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Additional DoF in Cooperative Radar-Communications Systems

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Abstract—A novel cooperative radar-communications system is proposed in this paper, where the communications base station acts as a bistatic radar receiver. Thanks to the cooperation between the two systems, the radar can benefit from an additional target channel. It is shown, using both analytical results and simulations, that the radar can improve its estimation performance in the proposed scenario. Radar waveform optimization based on Cramér-Rao Bound minimization is also presented in this paper. Using the obtained power allocation solutions it is shown that in the proposed cooperative scenario the radar gains an additional degree of freedom for optimally using its power budget.

I. INTRODUCTION

The need for spectrum sharing and coexistence among radar and wireless communications systems was pointed out in [1]. Proposals regarding a shift in spectrum regulation by the US spectrum regulator FCC have been made already [2]. The coexistence problem has also been considered in the research community due to a considerable number of wireless systems that may have to operate in the same spectral bands, for example: LTE, 5G, WiFi, Citizens Broadband Radio Service and S-band radars. A review on coexistence among radar and communications systems as well as a view on the future of these systems is presented in [3]. Many have focused their efforts on proposing coexistence methods, for example [4]–[18]. As pointed out in [3], two main approaches are considered in the literature: addressing only the coexistence problem and considering cooperation between the two systems as well. In a cooperative scenario, there is some exchange of information between the two systems for their mutual benefit. Various radar waveform optimization strategies have been proposed for radar-communications coexistence, for example [3], [19]–[25]. Also, different criteria have been used for radar waveform optimization, for example SINR at the radar receiver [19], mutual information (MI) [6], [26], or probability of detection [7].

In this paper a cooperative radar-communications system is considered where the communications base station (BS) is capable of receiving and processing radar emissions. This resembles the joint multiple access channel (MUDR) coexistence topology identified in [3] and considered in [27], where the radar simultaneously acts as a communications receiver. Nevertheless, in this paper the communications BS acts also as a bistatic radar receiver, which can help the radar to improve its target parameter estimation performance. In particular, target time delay estimation is considered in this paper. Cramér-Rao Bound (CRB) for target time delay estimation is analytically derived for the proposed cooperative scenario and shown to be lower than the one for the case when the radar operates alone. This is due to the additivity property of the Fisher Information (FI). Using simulations it is shown that also the actual estimation error in terms of Root Mean Squared Error (RMSE) is lower in the cooperative scenario. Radar waveform optimization based on CRB minimization in the proposed cooperative scenario is also proposed in this paper. The obtained power allocation solutions demonstrate that in this scenario the radar allocates its power based on the quality of two channels, rather then just one, as in the case of a radar operating alone. This provides an additional degree of freedom (DoF) for the radar to optimally use its power budget.

II. SYSTEM MODEL AND PROCESSING

A cooperative radar-communications system scenario is considered in this paper. Both systems operate in the same bandwidth and both use OFDM waveforms with \( N \) subcarriers. Moreover, it is assumed that the communications system BS is able to receive and process radar emissions. The goal of the radar is to improve its target time delay estimation performance with the help from the BS which acts as a bistatic receiver. This provides an additional DoF for the radar to optimally use its power budget. The system model considered in this paper is introduced in Fig. 1. The two channels through which the target is observed are called in this paper the monostatic and the bistatic channels. On the monostatic channel the radar signal is reflected off the target back at the radar, while on the bistatic one it is reflected at the BS. The radar signal reflected off the target also interferes with the user of the communication system. The exchange of information between the two systems is done through a...
connecting link. Among others, for example in this paper, the transmitted radar signal and the measurements performed by the BS.

The frequency domain signal received at the radar can be expressed using a matrix notation as:

\[ y_{\text{rad}} = y_m + v, \]

where \( y_m \) is the radar signal received on the monostatic channel and \( v \) is the radar receiver noise. All are vectors of size \( N \), the number of subcarriers in the OFDM signal. The signal received at the BS is composed of the radar signal reflected off the target as well as the uplink transmission from the communications users. Consequently, it can be expressed as:

\[ y_{\text{BS}} = y'_b + y_{\text{com}} + w, \]

where \( y'_b \) is the radar signal received on the bistatic channel, \( y_{\text{com}} \) represents the uplink communications signals and \( w \) the BS receiver noise. It is assumed that the BS can employ Successive Interference Cancellation (SIC) [9], [27], [28]. This approach is applicable if the transmitted communications and radar signals are digital, with the radar symbols drawn from a finite alphabet, for example polyphase P3 and P4 codes. It is assumed that the systems are cognitive and constantly sense and learn their environments with the purpose of adapting their transmission based on a certain goal. Also, in this cooperative scenario the transmitted radar signal is available at the BS through the connecting link illustrated in Fig. 1. Consequently, the bistatic radar signal can be reconstructed at the BS based on a predicted delay, similarly as considered in [9], [27]. The reconstructed radar target return is removed from the signal at the BS and the communication signal, free from radar interference, is obtained.

The geometry of the cooperative system, in terms of the delays experienced by the radar signals, is presented in Fig. 2. It is considered that \( \tau \) is the round-trip delay from the radar to the target, while \( \tau_b = \tau/2 + \tau_x \) is the delay experienced by the radar signal reflected off the target that arrives at the BS. Also, \( \tau_d \) is the delay corresponding to a direct path transmission between the radar and the BS. It was shown in [11], based on [29], that the following is true:

\[ \tau_x = \frac{\tau_b^2 - 2\tau_b\tau_d \sin \alpha - \tau_d^2}{2(\tau_b - \tau_d \sin \alpha)}, \]

where the angle \( \alpha \) represents the complement of the angle that the radar beam is making with the horizontal axis. As \( \tau_b \) is used for the reconstruction of the radar signal, \( \tau_d \) and angle \( \alpha \) are assumed known, also \( \tau_x \) can be obtained. This delay can be compensated for in the reconstructed bistatic radar target return. Consequently, the reconstructed radar signal at the BS can be written as:

\[ y'_{\text{rec}} = y'_b + e' = R\Gamma'_b h_b + e', \]

where \( R \) is an \( N \times N \) diagonal matrix containing the vector \( r \) of frequency domain transmitted radar symbols, \( \Gamma'_b \) is an \( N \times N \) diagonal matrix containing the phase shifts of each subcarrier due to the target delay, \( h_b \) is the \( N \times 1 \) frequency response vector of the target on the bistatic channel and \( e' \) is the \( N \times 1 \) residual error vector associated with the reconstruction process. The diagonal matrix \( \Gamma'_b \) is given by \( \Gamma'_b{k,k} = \exp(-j2\pi k\Delta f(\tau/2 + \tau_x)), \) with \( k = 0 \ldots N - 1 \) the subcarrier index and \( \Delta f \) the intercarrier spacing. After compensating for \( \tau_x \) by multiplying the reconstructed signal in (4) with an \( N \times N \) diagonal matrix whose elements are given by \( [A_{\text{comp}}]_{k,k} = \exp(j2\pi k\Delta f \tau_x) \) for \( k = 0 \ldots N - 1 \), the following is obtained:

\[ y_{\text{rec}} = R\Gamma_b h_b + e, \]

where \( [\Gamma_b]_{k,k} = \exp(-j\pi k\Delta f \tau) \).

The measurements obtained at the BS in (5) can be made available to the radar using the connecting link. Consequently, the following set of measurements is considered at the radar:

\[ \begin{cases} y_{\text{rad}} = R\Gamma_m h_m + v \\ y_{\text{rec}} = R\Gamma_b h_b + e \end{cases}, \]

where \( \Gamma_m \) is an \( N \times N \) diagonal matrix containing the phase shifts for each subcarrier due to the round-trip target delay, with elements \( [\Gamma_m]_{k,k} = \exp(-j2\pi k\Delta f \tau) \), and \( h_m \) the \( N \times 1 \) frequency response vector of the target for the monostatic radar channel.

The assumptions considered in this paper can be summarized as follows:

- The target is static over the observation interval (OFDM symbol length).
- The distance between radar and BS is known, as well as their locations.
- The BS is capable of receiving and processing radar emissions.
- The angle of the directional radar antenna (or main beam) is known.

III. CRB FOR TARGET TIME DELAY ESTIMATION

In the following, the CRB for target time delay estimation is derived based on the independent measurements obtained in (6). It is reasonable to assume that for a cognitive radar the target frequency responses \( h_m \) and \( h_b \) are either known or can be reliably estimated [30]. Consequently, the only unknown parameter in (6) is the target time delay \( \tau \) found in the diagonal matrices \( \Gamma_m \) and \( \Gamma_b \).

The measurements vector \( y = [y_{\text{rad}}^T \ y_{\text{rec}}^T]^T \) is assumed to follow a complex Gaussian distribution \( y \sim \mathcal{CN}(m(\tau), \Sigma) \), where the mean vector and the covariance matrix:

\[ m(\tau) = \begin{bmatrix} R\Gamma_m h_m \\ R\Gamma_b h_b \end{bmatrix} \quad \text{and} \quad \Sigma = \begin{bmatrix} \sigma_m^2 \mathbb{I} & 0 \\ 0 & \sigma_b^2 \mathbb{I} \end{bmatrix}. \]
are of size $2N \times 1$ and $2N \times 2N$ respectively, with $\sigma^2$ and $\sigma^2_v$ the variances for the independent noise and reconstruction error respectively. As the reconstruction error depends on the noise at the BS it is independent from the noise at the radar receiver which is in a different location. For the considered case, the general FI expression for the complex Gaussian density function presented in [31] can be used:

$$F I(\tau) = 2 \mathbb{R} \left\{ \frac{\partial m^H (\tau)}{\partial \tau} - \sum_k \frac{\partial m(\tau)}{\partial \tau} \right\}.$$  

(8)

Two cases are presented in the following: the cooperative case discussed in this paper and the traditional case, where the radar operates without the help of the communications BS. After appropriate simple calculations the FI in the cooperative case is obtained as:

$$F I_{cp}(\tau) = 2 \left[ \frac{(2\pi \Delta f)^2}{\sigma^2} h_m^H D^2 R^H R h_m + \frac{(\pi \Delta f)^2}{\sigma^2_v} h_b^H D^2 R^H R h_b \right].$$

(9)

where $D = \text{diag}(\{0, \ldots, N-1\})$, and from which the CRB for target time delay estimation can be obtained as $\text{CRB}_{cp}(\tau) = F I_{cp}(\tau)^{-1}$. Similarly, the FI for the radar only case is obtained as:

$$F I_{ro}(\tau) = 2 \left[ \frac{(2\pi \Delta f)^2}{\sigma^2} h_m^H D^2 R^H R h_m \right],$$

(10)

from which $\text{CRB}_{ro}(\tau) = F I_{ro}(\tau)^{-1}$.

It is observed that a positive term is added to the FI for the cooperative case of (9) in comparison to the FI for the radar only case of (10). This term is due to the measurements provided by the communications BS acting as a bistatic radar receiver. A larger FI is thus obtained in the cooperative case and consequently also a lower CRB. This shows that theoretically a better estimation performance is obtained in the cooperative case.

IV. RADAR WAVEFORM OPTIMIZATION

In the following, radar waveform optimization in the proposed cooperative radar-communications system is also introduced. The objective function to be minimized is the CRB for the target time delay estimation derived in Section III, which is equivalent to maximizing the FI. It is typical for scenarios involving coexistence among radar and communications systems to limit the interference caused to the other system. Consequently, similar to several others like [6], [11], [26], the communications system imposes a maximum transmitted power for the radar system. This is employed in the radar waveform optimization problem as a power constraint per subcarrier and comes from a minimum data rate guaranteed for the communications users. This minimum data rate guaranteed for the communications users is the cooperation incentive for the communications system. Examples of how the power constraint can be obtained are provided in [6], [7], [26] for example. Another constraint employed in the radar waveform optimization problem is on the total transmitted radar power.

Considering the maximization of the FI in (9) instead of minimizing the CRB, the objective function of the radar waveform optimization problem can also be written as:

$$F I_{cp}(\tau) = 2 \sum_{k=0}^{N-1} \frac{(2\pi \Delta f)^2}{\sigma^2} h_m^H |k^2| \left| h_m[k] \right|^2 |r[k]|^2 + \sum_{k=0}^{N-1} \frac{(\pi \Delta f)^2}{\sigma^2_v} k^2 \left| h_b[k] \right|^2 |r[k]|^2.$$  

(11)

Consequently, after few simplifications, the radar waveform optimization problem for the cooperative case can be stated as follows:

$$\begin{align*}
&\text{maximize} \quad \sum_{k=0}^{N-1} k^2 \left( \frac{|h_m[k]|^2}{\sigma^2} + \frac{|h_b[k]|^2}{\sigma^2_v} \right) |r[k]|^2 \\
&\text{subject to} \quad \sum_{k=0}^{N-1} |r[k]|^2 \leq P_T \\
&\qquad \qquad |r[k]|^2 \leq u[k], \forall k = 0 \ldots N - 1
\end{align*}$$

(12)
where $P_t$ is the total transmitted radar power, while $u[k]$ is the power constraint imposed by the communications system on the $k$th subcarrier. This optimization problem is recognized as a Linear Programming (LP) which can be solved in Matlab using CVX. The solution is a power allocation per subcarrier. Similarly for the radar only case, the radar waveform optimization problem can be stated as in (12) with the appropriate objective function based on (10). The radar waveform optimization problems in both cooperative and radar only cases are solved and the optimum power allocations are presented in Fig. 3 for a symmetric spectrum, $D = \text{diag}(-N/2, \ldots, N/2-1)$. It is observed that there is a trade-off between allocating power based on channel gain or subcarrier index. This is true for both solutions and it is typical to CRB based waveform optimization [32]–[34]. Moreover, it is observed that in the cooperative case, power is allocated based on the channel gain of both monostatic and bistatic channels. Intuitively, power is allocated to the subcarriers that experience a larger channel gain and lower noise and interference levels in order to benefit the estimation performance. This is due to the additional available DoF brought by the bistatic channel for the radar to optimally use its power budget.

V. ESTIMATION PERFORMANCE EVALUATION

In this section, the performance of the optimized waveforms in Section IV is investigated in terms of the achieved RMSE for target time delay estimation. The estimation error for the cooperative and radar-only cases is compared and the benefit of cooperating with the BS in the proposed scenario is demonstrated.

In order to determine the actual estimation performance an estimator is derived in the following. The joint density of the measurements in (6), given the assumptions in this paper, is:

$$f(y; \theta) = \frac{1}{\pi \det(\Sigma)} \exp\left(-\frac{(y - m)^H \Sigma^{-1} (y - m)}{2}\right),$$

where $\det(\Sigma)$ is the determinant of $\Sigma$ and the corresponding log likelihood function is:

$$L(y; \theta) = -\ln(\pi \det(\Sigma)) - (y - m)^H \Sigma^{-1} (y - m).$$

The negative of the log likelihood function, after ignoring the terms that do not depend on $\tau$, can be written as:

$$l(y; \tau) = \frac{1}{\sigma^2} (y_{\text{rad}} - m_{\text{rad}}(\tau))^H (y_{\text{rad}} - m_{\text{rad}}(\tau)) + 1$$

$$\frac{1}{\sigma^2} (y_{\text{rec}} - m_{\text{rec}}(\tau))^H (y_{\text{rec}} - m_{\text{rec}}(\tau)),$$

where $m_{\text{rad}}(\tau) = R\Gamma_m(\tau) h_{\text{rad}}$ and $m_{\text{rec}}(\tau) = R\Gamma_b(\tau) h_{\text{rec}}$. Consequently, the estimate of the target time delay can be obtained as:

$$\hat{\tau} = \arg \min_\tau l(y; \tau).$$

Finding the exact MLE in (16) is not trivial, thus a grid search is performed to obtain $\hat{\tau}$ from its discrete-time model $\tau_p = p \frac{1}{P} \Delta \tau$, with $p = 0 \ldots P - 1$, similarly to [11].

VI. CONCLUSIONS

In this paper a particular example of a cooperative radar-communications system is proposed, where the communications BS is capable of receiving and processing radar emissions. As a result of the BS capability, as well as the cooperation between the two systems, the radar can improve its target parameter estimation performance. This is thanks to the communications BS which can provide extra measurements of the target.

In particular, the target time delay estimation is considered in this paper. It is demonstrated, using analytical results and simulations, that the radar benefits from the proposed cooperative scenario. The expressions for the FI and CRB are analytically derived and it is shown that the FI is increased. This is due to an additive positive term which increases the FI for the cooperation case. In simulations it is shown that the CRB for the cooperative case is lower than the CRB for the radar only case. A significant difference between the two
DoF is available for the radar to optimally use its power towards the edge of the spectrum. The average estimation performance improvement is demonstrated by comparing the average RMSE obtained in both cases as well.

Based on the derived theoretical bounds, namely CRB in this paper, radar waveform optimization is also proposed in the considered cooperative scenario. The solutions to the optimization problems are power allocations over subcarriers. It is shown that in the proposed cooperative case an additional DoF is available for the radar to optimally use its power budget. Also, the trade-off between power allocation based on subcarrier gain or subcarrier index is shown.

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