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Quantum backscatter communication with photon number states

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Abstract—Backscattered signals are always obscured by the unavoidable channel noise. However, by exploiting quantum physics recent protocols had been developed to enhance the probability of detecting backscattered signals in a very noisy environment [1], [2]. In this paper we propose a new detection scheme that is simpler in nature than the sum frequency receiver that was proposed for the quantum illumination protocol [3]. Signals are generated using spontaneous parametric down conversion (SPDC) and are transmitted via the simple modulation technique of on-off keying (OOK), while the receiver design will rely upon the purely non-classical Hong-Ou-Mandel (HOM) effect.

Index Terms—Quantum Metrology, Quantum Backscatter Communication(QBC), Entanglement, Hong-Ou-Mandel (HOM) effect, N00N state.

I. INTRODUCTION

The need to transmit very weak signals over noise contaminated channels is crucial to the successful implementation of backscatter communication systems. Nonetheless, the detection of these fragile signals is limited classically by shot noise [4]. Such deficit can be redeemed by engineering communication systems that are governed by the laws of quantum physics [5]. Quantum entanglement, whereby distant systems exhibit non-local measurement correlations, holds the keys to realizing precision measurements that go beyond the classical shot noise limit. By harnessing these non-classical correlations recent protocols had been developed to detect targets submersed in noisy and lossy environment [6], while recently the same concept was deployed to detect backscattered communication signals in the microwave regime [1], [2]. Moreover, it had been shown that target detection using entangled photon pairs is optimal under the same environmental assumptions [6]–[8].

The concept of quantum backscatter communication (QBC) bears a great resemblance to that of the quantum radar [9], since the objective of both systems is to detect faint signals in the presence of overwhelming noise. Accordingly, protocols designed for one of the two systems can be used for the other as well.

Building upon these efforts, the goal of this article is to take advantage of quantum entanglement to enhance the detection probability of microwave backscattered signals.

Fig. 1. Quantum backscattering using photon number states.

The most authentic way to generate entangled photon pairs is the process of spontaneous parametric down-conversion (SPDC) [10], by which a strong pump photon is converted into a pair of lower energy ones. These downconverted photons are usually denoted as the *signal* and the *idler*. The generated pairs are characterized by being physically indistinguishable, on top of that, they display measurement correlations in a manner that can not be explained by classical physics.

Utilizing this valuable resource, our transmitter will be a microwave entanglement generator, where the signal photon is sent to interact with the backscatter antenna, while the idler one will be retained at the receiver. Nevertheless, the probing intervals are limited by the ability of our source to generate independent entangled photon pairs, where for the chosen frequency range the number of generated modes is at most $\approx 10^5$ [1].

To make things practical, we need an efficient receiver architecture that is able to detect entangled photon pairs. Using a simple balanced beamsplitter we can achieve this goal by exploiting the non-classical Hong-Ou-Mandel (HOM) phenomenon [11]. When two indistinguishable photons impinge on a balanced beamsplitter from two different ports, they coalesce and exit the beamsplitter from the same port. Experimentally this can be verified by counting coincidence events. The two physical processes that lead to a coincidence count are either a reflection-reflection or a transmission-transmission. On a balanced beamsplitter these two processes destructively interfere as they become indistinguishable, thus leading to a zero coincidence rate. This result was first demonstrated by Hong, Ou and Mandel [11]. Recently, a similar effect was witnessed in the microwave domain [12], consequently, we can take advantage of this result to design our receiver, in the

sense that, we first receive the backscattered signal photon, then we match it with the retained idler. After that we make the decision based on the coincidence statistics. The two photon interference effect that takes place inside the HOM receiver is very similar to the idea of matched filtering [13]. We believe that this receiver architecture is advantageous over the sum frequency receiver [3] due to its simplicity.

Assuming a Rayleigh-fading backscatter channel having uniformly distributed phase offset, we will only consider probing our system with photon number states, since all phase information are typically randomized in a Rayleigh fading environment, besides, the sum-frequency receiver of quantum illumination offers no performance advantage for Rayleigh-fading targets [14]. Moreover, unlike Gaussian states, photon number states are characterized by being orthogonal, hence they are easier to discriminate.

So, this paper is organized as follows. Sec.II is devoted to theory and model description. In Sec.III, we discuss the performance of the proposed model. Sec.IV will be our conclusion.

II. MODEL DESCRIPTION

Figure 1 depicts the main building blocks of our backscatter communication system. At the transmitter we consider an entanglement source that is able to continuously generate correlated photon pairs, where each pump photon is disintegrated into a signal and idler ones. The down-converted photons conserve both energy and momentum [10]

$$\hbar\omega_{pump} = \hbar\omega_s + \hbar\omega_i \quad (1)$$

$$\hbar k_{pump} = \hbar k_s + \hbar k_i$$

where $\hbar\omega_{pump}$, $\hbar\omega_s$, $\hbar\omega_i$ are the pump, signal and idler energies respectively, while $\hbar k_{pump}$, $\hbar k_s$, $\hbar k_i$ are their associated momenta. The previous relations suggest that a multitude of different photon pairs can satisfy the conservation conditions [10]. The output two-photon entangled state can be written as [15], [16]

$$|\psi\rangle = \int \mathcal{M}(\omega_s, \omega_i) \hat{a}^\dagger(\omega_s) \hat{a}^\dagger(\omega_i) |0_s, 0_i\rangle d\omega_s d\omega_i. \quad (2)$$

where $\hat{a}^\dagger(\omega_s)$, $\hat{a}^\dagger(\omega_i)$ are the creation operators of the signal and idler modes respectively, $|0_s, 0_i\rangle$ is the signal and idler two mode vacuum state, and $\mathcal{M}(\omega_s, \omega_i)$ is the two-photon spectral amplitude [15]. According to energy conservation, $\mathcal{M}(\omega_s, \omega_i)$ can be written as [16]

$$\mathcal{M}(\omega_s, \omega_i) \approx \delta(\omega_{pump} - (\omega_s + \omega_i))g(\nu). \quad (3)$$

where $\nu = \frac{\omega_s - \omega_i}{2}$, $\omega_s = \omega_0 + \nu$, $\omega_i = \omega_0 - \nu$, $\omega_0 = \frac{\omega_{pump}}{2}$, and $g(\nu)$ is the spectral density function of the joint two-photon state [16].

The two-photon coherence time τ_2 is determined by the width $\Delta\nu$ of the power spectrum $|g(\nu)|^2$

$$\tau_2 \approx \frac{1}{\Delta\nu} \quad (4)$$

Turning our attention to the HOM effect, for a given value of ν , coincidence counts will only take place if and only if the two input photons are both transmitted or reflected. Hence, the corresponding probability amplitudes will be [16]

$$\mathcal{A}_{tt} = t^2 g(\nu) e^{i\phi_{tt}(\nu)}. \quad (5)$$

and

$$\mathcal{A}_{rr} = r^2 g(\nu) e^{i\phi_{rr}(\nu)}. \quad (6)$$

where $\mathcal{A}_{tt}$, $\mathcal{A}_{rr}$ are the amplitudes of the transmission-transmission and reflection-reflection processes respectively, t and r are the complex transmission and reflection coefficients of the beamsplitter and $\phi_{tt}(\nu)$, $\phi_{rr}(\nu)$ are respectively the phases of the aforementioned processes.

Then at a balanced beamsplitter, the amplitudes transform as follows

$$\mathcal{A}_{tt} = \frac{1}{2} g(\nu) e^{i\phi_{tt}(\nu)}. \quad (7)$$

and

$$\mathcal{A}_{rr} = \frac{-1}{2} g(\nu) e^{i\phi_{rr}(\nu)}. \quad (8)$$

Therefore the total probability of getting a coincidence click is [16]

$$\begin{aligned} \mathcal{P}_{coinc} &= \int |\mathcal{A}_{tt}(\nu) + \mathcal{A}_{rr}(\nu)|^2 d\nu. \\ &= \int |g(\nu)|^2 \sin^2\left(\frac{\Delta\phi(\nu)}{2}\right) d\nu. \end{aligned} \quad (9)$$

where

$$\Delta\phi(\nu) = \phi_{tt}(\nu) - \phi_{rr}(\nu) \quad (10)$$

The propagation phases $\phi_{tt}(\nu)$, $\phi_{rr}(\nu)$ depend on the down-converted frequencies and the distances covered by the two photons. With regard to Fig.1 we can define the total distances travelled by the signal and idler photons from their point of creation till hitting the coincidence counter as follows; first for the transmission-transmission process we have

$$\begin{aligned} \mathcal{L}_{stot}^{tt} &= \mathcal{L}_s + \mathcal{L}_2 \\ \mathcal{L}_{itot}^{tt} &= \mathcal{L}_i + \mathcal{L}_1 \end{aligned} \quad (11)$$

where $\mathcal{L}_s$, $\mathcal{L}_i$ are the distances taken by the signal and idler photons from their point of creation till the beamsplitter interface, while $\mathcal{L}_2$, $\mathcal{L}_1$ are the distances from the beamsplitter to the detectors D_2 and D_1 respectively. As for the reflection-reflection process we have

$$\begin{aligned} \mathcal{L}_{stot}^{rr} &= \mathcal{L}_s + \mathcal{L}_1 \\ \mathcal{L}_{itot}^{rr} &= \mathcal{L}_i + \mathcal{L}_2 \end{aligned} \quad (12)$$

where distances are defined in exactly the same manner as equation (11).

Accordingly, we can write the phase of the transmission-transmission processes as

$$\phi_{tt}(\nu) = k_s \mathcal{L}_{stot}^{tt} + k_i \mathcal{L}_{itot}^{tt} \quad (13)$$

where $k_s = \frac{\omega_s}{c}$, $k_i = \frac{\omega_i}{c}$ are the wave vectors of the signal and idler photons respectively and 'c' is the photon speed. While in the reflection-reflection process the input beams exit the beamsplitter in different modes, hence the phase change corresponding to this process is written as

$$\phi_{rr}(\nu) = k_s \mathcal{L}_{i_{tot}}^{rr} + k_i \mathcal{L}_{s_{tot}}^{rr} \quad (14)$$

Writing equations (13) and (14) as a function of the signal and idler frequencies, then substituting the resulting expressions into equation (10) gives us [16]

$$\Delta\phi(\nu) = 2\nu \frac{\Delta\mathcal{L}}{c} \quad (15)$$

where $\Delta\mathcal{L} = \mathcal{L}_s - \mathcal{L}_i$. In terms of the time delay between the two photons Δt we can write the previous expression as

$$\Delta\phi(\nu) = 2\nu\Delta t. \quad (16)$$

Therefore, the final form of the coincidence probability will be

$$\mathcal{P}_{coinc} = \int |g(\nu)|^2 \sin^2(\nu\Delta t) d\nu. \quad (17)$$

Assuming a Gaussian spectral density function $g(\nu)$, where $\tau_2 \approx \frac{1}{\Delta\nu}$ is the two photon coherence time, the coincidence probability can be evaluated as follows

$$\begin{aligned} \mathcal{P}_{coinc} &= \frac{\tau_2}{\sqrt{\pi}} \int e^{-\tau_2^2 \nu^2} \sin^2(\nu\Delta t) d\nu \\ &= \frac{1}{2} \frac{\tau_2}{\sqrt{\pi}} \int e^{-\tau_2^2 \nu^2} d\nu - \frac{1}{2} \frac{\tau_2}{\sqrt{\pi}} \int e^{-\tau_2^2 \nu^2} \cos(2\nu\Delta t) d\nu \\ &= \frac{1}{2} - \frac{1}{2} \frac{\tau_2}{\sqrt{\pi}} \Re \left(\int e^{-\tau_2^2 \nu^2} e^{2i\nu\Delta t} d\nu \right) \\ &= \frac{1}{2} - \frac{1}{2} e^{-\left(\frac{\Delta t}{\tau_2}\right)^2} \end{aligned} \quad (18)$$

Plotting the coincidence probability against the time delay between the signal and idler photons gives the famous HOM dip [11]

Fig. 2. The coincidence probability as a function of the time delay between the two photons.

As can be seen in Fig.2, a null coincidence probability occurs when $\mathcal{L}_s = \mathcal{L}_i$. This means that we should compensate the introduced delay due to the interaction of the signal photon

with both the propagation channel and the backscatter antenna in order to preserve the two photon indistinguishability, otherwise, the two photons will become path-distinguishable and each one of them will behave independently at the balanced beamsplitter. Therefore, based on the coincidence statistics our HOM receiver will have the ability to decide whether to acknowledge the successful receipt of the signal photon or not. The generated two-photon state after the HOM interference effect is expressed as

$$|\gamma\rangle_{N00N} = \frac{1}{\sqrt{2}}(|2n_s, 0_i\rangle + |0_s, 2n_i\rangle) \quad (19)$$

such that $n = (1, 2, ..etc.)$.

For $n = 1$ this state will be denoted as the two-photon N00N state, where all the involved photons bunch into either of the two available modes. In general, the average number of photons per each mode is

$$\langle \hat{a}^\dagger \hat{a} \rangle_s = \langle \hat{a}^\dagger \hat{a} \rangle_i = n \quad (20)$$

where $\hat{a}^\dagger \hat{a}$ is the number operator.

III. PERFORMANCE ANALYSIS

In this section we describe briefly our channel model and explore the limits and practicality of our receiver design.

A. Channel Model

The dynamical evolution of backscattered signal photons inside a propagation channel can be modeled in terms of a simple beamsplitter transformation [17], such that the signal is mixed with the background thermal bath

$$\hat{a}_{\text{input signal}} = \sqrt{\alpha} \hat{a}_{\text{output signal}} + \sqrt{1 - \alpha} \hat{a}_{\text{Thermal noise}} \quad (21)$$

where $\hat{a}$ is the annihilation operator of the respective fields and α is the transmission efficiency of the channel. In light of this simple model we assumed that the backscattered signal is either transmitted or totally replaced by environment noise. This assumption is a bit pessimistic, since in real transmission scenarios there is a significant chance that a fraction of the supposedly lost signal is received despite the presence of tremendous noise. Furthermore, we will consider a very simple receiver design, such that our detectors will not be equipped with a noise filtering mechanism¹. This is quite different from Lloyd's original proposal [6] where he considered band-limiting the detection window.

B. Practical Limits

In classical decision theory we usually employ the Chernoff bound to set an upper limit to error probability.

Since states of an open quantum system are best described as density operators [16], a quantum version of the Chernoff bound had been considered to discriminate between different quantum states [19]. In the present analysis the probability that our HOM receiver makes a wrong decision with regard to

¹Our approach can be related to Von Neuman's in his famous paper on probabilistic logic [18], where he considered the possibility of building a functioning computing machine out of unreliable components.

two possible states $\hat{\rho}_0$ & $\hat{\rho}_1$ is upper bounded by the quantum Chernoff bound (QCB)

$$\mathcal{P}_{\text{HOM}} \leq \mathcal{P}_{\text{QCB}} = \frac{1}{2} \inf_{0 \leq s \leq 1} \text{Tr}[\hat{\rho}_0^s \hat{\rho}_1^{1-s}] \quad (22)$$

and for M copies of the quantum states the previous relation can be generalized to

$$\mathcal{P}_{\text{HOM}} \leq \mathcal{P}_{\text{QCB}} = \frac{1}{2} \left(\inf_{0 \leq s \leq 1} \text{Tr}[\hat{\rho}_0^s \hat{\rho}_1^{1-s}] \right)^M \quad (23)$$

Assuming a channel corrupted by thermal noise, the optimum is attained at $s = \frac{1}{2}$ [20]. Hence, the two different states to be discriminated by the HOM receiver are

$$\begin{aligned} \hat{\rho}_0 &= \hat{\rho}_{\text{Th}}(\bar{n}_{\text{Th}}) \otimes \text{Tr}_{\text{signal}}[|\gamma\rangle \langle \gamma|_{N00N}] \\ &= \hat{\rho}_{\text{Th}}(\bar{n}_{\text{Th}}) \otimes \frac{1}{2} (|2n\rangle \langle 2n| + |0\rangle \langle 0|)_{\text{idler}} \\ \hat{\rho}_1 &= \eta |\gamma\rangle \langle \gamma|_{N00N} + (1 - \eta) \hat{\rho}_0 \end{aligned} \quad (24)$$

where $\hat{\rho}_{\text{Th}}(\bar{n}_{\text{Th}}) = \sum_{l=0}^{\infty} \frac{\bar{n}_{\text{Th}}^l}{(1 + \bar{n}_{\text{Th}})^{1+l}} |l\rangle \langle l|$ is the thermal state, $\text{Tr}_{\text{signal}}[|\gamma\rangle \langle \gamma|_{N00N}]$ is the partial trace operation over the signal basis, while $\bar{n}_{\text{Th}} = \left[\exp(\frac{-\hbar\omega}{k_b T}) [1 - \exp(\frac{-\hbar\omega}{k_b T})]^{-1} \right]$ is the average number of photons, such that $\hbar$ is Plank's constant, ω is the photon frequency, k_b is Boltzman's constant and T is the temperature in Kelvins.

The first state arises when the backscatter antenna is not transmitting, where in this situation the signal photon is considered lost and the HOM receiver will only mix the retained idler with thermal noise. The second state is received when the backscatter antenna is transmitting, however the HOM receiver will only be able to detect the entangled N00N state with probability η , while with probability $1 - \eta$ it will receive thermal noise. The parameter η reflects the transmission efficiency of our channel, where a noisy and lossy channel can be modeled as a simple beamsplitter with $\sqrt{\eta}$ as its transmission coefficient [17].

We are now ready to calculate the error probability according to the optimized version of equation (22)

$$\begin{aligned} \mathcal{P}_{\text{QCB}} &= \frac{1}{2} \left(\text{Tr}[\hat{\rho}_0^{\frac{1}{2}} \hat{\rho}_1^{\frac{1}{2}}] \right)^M \\ &= \frac{1}{2} \left((\text{Tr}[\hat{\rho}_0^{\frac{1}{2}} (\eta \hat{\rho}_{N00N})^{\frac{1}{2}} + ((1 - \eta) \hat{\rho}_0)^{\frac{1}{2}} (\hat{\rho}_0)^{\frac{1}{2}}]) \right)^M \\ &= \frac{1}{2} \left[\sqrt{\frac{\eta e^{\frac{-n\hbar\omega}{k_b T}} (1 - e^{\frac{-\hbar\omega}{k_b T}})}{2}} \cosh\left(\frac{n\hbar\omega}{2k_b T}\right) + \sqrt{1 - \eta} \right]^M \\ \mathcal{P}_{\text{HOM}} &\leq \\ \mathcal{P}_{\text{QCB}} &= \frac{1}{2} \left[\sqrt{\frac{\eta e^{\frac{-n\hbar\omega}{k_b T}} (1 - e^{\frac{-\hbar\omega}{k_b T}})}{2}} \cosh\left(\frac{n\hbar\omega}{2k_b T}\right) + \sqrt{1 - \eta} \right]^M \end{aligned} \quad (25)$$

where in the second line we made use of the fact that the number states are orthogonal, hence we can write $\hat{\rho}_1$ in the following diagonal representation $\sum_i |\psi\rangle_i \langle \psi|_i$, consequently taking the square root of $\hat{\rho}_1$ is now feasible.

As pointed out earlier in this article, in the presence of very high noise, using entangled photon pairs to probe the backscatter antenna is advantageous over direct probing with uncorrelated ones.

To witness this advantage we now proceed with calculating the error probability in the case of direct transmission with non-entangled number states. The two possible states in this situation read off as

$$\begin{aligned} \hat{\rho}_0 &= \hat{\rho}_{\text{Th}}(\bar{n}_{\text{Th}}) \\ \hat{\rho}_1 &= \eta |n\rangle \langle n| + (1 - \eta) \hat{\rho}_0 \end{aligned} \quad (26)$$

where these two states arise by following exactly the same reasoning that was developed for the case of N00N states. Therefore the optimized error probability will be

$$\begin{aligned} \mathcal{P}_{\text{QCB}} &= \frac{1}{2} (\text{Tr}[\hat{\rho}_0^{\frac{1}{2}} \hat{\rho}_1^{\frac{1}{2}}])^M \\ &= \frac{1}{2} ((\text{Tr}[\hat{\rho}_0^{\frac{1}{2}} (\eta |n\rangle \langle n|)^{\frac{1}{2}} + ((1 - \eta) \hat{\rho}_0)^{\frac{1}{2}} (\hat{\rho}_0)^{\frac{1}{2}}])^M \\ \mathcal{P}_{\text{QCB}} &= \frac{1}{2} \left[\sqrt{\eta (1 - e^{\frac{-\hbar\omega}{k_b T}})} \sqrt{e^{\frac{-n\hbar\omega}{k_b T}}} + \sqrt{1 - \eta} \right]^M \\ \mathcal{P}_{\text{HOM}} &\leq \\ \mathcal{P}_{\text{QCB}} &= \frac{1}{2} \left[\sqrt{\eta (1 - e^{\frac{-\hbar\omega}{k_b T}})} \sqrt{e^{\frac{-n\hbar\omega}{k_b T}}} + \sqrt{1 - \eta} \right]^M. \end{aligned} \quad (27)$$

Assuming a very low signal-to-noise ratio and an extremely small transmission efficiency η , Fig.3 shows that entangled photon pairs perform better in a lossy environment, since part of the information about the signal photon is encoded in its joint probability distribution with the idler and can be exploited at the receiver despite the presence of extreme noise. On the other hand, using uncorrelated photon number states is less advantageous due to the absence of correlation information at the receiver, hence there is no compensation for the extreme loss caused by the environment, however when we increased the signal to noise ratio it can be seen that the uncorrelated number states outperform the entangled ones.

Fig.4 displays the performance of the entangled N00N states under different environment temperatures, where for a fixed number of photons, increasing the temperature decreased the signal to noise ratio and consequently a better performance was achieved.

In Fig.5 we again assume the same specifications of Fig.3 but with an increased channel efficiency, this had the major consequence of decreasing the number of photon pairs needed to replicate the results of Fig.3

To summarize, the performance advantage of entangled N00N states over uncorrelated number states is always granted by the successful generation and measurement of a large number of entangled pairs, which is a major challenge, while the transmission efficiency of the channel will dictate the number of these pairs to achieve a desirable error limit.

Fig. 3. Bit error probability for both a number state and an entangled N00N state of 20 photons at 5.5 GHZ, where $\eta \approx 0.043$. The average number of noisy microwave photons is equal to that of room temperature. The advantage of the N00N states was lost when we increased the number of photons to 300.

Fig. 4. Bit error probability for N00N states at different temperatures.

Fig. 5. Bit error probability for both number states and entangled N00N states with the same specifications as Fig.3 but with different $\eta \approx 0.1$.

C. Feasibility Of The Detection Process

In this section we draw a comparison between our detection mechanism and that of the sum frequency receiver, in such a way that more attention will be paid to the inner physics of each approach. In the HOM receiver the detection procedure is reckoned by establishing a two-photon interference effect that can be manifested by using a lossless balanced beamsplitter. Such a beamsplitter transformation $\hat{U}$ can be described quantum mechanically by the following linear interaction Hamiltonian [21], [22]:

$$\hat{U} = \exp \theta (\hat{a}_s^\dagger \hat{a}_i - \hat{a}_s \hat{a}_i^\dagger) \quad (28)$$

where $\hat{a}_s^\dagger(\hat{a}_s)$, $\hat{a}_i^\dagger(\hat{a}_i)$ are the creation (annihilation) operators of the signal and idler modes respectively, while θ is the beamsplitter mixing angle, such that, θ is equal to $\frac{\pi}{4}$ for a balanced beamsplitter. From a phenomenological perspective, a quantum beamsplitter creates photons in one mode while simultaneously destroys them in the other. The previous beamsplitter operator is unitary $\hat{U}^\dagger \hat{U} = \hat{I}$, where $\hat{I}$ is the identity operator, furthermore it conserves the total number of photons,

$$\hat{U}(\hat{a}_s^\dagger \hat{a}_s + \hat{a}_i^\dagger \hat{a}_i) \hat{U}^\dagger = \hat{a}_s^\dagger \hat{a}_s + \hat{a}_i^\dagger \hat{a}_i \quad (29)$$

where $\hat{a}_s^\dagger \hat{a}_s$, $\hat{a}_i^\dagger \hat{a}_i$ are the number operators of the signal and (idler) modes respectively. Hence, we can conclude that the HOM receiver is a unitary linear receiver that preserves the total number of photons. On the other hand, the interaction Hamiltonian describing the sum frequency receiver is expressed as [2], [3] :

$$\mathcal{H}_{int} = \hbar g (\hat{a}_{pump}^\dagger \hat{a}_s \hat{a}_i + \hat{a}_{pump} \hat{a}_s^\dagger \hat{a}_i^\dagger) \quad (30)$$

where $\hat{a}_{pump}^\dagger(\hat{a}_{pump})$ are the creation (annihilation) operators of the pump photon respectively, while $\hat{a}_s^\dagger(\hat{a}_s)$ and $\hat{a}_i^\dagger(\hat{a}_i)$ are the same as defined in eqn.(28). The parameter 'g' is the so called nonlinear coupling strength, which suggests the need for a material that exhibits a kerr non-linearity in order to realize the Hamiltonian of eqn.(30), by which, a pump photon will be created at the expense of annihilating a signal and an idler ones. Besides the fact that kerr non-linearities are typically very weak [23], the aforementioned Hamiltonian doesn't conserve the total number of photons as it doesn't commute with the total number operator $\hat{\mathcal{N}}_{tot} = \hat{a}_{pump}^\dagger \hat{a}_{pump} + \hat{a}_s^\dagger \hat{a}_s + \hat{a}_i^\dagger \hat{a}_i$

$$[\mathcal{H}_{int}, \hat{\mathcal{N}}_{tot}] = \hbar g (\hat{a}_{pump}^\dagger \hat{a}_s \hat{a}_i - \hat{a}_{pump} \hat{a}_s^\dagger \hat{a}_i^\dagger). \quad (31)$$

Putting these together, it can be said that the physics of the sum frequency receiver is more complicated than that of the HOM receiver. Furthermore, the channel design of the HOM receiver doesn't require a phase stabilization mechanism, since in this case we would be probing the backscatter antenna with photon number states. Contrarily, the sum frequency receiver channel needs to be phase-stabilized first, as in such case the backscatter antenna would be probed with coherent squeezed

states, thus adding more complex arrangements to ensure the success of the protocol. To conclude, the linearity of the HOM receiver holds a great potential for realizing quantum backscatter communication and is indeed worth pursuing for the simplicity it displays.

IV. CONCLUSION

Inspired by the recent successes of quantum physics in metrology and communication, in this paper we proposed a receiver architecture built upon the HOM interference effect. Our design is easily implementable compared to Quantum illumination's sum-frequency receiver. The proposed model showed an enhanced error probability over direct transmission in noisy and lossy environment, which is the usual setting of a backscatter communication system. Despite the innocuousness of our design, the whole detection procedure relies upon a very intricate process that is extremely sensitive to time delays. Such limitation, could pose a real threat to the success of our protocol. Solutions to this problem will be considered when we physically realize our model.

APPENDIX

In this appendix we derive equation (25). A similar approach can be used to derive (27) as well. In equation(25), the linearity of the trace operation implies that we can calculate each term separately, hence for the first term we have

$$\begin{aligned} & \text{Tr}[\hat{\rho}_0^{\frac{1}{2}}(\eta\hat{\rho}_{N00N})^{\frac{1}{2}}] \\ &= \sqrt{\eta} \sum_{l=0}^{\infty} \left(\frac{\bar{n}_{\text{Th}}^l}{(1+\bar{n}_{\text{Th}})^{1+l}} \right)^{\frac{1}{2}} |l\rangle \langle l| \otimes \\ & \quad \frac{1}{\sqrt{2}} (|2n\rangle \langle 2n| + |0\rangle \langle 0|)_{id} [\hat{\rho}_{N00N}] \end{aligned}$$

and since $\hat{\rho}_{N00N}$ is a pure state

$$\begin{aligned} &= \sqrt{\eta} \langle \gamma |_{N00N} \sum_{l=0}^{\infty} \left(\frac{\bar{n}_{\text{Th}}^l}{(1+\bar{n}_{\text{Th}})^{1+l}} \right)^{\frac{1}{2}} |l\rangle \langle l| \\ & \quad \otimes \frac{1}{\sqrt{2}} (|2n\rangle \langle 2n| + |0\rangle \langle 0|)_{id} |\gamma_{N00N}\rangle \end{aligned}$$

from the orthogonality of number states

$$\begin{aligned} &= \left[\frac{\sqrt{\eta}}{2\sqrt{2}} \left(\frac{\bar{n}_{\text{Th}}^n}{(1+\bar{n}_{\text{Th}})^{\frac{1}{2}+n}} \right) + \left(\frac{1}{(1+\bar{n}_{\text{Th}})^{\frac{1}{2}}} \right) \right] \\ &= \frac{\sqrt{\eta}}{2\sqrt{2}(1+\bar{n}_{\text{Th}})} \left[1 + \left(\frac{\bar{n}_{\text{Th}}}{(1+\bar{n}_{\text{Th}})} \right)^n \right] \\ &= \frac{(\bar{n}_{\text{Th}})^{\frac{n}{2}} \sqrt{\eta}}{(1+\bar{n}_{\text{Th}})^{\frac{n+1}{2}} \sqrt{2}} \\ & \quad \left[\frac{1}{2} \left(\frac{(1+\bar{n}_{\text{Th}})}{\bar{n}_{\text{Th}}} \right)^{\frac{n}{2}} + \left(\frac{(1+\bar{n}_{\text{Th}})}{\bar{n}_{\text{Th}}} \right)^{\frac{-n}{2}} \right] \end{aligned}$$

$$\text{then by writing } \bar{n}_{\text{Th}} \text{ as } \left[\exp\left(\frac{-\hbar\omega}{k_b T}\right) [1 - \exp\left(\frac{-\hbar\omega}{k_b T}\right)]^{-1} \right]$$

we get the final form of equation (25)

$$= \left[\sqrt{\frac{\eta e^{\frac{-n\hbar\omega}{k_b T}} (1 - e^{\frac{-\hbar\omega}{k_b T}})}{2}} \cosh\left(\frac{n\hbar\omega}{2k_b T}\right) \right]$$

while the second part of the trace, $\text{Tr}[\hat{\rho}_0^{\frac{1}{2}}((1-\eta)\hat{\rho}_0)^{\frac{1}{2}}]$, yields $\sqrt{1-\eta}$ since the infinite sum over the the thermal states converges to unity.

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