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Evidence for stabilised entanglement of massive mechanical oscillators

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Quantum entanglement is a phenomenon whereby systems cannot be described independently of each other, even though they may be separated by an arbitrarily large distance. It is at the heart of some of the most famous debates in the development of quantum theory [1]. Nonetheless, entanglement nowadays has a solid theoretical and experimental foundation, and it is the crucial resource behind many emerging quantum technologies. Entanglement has been demonstrated for microscopic systems, such as with photons [2–5], ions [6], and electron spins [7], and more recently in microwave and electromechanical devices [8–10]. For macroscopic objects [8–14], however, entanglement becomes exceedingly fragile towards environmental disturbances. A major outstanding goal has been to create and verify entanglement of the centre-of-mass motion of macroscopic objects. Here, we carry out such an experimental demonstration, with the moving bodies realized as two micromechanical oscillators coupled to a microwave-frequency electromagnetic cavity that is used to create and stabilise the entanglement of the centre-of-mass motion of the oscillators [15–17]. We infer the existence of entanglement in the steady state by combining measurement of correlated mechanical fluctuations with an analysis of the microwaves emitted from the cavity. Our work qualitatively extends the range of entangled physical systems, with implications in quantum information processing, precision measurement, and tests of the limits of quantum mechanics.
There exist several proposals for how cavity optomechanical setups could be used to entangle the motional quantum states of two massive mechanical oscillators, see e.g., Refs. [15, 18–22]. In such setups, two movable mirrors are incorporated into a resonant optical cavity, and radiation pressure forces inside the cavity can be tailored such that the motion of the mirrors becomes highly correlated and even entangled. This approach does not require any direct interaction between the moving masses, allowing them to be spatially separated. A correlated reduction of noise below the thermal level in the motion of two mechanical oscillators has previously been demonstrated in experiment [23, 24], but not near the quantum level as required for entanglement.

Our work is based on a series of proposals [15, 20, 21] for using reservoir engineering to stabilise two cavity-coupled mechanical oscillators into a steady state that is entangled. The recipes are extensions of an approach used in recent work to squeeze the motion of a single oscillator [25–27]. An oscillator with frequency $\omega_1$ and position operator $x_1(t) = X_1(t) \cos \omega_1 t + P_1(t) \sin \omega_1 t$, is squeezed if the variance of either quadrature amplitude operators $X_1$ or $P_1$ is smaller than the quantum zero-point fluctuation level. Introducing a second oscillator with frequency $\omega_2$ and position operator $x_2$ expressed in terms of quadratures as above, one can introduce four collective quadrature operators, $X_\pm = \frac{1}{\sqrt{2}} (X_2 \pm X_1)$ and $P_\pm = \frac{1}{\sqrt{2}} (P_2 \pm P_1)$. The state corresponding to either the variances of $X_+ \text{ and } P_-$, or $X_- \text{ and } P_+$, being reduced below the quantum zero-point fluctuations level is a canonical entangled state known as the two-mode squeezed state. Specifically, the state is entangled if $\langle X_2^+ \rangle + \langle P_2^- \rangle < 1$, see Fig. 1e, a criterion commonly referred to as the Duan inequality [28]. While such Gaussian states exhibit a positive-definite Wigner function, they can in principle have arbitrarily large amounts of entanglement, and they can lead to violations of local realism [29, 30], and are essentially the state considered in the EPR gedanken experiment [1].

In our experiment, a single driven cavity mode is used both to prepare a correlated state of two mechanical oscillators as well as to directly measure fluctuations in the $X_+$ collective quadrature (via a two-mode back-action evading measurement) [15, 17]; we find $\langle X_2^+ \rangle \simeq 0.41 \pm 0.04$. While a direct measurement of $P_-$ is not possible, by constraining system parameters and analyzing the full output spectrum of our cavity, we are also able to infer its fluctuations, $\langle P_2^- \rangle \simeq 0.42 \pm 0.08$ (see e.g. Ref. [25]). We hence have an entangled state with $\langle X_2^+ \rangle + \langle P_2^- \rangle \simeq 0.83 \pm 0.13 < 1$ of the centre-of-mass motion of two oscillators each consisting of approximately $10^{12}$ atoms.

As shown in Fig. 1b, we use a microwave-frequency realization of cavity optomechanics involving two micromechanical drum oscillators [31], and a superconducting on-chip circuit acting as the electromagnetic cavity (frequency $\omega_c$). The oscillators’ positions affect the total capacitance, and hence modulate the frequency of the cavity. This creates an effective radiation pressure interaction as with optical cavities and mirrors. In order to generate two-mode squeezing and entanglement, we pump the system with two strong pump microwave tones at the frequencies $\omega_- = \omega_c - \omega_1$ and $\omega_+ = \omega_c + \omega_2$, below and above (respectively) the cavity frequency as shown in Fig. 1b,c.

To describe this system, we initially assume for simplicity that the two oscillators have equal single-photon radiation-pressure coupling strengths $g_0$, and that the pumps are applied at the red and blue sideband frequencies $\omega_- = \omega_c - \omega_1$ and $\omega_+ = \omega_c + \omega_2$, respectively. Details of the derivations, including non-idealities, are discussed in the Supplementary Information. The strong pumps enhance the radiation-pressure interaction, yielding many-photon coupling rates
\( G_\pm = g_0 \alpha_\pm \), where \( \alpha_\pm \) are the field amplitudes induced in the resonator due to the pump tones at \( \omega_\pm \). We also introduce the mechanical Bogoliubov modes which are obtained by a two-mode squeezing transformation on the original mechanical annihilation operators, viz. \( \beta_1 = b_1 \cosh r + b_2^* \sinh r \), and \( \beta_2 = b_2 \cosh r + b_1^* \sinh r \), where \( \tanh r = G_+/G_- \). Defining \( \Omega = (\omega_2 - \omega_1)/2 \), and working in a rotating frame (at \( \omega_c + \Omega \) for the cavity, \( (\omega_2 + \omega_1)/2 \) for each mechanical oscillator), the linearized optomechanical Hamiltonian is:

\[
H = -\Omega a^\dagger a + \Omega \left( \beta_1^\dagger \beta_2 - \beta_1^\dagger \beta_1 \right) + \mathcal{G} \left[ a^\dagger (\beta_1 + \beta_2) + a \left( \beta_1^\dagger + \beta_2^\dagger \right) \right].
\]

Here \( \mathcal{G} = \sqrt{G_+^2 - G_-^2} \) is an effective optomechanical coupling rate. This Hamiltonian is essentially that in Ref. [15], but with the pump tones set as in Ref. [21]. It describes cooling of the Bogoliubov modes by cavity cooling towards their ground state, which corresponds to a stabilised, two-mode squeezed state of the bipartite mechanical system. In contrast to dynamical protocols, e.g. Refs. [10, 13], the system hence stays entangled indefinitely. Here, non-degenerate mechanical frequencies are essential, so that both Bogoliubov modes are efficiently cooled by different frequency components of the cavity. Asymmetries in the single-photon couplings \( g_1 \) and \( g_2 \) for the two oscillators, or non-zero detunings of the pump tones (by the amounts \( \delta_\pm \), see Fig. 1b) from the mechanical sidebands, introduce additional terms in the Hamiltonian Eq. (1), and one might expect that the steady-state entanglement is reduced. However, we find numerically that one can greatly compensate for asymmetries in couplings by optimising the pump detunings \( \delta_\pm \neq 0 \).

As displayed in Fig. 1b, an essential part of the entanglement verification strategy consists of two-mode back-action evading (BAE) detection [16, 17] operated in the same cavity mode, allowing for mapping the mechanical motion to the output field. This involves two relatively weak probe tones applied at \( \omega_{d\pm} \approx \omega_c \pm (\omega_2 + \omega_1)/2 \), approximately in the middle of the sideband frequencies. In order to preserve the same rotating frame for creation and detection of the two-mode squeezing, we strictly require \( \omega_{d+} - \omega_{d-} = \omega_+ - \omega_- \), ideally up to complete phase coherence between the tones. In a manner similar to the pumps, the probes induce effective couplings \( g_\pm = g_0 \alpha_\pm^d \), with the amplitudes \( \alpha_\pm^d \), which in the ideal two-mode back-action evading case are equal. Since we are using the same cavity mode for both creating the entanglement via the pumps and detecting it, the pump spectra and probe spectra need to be independent. This is achieved by ensuring that the mechanical contributions to the output cavity spectrum from the pump and probe tones have negligible spectral overlap. Hence, the faithful reconstruction of the \( X_+ \) collective quadrature spectrum from the probe signal is possible. In contrast to BAE detection of single-mode squeezing [25], both the pumps and probes can be set to optimal frequencies for the creation and detection of two-mode squeezing (see Fig. 1b-d).

In our device the two oscillators are far separated by 600 microns, they have no direct coupling, and the system is well described by Eq. (1). We use the fundamental drum modes of the oscillators with the resonance frequencies \( \omega_1/2\pi \simeq 10.0 \) MHz and \( \omega_2/2\pi \simeq 11.3 \) MHz, and linewidths \( \gamma_1/2\pi \simeq 106 \) Hz and \( \gamma_2/2\pi \simeq 144 \) Hz, respectively. The microwave cavity, with the frequency \( \omega_c/2\pi \simeq 5.5 \) GHz, has separate input and output ports. All the input signals are applied through a port coupled weakly at the rate \( \kappa_E/2\pi \simeq 60 \) kHz, whereas the output is strongly coupled at...
\( \kappa_{\text{ext}} / 2\pi \simeq 1.13 \text{ MHz} \). The cavity also has internal losses at the rate \( \kappa_1 / 2\pi \simeq 190 \text{ kHz} \), and all the loss channels sum to the total linewidth \( \kappa / 2\pi \simeq 1.38 \text{ MHz} \). We find that our fabrication process can produce basically identical single-photon couplings, \( g_1 / g_2 \simeq 0.98 \) for two oscillators of different frequencies, in fact, this is more than sufficient for the purpose of generating entanglement since numerically we find that an asymmetry up to \( \sim 20 \% \) can be compensated via detunings.

The motion of the mechanical oscillators is measured via the power scattered from the applied microwave tones, both pumps and probes, by their interaction with the oscillators. We collect this weak signal using standard techniques, including a low-noise cryogenic microwave amplifier followed by room-temperature signal analysis. A sequence of calibrations, described in detail in the Supplementary Information is important for the experiment. First, based on a standard thermal calibration using a single red-detuned tone, the mechanical modes are found to thermalize down to the equilibrium phonon occupation numbers \( n^T_1 \simeq 41 \) and \( n^T_2 \simeq 30 \) for oscillators 1 and 2, respectively, at the base temperature \( \simeq 14 \text{ mK} \) of the dry dilution refrigerator. These values imply the initial variances of the collective quadratures \( \langle X_2^\pm \rangle^T \) and \( \langle P_2^\pm \rangle^T \simeq 36 \).

We proceed with standard sideband cooling of each mechanical oscillator separately using a single red-detuned pump (see Fig. 2 for data from oscillator 1). This allows characterization of the behaviour of the system under intense pumping. Importantly, it calibrates the gain of the detection system for the later interpretation of the spectrum under two-tone pumping, as well as the effective coupling of the red-detuned tone. The goal of calibrating the probe tones is to use the total power in the probe spectra as a straightforward thermometer for the quadratures. Similar to the single red tone case, we run a thermal calibration with both probe tones on that allows us to determine the collective occupation number measured at a small probe power. Subsequently, a power sweep calibration of the probes connects a given signal strength to the quadrature variance.

Next we discuss the main experiment that uses two pairs of tones, namely the pumps and the probes. The pump tones are used to create entanglement, and the probe tones enable measurement of quadrature variances of the collective mechanical state. First, we focus on the spectrum due to the pump tones. In Fig. 2 we display the pump output spectra \( S_{\text{out}}[\omega] \) from the cavity under several pumping conditions, given in absolute units determined via the gain calibration. This spectrum is one piece of information available for characterizing the entanglement of the two oscillators, since it allows for the inference of the effective temperatures of the reservoirs associated with the cavity and the two oscillators. Our theoretical modeling uses standard input-output theory for electromagnetic cavities, treating the pump and probe tones as effectively belonging to independent modes. The variances of the collective quadratures, with a given set of parameters, can be evaluated within the same framework.

Now consider the spectrum resulting from the probe tones. The ability to control the relative phase \( \phi \) of the two probe tones allows us to infer the variance of a general collective quadrature \( X_+^\phi = X_+ \cos \phi + P_+ \sin \phi \). Written in terms of the spectra \( S_{X_+}[\omega] \) and \( S_{P_+}[\omega] \) of \( X_+ \) and \( P_+ \), respectively, the measured spectrum is then proportional to \( S_{X_+^\phi}[\omega] = S_{X_+}[\omega] \cos^2 \phi + S_{P_+}[\omega] \sin^2 \phi \). In the measurement, the probe signal is visible as peaks on top of the pump spectrum at the frequencies \( \sim \omega_c \pm \Omega \) on either side of the cavity. In order to obtain the probe spectrum as
sketched in Fig. 1d, we subtract the background measured in the absence of the probe tones. In Fig. 3 we display the measured probe spectra $S^d_{\text{out}}(\omega)$ corresponding to dataset A. The probe power at $g_\pm/2\pi \simeq 40$ kHz was kept much smaller than the pump power. The theoretical model, shown with the same parameters for all curves, is in excellent agreement with the experiment, including the positions and markedly non-Lorentzian lineshapes of the peaks, all of which depend on the phase. The unusual shapes physically arise due to the fact that the two oscillators are pumped in a very unequal manner, and they exhibit individual optical springs which add up to the collective spectrum. We infer the quadrature variance $\langle (X^\phi)^2 \rangle$ from the total integrated area of the peaks. This method, as opposed to relying on the particular shapes of the peaks, is insensitive to the phase drift of the microwave sources occurring during the data acquisition. Within a typical integration time for one curve of approximately 30 minutes, $\phi$ can drift several degrees, leading to slight departures from the theoretically determined curves.

From our modeling (see Supplementary Information), we confirm that the probe peak area faithfully reproduces the quadrature variances following our calibration as described above. This rigorously holds if the probe powers are perfectly matched, i.e., $\alpha^d_+ = \alpha^d_-$, which was calibrated without the pump tones. However, for the theory fits in Fig. 3 we need to assume an imbalance of $g_-/g_+ \simeq 1.055$, which we attribute to a shift of the cavity frequency when the strong pumps are present, causing a reduction of the blue probe tone in the system involving an input filter with a steep slope at the blue side. The reduced $g_+$, given that $g_-$ stays constant, means that the probe area will under-estimate the quadrature variance, here by $\simeq 18\%$, a number obtained numerically from the model. We will hence scale up the quadrature variances inferred from the probe areas by this percentage. A similarly low blue pump power as compared to the best estimate is observed in the pump spectra as well, but the red pump matches the calibration, supporting the handing of the imbalance as described. We emphasize that the probe inference does not assume anything about the mechanical oscillators or the dynamics induced by the pumps, but only assumes an understanding of the dynamics associated with the probe tones.

In Fig. 4a,b we display a measurement of the $X^\phi_+$ quadrature variance, with 95% statistical confidence intervals. In the optimal case of $\phi \simeq 4^\circ$ minimizing the variance in Fig. 3 we obtain $\langle (X^\phi_+)^2 \rangle \simeq 0.41 \pm 0.04$; that is 0.9 dB below vacuum. Several points fall well below the quantum zero-point noise level in both datasets A and B. Since the best theoretical fit to the measured probe spectra is obtained with dataset A, we base our main claims on this data. With dataset B, the pump spectrum shows good agreement, while we believe the probe spectrum was subject to larger phase drifts during the data acquisition.

Now consider the measurement of the variance of the $P_-$ quadrature, needed for examining the Duan criterion and verifying quantum entanglement. The two-mode BAE probe detection, as mentioned, does not couple to $P_-$ or $X_-$. We therefore use the other source of information available, namely the pump spectrum, and then combine this information with that provided by the probe detection. A least-squares fit to an analytical expression describing the pump spectrum, using the three bath temperatures as adjustable parameters, combined with the aforementioned calibrations, allows the evaluation of these variances. The fits are shown in Fig. 2 displaying an excellent agreement to the experiment. For dataset A, we obtain the variance $\langle P_-^2 \rangle \simeq 0.45 \pm 0.08$. For the $X_+$ quadrature, we similarly
get $\langle X^2 \rangle \simeq 0.46 \pm 0.08$, close to the value obtained from the direct BAE detection method described above. Given our knowledge of system parameters and the dynamics of this scheme, the two quadratures are expected to have variances within 5% of one another (see Supplementary Information), providing additional evidence for the value of $\langle P^2 \rangle$ based on BAE detection.

The error analysis of the probe measurement uses straightforward error propagation of the experimental calibrations, and of a statistical error from integrating the probe peak area. For the pump spectrum, the analysis is complicated because it involves more parameters, some of which can sensitively affect the steady-state entanglement. Here we adopt an error analysis method known as the Bayesian Monte Carlo method, similar to Ref. [25], to rigorously infer the parameters including uncertainties and correlations. The method generates a sample of the parameter distribution for which the theory model agrees with the measured pump spectra within the statistical uncertainty. We sample the posterior distributions of all parameters, and use the distributions to estimate the confidence limits of the $P_-$ quadrature variance. We obtain that at 96% probability, $\langle P^2 \rangle < 0.5$ in case of the data in Fig. 3 (dataset $A$). This approach also yields the most likely value $\langle P^2 \rangle \simeq 0.42 \pm 0.08$ that agrees with the values obtained above, but is determined independently.

The best estimate of the Duan quantity is given by combining all the information, namely $\langle X^2 \rangle$ from probe detection, and that for $\langle P^2 \rangle$ as explained above, giving $\langle X^2 \rangle + \langle P^2 \rangle = 0.83 \pm 0.13$ for dataset $A$ and similarly $0.72 \pm 0.18$ for dataset $B$, where the error bars give the worst-case combination of the individual measurements. The fluctuations hence satisfy the Duan bound for entanglement $\langle X^2 \rangle + \langle P^2 \rangle < 1$ with more than 2σ confidence.

The entangled mechanical oscillators combined with phase-sensitive measurement systems can find practical use in precise reconstruction of classical resonant forces, having implications in quantum metrology. Fundamentally, the entanglement of massive mechanical oscillators establishes a new regime for experimental quantum mechanics. In the future one could demonstrate quantum teleportation of motional states or, if one could make measurements of phonon number, test Bell-CHSH type inequalities [29] with massive mechanical objects.


Supplementary Information is available in the online version of the paper.

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Author contributions M.A.S. initiated the project and was involved in all subsequent stages. C.F.O.-K. carried out the measurements. C.F.O.-K. and E.D. analyzed the data. E.D. and J.-M.P. designed and fabricated the devices. M.J.W., A.A.C., F.M. and M.A. developed the theory. All authors participated in writing the paper.

Author Information Reprints and permissions information is available at www.nature.com/reprints. The authors declare no competing financial interests. Readers are welcome to comment on the online version of the paper. Correspondence and requests for materials should be addressed to M.A.S. (mika.sillanpaa@aalto.fi).

Data availability The data that support the findings of this study are available from the corresponding author upon reasonable request.
FIG. 1. Creating and detecting motional entanglement. a, Schematic of two vibrating mirrors coupled via an electromagnetic cavity (left), and a micrograph showing the microwave optomechanical device consisting of a superconducting transmission line resonator, whose opposite ends are connected by two mechanical drum-type oscillators marked with arrows (right). The device is fabricated on a quartz chip out of aluminium. b, Spectral picture of the involved microwave frequencies showing the pump tones (red and blue), and the probe tones (grey). The bare cavity response function is illustrated in green. c, The strong pumps applied at the frequencies $\omega_\pm$ create all-mechanical entanglement, as well as carry out information as incoherently scattered microwave light (pump spectrum). d, Two additional weak probe tones are applied at the frequencies $\omega_{d\pm}$ in order to reconstruct the $X_+$ collective mechanical quadrature using a two-mode back-action evading measurement (probe spectrum). The curved arrows indicate the involved sideband processes that scatter phonons of frequencies $\omega_1$ (yellow) or $\omega_2$ (pink). e, Illustration of the correlations in two-mode squeezing in terms of fluctuations (shaded) of the quadrature amplitudes. Left, the sum of $X$ quadratures of the two oscillators fluctuates less than the zero-point level (circle). Right, the difference of $P$ quadratures is similarly localized.

FIG. 2. Pump spectra under two-tone driving. a, The pump amplitudes are $G-/2\pi \simeq 278$ kHz, $G+/2\pi \simeq 166$ kHz (labelled dataset A in the following). b, Dataset B, having higher pump powers $G-/2\pi \simeq 332$ kHz, $G+/2\pi \simeq 210$ kHz. The blue points, shown for reference, are the sideband cooling calibrations run for oscillator 1. Theory curves are given by the solid lines. The remaining parameters are listed in the Supplementary Information.

FIG. 3. Two-mode back-action evading readout. The probe output spectrum recorded with the same parameters as the pump spectrum in Fig. 2a (dataset A), i.e., $G-/2\pi \simeq 278$ kHz, $G+/2\pi \simeq 166$ kHz. The left and right peaks peaks correspond to the respective peaks in the sketch in Fig. 1d. The probe phases are written in the panels, and the solid lines are theoretical predictions. The four lowermost panels exhibit two-mode squeezing below the quantum zero-point fluctuation level. The two uppermost panels, having a different vertical scale, present hot quadratures.

FIG. 4. Fluctuations of collective quadratures. a, The $X^\phi_+$ quadrature variance measured by the probe spectra. The black circles come from the dataset A, and red circles correspond to dataset B. b, Zoom-in of a. c, The Duan quantity for entanglement as a function of the probe tone phase. d, The Duan quantity, for the optimal value of $\phi$, as a function of the strength of the red-detuned pump tone. The solid lines coloured similar to the datasets are theoretical fits using the bath temperatures determined by the pump spectra. The blue horizontal line marks the quantum zero-point fluctuations level. The error bars denote $2\sigma$ statistical confidence.
a

\[ x_1(t) \overset{\text{spring}}{\leftrightarrow} x_2(t) \]

b

**pump and probe**

\[ \omega_- \rightarrow |\delta_-| \quad \omega_+ \rightarrow |\delta_+| \]

\[ \omega_{c-\omega_2} \quad \omega_{c-\omega_1} \quad \omega_c \quad \omega_{c+\omega_1} \quad \omega_{c+\omega_2} \]

Frequency

**pump (entangle)**

\[ \omega_- \rightarrow \omega_{d-} \quad \omega_+ \rightarrow \omega_{d+} \]

\[ \omega_{c-\omega_2} \quad \omega_{c-\omega_1} \quad \omega_c \quad \omega_{c+\omega_1} \quad \omega_{c+\omega_2} \]

Frequency

**probe (readout)**

\[ \omega_- \rightarrow \omega_{d-} \quad \omega_+ \rightarrow \omega_{d+} \]

\[ \omega_{c-\omega_2} \quad \omega_{c-\omega_1} \quad \omega_{c-\omega_1} \quad \omega_{c+\omega_1} \quad \omega_{c+\omega_2} \]

Frequency

**e**

\[ X_2 \]

\[ x_{zp} \]

\[ X_1 \]

\[ \Delta X_- \quad \Delta X_+ \]

\[ P_2 \]

\[ p_{zp} \]

\[ P_1 \]

\[ \Delta P_- \quad \Delta P_+ \]
\[
S_{\text{out}[\omega]}(\omega - (\omega_- + \omega_+)/2 (2\pi \text{ MHz}))\]