ratio of the embankment is denoted as d_s and is determined as the ratio between the maximum vertical displacement of the granular embankment ($y_{s \max}$) and the embankment height as follows:

$$d_s = \frac{y_{s \max}}{h_m} \quad (12)$$

With respect to each numerical simulation, the weight of the granular material progressively increases until the right granular mass is obtained to simulate a progressive subsoil settlement. At the end of the numerical process, the efficacy and the maximal vertical displacement of the granular embankment are noted.

230 **General trends obtained for different embankment heights and subsoil stiffness**

231 **Figure 9. Efficacy versus subsoil stiffness for the material density M**

232 Initially, comparisons between the two numerical models were performed by considering the efficacy rela-
 233 tive to the soft soil stiffness (K_s). Figure 9 shows typical curves of the efficacy relative to the subsoil stiffness
 234 obtained for various heights of the embankment with FDM and DEM. With respect to each tested case, an
 235 optimum value of the efficacy is determined as a function of the subsoil stiffness. With respect to high values
 236 of the subsoil stiffness, the load resulting from the weight of the granular embankment is mainly supported by
 237 the subsoil because the subsoil is sufficiently stiff to support the applied load. When the stiffness of the subsoil
 238 decreases, the load transmitted to the piles increases until a maximal value is reached at which the piles begin
 239 to punch the granular embankment. During the process, shear bands are formed and developed inside the
 240 granular layer. After reaching the maximal value of the efficacy, the efficacy reduces significantly with de-
 241 creases in the subsoil stiffness. The reduction in efficacy results from the softening behavior of the granular
 242 materials. Progressive failure occurs during the development of the shear bands inside the granular layer. The
 243 stress mobilized in shear bands reduces owing to the degradation in the shear strength, and this reduces the
 244 forces that are transferred to the piles.

Figure 10. Efficacy relative to the shearing ratio for the material density M

Similar trends are observed while investigating the efficacy relative to the shearing ratio (see Figure 10). High values of subsoil stiffness decrease the efficacy and shearing ratio. For cases with low displacement, a decrease in the subsoil stiffness increases the efficacy as well as the vertical displacements (and thereby the shearing ratio). These trends are observed both for the FDM and DEM despite the disparity obtained between the two numerical models. For cases with high displacement when the shearing ratio approximately exceeds 0.2 (corresponding to low values of the subsoil thickness), we observe (Figure 10) that the granular material can move freely (i.e., the influence of the pile cap boundary condition is absent), and thus the embankment begins to slide in the DEM. At this stage, a significant reduction in the load transfer efficacy is observed in the discrete model although only a slight reduction is observed in the continuum model. Consequently, given a high displacement, the continuum model predicts lower stress distributions on the subsoil relative to the DEM model and lower settlements for low subsoil stiffness.

Figure 11. Stress distribution on the pile cap ($h_m=1.5$ m and $E_{oed} = 0.2$ MPa)

Figure 12. Stress distribution on the soft soil ($h_m=1.5$ m and $E_{oed} = 0.2$ MPa)

With respect to the reference case ($h_m=1.5$ m and $E_{oed} = 0.2$ MPa), the efficacies of the load transfers equal 0.28 and 0.16 for the DEM and FDM, respectively. Therefore, the displacements of the subsoil and vertical load acting on the supporting soil in FDM are lower than those of the DEM. To investigate the load transfer within the granular embankment, the stress distribution on the pile is shown in Figure 11. In DEM, the stress is calculated by dividing the granular embankment forces acting on the elastic springs with the representative areas of the springs. In FDM, the stress is directly extracted from the interface between the soft soil and embankment layer. Generally, the stress observed in the discrete model gradually increases from the center of the pile to the corner. Conversely, a high stress concentration is observed in the corner of the pile in the continuum model albeit not in the discrete model. In the FDM, the displacement field is restricted by the

268 compatibility condition. Therefore, as opposed to sliding relatively as observed in the DEM, the FDM prevents
 269 the embankment material from sliding at the corner of the pile. When the pile punches the granular soil, it
 270 induces a high stress concentration. The initial analysis reveals that the corner of the pile cap represents the
 271 singularity for the FDM. The singularity introduces a force concentration that is not observed with DEM.
 272 Therefore, DEM allows a better force distribution owing to its discrete nature. Figure 12 illustrates the vertical
 273 stresses of the embankment weight on the soft soil for the reference case ($h_m=1.5$ m and $E_{oed} = 0.2$ MPa). As
 274 shown in the figure, the stress on the soft soil is almost equally distributed at 20 kPa for the continuum model
 275 and 18 kPa for the discrete model (a value of 23.25 kPa is obtained without piles). With the FDM model, it is
 276 noted that a peak on the stress distribution near the piles occurs owing to similar mechanisms that can affect
 277 the load distribution over the piles and potentially affect the efficacy value.

278 **Figure 13. Settlement of FDM and DEM simulations ($h_m=1.5$ m and $E_{oed} = 0.2$ MPa)**

279 With respect to the reference case, the higher value of stress distribution on the soft soil that is observed in
 280 the continuum model also results in a settlement exceeding that the discrete model as shown in Figure 13. In
 281 the discrete model, the settlement is calculated from the displacement of the subsoil layers. Given the high
 282 fluctuation in the subsoil's elevation, the settlement is considered as the average value of the subsoil layer
 283 displacement. In the continuum model, the settlement is extracted from the interface between the soft layer
 284 and embankment layer. The performed numerical simulations indicate that the stiffness of the interface has a
 285 very low effect on the value and distribution of the stress as well as the settlement of the embankment layer.

286 The pile settlement is negligible owing to the high stiffness of the pile. With respect to the discrete model, the
 287 settlement under the embankment center is approximately 0.09 m while the settlement in the continuum model
 288 is approximately 0.1 m for the reference case. The stress in the continuum model exceeds that in the discrete
 289 model, and thus the settlement in the continuum model exceeds that in the discrete model.

290 **Figure 14. Contour displacement of the discrete model (left) and continuum model (right)**

291 **($h_m=1.5$ m and $E_{oed} = 0.2$ MPa)**

292 Figure 14 illustrates the displacement field of the discrete model (a) and continuum model (b). A similar
 293 displacement field is observed for these two models. A high vertical displacement is observed in the middle
 294 of the two piles and above the soft soil layer. The displacements significantly reduce to zero near the top of
 295 the pile. The propagation of the displacement field exhibits a punching failure mechanism in both cases. When
 296 the pile begins to penetrate the granular layer, the efficacy of the material begins to reduce with increases in
 297 the displacement.

298 Based on the equilibrium models, the load transfer mechanism within the granular embankments is observed
 299 as a change in the orientation of the main stresses. In order to evaluate the changes in the load transfer mech-
 300 anisms, different cases are presented in Figure 15 and Figure 16. The principal stresses within the granular
 301 embankments between two piles are as follows: Case (a) involves an extremely soft soil ($K_s=0.05$ MPa) and

indicates a strong punching of the granular embankment by the pile; case (b) corresponds to the maximal efficacy obtained in DEM ($K_s=0.2$ MPa), and case (c) corresponds to a relatively rigid subsoil ($K_s=1$ MPa).

With respect to the DEM, the stress tensor within a volume V of particles is calculated using Eq.(13) (Weber, 1966) by considering the contact forces defined at all contact points included in volume V . Specifically, N_c denotes the number of contact points in V , f_i denotes the projection of the contact force f on the i -axis, and l_j denotes the projection of the branch vector l on the j -axis with $i = x, y, z$ and $j = x, y, z$. The branch vector l is defined by the vector linking the centers of the clumps in contact as follows:

$$\sigma_{ij} = \frac{1}{V} \sum_{\alpha=1}^{N_c} f_{\alpha}^i l_{\alpha}^j \quad (13)$$

Figure 15. Principal stresses (Discrete element method) within the granular embankment for different values of the subsoil stiffness: (a) $K_s = 0.05$ MPa, (b) $K_s = 0.2$ MPa and (c) $K_s = 1$ MPa. Values computed for a thin slice of grains between two piles.

Figure 16. Principal stresses (Finite difference method) within the granular embankment for different values of the subsoil stiffness: (a) $K_s = 0.05$ MPa, (b) $K_s = 0.2$ MPa and (c) $K_s = 1$ MPa. Values computed between two piles for the initial FDM mesh.

In the discrete model, the load transfer mechanism occurs for extremely low displacements of the subsoil ($K_s = 1$ MPa) and reaches a maximum value of the subsoil stiffness ($K_s = 0.2$ MPa). In the case of extremely soft soil, the punching mechanism induces an extremely high displacement of the subsoil, the sliding of the granular material around the pile (for DEM), and a very disturbed load transfer mechanism (both for DEM and FDM). An advantage of DEM involves detecting the aforementioned behavior in extremely high displacements.

In the example, the punching failure is noted in both the continuum and discrete models. However, the discrepancy between discrete model and continuum model is observed when a relative sliding (large displacement) occurs between the granular material and stiff pile. The compatibility condition in the continuum modeling prevents the sliding, and thus a very high stress concentration is observed in the corner of the pile as the displacement is prevented. Conversely, the discrete model allows the granular embankment to slide along the pile shaft, and stress concentration in the corner of the pile is absent. This is the main difference that explains part of the difference in the stress distribution and settlements on the soft soil.

Effect of the porosity state of materials

Figure 17. Maximum efficacy relative to the peak friction angle

The porosity of the material reflects the initial density state of the embankment material and its mechanical behavior (especially the peak friction angle). The numerical triaxial test simulations indicated that the peak friction angle increases with decreases in the porosity with the peak friction angle $\phi_p = 34^\circ$, 40° , and 46° for loose (L), medium (M), and dense (D) granular materials, respectively. Figure 17 shows the influence of the peak friction angle on the maximum efficacy. A higher value of the maximum efficacy is observed if the granular embankment materials are denser (higher peak friction angle). With respect to a denser material (D), the granular materials exhibit a strong shearing resistance, behave in a dilative manner, and induce a high degradation in the shear strength under shearing (Figure 18) based on the triaxial test behavior. In contrast,

with respect to the looser material (L), the granular materials exhibit poor shearing resistance and behave in a less dilative manner. A significant softening behavior is not observed for these materials (Figure 19). A comparison of DEM and FEM reveals a similar trend and similar results for the maximal efficacy, especially in the case of dense material irrespective of the embankment height.

Figure 18. Efficacy relative to the shearing ratio for the dense material (D)

Figure 19 Efficacy relative to the shearing ratio for the loose material (L)

The influence of the shearing ratio on the efficacy for three different porosity materials (D, M, and L) in the discrete model and continuum model is analyzed by varying the embankment heights. Figure 18 and Figure 19 present the results for the dense material (D) and loose material (L), respectively, while the results for the medium material (M) are shown in Figure 10. With respect to the dense and medium materials (D and M), the

efficacy increases with the shearing ratio and reaches a peak at a shearing ratio of approximately 0.1. After reaching the peak, the efficacy reduces with increases in the shearing ratio due to shearing resistance degradation. With respect to the looser material (L), there is no reduction in efficacy in the continuum model while the discrete model exhibits a slight decline in the efficacy. This is potentially because the soil-arching mechanism strongly depends on the shearing behavior of the embankment materials and the degradation of the shear strength results in a reduction in the efficacy under high displacement. The different results for the DEM and FDM in terms of the loose material could be caused by porosity changes. In the discrete model, the granular material is considered as porous media, and an evolution of the porosity is observed in the granular material to a denser state. This leads to a slight decrease in the efficacy when shearing. Conversely, for the continuum model the considered constitutive model does not consider the porosity variation, and therefore no evolution of porosity occurs. Therefore, a reduction in the efficacy does not occur.

Effect of embankment height and subsoil stiffness

Figure 20. Maximum efficacy relative to the embankment height

Figure 20 shows that the maximum efficacy increases with the embankment height in the both discrete and continuum models. It indicates that soil arching develops strongly in the embankment fill with the embankment height. In the study, the influence of the subsoil stiffness (subgrade reaction) is investigated in the range

367 from 0.05 to 1 MPa for different types of embankment materials. Figure 21 and Figure 22 depict the results
 368 of the effect of the subsoil stiffness for the low embankment ($h_m = 0.75$ m) and high embankment ($h_m = 2.25$
 369 m), respectively. Evidently, the denser materials exhibit higher efficacy. This again confirms that the shearing
 370 behavior plays a critical role in the soil-arching mechanism. Denser materials exhibit a higher shearing re-
 371 sistance as observed in the triaxial test, and this induces higher efficacy in the piled embankment systems.
 372 With respect to the low embankment, the efficacy of the discrete model exceeds that of the continuum model
 373 in which no soil arching occurs in the loose material (L). This indicates the complexity in reproducing the load
 374 transfer mechanisms with the continuum model for low embankment thicknesses. In contrast to the low em-
 375 bankment, the discrete and continuum models of the high embankment exhibit a similar prediction of efficacy,
 376 especially for dense materials.

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Figure 21. Efficacy for the low embankment ($h_m = 0.75$ m)

Figure 22. Efficacy for the high embankment ($h_m = 2.25$ m)

Figure 23. Shearing ratio relative to soft soil stiffness for the dense material (D)

Figure 24. Shearing ratio relative to soft soil stiffness for the loose material (L)

Figure 23 and Figure 24 show a correlation between the soft soil stiffness and shearing ratio for the continuum and discrete models for different embankment heights for the dense material D and loose material L, respectively. A similar trend is observed for the rate of the shearing ratio with soft soil stiffness for all the cases. For each studied case, a high soft soil stiffness induces low displacements in the soft soil and consequently low load transfer to the piles.

As previously mentioned, a full-arch effect is obtained prior to the punching of the embankment by the piles. In the high displacement domains (for shearing ratio $d_s > 0.1$ and soft soil stiffness $K_s < 0.1$ MPa), the embankment begins to fail at an excessive shearing rate. It is noted that in the domain corresponding to 0.1 MPa $< K_s < 0.2$ MPa, the shearing rate changes significantly and independently with respect to the embankment height and to the porosity state of materials. In the large displacement domain, the efficacy of the embankment begins to decrease as shown in Figure 10, Figure 18, and Figure 19. Therefore, the high reduction in the efficacy is related to the changes in the shearing mode mechanism in the embankment layer.

Deduced mechanisms

Figure 25 Schematic of the punching failure (ASIRI, 2012)

The results indicate that the load transfer mechanisms significantly depend on the height of the embankment. With respect to high values of the embankment thickness, a fully arching mechanism can develop within the granular layer until the subsoil layer can support a reasonable part of the overload. With respect to a very

low subsoil stiffness, the arching mechanism is strongly disturbed leading to very high vertical settlements and this is generally not in accordance with the reliability of the structure. A comparison of the two numerical models suggests that similar values of the efficacy are obtained for high values of granular layer thickness and subsoil stiffness.

Conversely, for low granular layer thicknesses, high vertical displacements and failures occur with respect to punching when the subsoil thickness is sufficiently low. The punching failure mechanism (Carlsson mechanism (Carlsson, 1987)) corresponds to a low embankment height in which the shear plane is mobilized from the corner of the pile cap to the upper surface of the embankment. Figure 25 shows the schematic of the punching failure for a thin embankment layer. The shear plane is planar, and the efficacy of the low embankments ($h_m \leq h^*$) with rectangular pile caps ($a \times a$) is calculated based on the ASIRI recommendations (ASIRI 2012; Chevalier et al. 2010) as follows:

$$E = \frac{W_p}{s^2 \gamma h_m} \quad (14)$$

where W_p denotes the embankment weight transfer to a single pile cap, and it is expressed as follows:

$$W_p = \gamma \left[a^2 h_m + 2 a h_m^2 \tan \theta + \frac{\pi}{3} h_m^3 \tan^2 \theta \right] \text{ for } h_m \leq h^* = \frac{s-a}{2 \tan \theta} \quad (15)$$

where h^* denotes the height of the granular layer for the areas over the pile joint in the granular mass. The efficacy is calculated since the shearing angle θ is known. Chevalier (Chevalier et al., 2010) indicated that the angle θ ranges from the critical state friction angle ϕ_{cs} to the peak friction angle ϕ_p for granular materials.

Table 4 Efficacy calculations for the low embankments ($h_m = 0.75, 1.5$ m)

Material	h_m	h^*	R_f	ϕ_p	W_p (kN)	Analytical Efficacy (E_a)	DEM Efficacy (E_{DEM})	FDM Efficacy (E_{FDM})
D	0.750	1.361	0.9	46°	19.940	0.179	0.209	0.192

M	0.750	1.652	0.9	40°	15.401	0.147	0.159	0.084
L	0.750	2.029	0.9	34°	12.109	0.122	0.123	0.069
D	1.500	1.361	0.9	46°	93.512	0.420	0.390	0.366
M	1.500	1.652	0.9	40°	67.693	0.324	0.287	0.289
L	1.500	2.029	0.9	34°	49.582	0.250	0.203	0.245

Figure 26. Comparison between numerical and analytical models ($h_m = 0.75, 1.5$ m)

In the study, the peak friction angle corresponds to the maximum shearing resistance of the materials. At this stage, the maximum efficacy is reached as described in the previous section. Therefore, the shearing angle θ can be considered as the peak friction angle at the same time when the piled embankment obtained the maximum efficacy.

Table 4 and Figure 26 compare the efficacy of predictions between the both discrete and continuum numerical models and the analytical models in ASIRI with $\theta = R_f \cdot \phi_p$ only for the embankment height $h_m = 0.75$ m and 1.5 m ($h_m < h^*$) and three different materials (i.e., loose L, medium M, and dense D). The R_f considers the rate of mobilization of the friction as a function of the shearing rate. The analytical solutions are based on the punching failure mechanisms in the ASIRI recommendation (Table 4) and indicate that a good agreement exists with the discrete model for the low embankment with respect to the studied cases. With the exception

of extremely low values of the embankment thickness, the results of the continuum numerical models for the maximal efficacy values due to the punching are significantly satisfactory.

CONCLUSIONS

Piled embankment systems are effective soft soil improvements that reduce settlements and construction time. In the systems, the soil-arching mechanism plays an important role in the effectiveness of the system. In the study, the soil-arching phenomenon is investigated by both the continuum and discrete models considering the shear strength degradation of the granular materials. In the discrete model, the shear strength degradation is naturally captured for the dense material (high friction angle and low porosity), while in the continuum model, a cap yield constitutive model enhanced with friction softening is employed. The soil-arching phenomenon is related to the shearing resistance of the embankment materials. It is reflected by the increases in efficacy with the embankment height and with higher-shear-strength materials. Additionally, the presence of shear strength degradation induced a reduction in the efficacy in the piled embankment system.

The discrepancy between the discrete and continuum models is mainly observed when a relative sliding occurs between the granular layers and the pile caps. The compatibility condition in the continuum model prevents sliding and induces a stress concentration in the corner of the pile in the continuum model. When compared with the discrete model, the continuum model does not exhibit soil arching for the loose materials and for a very thin embankment ($h_m = 0.75$ m) because the continuum model does not capture the evolution of the porosity as observed in the discrete model.

The results of the parametric studies indicate that the efficacy begins to decline when the granular material reaches the domain for the shearing ratio values ranging from 0.1 to 0.2. This domain may be not influenced by the embankment heights or porosity states of the granular material although it is affected by the stress–strain response of materials. With respect to the low embankments, the maximal soil transfer mechanism is

related to a punching failure mechanism, and it is confirmed by a good agreement of the efficacy predictions between the discrete and continuum numerical models and the analytical solutions proposed in ASIRI (2012).

Given the present knowledge, when the subsoil stiffness significantly contributes to support the load embankment, the numerical models constitute the optimal method to estimate the vertical settlements of the granular embankment given the analytical formulation considering the coupled phenomenon of load transfer between the subsoil and piles. Further numerical analyses should be performed to investigate the aforementioned phenomena.

From a practical point of view, the FDM methods are easy to use and more accessible in engineering practice. With respect to high-thickness embankments that are subjected to small deformations, the FDM numerical results are equivalent to those of the DEM and appear significantly relevant. The relatively low computation time of this method relative to that of the DEM implies that the method is presently preferable for geotechnical applications involving several piles. Conversely, the DEM appears necessary when fine representations of discontinuities and large displacements are required (i.e., low-thickness embankments and great relative displacements between the granular embankments and piles). The use of the DEM requires a considerable expertise to build the numerical model and analyses the results. It is expected that this method will be commonly used for engineering given continuous progress in informatics calculations and the current high diffusion of the DEM codes. A comparison between the two methods should be achieved for cyclic, seismic, and dynamic applications.

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