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Mushroom High-Impedance Metasurfaces for Perfect Absorption at Two Angles of Incidence

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Abstract—In this paper, we study the possibility to achieve perfect absorption of TM-polarized plane electromagnetic waves for two incidence angles at the same frequency. The effect was obtained in a high-impedance surface (HIS) also called a mushroom metasurface due to a combination of two types of dissipation losses inside each unit cell of the HIS, dielectric losses and resistance of lumped loads. The dual-angle perfect absorption is a narrow-band resonance effect. It was shown analytically that one can completely suppress the reflection at two predefined angles of incidence. This result was confirmed by numerical simulations of a practical (realistic and affordable) structure.

Index Terms—Mushroom, Metasurface, Perfect Absorber.

I. INTRODUCTION

In many of recently published papers the term perfect absorbers defines electrically thin sheets capable of total absorption at a given frequency for a given angle of incidence and a given polarization. Perfect absorbers can be implemented as impenetrable metasurfaces (MSs), two-dimensional periodic structures composed of very subwavelength unit cells. A detailed review of various metasurface-based absorbers and their operational principles can be found in [1]. One of the first absorbing MSs was proposed in [2]. This was a lossy high-impedance surface (HIS) also called a mushroom-type metasurface. Mushroom-type MSs were introduced in 1999 in [3]. They consist of a dense periodic grid of metal patches on a metal-backed dielectric slab. Centers of every patch may be connected to the ground plane by metal pins that makes a unit cell of this MS look like a mushroom. At the resonance a low-loss mushroom HIS behaves as a magnetic wall because its surface impedance tends to infinity. In the resonance band the electric field of the incident wave is locally enhanced and concentrates in the gaps between adjacent patches. If such a HIS is covered by a resistive sheet as in [2], or the dielectric of its substrate is lossy, or patches are performed of a resisting metal, this concentration of the electric field results in resonant absorption of an incident wave [4], [5]. The same result was achieved using resistors connecting adjacent patches [6].

Perfect absorption means that the impedance matching condition is satisfied: the homogenized surface impedance of the HIS is real and equal to 120π Ohm [1]. This condition can be only achieved at the magnetic mode resonance of unit cells for particular amount of their intrinsic loss. The initial unit-cell geometry can be modified to broaden the resonance bandwidth. Many efforts have been made to suppress angular sensitivity of absorption, i.e. to make the resonant frequency of a MS insensitive to the incidence angle. Wide-angle operation may be realized in arrays of specially shaped separated particles (see e.g in [7]–[9]). Moreover it is possible to extend the angular range of mushroom-type absorbers. Thus in [5] it was proposed to use high-$\epsilon$ substrates. In all the above mentioned designs the angular spectrum of absorption was still limited. Typically an absorbing HIS is optimized for perfect absorption at the normal incidence, while the reflectance increases versus the incidence angle, and the target is to reduce this increase as possible. Despite that this increase can be made small with the previously employed structures, in these structures it is impossible to reach perfect absorption at two predefined angles of incidence. In this work we modify the mushroom HIS to reach this new goal at least for TM-polarized waves. As it is shown in the following sections (analytically and numerically) the suggested structure allows perfect absorption for two given incidence angles due to an additional degree of freedom granted by lumped loads.

II. ON THEORY OF PERFECT ABSORBERS BASED ON A MUSHROOM STRUCTURE

We consider a MS whose geometry is illustrated by Fig. 1. The structure is a mushroom-type HIS consisting of an array of square metal patches located over a metal-backed dielectric substrate of the strongly subwavelength thickness $h \ll \lambda$, where $\lambda$ is the wavelength in free space. With high accuracy the patches and the ground plane can be assumed to be perfect electric conductors (PEC). Each patch is connected to the ground plane by a via (a thin cylindrical PEC wire of the radius $r_0 \ll h$) and a lumped element with the complex impedance $Z_{\text{Load}}$. The structure has the period $a \ll \lambda$, and the gap widths between adjacent patches are very narrow ($w \ll a$). A mushroom structure with loaded vias and no losses has been first suggested in [10], but targeted, however, to use in artificial magnetic conductor (AMC) substrates. In this work...
we for the first time employ a similar structure as a dual-angular perfect absorber by introducing specially engineered losses. First, our substrate can have the complex permittivity $\varepsilon_h = \varepsilon_h' - j\varepsilon_h''$. Second, our loads may contain the resistive part $Z_{\text{Load}} = R_{\text{Load}} + jX_{\text{Load}}$. These additional degrees of freedom enable the perfect absorption at the same frequency $\omega_0$ for two predefined incidence angles $\theta_{1,2}$.

Since $\omega \ll \lambda$, the square lattice of vias in the structure can be considered as a wire medium (WM) slab of the thickness $h$, characterized by the non-local effective permittivity tensor $\hat{\varepsilon}$. The WM slab is bounded at $z = -h$ by the ground plane and at $z = 0$ by the array of patches As shown in [10], the whole structure can be homogenized using this non-local model of the WM slab and the generalized additional boundary conditions (GABCs) [11] for its boundaries. When the structure is illuminated by a TM-polarized plane wave at the angle $\theta$, two eigenmodes (the TM- and the TEM-polarized plane waves) are excited inside the WM slab [12]. The presence of these two waves in the slab requires GABCs to find their amplitudes and finally the surface impedance $Z_{\text{TM}}$ of the MS for the given incidence angle $\theta$. $Z_{\text{TM}}$ is defined as the ratio of the tangential components of the electric and magnetic fields in the plane $z = 0$. In the upper half-space ($z > 0$) the electromagnetic field is the sum of the incident and the reflected waves, whereas the reflection coefficient $R_{\text{TM}}$ is fully determined by $Z_{\text{TM}}$.

It is possible to describe the perfect absorption regime $R_{\text{TM}}(\omega_0) = 0$ in two different languages both involving $Z_{\text{TM}}$. The first one operates with surface eigenmodes, whose dispersion equation for the TM-polarization is $n^2 = 1 - z_{\text{TM}}^2$ [13]. Here $n = q/k_0 \equiv q/\omega\sqrt{\varepsilon_0\mu_0}$ is the slow-wave factor of the surface wave with wave number $q$ and $z_{\text{TM}} = Z_{\text{TM}}\sqrt{\varepsilon_0/\mu_0} \equiv Z_{\text{TM}}/\eta$ is the dimensionless surface impedance (normalized to that of free space) depending on both $\omega$ and $n$. In presence of losses $\tau_{\text{TM}}(\omega, n)$ can be engineered so that the eigenmode equation can be satisfied for real $n \leq 1$ at a specific frequency $\omega_0$. This is the leaky mode solution arising for TM-waves namely due to losses (in lossless structures only TE-polarized surface waves can be leaky). Excitation of the surface eigenmode in the electrically thin layer by an incident plane wave implies strong concentration of the electromagnetic energy in it. If there are dissipation losses inside the layer, the incident field power can be completely absorbed. This is the language of work [13]. In the language of radio engineers the eigenmode equation $n^2 = 1 - z_{\text{TM}}^2$ is the impedance matching condition. Indeed, substitution $n = \sin \theta$ where $\theta$ is the incidence angle transforms this equation into $\tau_{\text{TM}} = \text{FS}$, where $\tau_{\text{FS}} = \cos \theta$ is the normalized wave impedance of a TM-polarized wave in free space referred to the plane $z = 0$ [14]. The matching implies $R_{\text{TM}}(\omega_0, \theta) = 0$. The last is the language of the work [1], where an alternative model for mushroom absorbers was developed without using surface eigenmodes. Instead, an absorber is replaced by an effective bianisotropic MS, described by electric and magnetic surface polarization sheets. Within the resonance band of the bianisotropic surface impedance parameter there is a frequency at which the waves produced by the induced electric and magnetic polarization sheets mutually cancel out at $z > 0$. This model also describes the local field concentration in the MS. The equivalence of the two above discussed models was proved in [1] in the special case of the normal incidence.

To understand why the loaded vias grant the perfect absorption for two incidence angles of a TM-wave at the same frequency an analytical exercise was done neglecting one of the waves (TM-mode) excited in the WM slab. This model gives a sufficient accuracy for $\tau_{\text{TM}}(\omega, n)$ if $\varepsilon_b^0 \gg 1$ when the spatial dispersion in the WM is weak [15]. In this case the surface impedance is described by formula (11) of paper [5], that in our notations can be written in the form:

$$Z_{\text{TM}} \approx \frac{j\omega L(1 - \sin \theta/\varepsilon_{\text{WM}})}{1 - \omega^2LC(1 - \sin \theta/\varepsilon_{\text{WM}})}$$ (1)

where $\varepsilon_{\text{WM}}$ is the complex permittivity of the wire medium slab composed of wires loaded by impedances $Z_{\text{Load}}$. This parameter is described by formula (12) of paper [16] yielding under our assumptions to $\varepsilon_{\text{WM}} = \varepsilon_b (1 - j(k_\text{p}h)^2/\varepsilon_{\text{Load}})$. In (1) $L = \mu_0 h$ is the effective inductance of the space between the patches and the ground plane, while $C = C^r - jC^\text{bi}$ is a complex value, proportional to $\varepsilon_b$. In the lossless case $C$ is the effective capacitance of the patch array [13]. It is possible to show analytically that at the frequency $\omega_0 = 1/\sqrt{LC}$ the condition $R_{\text{TM}} = 0$ can be exactly satisfied for the two angles: $\theta_1 = 0$ (if $C^r\eta^2 = L$) and $\theta_2 \neq 0$ (by tuning $Z_{\text{Load}}$). The solution granting $R_{\text{TM}}(\omega_0, n)$ for $n \neq 0$ is compatible with the condition $C^r\eta^2 = L$ if $\text{Re}(Z_{\text{Load}}) > 0$ and $\text{Im}(Z_{\text{Load}}) < 0$, i.e. the loads of wires comprise the resistive and capacitive parts. In the further theoretical calculations we use a more accurate spatially dispersive model of the mushroom structure with loaded vias including both TEM- and TM-waves and describing the complex loads of wires with GABCs [10].

### III. Idealized Dual-Angle Absorber

When optimized for the normal incidence, perfect MS absorbers without loads in the vias may still manifest quite high absorption also at the oblique incidence [1], which is, however, imperfect and degrades along with the incidence angle. In the considered structure depicted in Fig. 1 by tuning the complex impedance $Z_{\text{Load}}$ of the lumped loads it is possible to recover perfect absorption at the desired angle $\theta = \theta_2 > 0^\circ$ for TM-polarization without damaging perfect absorption at $\theta = \theta_1 = 0^\circ$. The corresponding complex impedance of the loads calculated analytically as a function of $\theta_2$ is shown in Fig. 2. The curves for the load resistance $R_{\text{Load}}$ and the reactance $X_{\text{Load}}$ correspond to the chosen frequency of perfect absorption $f_0 = 4.68$ GHz and the following parameters of the MS: $a = 10$ mm, $w = 0.5$ mm, $r_0 = 0.3$ mm and are given for four values of the thickness $h$: 1, 2, 4 and 4.5 mm. For each $h$ the absorber was tuned at the normal incidence by choosing the proper complex permittivity $\varepsilon_h$, namely: $\varepsilon_b = 11.4 - j1.2$, $5 - j1.6$, $1.9 - j1.1$, and $1.6 - j1.0$, respectively.

From Fig. 2 one can see that the larger is the second angle of perfect absorption $\theta_2$, the smaller is the required load resistance. Also, the needed load reactance monotonously grows. In fact, without any vias and loads it can be shown that at grazing angles the surface impedance of a mushroom MS drops down, but not as dramatically as the TM-polarized
wave characteristic impedance $Z_{FS}^{TM} = \eta \cos \theta$. The presence of the loads connecting adjacent patches through the ground helps further reducing the surface resistance at grazing angles. Moreover, the appropriate load reactance recovers the resonance (the surface impedance is real at $\theta = \theta_2$).

In order to illustrate the dual frequency absorption regime for $h = 2$ mm, the data of Fig. 3(b) was used for two particular cases: $\theta_2 = 45^\circ$ and $\theta_2 = 60^\circ$. The corresponding analytically calculated reflection coefficients as functions of the incidence angle are shown in Fig. 3 with solid curves. To prove these results a unit cell of the considered structure was simulated numerically using CST Microwave Studio 2015 (Frequency Domain Solver with periodic boundary conditions). The numerically calculated angular dependences of the TM-wave reflection coefficient are given in Fig. 3(a) ($\theta_2 = 45^\circ$) and Fig. 3(b) ($\theta_2 = 60^\circ$) with dashed curves. In fact, the dashed curves in Fig. 3 were obtained after small variation of the parameter values predicted by the theory. Namely, in numerical simulation a small frequency shift occurs with respect to the theory: the dashed curves correspond to the frequency $f_0 = 4.48 \text{ GHz}$, while the solid ones were obtained at $f_0 = 4.68 \text{ GHz}$. Moreover, to precisely recover the dual-angle perfect absorption regime a small variation in the complex substrate permittivity $\varepsilon_h$ and the load impedance was made, in accordance to Table I. These mismatches between the theory and the simulation can be attributed to the near-field interaction between the patches and the ground plane modifying the grid impedance of the patch array. This interaction was not taken into account in the analytical model. As can be concluded from Fig. 3 and Table I, the GABCs approach though giving some deviations from full-wave numerical simulation, still correctly predicts the angular behavior of the reflection coefficient and provides a good first-order approximation for the MS parameters. In the following section we propose a practical absorber design based on a printed-circuit board (PCB) with surface-mounted elements. The parameters were estimated using the same theory, while further optimization was made by numerical simulation.

IV. PRACTICAL DUAL-ANGLE ABSORBER

In this section we propose and numerically study a practical realization of a dual-angle perfect absorber operating similarly to the idealized one studied in the previous section. In the practical design the HIS is represented by a standard PCB on the $h = 1.5$-mm-thick FR-4 grounded substrate with the complex permittivity of $\varepsilon_h = 4.3 - j0.1$. The array of square copper patches with the period of $a = 8$ mm and the separation of $\omega = 0.1$ mm is printed on the top layer of the PCB. The geometry of the structure is depicted in Figs. 4 and 5. In Fig. 5 only the top PCB layer with patches (yellow color represents copper) is shown. Note that one of the patches appears as a yellow grid to make the interior of a unit cell visible. Since the losses in this substrate are insufficient we added lossy surface-mounted elements (SMD resistors) $R_2$ connecting each pair of adjacent patches on the top layer. In the analytical description of the structure the capacitive grid impedance of the patch array should be modified by connecting the resistance $R_2$ in parallel to the capacity of $Z_g$ [6]. As follows from the theory, the above parameters allow perfect absorption of a normally incident wave at 5.2 GHz with $R_2 = 430$ Ohm. In order to obtain also perfect absorption at 45°, each patch was connected to the ground plane through a metal via of the radius $r_0 = 0.3$ mm and four identical lumped elements mounted on the bottom PCB layer. Each element is composed of the resistor $R_1$ and the capacitor $C_1$ connected is series (see Fig. 4). Such connection of lumped elements means that the parameters of the practical absorber

![Fig. 2. Complex load impedance $Z_{Load}$ required for perfect absorption at $\theta = \theta_2$ in addition to perfect absorption at the normal incidence (as function of $\theta_2$) for $h = 1$ mm (a); $h = 2$ mm (b); $h = 4$ mm (c); $h = 4.5$ mm (d).](image)

![Fig. 3. Reflection coefficient of the idealized dual-angle absorber with substrate thickness $h = 2$ mm vs. the incidence angle: (a) optimized for $\theta_2 = 0^\circ$ and $\theta_2 = 45^\circ$; (b) optimized for $\theta_1 = 0^\circ$ and $\theta_2 = 60^\circ$.](image)

![Fig. 4. Cross-sectional view of the practical absorber structure.](image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Theoretical</th>
<th>Numerically optimized</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_0$, GHz</td>
<td>4.68</td>
<td>4.48</td>
</tr>
<tr>
<td>$\varepsilon_h$</td>
<td>5.000 – 1.16j</td>
<td>5.004 – 1.06j</td>
</tr>
<tr>
<td>$Z_{Load}$ for $\theta_2 = 45^\circ$, Ohm</td>
<td>6.47 – 2.77j</td>
<td>6.887 – 19.85j</td>
</tr>
<tr>
<td>$Z_{Load}$ for $\theta_2 = 60^\circ$, Ohm</td>
<td>6.15 + 0.80j</td>
<td>5.444 – 15.46j</td>
</tr>
</tbody>
</table>

![Table I. Theoretical and numerically optimized parameters of idealized structure](image)
and the analytical model parameters relate to each other as:

\[ R_{\text{Load}} = R_1/2 \text{ and } X_{\text{Load}} = 1/2\omega C_1. \]

The parameters of the SMD elements \( R_1 \) and \( C_1 \) required for perfect absorption at \( \theta_2 = 45^\circ \) were found analytically and listed in the second column of Table II. The theoretical reflection coefficient as a function of the incidence angle is shown with a solid line in Fig. 6. On the next step the practical absorber with the same parameters was numerically simulated, which resulted in imperfect absorption at \( 45^\circ \). However, by a fine parametric optimization based on this first approximation the expected dual-angular perfect absorption was restored. The final SMD parameters are given in the third column in Table II. For this case the reflection coefficient depending on the incidence angle is depicted with the dashed curve in Fig. 6. The difference between the theory and the simulation in what concerns \( Z_{\text{Load}} \) can be explained by unaccounted near-field interactions changing the grid impedance of the patch array.

V. Conclusion

In this work we have shown that a mushroom-type MS with a lossy dielectric substrate and a vias with complex impedance loads may serve as a dual-angle perfect absorber for TM-polarized electromagnetic waves. The dual-angle perfect absorption was shown analytically using the approach based on the existing theory taking into account the spatial dispersion in the array of vias (through GABCs) and confirmed numerically using CST Microwave Studio. A rather good correspondence between our theory and simulation was achieved. One of two incidence angles of perfect absorption can be zero, while the other one (for oblique incidence) can be predefined. A practical bi-layer PCB design of a dual-angle perfect absorber using SMD lumped elements was proposed and studied numerically taking into account losses in the metal.

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