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Global Sea Level Linked to Global Temperature

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We propose a simple relationship linking global sea level variations on time scales of decades to centuries to global mean temperature. This relationship is tested on synthetic data from a global climate model for the past millennium and for the next century. When applied to observed data of sea level and temperature for 1880-2000, and taking into account known anthropogenic hydrologic contributions to sea level, the correlation is greater than 0.99, explaining 98% of the variance. For future global temperature scenarios of the IPCC 4th Assessment Report, the relationship projects a sea level rise ranging from 75 to 190 cm for the period 1990-2100.

Sea level rise is amongst the potentially most serious impacts of climate change. Yet sea level changes cannot yet be predicted with confidence using models based on physical processes, since the dynamics of ice sheets and glaciers and to a lesser extent also that of oceanic heat uptake is not sufficiently understood. This is seen, e.g., in the fact that observed sea level rise exceeded that predicted by models by about 50% for the periods 1990-2006(1) and likewise for 1961-2003(2). The last IPCC assessment report did not include rapid ice flow changes in its projected sea level ranges, arguing that these could not yet be modelled, and consequently did not present an upper limit of the expected rise(2).

This has caused considerable recent interest in semi-empirical approaches to projecting sea level rise(3-8). These are based on using an observable that climate models can predict with confidence, namely global mean temperature, and establish with the help of observational data how this is linked to sea level. Here we present a substantial extension and improvement to the semi-empirical method proposed by
Rahmstorf (3) (henceforth called R07). We test it on synthetic as well as real data and apply it to obtain revised sea level projections up to the year 2100.

**Methods**

Rahmstorf originally proposed that the initial rate of sea level rise in response to a large, rapid warming could be approximated by

\[ \frac{dH}{dt} = a(T - T_0). \] (1)

Here \( T_0 \) is a base temperature at which sea level is in equilibrium with climate, so that the rate of rise of sea level \( H \), \( \frac{dH}{dt} \), is proportional to the warming above this base temperature. \( T_0 \) and \( a \) are to be determined from data. For the ice melt contribution (glaciers and ice sheets) this corresponds to an approach commonly used in ice modelling, where the rate of mass loss is assumed to be proportional to the temperature increase above a threshold value (9).

Eq. 1 is based on the assumption that the response time scale \( \tau \) of sea level is long compared to the time scale of interest (typically ~100 years). Grinsted et al (8) have recently proposed to model the link of sea level and temperature with a single finite time scale. For the fit to instrumental temperature and sea level data they obtain a time scale of just over 1,000 years and thus essentially replicate the results of R07, albeit using a different sea level data set. However, some components of sea level adjust quickly to a temperature change, e.g. the heat content of the oceanic surface mixed layer. Therefore, we here propose to extend the semi-empirical method by a rapid-response term:

\[ \frac{dH}{dt} = a(T - T_0) + b \frac{dT}{dt} \] (2)

This second term corresponds to a sea level response that can be regarded as “instantaneous” on the time scales under consideration, as it implies \( H \sim T \). Given the two time scales represented, we will call this the dual model.
Results

Testing the dual model on simulated future sea level rise

R07 presented a test of his method on future sea level projections from a coupled climate model (10) and found the method was unable to capture a levelling off of the rate of sea level rise in the second half of the 21st century found in the climate simulation. We repeat this test with the dual model (Fig. 1). The parameters are fitted to the global temperature and sea level output from the climate model for 1880-2000, resulting in $a = 0.080 \pm 0.017 \text{ cm K}^{-1} \text{ a}^{-1}$, $b = 2.5 \pm 0.5 \text{ cm K}^{-1}$ and $T_0 = -0.375 \pm 0.026 \text{ K}$ (temperature relative to the reference period 1951-1980). Sea level for 2000-2100 is then computed from global temperature using Eq. 2 and compared to the sea level simulated by the climate model (which includes a 3-D ocean general circulation model) (10). The fact that the rate of rise levels off after 2050 is captured well by the dual model. This is related to temperatures rising less than exponentially. As seen from Eqs. 1 and 2, the dual model is equivalent to the R07 model in case of an exponential temperature increase.

Note that this test is for the thermal expansion component of sea level rise only, since only that is captured in the climate model. In this case, we expect the rapid response of sea level (second term) to be due to heat uptake of the surface mixed layer of the ocean, for which $\Delta H = h \cdot \alpha \cdot \Delta T$, where $h$ is the mixed layer depth and $\alpha$ is the mean thermal expansion coefficient. Hence, $b = h \cdot \alpha$. For a typical value $\alpha = 2.5 \times 10^{-4} \text{ K}^{-1}$ (sea water at 20 ºC), the value we found for $b$ corresponds to a mixed layer depth of $h = 100 \text{ m}$. This physically plausible result is evidence for the validity of the method.

Testing the dual model for the past millennium

We performed a similar test on a model simulation of the past millennium, where the climate model was forced by solar variability, volcanic activity, changes in greenhouse gas concentration and tropospheric sulphate aerosols. This simulation, along with the forcing and a range of other models, was published in the IPCC 4th Assessment Report (2) (AR4; Fig. 6.14 thereof) and compared with paleo-climatic data. We applied...
the dual model fit discussed in the previous section, using the parameters a and b obtained there to predict sea level variations for the time period 1000 to 2000 AD (Fig. 2). A small adjustment was applied visually to T₀ to make long-term sea level rise match – any small error in this parameter will lead to a drift in sea level that accumulates over time and becomes an issue over multiple centuries.

The figure shows that the single-term model of R07 can capture the 20th-Century sea level rise but not the short-term variability. The latter is to be expected since basic assumptions behind this approximation are not fulfilled here; the method is used outside its range of applicability. In contrast, the new dual model also captures the short-term response and performs well when compared to the sea level variations simulated by the climate model, explaining 82% of sea level rate variance. Even the brief negative excursions of the rate of sea level change due to volcanic eruptions (which in this model are implemented as a globally-averaged forcing) are reproduced faithfully. A mismatch in the bottom panel in the first few centuries is due to a small offset in the rate there – adjusting T₀ can remove this offset either for the first or the second half of the millennium, but not both. This points to an inherent limitation of assuming an infinite adjustment time scale in the slow term, an assumption that breaks down beyond about 500 years. This limitation does not affect our fit to the instrumental data and the future projections discussed later, both of which are limited to the 100-year time frame. A good performance is also found for millennial simulations with two other climate models, the ECHO-G model (11) and the ECBilt-CLIO model (12) (see supplementary information).

**Testing the dual model on observed data**

The third and most interesting test is the application to observed data of global temperature and sea level for 1880-2000, since this covers the full climate-related sea level response and not just thermal expansion. The IPCC AR4 concludes that thermal expansion can explain roughly 25% of observed sea level rise during 1961-2003 and 50% for 1993-2003, but with considerable uncertainty. The remainder is mostly due to ice melt; climate-induced changes in land water storage played a minor, but not negligible role(13). A recent analysis concludes that during 2003-2008, the split was 20% thermal and 80% ice melt(14). While a five-year period is likely to be dominated
by natural variability, there is some reason for concern that ice-melt could take an increasingly larger share in the course of this century.

For temperature we use the NASA GISS data set(15) because it has the best global coverage. For sea level we use the data of Church and White(16) as adopted by IPCC. These sea level data are already corrected for post-glacial rebound; since this is a constant linear trend it does not affect our proposed link to temperature. We further corrected the sea level data for the amount of water stored in man-made reservoirs (see SOM), since these cause a small sea level drop not related to climate which must be excluded from the sea level changes linked to global temperature as considered in Eq. 2. Possible effects of groundwater mining are considered below.

Fig. 3 shows the remarkably close link of global temperature and the rate of sea level rise we find for 1880-2000. In particular, it shows that the rate of sea level rise increased up to 1940 in line with rising temperatures, then stagnated up to the late 1970s while global temperature also remained nearly level, followed by another rise that continues until today. Note that the parameters a and $T_0$ are essentially determined by making mean value and trend agree over the data period. Any agreement of the R07 model (grey line) with observed sea level beyond the linear trend is thus not fitted but an independent test of the concept. The near-perfect fit of the dual model arises because almost all of the remaining misfit of the R07 model is proportional to $dT/dt$.

Fig. 4 shows the observations-based rate of sea level rise as a function of the right-hand-side of Eqs. 1 and 2. The parameter values obtained are $T_0 = -0.41 \pm 0.03$ K, $a = 0.56 \pm 0.05$ cm a$^{-1}$ K$^{-1}$ and $b = -4.9 \pm 1.0$ cm K$^{-1}$. The linear fit is clearly better for the dual model: the Pearson correlation coefficient $r = 0.992$ for the dual model (0.96 for the R07 model even when the artificial reservoir correction is applied). This is to be expected since it contains one additional free parameter. Whether it still is the preferred model when this extra degree of freedom is taken into account can be tested with the Akaike information criterion, a standard statistical technique for such cases (see SOM). The analysis shows that the dual model is the preferred model, suggesting also here that the second term is physically meaningful.
Another semi-independent test is provided by the satellite sea level record updated from (14) which started in 1993 and now provides 16 years of data (up including 2008), with a linear trend of 3.4 mm/year (including post-glacial rebound adjustment). When the reservoir correction is applied this yields 3.6 mm/year, and the extra point is shown as open circle in Fig. 4.

Remarkably, the value we find for b is negative. We can think of two possible physical explanations. The first is that the initial rapid sea level response to a warming is indeed negative. A possible mechanism for this is higher evaporation from the sea surface and subsequent storage of extra water on land e.g. in form of soil moisture (17, 18). Note, however, that such a negative effect would have to be large: it would need to compensate the b of +2.5 cm K\(^{-1}\) found earlier for thermal expansion and thus would need to be three times as large as this to cause the overall negative b value we found. It is hard to see how the very large amount of water needed to be stored on land could remain inconspicuous.

The second possibility is of a positive, but time-lagged sea level response. That a negative b corresponds to a lag is easily seen for the example of a steady linear temperature rise with rate c starting at time t=0. The solution then is H= 1/2 act (t+2b/a). This is the same parabola as for the R07 model (H = 1/2 a c t\(^2\)), except that its origin is shifted by (b/a, b\(^2\)c/2a), see Fig. 5. For c = 0.01 K a\(^{-1}\) (i.e., an idealised 20th-Century warming) and the parameter values found above, this shift is 12 years and -0.25 cm. The short transient sea level offset of a maximum of -2.5 mm is too small to measure and of no consequence on the longer time scales (>15 years) considered here. However, the implied time lag of 12 years in this idealised case is permanent and significant.

We tested for this by implementing a time lag directly in Eq. 2:

\[
dH(t)/dt = a (T (t+\tau)-T_0) + b dT (t+\tau)/dt
\]

and subsequently finding the best fit for T\(_0\), a, b and \(\tau\). When the resulting Pearson correlation is plotted as a function of b and \(\tau\) (see Supplementary Information), a linear dependence between the two is seen, with two optimal solutions: one for zero \(\tau\) and b = -5 cm K\(^{-1}\), and another one b = 4.5 cm K\(^{-1}\) and \(\tau\) = 13 years. Choosing the value b = 2.5
cm K\(^{-1}\) from our model simulations for the thermal expansion effect corresponds to \(\tau = -11\) years, close to this optimum. The two parameters \(b\) and \(\tau\) cannot be unambiguously separated by the statistical fit since their effect on \(H\) is so similar. Thus, the most plausible physical interpretation of our statistical fit is that the negative value of \(b\) results from a positive ocean mixed layer response combined with a lag of over a decade in the response of the ocean-cryosphere system.

Several mechanisms could be envisaged for a delayed onset of sea level rise after warming. For example, mass loss of ice sheets can be caused by warm water penetrating underneath ice shelves, triggering their collapse and subsequent speed-up of outlet glaciers banked up behind the ice shelf(19). We cannot explore the causes of delay in more detail here, but note that the statistical result is robust irrespective of its causes.

The quality of fit found also independently confirms the quality on interdecadal time scales of both the global temperature and sea-level time series used. Any erroneous long term trend in temperatures or acceleration in sea level would cause conspicuous misfits. The reservoir adjustment(20) is a major contributor to the success of the fit. Given this is a known and valid correction this further corroborates the physical basis of the agreement we find.

**Other non-climatic sea level contributions**

We have corrected the sea level data for the reservoir storage component, but a further non-climatic effect of relevant magnitude is the mining of groundwater for human uses in arid regions (21). No time series of this is available, so it cannot be included in the above analysis. We consider it an uncertainty and test the sensitivity of our results to this term.

Estimates differ considerably, but in recent decades groundwater mining could have contributed 0.2-0.3 mm/yr to sea level (21). We consider two scenarios for the time evolution leading up to this high-end estimate: a linear and an exponential increases of the pumping rate, starting at zero in the year 1870, see SOM. Remarkably and contrary
to the reservoir storage correction, neither scenario significantly affects the quality of fit found. For all cases, fit parameters remain within the 1σ uncertainty of their values found in the previous section. The temperature sensitivity of long-term sea level change $a$ is reduced by 6-7%. The impact on future sea level projections is reduced by the fact that groundwater mining for irrigation purposes is likely to increase in future and in this sense behaves like the climate-related component of sea level rise. This is in contrast to the reservoir correction included above, since the potential for additional water storage on land is limited and reservoir-building has declined and almost come to an end (20).

**Projections of future sea level**

After Eq. 2 has passed a threefold test with simulated and observed sea level data, we will apply it to the 21st century using global temperature projections from the IPCC AR4. We use the emulated global temperature data for 1880-2100 for 6 emission scenarios, 3 carbon cycle feedback scenarios and 19 climate models, as shown in Fig. 10.26 of the AR4(2). For these 342 temperature scenarios sea level was computed using Eq. 2, assuming equal average temperatures across all models for the period 1880-1920, close to pre-industrial temperatures. This reference period leads to slightly different temperatures (and hence rates of sea level rise) in 1990 according to the climate sensitivity of the respective model; more sensitive models show greater warming and greater sea level rise consistently for the 20th and 21st centuries.

For each of the six emission scenarios, we computed the mean sea level curve across all 19 models, using the standard carbon cycle setting. These are the solid lines shown in Fig. 6 for three of the emissions scenarios. The coloured uncertainty bands for each scenario encompass one standard deviation from the model mean, using the high carbon cycle setting for the upper uncertainty limit and the low carbon cycle setting for the lower limit, as is done in the AR4 for temperatures. The additional grey uncertainty band shows an added ±7%, representing one standard deviation of the uncertainty of the fit shown in Fig. 4. The temperature and sea level ranges for all 6 emission scenarios are compiled in Table 1.
Overall, sea level projections range from 75 to 190 cm for the period 1990-2100. In the two sensitivity scenarios for groundwater mining discussed above, the lowest climate-related sea level rise drops from 81 cm (see Table) to 75 - 76 cm. However, as mentioned above, groundwater-mining will continue to raise sea level. Even without further increase, i.e. at a constant mining rate of 0.3 mm/year, this would add 3.3 cm to the projection, increasing it to 78-79 cm. Hence, even considering high-end estimates for the effect of past groundwater mining all scenarios remain well within the overall range shown in Fig. 6.

The model averages for all emission scenarios are remarkably close together. This is mostly due to the fact that sea level rise integrates the temperature rise over time in the first term of Eq. 2, so that a temperature increment in 1999 has hundred times the effect on final sea level compared to the same increment in 2099. Temperatures in the various emission scenarios are still close together in the first half of the century. The second term in Eq. 2 furthermore implies a time lag, so that emissions reductions (as in scenario B1) only slow down sea level rise after over a decade delay. These results suggest that emissions reductions early in this century will be much more effective in limiting sea level rise than reductions later on. This effect can be seen when comparing scenarios A1B and A2, which produce the same sea level rise by 2100 despite A1B being 0.8 °C cooler then. This is due to A1B being slightly warmer early on in the 21st century.

Another interesting aspect of these projections is that the thermal share in the rate of sea level rise declines over the 21st century, if we take the parameters (a,b) fitted to the climate model simulation above to represent thermal expansion. For the period 1961-2003, the thermal share is ~ 30%, as compared to 25% estimated in the AR4 and 40% by Domingues et al(22). In our projection it gradually declines to ~ 20% in the latter half of the 21st Century. This is directly linked to the fact that thermal expansion is associated with positive b while total sea level has a negative b, corresponding to a delay in the ice response. Qualitatively we consider this decline in the thermal share and increasing importance of ice melt a robust result. This is our key difference to the IPCC AR4, where the ice melt share is assumed to diminish with thermal expansion contributing between 55 and 70% of the total sea level rise over the 21st century.
Discussion: Implications for the future

If our method presents a reasonable approximation of the future sea level response to global warming, then for a given emission scenario sea level will rise around three times as much by 2100 as the projections (excluding rapid ice flow dynamics) of the IPCC AR4 have suggested. Even for the lowest emission scenario (B1), sea level rise is then likely to be around one meter; for the highest, it may even come closer to two meters.

Uncertainties remain, however. While the thermal expansion response has been tested on simulated data, it is less clear whether the information contained in the 120 years of observational data about the ice response is sufficient to describe the future ice melt contribution out to the year 2100. The key question then is: will the ice melt response observed so far, as captured in our dual model, over- or underestimate future sea level rise? On one hand, the surface area of mountain glaciers vulnerable to melting will decrease in future as glaciers disappear. On the other hand, more ice higher up in mountains and particularly the big continental ice sheets will increasingly become subject to melting as temperatures warm. The net effect – an increasing or decreasing surface area subject to melting – is not easily determined without detailed regional studies. In addition, highly non-linear responses of ice flow may become increasingly important during the 21st century. These are likely to make our linear approach an under-estimate. Overall, we judge it more likely than not that sea level could rise faster than suggested by the simple projection based on Eq. 2.

How much faster? Pfeffer et al.(23) provided an independent estimate of maximum ice discharge based on geographic constraints on ice flow; they concluded that sea level rise in the 21st century is very unlikely to exceed 200 cm. If this estimate is correct, a non-linear dynamical ice sheet response may not change our estimate upwards by very much.

In order to limit global sea level rise to a maximum of one meter in the long run (i.e., beyond 2100), as proposed recently as a policy goal(24), deep emissions reductions will be required. Likely these would have to be deeper than those needed to limit global warming to 2 °C, the policy goal adopted by the European Union and other countries. Our analysis further suggests that emissions reductions need to come early in this century to be effective.
Acknowledgements

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References


Figure Legends

**Figure 1.** Top: The rate of sea level rise from a climate model simulation (red) compared to that predicted by Eq. 1 (grey) and Eq. 2 (blue) based on global mean temperature from the climate model. Shaded areas show the uncertainty of the fit (one standard deviation). The parameter calibration period is 1880-2000, while 2000-2100 is the validation period. Bottom: The integral of the curves in the top panel, i.e. sea level proper.
Figure 2. (Top) Rate of sea level rise for the last millennium from a climate model simulation (red), compared to that predicted by Eq. 1 (grey) and Eq. 2 (blue) based on global mean temperature from the climate model. Note that parameter calibration was done for the model.
simulation shown in Fig. 1 for 1880-2000 only (but see below for $T_0$). **(Bottom)** Sea level for the last millennium, obtained by integration. A correction to the $T_0$ parameter of -0.13K (corresponding to a constant sea level trend of $a \cdot T_0 = -0.1$ mm/yr) was applied in order to make the long term sea level trends match better after the year 1400.

![Graph](image)

**Figure 3.** Top: Observations-based rate of sea level rise (with tectonic and reservoir effects removed, red) compared to that predicted by Eq. 1 (grey) and Eq. 2 (blue with uncertainty estimate) using observed global mean temperature data. Also shown is the estimate from Eq. 2 using only the first half of the data (green) or the second half of the data (light blue). Bottom: The integral of the curves in the top panel, i.e. sea level proper. In addition to the smoothed sea level used in the calculations, the annual sea level values (thin red line) are also shown. The dark blue prediction by Eq. 2 almost obscures the observed sea level due to the close match.
Figure 4. Blue crosses show 15-year averages of global temperature (relative to 1951-1980) versus the rate of sea level rise, with their linear least-squares fit, as in R07 but using 15-year bins. The green crosses show the adjustment to sea level induced by the reservoir correction(20), leading to a steeper slope. The red dots show the expression $(T + b/a \frac{dT}{dt})$ appearing on the right hand side of Eq. 2. This increases the slope again and leads to a much tighter linear fit. The open red circle is a data point based on the satellite altimetry data for 1993-2008.

Figure 5. Schematic of the response to a linear temperature rise. (a) Temperature. (b) First term on the right hand side of Eq. 2. (c) Second term. (d) Total sea level response. (e) Comparison to the case $b=0$ (Eq. 1), showing that $b<0$ primarily corresponds to a time lag in the sea level response.
Figure 6. Projection of sea level rise from 1990-2100, based on IPCC temperature projections for three different emission scenarios (labelled on right, see text for explanation of uncertainty ranges). The sea level range projected in the IPCC AR4(2) for these scenarios is shown for comparison in the bars on the bottom right. Also shown is the observations-based annual global sea level data(16) (red) including artificial reservoir correction(20).
Table Legends

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Table 1. Temperature ranges and associated sea level ranges by the year 2100 for different IPCC emission scenarios. The temperatures used are taken from the simple model emulation of 19 climate models as shown in Fig. 10.26 of the IPCC AR4; they represent the mean ± one standard deviation across all models, including carbon cycle uncertainty. The sea level estimates were produced using Eq. 2 and 342 temperature scenarios and are given here excluding the uncertainty of the statistical fit, which is approximately ± 7% (one standard deviation).
Supporting Information

**Data sources.** Global mean sea level data from Church and White(1) and global temperature anomaly data from GISTemp(2) were processed as annual means, smoothed with a 15-year smoothing period using Singular Spectrum Analysis, SSA(3) in order to reduce the extraneous impact of short-term variability. After that, data were binned into 15 year bins where not otherwise stated.

**Statistical analysis.** The dual model fit was performed using a modified version of Eq. 2:

\[
\frac{dH}{dt} = a ((T - T_0) + \lambda \frac{dT}{dt}),
\]

with \(\lambda = b/a\); \(\lambda\) was varied iteratively to maximize the Pearson correlation \(r\) of the linear fit yielding \(T_0\) and \(a\). Error bounds plotted in Figs. 1 and 3 include the error contribution of the linear fit but not that of \(\lambda\). The autocorrelation introduced by smoothing was accounted for. The bounds represent one standard deviation and agree well with independent jackknife estimates. Standard deviations of fit parameters stated in the text were obtained by the delete-one jackknife applied to eight 15-year data bins.

**Artificial reservoir correction.** The reservoir correction derived by Chao *et al.*(4) was used in a simple analytical form:

\[
\Delta H = 1.65 \text{ cm} + (3.7 \text{ cm} / \pi) \arctan \left(\frac{t-1978}{13}\right),
\]
the integral of a Cauchy-Lorentz distribution approximating reservoir construction activity.

**Ground water extraction sensitivity analysis.** In the absence of a time series for this as we have for the artificial reservoir correction, we considered two educated guesses: a linear increase in mining rate leading to

\[ \Delta H = -\alpha (t - 1870)^2 / \tau, \]

with \( \alpha = 0.015 \text{ cm/a} \) and \( \tau = 132 \text{a} \); and an exponential increase leading to

\[ \Delta H = -\alpha \tau \exp \left( \frac{(t - 1870) / \tau}{\tau} \right) + \alpha (t - 1870) + \alpha \tau, \]

with \( \alpha = 0.00118 \text{ cm/a} \) and \( \tau = 40 \text{a} \). (Longer e-folding time scales do not affect the basic outcome.)

Both are designed to start from a rate of zero and produce a rate of 0.3 mm/a by 2000; the latter estimate is considered as “medium confidence”\(^5\).

<table>
<thead>
<tr>
<th>Scenario</th>
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<th>( b )</th>
<th>( r )</th>
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<td>0.992</td>
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<td>-5.0</td>
<td>0.989</td>
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**Table S1.** Ground water extraction effect on estimated parameters of fit. \( r \) is the Pearson correlation coefficient of fit.
The main effect is to reduce \( a \) by about 6-7%, well within its 1\( \sigma \) uncertainty, while the quality of fit as expressed by \( r \) remains largely unaffected. Note that if the ground water extraction function were strictly proportional to sea level rise, then the effect on quality of fit would vanish exactly.

Groundwater mining has two opposing effects on our future projections: it reduces the projections by affecting the parameters as shown in the table (which we call the *indirect* effect), and it increases the projections as groundwater mining is likely to continue (and increase) in future, adding to sea level rise (the *direct* effect).

At the upper limit of our projections, the indirect effect reduces the high-end of the A1FI scenario from 179.1 cm to 164.4 cm (linear case) or 165.6 cm (exponential case). If we assume mining continues to increase in future as in the linear case, then this would add another 4.4 cm over 1990-2100 (as compared to 1.9 cm over 1870-2000). A constant mining rate of 0.3 mm/a would add 3.3 cm, while a further exponential increase would add 13.8 cm. The latter number we consider unrealistically high. Adopting 4.4 cm as a plausible low estimate, the total high-end projection would be 168.8 cm, a reduction by 6%, again within our stated 7% 1\( \sigma \) uncertainty. A fully analogous analysis applies at the lower limit of our projections.

Hence we conclude that groundwater mining, although its contribution is rather uncertain, is likely to have only a minor effect on our projections.

**Akaike information criterion.** In order to compare the performance of both the original model, and the dual model, in reconstructing sea level over 1880-2000, we first computed the residual sum of squares (RSS) for both fit alternatives, re-centering to
eliminate the arbitrary integration constant. We used the Akaike Information Criterion (6) in its small-sample version:

$$\text{AIC}_c = 2K + n \ln \left( \frac{\text{RSS}}{n} \right) + 2K \frac{(K+1)}{(n-K-1)}.$$

Here $n=8$ is the sample size and $K$ the number of model parameters, including the intercept $T_0$, the integration constant $H(t_0)$, and the error variance. We find:

<table>
<thead>
<tr>
<th></th>
<th>Original model (a only)</th>
<th>Dual model (a and b)</th>
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</thead>
<tbody>
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<td>$K$</td>
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<td>5</td>
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<td>RSS (cm$^2$)</td>
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<td>AIC$_c$</td>
<td>5.11</td>
<td>4.75</td>
</tr>
</tbody>
</table>

The preferred model is that with the smallest AIC$_c$ value. Both models are statistically plausible, their Akaike weights being 55% and 45%. What this means is that the extra parameter in the dual model does not lead to overfitting. Model selection, as pointed out in (6), should not be based on statistics only, but also consider the physics of the process being modeled.

Akaike used in the above form assumes independently distributed errors for the models compared; its applicability becomes questionable (6) if reality differs strongly from this assumption. We performed an independence check on the residuals both from the original and from the dual fit model using the Ljung-Box test. It was found that the original model fit had significantly auto-correlated residuals, with a Ljung-Box statistic of 8.236 for three degrees of freedom, yielding $p=0.04$. The dual model fit, however, had its most significant Ljung-Box statistic, 5.232, at four DoF, for $p=0.26$. Figure S1 shows both the residuals of the two fits and their respective autocorrelation functions.
**Cross-validation.** A further test of the robustness of our result is using part of the data to predict the part that was withheld, as in Rahmstorf(7). We did this with the observational data 1880-2000, using first the first half (only four 15-year bins) and then the second half, to predict sea level for the other half. The results are included in Fig. 3; numerical results below. With only four calibration data points, performance is naturally degraded; even so, the error in sea level in the year 2000 is only 3 cm.

<table>
<thead>
<tr>
<th></th>
<th>$T_0$ (K)</th>
<th>$a$ (cm a$^{-1}$ K$^{-1}$)</th>
<th>$b$ (cm K$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Full data</strong></td>
<td>0.56</td>
<td>0.56</td>
<td>-4.9</td>
</tr>
<tr>
<td><strong>First half</strong></td>
<td>0.37</td>
<td>0.72</td>
<td>-6.2</td>
</tr>
<tr>
<td><strong>Last half</strong></td>
<td>-0.55</td>
<td>0.42</td>
<td>-3.2</td>
</tr>
</tbody>
</table>

A more powerful cross-validation, though for model-generated data, is that shown in Fig. 2, where calibration for 120 years is used to predict sea level changes over a millennium.

**Aliasing of $\tau$ and $b$.** To demonstrate this effect we start from the Taylor expansion up to the second time derivative of temperature:

$$\frac{dH}{dt} = a T (t+\tau) + b \frac{dT}{dt} (t+\tau) =$$

$$= a T (t) + (a \tau + b) \frac{dT}{dt} (t) + \left(\frac{1}{2} a \tau^2 + b \tau\right) \frac{d^2T}{dt^2}.$$

From this we see that the expansion in delayed temperature $T$ and temperature rate $dT/dt$, with coefficients $a$ and $b$, is equivalent to an expansion in undelayed $T$ and $dT/dt$,
but with coefficients $a$ and $b'=(a\tau+b)$, provided that the second-derivative term vanishes:

$$\left(\frac{1}{2} a\tau^2+br\right) = 0.$$  

This happens trivially for $\tau=0$, and nontrivially for $\tau=-2b/a$, for which $b'=-b$.

To test this hypothesis, we produced a plot (Figure S2) of Pearson correlation values for delayed dual model fits to the observation data, on the space of $(b,\tau)$ value pairs. In this computation we used only 105 years of data to accommodate the delay $\tau$. Inspection of the plot shows that there are indeed two equivalent maxima: one corresponding to $\tau=0$ and $b = -4.5$ cm K$^{-1}$, and the other to $\tau = -13$ years and $b = +4.5$ cm K$^{-1}$. We propose that the latter solution is the physically realistic one: a combination of a positive $b$ coefficient due to thermosteric ocean surface layer expansion, offset by a large delay of the response masquerading as a negative $b$ coefficient. This aliasing works only for certain ratios of $a$, $b$ and $\tau$, and only thanks to the powerful smoothing (15 years) used on the data, making further terms in the above Taylor expansion negligible.

**Millennium fits to other models.** We performed least-squares parameter fits to data generated by the models ECHO-G (8) and ECBilt-CLIO (9). Results are displayed in Figures S3 and S4.
List of Supplementary figures

**Figure S1.** (Left) sea level residuals after fit for the original (a only) model (red) and for the dual (a and b) model (blue). Drawn curves: annual residuals after re-centering. Bullets and dotted lines: 15-year binned values. (Right) sea level residuals autocorrelations, original (a only) model (red) and dual (a and b) model (blue).
Figure S2. Pearson correlation $r$ as a function of delay $\tau$ and coefficient $b$, computed for a dual model fit to sea level data truncated to 1895-2000.
**Figure S3.** Millennium fit to data from ECHO-G. Fit parameters: $a = 0.05 \text{ cm/a/K}$, $b = 2.2 \text{ cm/a}$, variance explained 66%.

**Figure S4.** Millennium fit to data from ECBilt-CLIO. Fit parameters: $a = 0.13 \text{ cm/a/K}$, $b = 1.8 \text{ cm/a}$, variance explained 65%.
List of supplementary tables

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$T_0$</th>
<th>a</th>
<th>b</th>
<th>r</th>
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<tbody>
<tr>
<td>none</td>
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<td>0.56</td>
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<td>0.50</td>
<td>-5.0</td>
<td>0.988</td>
</tr>
</tbody>
</table>

Table S1. Ground water extraction effect on estimated parameters of fit. $r$ is the Pearson correlation coefficient of fit.

References


