This is an electronic reprint of the original article.
This reprint may differ from the original in pagination and typographic detail.

Kumar, G.; Kalita, K.; Tammi, K.

Analysis of Bridge Currents and UMP of an Induction Machine With Bridge Configured Winding Using Coupled Field and Circuit Modeling

Published in:
IEEE Transactions on Magnetics

DOI:
[10.1109/TMAG.2018.2854666](https://doi.org/10.1109/TMAG.2018.2854666)

Published: 01/01/2018

Document Version
Peer reviewed version

Published under the following license:
Unspecified

Please cite the original version:
Kumar, G., Kalita, K., & Tammi, K. (2018). Analysis of Bridge Currents and UMP of an Induction Machine With Bridge Configured Winding Using Coupled Field and Circuit Modeling. *IEEE Transactions on Magnetics*, 54(9), [8104416]. <https://doi.org/10.1109/TMAG.2018.2854666>

Analysis of Bridge Currents and UMP of an Induction Machine with Bridge Configured Winding using Coupled Field and Circuit Modeling

Gaurav Kumar¹, Karuna Kalita¹, and Kari Tammi², *Member, IEEE*,

¹Department of Mechanical Engineering, Indian Institute of Technology Guwahati, Guwahati, 781039 INDIA

²Department of Mechanical Engineering, Aalto University, Espoo, 02150 FINLAND

Unbalanced magnetic pull (UMP) reduction is an important aspect of high-speed electrical machines and it can be reduced using a specialized winding scheme called bridge configured winding (BCW). A bridge configuration winding has two parallel paths in each phase. The mid points of these two parallel paths can be short-circuited to achieve passive control of the unbalanced magnetic pull (UMP) or a power supply can be added in between these points to achieve active control of UMP. Moreover, it is important to understand the behavior of UMP in different eccentricity conditions for its active suppression. A finite element modeling of bridge configured induction machine has been developed to study the effects of different eccentricity conditions on UMP and bridge currents. A generalized circuit equation has been coupled with the field equation for the implementation of bridge configuration winding. All of the three kinds of eccentricity conditions have been simulated. In addition, two experimental setups have been developed to study the UMP induced vibration and to study the effect of eccentricity on bridge currents.

Index Terms—Bridge configured winding (BCW), finite element method (FEM), induction machine, unbalanced magnetic pull (UMP).

I. INTRODUCTION

THE Unbalanced magnetic pull (UMP) in an electrical machine has been studied by various authors since the beginning of 20th century. The eccentric position of a rotor makes magnetic field distribution asymmetric and as a result, a force acts on the rotor in the direction of the shortest air gap. The transverse force generated due to the asymmetric distribution of the magnetic field is called UMP. The asymmetry in the air gap magnetic field can occur due to the faults such as eccentric rotor positions and it can also be created purposefully to generate a controllable transverse force on the rotor. These generated transverse force can be utilized for the suppression of vibration as well as for the development of bearingless machines. The UMP can be suppressed either passively or actively. Generally parallel winding [1] and damper winding [2] help in passive suppression of UMP. The dual set of winding scheme can be used for passive [3] as well as active suppression of UMP [4]. In the dual set of winding scheme the main winding is used for the torque production and the auxiliary winding is used to create asymmetry in the air gap magnetic field to generate transverse force on the rotor. The ultimate utilization of dual set of winding scheme is the development of bearingless machines [10]. Khoo [5] developed a single set of winding scheme called bridge configured winding scheme which has the capability to suppress the unbalanced magnetic pull actively as well as passively. For the generation of transverse force in an electrical machine of p pole pair, $p \pm 1$ pole pair magnetic field is needed. In a bridge configured winding, each phase of the winding is divided into 4 groups which are connected as a Wheatstone bridge in such a way that an additional supply

source across the bridge points of the winding would generate magnetic fields of $p \pm 1$ pole pairs. The advantage of BCW winding scheme is that the same winding can be used for the torque as well as transverse force generation. Khoo, Kalita, and Garvey [6] practically implemented BCW scheme on a brushless permanent magnet motor and showed its capability to generate controllable forces. Kalita, Natesan, Kumar, and Tammi [7] demonstrated partial active vibration control capability of BCW scheme in a 4-pole induction motor. Natesan, Kumar, Kalita and Rahman [17] developed a 37 kW bridge configured induction machine and showed experimentally that an eccentric rotor creates a potential difference across the bridge points of the bridge configured winding. When these bridge points are short circuited a current flows in the winding which passively controls UMP. To understand the behavior of bridge configured winding it is necessary to analyse the bridge currents in different eccentricity conditions.

There are no studies in the literature which demonstrate the effect of the different eccentricity conditions on the bridge currents and generated UMP in an electric machine with bridge configured winding. The finite element (FE) modeling is an efficient modeling technique for the investigation of machine behavior [14], [15]. Recently few authors have used finite element modeling for the investigation of unbalanced magnetic pull in different eccentricity conditions and the effect of parallel paths on unbalanced magnetic pull [11] - [13]. A few work has been carried out on FE modeling of bridge configured winding to analyse the bridge currents in static eccentricity condition [8]. The developed model does not predict the correct behavior of the bridge currents as well as UMP in the static eccentricity condition. It has been shown that in static eccentricity condition bridge currents have $f_s \pm f_r$ frequency components, where f_s is the frequency of the main supply voltage and f_r is rotor rotational frequency. However

in the static eccentricity condition the bridge current should have only f_s frequency component. It happens because in [8] a transformation matrix has been used for the implementation of bridge configuration winding, which enforces equal bridge currents in all the four legs of a particular phase. This is not a proper modeling of the bridge configuration winding scheme.

The aim of the author is to develop a simulation model which can be used to study bridge configured winding for the development of a bearingless machine. Hence, to understand and investigate the actual behavior of the bridge currents and UMP in different eccentricity conditions in bridge configured winding, a FE model of 3.7 kW bridge configured induction machine has been developed. A generalized circuit equation has been used to implement the bridge configuration winding in the FE model. The term generalized circuit means that the same numerical model can be used for the active as well as passive vibration control analysis, i.e. when bridge points are short circuited or bridge points are connected with an external isolated supply source. However the present analysis is based on two conditions, (i) when the bridge points are open (bridge OFF condition) and (ii) when the bridge points are short circuited (bridge ON condition). The analysis for the external supply through the bridge points has not been considered in this paper. The material nonlinearity has not been considered in the developed FE model. Air gap stitching method has been used for the implementation of rotor rotation, the static, the dynamic and the mixed eccentricity conditions. Considering the complexity in combining rotor transverse motion in commercial softwares, this FE model has been developed in MATLAB environment. However in this work transverse vibration of the rotor has not been coupled with the magnetic field and circuit equation. Two experimental setups have been developed to analyse the bridge currents and vibration generated due to eccentric rotor. The type of eccentricity present in the experimental setup is not known because no known external eccentricity has been provided and only the eccentricity due to the mechanical assembly faults is present in the system. The measured bridge currents and vibration behavior obtained from these experimental setups have been compared with the numerical investigation to establish the type of eccentricity present in the system.

The rest of this paper is organized as follows. In section II, fundamental equations of an electrical machine have been discussed. In section III, finite element modeling of BCW machine has been presented in detail. In section IV, numerical investigation of BCW machine in different eccentricity conditions have been presented. In section V, experimental investigation has been carried out to have a comparison with the numerical investigation.

II. FUNDAMENTAL EQUATIONS

This work is restricted to 2D field analysis, where the current density ($\vec{J}$) and the magnetic vector potential ($\vec{A}$) are z -directed (parallel to the axial length of the machine) and independent of z ,

$$\vec{J} = J(x, y, t) \vec{k} \quad (1)$$

$$\vec{A} = A(x, y, t) \vec{k} \quad (2)$$

where x and y are the cartesian coordinates, $\vec{k}$ is the unit vector parallel to the z -axis and t is the time. Since, vectors $\vec{J}$ and $\vec{A}$ have only one component, they can be treated as scalars.

The 2D approximation of field equation in x - y plane can be written as

$$\frac{\partial}{\partial x} \left[\frac{1}{\mu} \frac{\partial A}{\partial x} \right] + \frac{\partial}{\partial y} \left[\frac{1}{\mu} \frac{\partial A}{\partial y} \right] - \sigma \frac{\partial A}{\partial t} - \sigma \text{grad} V = 0 \quad (3)$$

where $\sigma \text{grad} V$ is the source of the excitation. This formulation is based on a squirrel cage induction machine, where two different types of conductors are used. The thick conductors (single conducting area) are used in the rotor and the thin conductors (multi turn coils) are used in the stator winding. The application of electrical continuity equation for the thick and the thin conductors gives $\text{grad} V = -U/l$, where U is the applied voltage. Hence, the 2D approximation of the field equation can be written as

$$\frac{\partial}{\partial x} \left[\frac{1}{\mu} \frac{\partial A}{\partial x} \right] + \frac{\partial}{\partial y} \left[\frac{1}{\mu} \frac{\partial A}{\partial y} \right] - \sigma \frac{\partial A}{\partial t} + \sigma \frac{U}{l} = 0 \quad (4)$$

we get the circuit equations for the thick and thin conductors by using $E + \frac{\partial A}{\partial t} = -\text{grad} V$ and Ohm's law, $J = \sigma E$, as (5) and (6) respectively.

$$U_r = R_r I_r + R_r \int_{s_r} \sigma \frac{\partial A}{\partial t} ds \quad (5)$$

$$U_s = R_s I_s + \frac{nl}{S_s} \int_{S_s} \sigma \frac{\partial A}{\partial t} ds \quad (6)$$

The subscripts r and s represent the rotor side and the stator side variables respectively and it has been used consistently in the entire article. The combined field equation considering the thick and the thin conductor excitation can be written as [9]

$$\frac{\partial}{\partial x} \left[\frac{1}{\mu} \frac{\partial A}{\partial x} \right] + \frac{\partial}{\partial y} \left[\frac{1}{\mu} \frac{\partial A}{\partial y} \right] - \sigma_r \frac{\partial A}{\partial t} + \frac{n_0}{S_s} I_s + \sigma_r \frac{U_r}{l} = 0 \quad (7)$$

where σ_r is the electrical conductivity of the rotor bar, n_0 is the no. of turns in the stator winding, S_s is the c/s area of the stator coil, l is the length of the rotor bar, U_r is the voltage across the rotor bar and μ is the permeability.

III. FINITE ELEMENT MODELING OF BCW ELECTRICAL MACHINE

In general any electrical machine can be divided into five domains for the purpose of numerical modeling i.e. the stator core domain, the stator winding domain, the air gap domain, the rotor core domain and the rotor conductor domain. The Galerkin method has been used for the application of FE discretization in (5), (6) and (7). The elemental matrices have been obtained for the field equation (7) for different domains of the machine. The elemental stiffness matrix for the entire domain, the elemental matrix for the contribution of the induced emf in the field equation due to rotor bar, the elemental matrix for the contribution of the excitation at the stator winding, and the elemental matrix for the contribution of the excitation at the rotor conductor can be represented as (8), (9), (10), and (11) respectively.

$$k_c^e = \iint_s \mathbf{v} (\{\mathbf{N}_{,x}\} [\mathbf{N}_{,x}] + \{\mathbf{N}_{,y}\} [\mathbf{N}_{,y}]) dxdy \quad (8)$$

$$c_r^e = \iint_s \sigma_r \{\mathbf{N}\} [\mathbf{N}] dxdy \quad (9)$$

$$p_s^e = \iint_s \frac{n_0}{S_s} \{\mathbf{N}\} dxdy \quad (10)$$

$$p_r^e = \iint_s \{\mathbf{N}\} \frac{\sigma_r}{l} dxdy \quad (11)$$

The elemental matrix for the rotor circuit equation and the elemental matrix for the stator circuit equation can be represented as (13) and (12) respectively.

$$q_r^e = R_r \sigma_r \iint_s \{\mathbf{N}\} dxdy \quad (12)$$

$$q_s^e = \frac{n_0 l}{S_s} \iint_s \{\mathbf{N}\} dxdy \quad (13)$$

where $\mathbf{v}$ is the magnetic reluctivity ($\mathbf{v} = \frac{1}{\mu}$) and $\{\mathbf{N}\}$ is a vector of the shape functions. Two types of elements have been used for the FE discretization. Eight noded quadrilateral elements have been used in the stator core, the stator coil, the rotor core and the rotor conductors, whereas six noded triangular elements have been used for the air gap modeling. For eight noded quadrilateral elements, the elemental stiffness matrix (8) can be written in natural coordinate system ($\xi - \eta$) as

$$k_n^e = \int_{-1}^{-1} \int_{-1}^{-1} \mathbf{v} \{\mathbf{gradN}\} [\mathbf{gradN}] |\mathbf{J}| d\xi d\eta \quad (14)$$

However for six noded quadrilateral elements, the elemental stiffness matrix (8) can be written in natural coordinate system as

$$k_n^e = \int_{-1}^1 \int_{-1}^1 \mathbf{v} \{\mathbf{gradN}\} [\mathbf{gradN}] |\mathbf{J}| d\xi d\eta \quad (15)$$

Similarly for eight noded quadrilateral element, the elemental stiffness matrix (9) to (12) can be written in natural

coordinate system as

$$c_r = \int_{-1}^{-1} \int_{-1}^{-1} \sigma_r \{\mathbf{N}\} [\mathbf{N}] |\mathbf{J}| d\xi d\eta \quad (16)$$

$$p_s = \frac{n_0}{S_s} \int_{-1}^{-1} \int_{-1}^{-1} \{\mathbf{N}\} |\mathbf{J}| d\xi d\eta \quad (17)$$

$$p_r = \int_{-1}^{-1} \int_{-1}^{-1} \frac{\sigma_r}{l} \{\mathbf{N}\} |\mathbf{J}| d\xi d\eta \quad (18)$$

$$q_s = \frac{n_0 l}{S_s} \int_{-1}^{-1} \int_{-1}^{-1} \{\mathbf{N}\} |\mathbf{J}| d\xi d\eta \quad (19)$$

$$q_r = R_r \sigma_r \int_{-1}^{-1} \int_{-1}^{-1} \{\mathbf{N}\} |\mathbf{J}| d\xi d\eta \quad (20)$$

After assembling all the elemental matrices the final assembled field equation can be written as (21). The assembled rotor and stator circuit equations can be written as (22) and (23) respectively. The stiffness matrix has the contribution from the entire domain, hence k^e has to be assembled for all the elements in the domain. However, c_r , p_r and q_r are applied to the rotor conductor domains only and p_s and q_s are applied in the stator coil domains.

$$[K]_{n \times n} \{A\}_{n \times 1} + [C_r]_{n \times n} \frac{\partial \{A\}_{n \times 1}}{\partial t} - [P_s]_{n \times z} [T]_{z \times u} \{I_s\}_{u \times 1} + [P_r]_{n \times v} \{U_r\}_{v \times 1} = 0 \quad (21)$$

$$[R_r]_{v \times v} \{I_r\}_{v \times 1} + [Q_r]_{v \times n} \frac{d\{A\}_{n \times 1}}{dt} = \{U_r\}_{v \times 1} \quad (22)$$

$$[R_s]_{u \times u} \{I_s\}_{u \times 1} + [Q_s]_{u \times n} \frac{d\{A\}_{n \times 1}}{dt} = \{U_s\}_{u \times 1} \quad (23)$$

where n represents the number of nodes in the domain, u is the number of stator circuit currents, v is the number of rotor conductors and z represents the number of coil side currents. In our analysis the typical value of n is 29,572. The transformation matrix used for the transformation of phase currents to the coil side currents is $[T]_{z \times u}$.

The BCW scheme has a double layer winding with two parallel paths and one additional source of power supply in each phase of the stator winding. The simulated motor is a 4-pole squirrel cage induction motor which has 36 stator slots and 26 rotor bars. In a conventional 3-phase machine the value of the stator circuit currents would be three. However in our analysis the value of the stator circuit current would depend on whether we use a simple parallel winding connection or a short circuited parallel winding connection with or without a power source. Hence for the inclusion of parallel paths and bridge connections, the field equation (21) and the stator circuit equation (23) have to be modified. To incorporate the parallel paths of the rotor bars of the squirrel cage along with the end rings, the rotor bar equation (22) has to be modified. This has been explained in details in next subsections.

A. Introduction to BCW winding

Fig. 1 represents the winding diagram of an induction machine with 36 stator slots and fig. 2 represents the coil connection diagram of the winding. Coils $(a1 - a3)$, $(aa4 - aa6)$ and $(aa1 - aa3)$, $(a4 - a6)$ form two parallel paths in phase A of the winding. Coils $(b1 - b3)$, $(bb4 - bb6)$ and $(bb1 - bb3)$, $(b4 - b6)$ form two parallel paths in phase B of the winding. Coils $(c1 - c3)$, $(cc4 - cc6)$ and $(cc1 - cc3)$, $(c4 - c6)$ form two parallel paths in phase C of the winding. For the explanation of the modeling of the circuit, twelve coil groups (W1-W12) have been formed. Coils $(a1 - a3)$, $(a4 - a6)$, $(aa1 - aa3)$, $(aa4 - aa6)$, $(b1 - b3)$, $(b4 - b6)$, $(bb1 - bb3)$, $(bb4 - bb6)$, $(c1 - c3)$, $(c4 - c6)$, $(cc1 - cc3)$, and $(cc4 - cc6)$ has been termed as coil group W1, W2, W3, W4, W5, W6, W7, W8, W9, W10, W11, and W12 respectively.

Fig. 1: Winding diagram of double layer bridge configuration winding

B. Stator winding circuit equation when bridges are open (bridge OFF condition)

Fig. 3 represents the star connected circuit diagram of a bridge configuration wound machine when bridge points are not connected and machine is supplied with a three phase star connected power source. We will refer this connection of the bridge as bridge OFF condition. The two bridge points of phase A are A_1 and A_2 , the two bridge points of phase B are B_1 and B_2 and the two bridge points of phase C are C_1 and C_2 respectively. The neutral points of the stator winding and the supply source are O and O' respectively. The three phase supply voltages are V_A , V_B , and V_C and the three phase currents of phase A, phase B, and phase C are i_A , i_B , and i_C respectively. The two parallel paths of phase A are $A - A_1 - O$ and $A - A_2 - O$ and i_1 and $i_A - i_1$ are the currents flowing in these parallel paths respectively. The two parallel paths of phase B are $B - B_1 - O$ and $B - B_2 - O$ and i_2 and $i_B - i_2$ are the currents flowing in these parallel paths respectively. The two parallel paths of phase C are $C - C_1 - O$ and $C - C_2 - O$

Fig. 2: Coil connection diagram of the winding

Fig. 3: Circuit connection when bridges are open

and i_3 and $i_C - i_3$ are the currents flowing in these parallel paths respectively.

The DC resistance of the each coil group including the end winding resistance is R and the end winding inductance of each coil group is L_{es} . The Kirchhoff's voltage law (KVL) has been used to write the circuit equations. There are five loops in the circuit shown in Fig. 3. Equations for these five loops have been written as (24) to (28). In addition to these equations, (29) can be written by imposing the Kirchhoff's current law (KCL) about the star point O of the winding. By

assembling (19) for coil group domains W1, W2, ..., W12, matrices $Q_1, Q_2, \dots, Q_{12}$ have been obtained respectively.

KVL for loop 1 ($O'AA_1OB_1BO'$) can be written as

$$[Q_1 + Q_4 - Q_5 - Q_8]_{1 \times n} \frac{d\{\mathbf{A}\}_{n \times 1}}{dt} + 2Ri_1 - 2Ri_2 + 2L_{es} \frac{di_1}{dt} - 2L_{es} \frac{di_2}{dt} = V_B - V_A \quad (24)$$

KVL for loop 2 ($O'BB_2OC_1CO'$) can be written as

$$[Q_6 + Q_7 - Q_9 - Q_{12}]_{1 \times n} \frac{d\{\mathbf{A}\}_{n \times 1}}{dt} + 2R(i_B - i_2) - 2Ri_3 + 2L_{es} \frac{d(i_B - i_2)}{dt} - 2L_{es} \frac{di_3}{dt} = V_C - V_B \quad (25)$$

KVL for loop 3 (AA_2OA_1A) can be written as

$$[Q_2 + Q_3 - Q_1 - Q_4]_{1 \times n} \frac{d\{\mathbf{A}\}_{n \times 1}}{dt} + 2R(i_A - i_1) - 2Ri_1 + 2L_{es} \frac{d(i_A - i_1)}{dt} - 2L_{es} \frac{di_1}{dt} = 0 \quad (26)$$

KVL for loop 4 (BB_1OB_2B) can be written as

$$[Q_5 + Q_8 - Q_6 - Q_7]_{1 \times n} \frac{d\{\mathbf{A}\}_{n \times 1}}{dt} - 2R(i_B - i_2) + 2Ri_2 - 2L_{es} \frac{d(i_B - i_2)}{dt} + 2L_{es} \frac{di_2}{dt} = 0 \quad (27)$$

KVL for loop 5 (CC_1OC_2C) can be written as

$$[Q_9 + Q_{12} - Q_{10} - Q_{11}]_{1 \times n} \frac{d\{\mathbf{A}\}_{n \times 1}}{dt} - 2R(i_C - i_3) + 2Ri_3 - 2L_{es} \frac{d(i_C - i_3)}{dt} + 2L_{es} \frac{di_3}{dt} = 0 \quad (28)$$

And KCL about point O can be applied as

$$i_A + i_B + i_C = 0 \quad (29)$$

Final circuit equation can be represented as

$$[Q] \frac{d\{\mathbf{A}\}}{dt} + [R] \{\mathbf{I}_s\} + [L_{es}] \left\{ \frac{d\mathbf{I}_s}{dt} \right\} = \{\mathbf{U}_s\} \quad (30)$$

where $\mathbf{I}_s = [i_1 \ i_A \ i_2 \ i_B \ i_3 \ i_C]^T$ and $\mathbf{U}_s = [V_A - V_B \ V_B - V_C \ 0 \ 0 \ 0 \ 0]^T$.

C. Stator winding circuit equation when bridges are closed (bridge ON condition)

Fig. 4 represents the star connected circuit diagram of a bridge configuration wound machine when the bridge points are closed or connected to a voltage source and the machine is supplied with a three phase star connected power source. We will refer this connection of bridge as bridge ON condition. An eccentric rotor generates a potential difference across the bridge points of the winding due to which, upon connecting these bridge points, a current flows across the bridge points which changes the dynamics of the machine [7]. An isolated controlled voltage source in current control mode can also be connected to these bridge points by which current can be injected in the winding to generate the desired force for bearingless operation or for the purpose of active vibration control. The aim of the present work is to study the effect of eccentric rotor when bridges are open and closed. However a

Fig. 4: Circuit connection when bridges are closed

generalized model has been developed considering a voltage supply across the bridge points. The DC resistance and the inductance of the coil connecting across the bridge points are R_b and L_b respectively. The voltage source connected between the bridge points A_1 and A_2 in phase A of the winding is V_{bA} . The voltage source connected between the bridge points B_1 and B_2 in phase B of the winding is V_{bB} . The voltage source connected between the bridge points C_1 and C_2 in phase C of the winding is V_{bC} . The bridge currents in phase A, phase B, and phase C of the winding are i_{bA} , i_{bB} , and i_{bC} respectively.

The bridge currents in the coil groups are shown in Fig. 4. Similar to the bridge open condition the Kirchhoff's voltage law has been used to write the circuit equations for eight loops as shown in (31) to (38) and the Kirchhoff's current law has been applied about the star point O as shown in (39).

KVL for loop 1 ($O'AA_1OB_1BO'$) can be written as

$$[Q_1 + Q_4 - Q_5 - Q_8]_{1 \times n} \frac{d\{\mathbf{A}\}_{n \times 1}}{dt} + 2Ri_1 - 2Ri_2 - Ri_{bA} + Ri_{bB} + 2L_{es} \frac{di_1}{dt} - 2L_{es} \frac{di_2}{dt} = V_B - V_A \quad (31)$$

KVL for loop 2 ($O'BB_2OC_1CO'$) can be written as

$$[Q_6 + Q_7 - Q_9 - Q_{12}]_{1 \times n} \frac{d\{\mathbf{A}\}_{n \times 1}}{dt} - 2Ri_2 - 2Ri_3 + 2Ri_B + Ri_{bB} + Ri_{bC} - 2L_{end} \frac{di_2}{dt} - 2L_{es} \frac{di_3}{dt} + 2L_{es} \frac{di_B}{dt} = V_C - V_B \quad (32)$$

KVL for loop 3 (AA_2OA_1A) can be written as

$$[Q_3 - Q_1]_{1 \times n} \frac{d\{\mathbf{A}\}_{n \times 1}}{dt} - 2Ri_1 + Ri_A - R_b i_{bA} - 2L_{es} \frac{di_1}{dt} + L_{es} \frac{di_A}{dt} = V_{bA} \quad (33)$$

KVL for loop 4 (**OA₁A₂O**) can be written as

$$[Q_2 - Q_4]_{1 \times n} \frac{d\{\mathbf{A}\}_{n \times 1}}{dt} - 2Ri_1 + Ri_A + 2Ri_{bA} + Rbi_{bA} - 2L_{es} \frac{di_1}{dt} + L_{end} \frac{di_A}{dt} + L_{es} \frac{di_{bA}}{dt} = V_{bA} \quad (34)$$

KVL for loop 5 (**B₁OB₂B₁**) can be written as

$$[Q_5 - Q_7]_{1 \times n} \frac{d\{\mathbf{A}\}_{n \times 1}}{dt} + 2Ri_2 - Ri_B + Rbi_{bB} + 2L_{es} \frac{di_2}{dt} - L_{es} \frac{di_B}{dt} = V_{bB} \quad (35)$$

KVL for loop 6 (**BB₁B₂B**) can be written as

$$[Q_8 - Q_6]_{1 \times n} \frac{d\{\mathbf{A}\}_{n \times 1}}{dt} + 2Ri_2 - Ri_B - 2Ri_{bB} - Rbi_{bB} + 2L_{end} \frac{di_2}{dt} - L_{es} \frac{di_B}{dt} - 2L_{es} \frac{di_{bB}}{dt} = -V_{bB} \quad (36)$$

KVL for loop 7 (**C₁OC₂C₁**) can be written as

$$[Q_9 - Q_{11}]_{1 \times n} \frac{d\{\mathbf{A}\}_{n \times 1}}{dt} + 2Ri_3 - Ri_C + Rbi_{bC} + 2L_{es} \frac{di_3}{dt} - L_{es} \frac{di_C}{dt} = V_{bC} \quad (37)$$

KVL for loop 8 (**CC₁C₂C**) can be written as

$$[Q_{12} - Q_{10}]_{1 \times n} \frac{d\{\mathbf{A}\}_{n \times 1}}{dt} + 2Ri_3 - Ri_C - 2Ri_{bC} - Rbi_{bC} + 2L_{es} \frac{di_3}{dt} - L_{end} \frac{di_C}{dt} - 2L_{es} \frac{di_{bC}}{dt} = -V_{bC} \quad (38)$$

And Kirchoff's current law about point **O** can be applied as

$$i_A + i_B + i_C = 0 \quad (39)$$

Final circuit equation for stator winding can be written as

$$[Q] \frac{d\{\mathbf{A}\}}{dt} + [R] \{\mathbf{I}_s\} + [L_{es}] \left\{ \frac{d\mathbf{I}_s}{dt} \right\} = \{\mathbf{U}_s\} \quad (40)$$

where $\mathbf{I}_s = [i_1 \ i_A \ i_2 \ i_B \ i_3 \ i_C \ i_{bA} \ i_{bB} \ i_{bC}]^T$ and $\mathbf{U}_s = [V_A - V_B \ V_B - V_C \ V_{bA} \ V_{bA} - V_{bB} \ V_{bB} - V_{bC} \ V_{bC} \ 0]^T$. If there is no external supply across the bridge points the supply voltage would be represented as $\mathbf{U}_s = [V_A - V_B \ V_B - V_C \ 0 \ 0 \ 0 \ 0 \ 0]^T$.

D. Rotor bar circuit equation

The Squirrel cage rotor conductors are connected in parallel and short-circuited with each other using the end rings. Fig. 5 represents the circuit connection of a squirrel cage rotor of an induction machine.

The rotor bar currents for the rotor bar 1 to rotor bar 26 are

Fig. 5: Parallel connection of the rotor bar conductors

$I_{b1}, \dots, I_{b26}$ respectively. The end ring effect has been modeled

with 26 end resistances (R_{er}) and 26 end inductances (L_{er}). The end ring currents for loop 1 to loop 26 are $I_{er1}, \dots, I_{er26}$ respectively. For each individual rotor bar the circuit equation can be written as

$$\begin{aligned} Q_{r1} \frac{dA}{dt} + R_r I_{b1} - U_{b1} &= 0 \\ &\vdots \\ Q_{r26} \frac{dA}{dt} + R_r I_{b26} - U_{b26} &= 0 \end{aligned} \quad (41)$$

where $Q_{r1}, \dots, Q_{r26}$ have been obtained by assembling (20) for rotor bar 1 to rotor bar 26 respectively. The DC resistance of each rotor bar is R_r and $U_{b1}, \dots, U_{b26}$ are the voltage drop across rotor bar 1 to rotor bar 26 respectively. The rotor bar circuit equation (41) can be written in matrix form as

$$[Q_r] \frac{d\{\mathbf{A}\}}{dt} + [R_r] \{\mathbf{I}_r\} - \{\mathbf{U}_r\} = 0 \quad (42)$$

The Kirchhoff's voltage law can be used to obtain the circuit equation for each loop as

$$\begin{aligned} 2R_{er} I_{er1} + 2L_{er} \frac{dI_{er1}}{dx} - U_{b1} + U_{b26} &= 0 \\ &\vdots \\ 2R_{er} I_{er26} + 2L_{er} \frac{dI_{er1}}{dx} - U_{b26} + U_{b25} &= 0 \end{aligned} \quad (43)$$

It can be written in matrix form as

$$2[R_{er}] \{\mathbf{I}_{er}\} + 2[L_{er}] \frac{d\{\mathbf{I}_{er}\}}{dt} + [M_2] \{\mathbf{U}_r\} = 0 \quad (44)$$

where $[R_{er}] = \begin{bmatrix} R_{er} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & R_{er} \end{bmatrix}$, $[L_{er}] = \begin{bmatrix} L_{er} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & L_{er} \end{bmatrix}$

, $[M_2] = \begin{bmatrix} -1 & 0 & 0 & \cdots & 1 \\ 1 & -1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & -1 \end{bmatrix}$, $\{\mathbf{I}_{er}\} = \begin{Bmatrix} I_{er1} \\ \vdots \\ I_{er26} \end{Bmatrix}$.

The connection between the rotor bar currents and the end ring currents can be written as

$$\begin{Bmatrix} I_{b1} \\ I_{b2} \\ \vdots \\ I_{b26} \end{Bmatrix} = \begin{bmatrix} -1 & 1 & 0 & \cdots & 0 \\ 0 & -1 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \cdots & -1 \end{bmatrix} \begin{Bmatrix} I_{er1} \\ I_{er2} \\ \vdots \\ I_{er26} \end{Bmatrix} \quad (45)$$

Which can be written in matrix form as

$$\{\mathbf{I}_r\} = [M_2]^T \{\mathbf{I}_{er}\} \quad (46)$$

The final connection equation of the rotor bar and the end ring has been obtained by combining (44) and (46) as

$$[M_2]^T [M_2] \{\mathbf{U}_r\} + 2[R_{er}] \{\mathbf{I}_r\} + 2[L_{er}] \frac{d\{\mathbf{I}_r\}}{dt} = 0 \quad (47)$$

E. Transformation matrix

It can be observed from (30) that in the case of bridge open condition there are six current variables available from

TABLE I: Current Transformation Group

Bridge Conditions	Coil Side Currents	Coil Currents	Group Coil Current	Circuit Currents
Open	72	36	12	6
Closed	72	36	12	9

the circuit equation which includes 3 phase currents and 3 branch currents. In bridge closed condition there are nine current variables available which includes 3 phase currents, 3 branch currents and 3 bridge currents as written in (40). A transformation matrix is needed for the coupling of the circuit current in the field equation as the size of the source distribution matrix, [Ps], in the field equation (21) is $n \times z$. The value of z is 72 as we have 72 coil sides. The size of source current, I_s in bridge open and bridge closed conditions are 6×1 and 9×1 respectively. Hence transformation matrices of sizes 72×6 and 72×9 are required to couple the circuit currents in the field equation for bridge open and bridge closed condition respectively.

In modeling of the thin coils, three terms have been used, coil sides, full coil and coil group. Each slot of the machine has two layers and consist of two different coil sides, as shown in Fig. 1. The two-coil sides of a coil are +a1 and -a1. The +ve sign indicates that the current is coming out of the plane and -ve sign indicates that the current is going into the plane. The coil groups W1, W2, W3 and W4 form phase A of the winding, the coil groups W5, W6, W7 and W8 form phase B of the winding and the coil groups W9, W10, W11 and W12 form the phase C of the winding as shown in Fig. 2. There are 72 coil side currents, 36 full coil currents and 12 coil group currents. There are 6 circuit currents in bridge open condition and 9 circuit currents in bridge closed or bridge supply condition. Fig. 6 represents the flow chart for conversion of circuit currents to coil side currents. Circuit currents are transformed to coil group currents and coil group currents are transformed to full coil currents and full coil currents are transformed to coil side currents. Conversion of group coil currents to full coil currents and coil side currents has same transformation matrix in bridge open and bridge supply or bridge closed condition. For the transformation

Fig. 6: Winding arrangement of BCW machine

from full coil currents to coils side currents, the following transformation has been used,

$$\begin{Bmatrix} +i_{a1} \\ -i_{a1} \\ \vdots \\ +i_{cc6} \\ -i_{cc6} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ -1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & -1 \end{bmatrix} \begin{Bmatrix} i_{a1} \\ \vdots \\ i_{cc6} \end{Bmatrix} \quad (48)$$

Which can be written as

$$\{i_{coil_side}\}_{72 \times 1} = [T_1]_{72 \times 36} \{i_{coil}\}_{36 \times 1} \quad (49)$$

Since each coil of the same coil group are connected in series so they must have the same current. For the transformation of phase A coil group currents to phase A full coil currents the following transformation has been used,

$$\begin{Bmatrix} i_{a1} \\ i_{a2} \\ i_{a3} \\ \vdots \\ i_{aa4} \\ i_{aa5} \\ i_{aa6} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} i_{w1} \\ \vdots \\ i_{w12} \end{Bmatrix} \quad (50)$$

Similarly, the transformation matrices for two other phases can also be written and can be combined as

$$\{i_{coil}\}_{36 \times 1} = [T_2]_{36 \times 12} \{i_{coil_group}\}_{12 \times 1} \quad (51)$$

For the case of bridge open condition the transformation from circuit currents to coil group current can be achieved as written in (52).

$$\begin{Bmatrix} i_{w1} \\ i_{w2} \\ i_{w3} \\ i_{w4} \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ -1 & 1 \\ 1 & 0 \end{bmatrix} \begin{Bmatrix} i_1 \\ i_A \end{Bmatrix} \quad (52)$$

Similar transformation matrices can be obtained for phase B and phase C. The combined transformation matrix can be written as

$$\{i_{coil_group}\}_{12 \times 1} = [T_o]_{12 \times 6} \{i_{circuit}\}_{6 \times 1} \quad (53)$$

In the case of bridge closed and bridge supply condition, the bridge currents pass through the bridge points in each phase. The transformation of circuit currents to coil group currents for phase A can be written as

$$\begin{Bmatrix} i_{w1} \\ i_{w2} \\ i_{w3} \\ i_{w4} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \begin{Bmatrix} i_1 \\ i_A \\ i_{bA} \end{Bmatrix} \quad (54)$$

Similar transformation matrices can be obtained for phase B and phase C. The combined transformation matrix can be written as

$$\{i_{coil_group}\}_{12 \times 1} = [T_c]_{12 \times 9} \{i_{circuit}\}_{9 \times 1} \quad (55)$$

So, the final transformation matrix for the bridge open condition can be written as

$$\{i_{coil_side}\}_{72 \times 1} = [T_{open}]_{72 \times 6} \{i_{circuit_current}\}_{6 \times 1} \quad (56)$$

where $[T_{open}]_{72 \times 6} = [T_1]_{72 \times 36} [T_2]_{36 \times 12} [T_o]_{12 \times 6}$

Similarly, the final transformation matrix for the bridge

closed or bridge supply condition can be written as

$$\{i_{coil_side}\}_{72 \times 1} = [T_{closed}]_{72 \times 9} \{i_{circuit_current}\}_{9 \times 1} \quad (57)$$

where $[T_{closed}]_{72 \times 9} = [T_1]_{72 \times 36} [T_2]_{36 \times 12} [T_c]_{12 \times 9}$

The combined discretized final field and circuit equations can be represented as

$$[K]_{n \times n} \{A\}_{n \times 1} + [C_r]_{n \times n} \frac{\partial \{A\}_{n \times 1}}{\partial t} - [P_s]_{n \times 72} [T]_{72 \times m} \{I_s\}_{m \times 1} + [P_r]_{n \times 26} \{U_r\} = 0 \quad (58)$$

$$[Q]_{x \times n} \frac{d\{A\}_{n \times 1}}{dt} + [R]_{x \times x} \{I_s\}_{x \times 1} + [L_{es}]_{x \times x} \left\{ \frac{dI_s}{dt} \right\}_{x \times 1} = \{U_s\}_{x \times 1} \quad (59)$$

$$[Q_r]_{26 \times n} \frac{d\{A\}_{n \times 1}}{dt} + [R_r]_{26 \times 26} \{I_r\}_{26 \times 1} - \{U_r\}_{26 \times 1} = 0 \quad (60)$$

$$[M_2]_{26 \times 26}^T [M_2]_{26 \times 26} \{U_b\}_{26 \times 1} + 2[R_{er}]_{26 \times 26} \{I_{er}\}_{26 \times 1} + 2[L_{er}]_{26 \times 26} \frac{d\{I_{er}\}_{26 \times 1}}{dt} = 0 \quad (61)$$

where m is 6 in the case of bridge open condition and m is 9 in bridge closed or bridge supply condition.

F. Time-discretization

The combined field and circuit equations for thick and thin conductors are shown in (58) to (61). This system of differential equations have been solved using Crank-Nicolson method. The Crank-Nicolson method is a second-order method, which is implicit in time and numerically stable. The time-discretization of (58) to (61) in matrix form can be written as shown in (62). The boundary condition has been applied by forcing the magnetic vector potential (A) to be zero at the outer boundary of the machine.

$$\begin{bmatrix} \left(\frac{[K]^{t+\Delta t}}{2\Delta t} + \frac{[C_r]}{2\Delta t} \right) & [P_s][T] & [0] & [P_r] \\ \frac{2}{\Delta t} [Q_s] & [R] + \frac{2}{\Delta t} L_{es} & [0] & [0] \\ \frac{2}{\Delta t} [Q_r] & [0] & [R_r] & [T_r] \\ [0] & [0] & [R] + \frac{2}{\Delta t} L_{er} & M_2^T M_2 \end{bmatrix} \{D\}^{t+\Delta t} = - \begin{bmatrix} \left(\frac{[K]^{t+\Delta t}}{2\Delta t} - \frac{[C_r]}{2\Delta t} \right) & [P_s][T] & [0] & P_r \\ -\frac{2}{\Delta t} [Q_s] & [R] - \frac{2}{\Delta t} L_{es} & [0] & [0] \\ -\frac{2}{\Delta t} [Q_r] & [0] & [R_r] & [T_r] \\ [0] & [0] & [R_{er}] - \frac{2}{\Delta t} L_{er} & M_2^T M_2 \end{bmatrix} \{D\}^t + \{S\}^{t+\Delta t} + \{S\}^t \quad (62)$$

$$\text{where } \{S\} = \begin{Bmatrix} \{0\}_{n \times 1} \\ \{U_s\}_{m \times 1} \\ \{0\}_{26 \times 1} \\ \{0\}_{26 \times 1} \end{Bmatrix} \text{ and } \{D\} = \begin{Bmatrix} \{A\}_{n \times 1} \\ \{I_s\}_{m \times 1} \\ \{I_r\}_{26 \times 1} \\ \{U_r\}_{26 \times 1} \end{Bmatrix}$$

G. Modelling of rotor eccentricity

To incorporate the rotor movement and the rotor eccentricity, the air gap stitching method has been applied. Fig. 7 represents the meshed region of the stator and the rotor domains.

Fig. 7: Meshed stator and rotor with unmeshed air gap

Fig. 8: Stitching in air gap region

It can be observed from Fig. 7 that the air gap region has been divided in four parts. Two parts of the air gap have been modeled with the stator region and one part of the air gap has been modeled with the rotor region. To incorporate the rotation and the eccentricity condition the remaining part of the air gap is stitched using six-noded triangular elements as shown in Fig. 8. The following algorithm has been used for the stitching of the air gap region. Suppose A, C, E, G, ... are

Fig. 9: Algorithm used for air gap stitching

the stator side nodes and B, D, F, ... are its mid-side nodes. Similarly, a, c, e, ... are the rotor side nodes and b, d, ... are its mid-side nodes as shown in Fig. 9. At first any two nodes from the stator side and the rotor side, for example Aa is connected arbitrarily. Now the shorter distance between Ac and aC is chosen to form the first triangle. Considering $aC < Ac$, $\triangle aAC$ can be formed. The mid-point of the aC is chosen for the mid-side node of aC. The next triangle is formed by choosing the shorter distance between aE and Cc. In the similar manner the whole domain is stitched. For the incorporation of rotation and eccentricity condition, one portion of the air gap is restitched at each time step. This process is carried out in three steps.

First of all rotor portion is rotated about the center point and then its location is shifted as per the eccentricity condition and after that stitching between the rotor side nodes and the stator side nodes is carried out. A rotor is said to be statically eccentric if the rotor centre does not coincide with the stator centre and the rotor rotates about the rotor centre. The rotor is said to be dynamically eccentric when the rotor centre does not coincide with the stator centre and the rotor rotates about its centre as well as whirls about the stator centre. A rotor is said to have mixed eccentricity if the rotor centre does not coincide with the stator centre and the rotor rotates about its centre as well as whirls about any other point in between the stator centre and the rotor centre.

IV. NUMERICAL INVESTIGATION

The coupled field and circuit equation shown in (62) has been simulated for two sec to calculate the magnetic vector potential at each node, the circuit currents, the rotor bar currents and the voltage across each rotor bar. The magnetic flux density in the air gap has been calculated from the obtained magnetic vector potential using the relationship, $B = \nabla \times A$ and subsequently the transverse forces (F_x, F_y) acting on the rotor have been calculated using the Maxwell stress tensor method as explained in [8]. A self developed preprocessor has been used to generate the mesh for the numerical simulation where the stator, rotor and air gap domain has 5760, 3536 and 1025 elements respectively. Time stepping scheme, Crank Nicolson method and MATLAB's inbuilt solver has been used for the simulation.

The results have been generated for two conditions, when (i) bridge connections of the winding are open or bridge OFF condition, (ii) bridge connections of the winding are closed or bridge ON condition. Table II shows the important parameters of the induction machine used for the simulation. The machine parameters have been calculated using the expressions explained in [18]. The simulation has been carried out for all three types of eccentricity conditions. For the investigation of the static eccentricity condition, the rotor has been shifted by a distance of $1e-4$ m along the x -axis which is 10 % of the air gap and the rotor rotates about this point. In case of the dynamic eccentricity the rotor center rotates about the center point of the stator with a whirling radius of $1e-4$ m and the rotor also rotates about its own center. Whereas, in the mixed eccentricity condition a static eccentricity of $0.5e-4$ m, which is 5% of the air gap, has been given along the x -axis, the rotor center rotates about this point with a whirling radius of $0.5e-4$ m and the rotor rotates about its own center as well. The bridge currents and the transverse forces acting on the rotor have been analyzed to study the effect of different eccentricity conditions. The bridge currents, and transverse forces acting on the rotor have been converted to the complex domain to investigate the various modes of the bridge currents and forces. Three phase bridge currents have been converted from a, b, c to $d, q, 0$ sequence using (63), where d, q represent the orthogonal components of current and 0 is the zero sequence component of the current. The space vector of the bridge currents can be written as $d + jq$. Similarly,

TABLE II: Machine Parameters

Parameters	Value
No. of poles	4
No. of stator slots	36
No. of rotor bars	26
Outer dia. of stator	160 mm
Outer dia. of rotor	103 mm
Air gap length	1 mm
Axial length of the motor	135 mm
Resistance per phase	6.8 Ω
End winding inductance	3.94 e-4 A/m
Resistance of each rotor bar	1 e-5 Ω
Rotor end ring resistance	2.49e-6 Ω
Rotor end ring inductance	1.54 E-8 A/m
Supply voltage	37 V, 50 Hz
Slip	0.02

transverse forces can be written in complex form as $F_x + jF_y$, where F_x is the force acting along x axis and F_y is the force acting along y axis. A full spectrum Fast Fourier Transform of space vector of bridge currents and transverse forces have been carried out to investigate the relationship between the bridge current modes and force modes. Generally in balanced system zero sequence component of current is negligible, however a substantial amount of zero sequence component of bridge current is present in bridge configured winding. Fast Fourier Transform of zero sequence component of bridge current has also been carried out to know its frequency components.

$$\begin{Bmatrix} d \\ q \\ 0 \end{Bmatrix} = \begin{bmatrix} \cos(0) & \cos(-\frac{2\pi}{3}) & \cos(\frac{2\pi}{3}) \\ -\sin(0) & -\sin(-\frac{2\pi}{3}) & -\sin(\frac{2\pi}{3}) \\ 0.5 & 0.5 & 0.5 \end{bmatrix} \begin{Bmatrix} a \\ b \\ c \end{Bmatrix} \quad (63)$$

Fig. 10: Bridge currents in static eccentricity condition.

Fig. 10 and Fig. 11 represent the bridge currents and full spectrum Fast Fourier Transform (FFT) of bridge currents space vector in static eccentricity condition respectively. Fig. 12 and Fig. 13 represent zero sequence component of bridge currents in static eccentricity condition and its FFT respectively. Fig. 14 and Fig. 15 represent the bridge currents and full spectrum FFT of space vector of bridge currents in

Fig. 11: Full spectrum FFT of space vector of bridge currents in static eccentricity condition.

Fig. 12: Zero sequence component of bridge currents in static eccentricity condition.

dynamic eccentricity condition respectively. Fig. 16 and Fig. 17 represent zero sequence component of bridge currents and its FFT in dynamic eccentricity condition respectively. Fig. 18 and Fig. 19 represent bridge currents and full spectrum FFT of space vector of bridge currents in mixed eccentricity condition. Fig. 20 and Fig. 21 represent zero sequence component of bridge currents and its FFT in mixed eccentricity condition respectively.

It is a known fact that the eccentric position of a rotor generates $p \pm 1$ pole pair magnetic field in the air gap, where

Fig. 13: FFT of zero sequence of bridge current in static eccentricity condition.

Fig. 14: Bridge currents in dynamic eccentricity condition.

Fig. 15: Full spectrum FFT of space vector of bridge currents in dynamic eccentricity condition.

p is the fundamental pole pair of the winding. However, these magnetic field components have different angular frequency in different eccentricity conditions. The angular frequency of these components in static and dynamic eccentricity are f_s and $f_s \pm f_r$ respectively [16]. The rotor frequency is given by $f_s(1-s)/p$, where s is the slip. The simulated bridge configured winding is a 4- pole winding, hence eccentric position of the rotor generates 2-pole and 6-pole magnetic fields in the air gap. Based on the above mentioned relationships it can be found that the angular frequency of the 2-pole and 6-pole magnetic fields in the static eccentricity condition is

Fig. 16: Zero sequence component of bridge currents in dynamic eccentricity condition.

Fig. 17: FFT of zero sequence of bridge currents in dynamic eccentricity condition.

Fig. 20: Zero sequence component of bridge currents in mixed eccentricity condition.

Fig. 18: Bridge currents in mixed eccentricity condition.

Fig. 21: FFT of zero sequence bridge currents in mixed eccentricity condition.

50 Hz, whereas in dynamic eccentricity condition the angular frequency of 2-pole magnetic field is 25.5 Hz and the angular frequency of 6-pole magnetic field is 74.5 Hz. These magnetic field components create potential difference across the bridge points of the bridge configured winding. When these bridge points are short circuited or in bridge ON condition current flows in the winding which has same frequency components as that of the eccentricity generated magnetic field.

In static eccentricity condition, the space vector of bridge currents have only supply frequency component (f_s) i.e. 50 Hz. In dynamic eccentricity condition the space vector of

bridge currents have $f_s \pm f_r$ components i.e. 25.5 Hz and 74.5 Hz. In mixed eccentricity condition the space vector of the bridge currents have combination of the static and dynamic eccentricity components and they are 25.5 Hz, 50 Hz and 74.5 Hz. In all eccentricity conditions the space vector of the bridge currents have both negative and positive mode. A substantial amount of zero sequence component of bridge currents is also present in all the eccentricity conditions, which has the same frequency components as that of the respective space vector of the bridge currents.

Fig. 19: Full spectrum FFT of space vector of bridge currents in mixed eccentricity condition.

Fig. 22: Full spectrum FFT of force in static eccentricity condition.

Fig. 23: Full spectrum FFT of force in dynamic eccentricity condition.

Fig. 24: Full spectrum FFT of force in mixed eccentricity condition.

Fig. 22, Fig. 23 and Fig. 24 represent the full spectrum FFT of force acting on the rotor in static eccentricity condition, dynamic eccentricity condition and mixed eccentricity condition respectively. The relationship between different bridge current modes and transverse force modes can be written as $\pm f_s \pm f_{bridge}$. Here, f_{bridge} is the frequency component of space vector of bridge currents. It can be observed in Fig. 22 that in the static eccentricity condition, when bridges are in OFF condition, the forces acting on the rotor has three components, 0 Hz and ± 100 Hz ($\pm 50 \pm 50$), whereas in bridge ON condition the double supply frequency component ($\pm 2f_s$) becomes almost negligible and only the DC component (0 Hz) of the force acts on the rotor. It can be seen in Fig. 23 that in the dynamic eccentricity condition, when bridges are in OFF condition the force acting on the rotor has six components, ± 24.5 Hz, ± 75.5 Hz and ± 124.5 Hz ($\pm 50 \pm 25.5$ and $\pm 50 \pm 74.5$), whereas in bridge ON condition most of the force component becomes negligibly small and only $+f_r$ (24.5 Hz) component of the force acts on the rotor. In the mixed eccentricity condition all the frequency components of the forces of static and dynamic eccentricity conditions are present. It can be observed in Fig. 24 that in bridge OFF condition the forces acting on the rotor have 0 Hz, ± 24.5 Hz, ± 100 Hz, ± 75.5 Hz and ± 124.5 Hz component whereas in bridge ON condition the forces acting on the rotor has 0 Hz component and $+f_r$ (24.5 Hz) component.

V. EXPERIMENTAL INVESTIGATION

Two experimental setups with two different types of rotor shafts have been used for the investigation of bridge currents and UMP generated vibration. For this purpose a 36-slots 3.7 kW and a 60-slots 37 kW 4 - pole induction machines have been re-wound with bridge configured winding scheme. The winding patterns of these two machines are same as that of the winding pattern of the machine used for the numerical investigation. Fig. 25 and Fig. 26 show the two developed experimental setups.

Fig. 25: Experimental setup with original rotor.

Fig. 26: Experimental setup with a long shaft, (1,2) - bearing housing at ND and D end, 3 - test machine, 4 - perforated disc, (5,6,7) - location for the rotor responses, 8 - panel board.

Fig. 25 represents a 3.7 kW 4-pole induction machine which is rewound using bridge configuration winding scheme where the original shaft of the machine has been used for the experiment. Since only a small portion of the shaft is out of the motor so an accelerometer has been placed on the body of the motor to measure the vibration signal of the motor. The rating of this experimental setup is same as that of the numerically simulated machine. Fig. 26 represents a 37 kW induction machine which is rewound using bridge configuration winding

scheme and the original shaft of the rotor has been replaced with a 1.5 meter long shaft to make the rotor system flexible so that all the frequency components of the force acting on the rotor can be observed. The eddy current sensors have been used to measure the displacement signals along x and y axes which have been placed at location 6 in the experimental setup as shown in Fig. 26. The bridge currents flowing through the bridge points of the motor have been measured using three current transducers. All the measurements have been recorded using NI-DAQ system and both motors have been operated in no load condition.

Similar to the numerical investigation, the measured bridge currents have been converted to space vector as explained in equation (63) and vibration measurements have been converted in complex domain as, $d_x + jd_y$ where d_x and d_y are the rotor vibration along x and y axis respectively. In the case of short rotor experimental setup, the machine has been supplied using a three phase auto transformer with a rated voltage of 415 V at 50 Hz (f_s). The motor has been operated at no load so the rotor rotational frequency (f_r) is very close to 25 Hz. Since the rotor is a rigid rotor, the bridge OFF and the bridge ON conditions do not have any effect on the accelerometer signals. So, the accelerometer data has been presented only for the bridge ON condition. It should be noted that in bridge OFF condition there would not be any flow of current through the bridge points so the bridge currents can only be recorded in bridge ON condition. Fig. 27 shows the bridge currents flowing through the bridge points of the machine and Fig. 28 shows the full spectrum FFT of space vector of measured bridge currents. The presence of ± 25 Hz ($f_s - f_r$) and ± 75 Hz ($f_s + f_r$) components in full spectrum FFT of space vector of bridge currents signifies the presence of dynamic eccentricity in the machine whereas the presence of 50 Hz (f_s) component represent the presence of static eccentricity condition in the machine. The presence of all three component of the bridge currents indicates that the machine is being operated in mixed eccentricity condition as shown in numerical investigation. Similar to numerical investigation zero sequence component is also present as shown in Fig. 29. Fig. 30 represents FFT of zero sequence component of the bridge currents which also signifies the presence of mixed eccentricity condition as all three frequency components are present. The acceleration data shows the presence of 0 Hz and 100 Hz component due to static eccentricity. Whereas 25 Hz, 75 Hz and 125 Hz components show the presence of dynamic eccentricity. The presence of these frequency components in combination as shown in Fig. 31 signifies the presence of mixed eccentricity condition in short rotor motor. In numerical investigation it has been observed that the bridge currents suppress almost all the frequency components of the UMP other than DC component and $f_s - f_r$ component. Whereas in this experimental setup bridge currents are not able to suppress motor vibration. It might be happening due to the rigid rotor.

So, to investigate further a rotor with a flexible shaft has been used for the experiment as shown in Fig. 26. Also to have a generalized effect the rotor vibration has been analyzed at two different supply frequencies i.e. 20 Hz and 30 Hz using a variable frequency drive. 20 Hz and 30 Hz have

Fig. 27: Measured bridge currents with original rotor.

Fig. 28: Full spectrum FFT of measured bridge currents when motor has its original shaft.

been chosen as main supply frequencies because shaft of the motor is flexible and it can not be operated at higher speeds. Again, the motor has been operated at no load condition so for supply frequencies (f_s) 20 Hz and 30 Hz, the rotor rotational frequencies (f_r) are very close to 10 Hz and 15 Hz respectively. Fig. 32 and Fig. 33 represent the bridge currents and full spectrum FFT of space vector of measured bridge currents in longer shaft rotor experimental setup when the main supply frequency is 20 Hz. All three components ± 10 Hz ($f_s - f_r$), ± 20 Hz (f_s) and ± 30 Hz ($f_s + f_r$) are present which indicates the presence of mixed eccentricity condition in the machine.

Fig. 29: Zero sequence bridge currents when motor has its original shaft.

Fig. 30: FFT of zero sequence bridge currents when motor has its original shaft.

Fig. 31: FFT of accelerometer data when motor has its original shaft.

Fig. 33: Full spectrum FFT of space vector of measured bridge currents when motor has a longer shaft.

Fig. 34: Zero sequence bridge currents when motor has a longer shaft.

Fig. 34 and Fig. 35 show the zero sequence current and its FFT respectively. FFT plot of zero sequence component of the bridge currents show the presence of all three frequency components. Similar behavior of the bridge currents have been observed when frequency of the main supply is 30 Hz.

Fig. 32: Measured bridge currents in experimental setup when motor has a longer shaft.

Fig. 36 and Fig. 37 represent the full spectrum FFT of the rotor vibration measured at location 6 of the experimental setup as shown in Fig. 26 when the main supply frequencies are 20 Hz and 30 Hz respectively. It can be observed from Fig. 36 and Fig. 37 that the rotor response has all the frequency

components as per the relation $\pm f_s \pm f_{bridge}$. f_{bridge} is ± 10 Hz, 20 Hz and ± 30 Hz when f_s is 20 Hz and f_{bridge} is ± 15 Hz, 30 Hz and ± 45 Hz when f_s is 30 Hz. The rotor response shows the presence of 0 Hz, ± 10 Hz, ± 30 Hz, ± 40 Hz, and ± 50 Hz frequency components when the bridge are in OFF condition and main supply frequency is 20 Hz. The rotor response shows the presence of 0 Hz, ± 15 Hz, ± 45 Hz, ± 60 Hz, and ± 75 Hz frequency components when bridges are in OFF condition and the main supply frequency is 30

Fig. 35: FFT of zero sequence bridge currents when motor has a longer shaft.

TABLE III: Summery of the investigation

Numerical Simulation							
Type of Eccentricity	Supply Freq.	Slip	Bridge Currents		UMP		
			Space Vector Component	Zero Sequence Component	Bridge OFF	Bridge ON	
Static Eccentricity	50 Hz	0.02	±50 Hz	50 Hz	0 Hz, ± 100 Hz	0 Hz	
Dynamic Eccentricity	50 Hz	0.02	± 24.5 Hz, ± 74.5 Hz	24.5 Hz, 74.5 Hz	± 24.5Hz, ± 75.5 Hz ± 124.5 Hz	24.5 Hz	
Mixed Eccentricity	50 Hz	0.02	0 Hz, ± 24.5 Hz, ± 74.5 Hz	0 Hz, 24.5 Hz, 74.5 Hz	0 Hz, ± 24.5 Hz, ± 75.5 Hz ± 100 Hz, ± 124.5 Hz	0 Hz, 24.5 Hz	
Experimental Measurements							
Type of Eccentricity	Supply Freq.	Slip	Type of Rotor	Bridge Currents		Motor Body/Rotor Vibration	
				Space Vector Component	Zero Sequence Component	Bridge OFF	Bridge ON
Not Known	50 Hz	No load	Short Rotor	± 25 Hz, ± 50 Hz, ± 75 Hz	25 Hz, 50 Hz, 75 Hz	0 Hz, 25 Hz, 75 Hz, 100 Hz, 125 Hz	0 Hz, 25 Hz, 75 Hz, 100 Hz, 125 Hz
Not Known	20 Hz	No load	Long Rotor	± 10 Hz, ± 20 Hz, ± 30 Hz	10 Hz, 20 Hz, 30 Hz	0 Hz, ± 10 Hz, ± 30 Hz ± 40 Hz, ± 50 Hz	0 Hz, 10 Hz
Not Known	30 Hz	No load	Long Rotor	± 15 Hz, ± 30 Hz, ± 45 Hz	15 Hz, 30 Hz, 45 Hz	± 0 Hz, ± 15 Hz, ± 45 Hz ± 60 Hz, ± 75 Hz	0 Hz, 15 Hz

Fig. 36: Full spectrum FFT of rotor response when main supply frequency is 20 Hz.

Fig. 37: Full spectrum FFT of rotor response when main supply frequency is 30 Hz.

Hz. Whereas, in bridge ON condition most of the vibration components are suppressed other than the DC component and $f_s - f_r$ component, which is 10 Hz and 15 Hz when the main supply frequency is 20 Hz and 30 Hz respectively.

In comparison with the numerical simulation all of the frequency components, be it bridge currents frequency component or vibration measurements frequency component of the experimental results demonstrate the presence of mixed eccentricity condition in the system. To have a clarity in the interpretation, the simulation and experimental results have been summarised in a table. Table III represents comparison of all the bridge currents, UMP and vibration measurement frequency components in all conditions.

VI. CONCLUSION

A FE modeling of a 36 slots 4 - pole induction machine with bridge configured winding has been carried out to study the effect of eccentric rotor on the bridge currents and the forces acting on the rotor. This model is able to provide correct behavior of bridge currents and UMP generated due to an eccentric rotor in all types of eccentricity conditions. Various modes of the bridge currents have been analysed by transforming it from a, b, c sequence to $d, q, 0$ sequence. A relationship of bridge current modes with respect to different eccentricity conditions have been established. Also, it has been shown that the zero sequence component of the bridge currents can not be ignored in bridge configured winding. The space vector of bridge currents show the presence of only one frequency component i.e. $\pm f_s$, two frequency components i.e. $\pm(f_s - f_r)$, $\pm(f_s + f_r)$ and three frequency components i.e. $\pm f_s$, $\pm(f_s - f_r)$ and $\pm(f_s + f_r)$ of the bridge currents for the case of the static, dynamic and mixed eccentricity conditions respectively. A relationship with respect to the main supply frequency component (f_s) and bridge currents frequency component (f_{bridge}) has been established. It has been shown that the developed UMP due to the eccentric rotor in bridge OFF condition has $\pm f_s \pm f_{bridge}$ frequency components. It has been also established that bridge currents suppresses all the UMP components other than DC component and $f_s - f_r$ component. It happens because eccentric rotor generates pulsating component of magnetic field which gets suppressed when bridges are in ON condition. Two experimental setups have been developed where bridge currents and UMP generated vibration have been analysed. In comparison with the numerical simulation, experimental behavior of bridge currents and UMP generated vibration show the presence of mixed eccentricity condition in both of the experimental setup. This behavior of experimental measurements in one way also verifies the capability of the developed code. The present study will be helpful in understanding the behavior of the bridge currents in different eccentricity conditions of the rotor and this would eventually be useful for the development of a bearingless machine using bridge configuration winding scheme.

APPENDIX A

List of symbols

x, y	Cartesian coordinates	L_b	Coil inductance of the bridge connection
t	Time	R_b	Coil resistance of the bridge connection
$\vec{J}$	Current density	i_{bA}, i_{bB}, i_{bC}	Bridge currents
$\vec{A}$	Magnetic vector potential	R_{er}	Rotor end ring resistance
$\vec{k}$	Unit vector parallel to z axis	L_{er}	Rotor end ring inductance
μ	Permeability	$I_{b1} \dots I_{b26}$	Rotor bar currents
ν	Magnetic reluctivity	R_r	Rotor bar resistance
V	Scalar potential	$U_{b1} \dots U_{b26}$	Voltage across each rotor bar
E	Electric field intensity	$I_{er1} \dots I_{er26}$	Rotor end ring currents
B	Magnetic field density	$[M_2]$	Transformation matrix for conversion of rotor end ring currents to rotor bar currents
σ_r	Electrical conductivity of the rotor bar	F_x, F_y	UMP along x and y axes
U_r	Voltage across the rotor bar	f_s	Main supply frequency
U_s	Voltage across stator coil	f_r	Mechanical frequency of the rotor
n_0	No. of turns in the stator winding	f_{bridge}	Frequency of the bridge currents
S_s	Cross sectional area of the stator coil	d_x, d_y	Displacement along x and y axes
l	Length of the rotor bar		
I_s	Stator coil current		
ξ, η	Natural coordinate system		
$k_c^e, c_r^e, p_s^e, p_r^e, q_r^e, q_s^e$	Elemental matrix in cartesian coordinate system		
$k_n^e, c_r, p_s, p_r, q_r, q_s$	Elemental matrix in natural coordinate system		
$\{N\}$	Shape function		
$ J $	Jacobian		
n	Number of nodes		
u	Number of stator circuit current		
z	Number of coil sides		
ν	Number of rotor conductors		
$[K]_{n \times n}$	Global stiffness matrix		
$[C_r]_{n \times n}$	Global matrix for induced emf due to the rotor bars		
$[P_s]_{n \times z}$	Global matrix for the stator coil excitations		
$[P_r]_{n \times \nu}$	Global matrix for the rotor conductor excitations		
$[T]_{z \times u}$	Transformation matrix for the transformation of phase currents to the coil side currents		
$\{I_s\}_{u \times 1}$	Stator circuit current vector		
$\{U_r\}_{\nu \times 1}$	Rotor bar voltage vector		
$\{I_r\}_{\nu \times 1}$	Rotor bar current vector		
$\{U_s\}_{u \times 1}$	Stator coil voltage vector		
$[R_r]_{\nu \times \nu}$	Rotor bar resistance matrix		
$[R_s]_{u \times u}$	Stator coil resistance matrix		
$[Q_s]_{u \times n}$	Global matrix of the induced emf for the stator circuit equation		
$[Q_s]_{\nu \times n}$	Global matrix of the induced emf for the rotor circuit equation		
$Q_1 \dots Q_{12}$	Global matrix of the induced emf for the stator circuit equation for coil groups $W_1 \dots W_{12}$		
R	Stator coil resistance		
L_{es}	End winding inductance of the stator coil		
V_A, V_B, V_C	Three phase voltages of the stator		
i_A, i_B, i_C	Three phase currents of the stator		
i_1, i_2, i_3	Coil currents in the parallel paths of the stator winding		

ACKNOWLEDGMENT

The authors would like to acknowledge the financial support of Ministry of Electronics and Information Technology, Govt. of India and Academy of Finland (decision #275687).

REFERENCES

- [1] A. Tenhunen, "Finite element calculation of unbalanced magnetic pull and circulating current between parallel windings in induction motor with non-uniform eccentric rotor," *Journal of system design and dynamics*, vol. 3, pp. 519-529, 2009.
- [2] D. G. Dorrell, J. Shek, M-F. Hsieh and M.A. Muller, "Damper winding in induction machines for reduction of unbalanced magnetic pull and bearing wear," *IEEE Trans. On Ind. Appl.*, Vol. 49, pp. 2206-2216, 2013.
- [3] A. Sinervo and A. Arkkio, "Eccentricity related forces in two pole induction motor with four pole stator damper winding analyzed using measured rotor orbits," *IEEE Trans. Magn.*, Vol. 49, pp. 3029-3037, 2013.
- [4] A. Laiho, A. Sinervo, J. Orivuori, K. Tammi, A. Arkkio and K. Zenger, "Attenuation of harmonic rotor vibration in a cage rotor induction machine by a self-bearing force actuator," *IEEE Trans. Magn.*, Vol. 45, pp. 5388-5398, 2009.
- [5] W. K. S. Khoo, "Bridge configured winding for polyphaser self-bearing machines," *IEEE Trans. Magn.*, Vol. 41, pp. 1289-1295, 2005.
- [6] W. K. S. Khoo, K. Kalita and S. D. Garvey, "Practical implementation of the bridge configured winding for producing controllable transverse forces in electrical machines," *IEEE Trans. Magn.*, Vol. 47, pp. 1712-1718, 2011.
- [7] K. Kalita, S. Natesan, G. Kumar and K. Tammi, "Vibration control in electrical machines using built-in actuator," *Proceedings of the 9th IFTOMM international conference on rotor dynamics*, PP. 1593-1603, 2015.
- [8] S. K. Rajkumar, K. Kalita, A. Banerjee and W. K. S. Khoo, "Electromagnetic analysis of a bridge configured winding cage induction machine using finite element method," *Progress In Electromagnetics Research B*, Vol. 48, PP. 347-373, 2013.
- [9] J. P. A. Bastos and N. Sadowski, *Electromagnetic modeling by finite element methods*, CRC Press, 2003.
- [10] A. Chiba, M. A. Rahman and T. Fukao, "Radial force in a bearingless reluctance motor," *IEEE Trans. Magn.*, Vol. 27, PP. 786790, 1991.
- [11] M. Donat, "Computational modelling of the unbalanced magnetic pull by finite element method," *Modelling of Mechanical and Mechatronics Systems*, Vol. 48, PP. 83-89, 2012.
- [12] D. G. Dorrell, A. Hermann and B. B. Jensen, "Analysis of unbalanced magnetic pull in wound rotor induction machines using finite element analysis - Transient, motoring and generating modes," *IECON 2013 - 39th Annual Conference of the IEEE Industrial Electronics Society*, Vienna, PP. 7307-7312, 2013.
- [13] A. Tenhunen, T. Benedetti, T. P. Holopainen and A. Arkkio, "Electromagnetic forces in cage induction motors with rotor eccentricity," *In Proceedings of IEMDC.*, 2003.

- [14] S. J. Salon, M. J. DeBortoli and R. Palma, "Coupling of transient fields, circuits, and motion using finite element analysis," *Journal of Electromagnetic Waves and Applications*, Vol. 4, No. 11, PP. 10771106, 1990.
- [15] V. N. Savov, Z. D. Georgiev and E. S. Bogdanov, "Analysis of cage induction motor by means of the finite element method and coupled system of field, circuit and motion equations," *Electrical Engineering*, Vol. 80, PP. 2128, Springer-Verlag, 1997.
- [16] O. Vitek, M. Janda, and V. Hajek, "Effects of eccentricity on external magnetic field of induction machine," *In Proceedings of 15th IEEE Mediterranean Electrotechnical Conference*, 2010.
- [17] S. Natesan, G. Kumar, K. Kalita and M. Rahman, "Effects of eccentricity on external magnetic field of induction machine," *In Proceedings of the 1st International and 16th National Conference on Machines and Mechanisms (iNaCoMM2013)*, IIT Roorkee, India, Dec 2013.
- [18] S. Kocman, T. Hrub, P. Penka and A. Neumann, "FEM model of asynchronous motor for analysis of its parameters," *In Proceedings of the 11th International Conference ELEKTRO 2016*, Trigan, 2016.

Gaurav Kumar received BE (VTU) in 2009, M.Tech degree in Machine Design from the Department of Mechanical Engineering IIT Guwahati in the year 2012. His area of interest involves rotordynamic and vibration analysis of electrical machines particularly AMB, Induction Machine and SRM. Currently he is pursuing his PhD in Indian Institute of Technology Guwahati, India.

Karuna Kalita received BE (Dibrugarh University), MTech (IIT Guwahati), and PhD (Nottingham University) in 1995, 2002, 2006. He is an Associate Professor in the Department of Mechanical Engineering at IIT Guwahati. He has worked for Converteam, UK (Currently GE) (2006-2008) and subsequently worked in the University of Nottingham as lecturer (2008-2010). His research interests include Rotor Dynamics, Modelling of Electromechanical Systems, Self-bearing machine, Gas foil bearings. He has published 16 journal papers, 23 conference

papers and two patents in Magnetic Bearings. He has guided one doctoral researcher and 14 masters students.

Kari Tammi (M15) was born on 1974. He received the M.Sc., Lic.Sc., and D.Sc. degrees from the Helsinki University of Technology in 1999, 2003, and 2007, respectively. He received a Teachers pedagogical qualification at the Hme University of Applied Sciences in 2017. He was a Researcher with CERN, the European Organization for Nuclear Research, from 1997 to 2000, and a Post-Doctoral Researcher with North Carolina State University, USA, from 2007 to 2008. From 2000 to 2015, he was a Research Professor, a Research Manager, the

Team Leader, and other positions at the VTT Technical Research Centre of Finland. He has been an Associate Professor with Aalto University since 2015. He currently serves in the Finnish Administrative Supreme Court as a Chief Engineer Counselor. He has authored over 60 peer-reviewed publications cited in over 1500 other publications. He is a member of the Finnish Academy of Technology. He serves as the Deputy Chair for IFTOMM Finland.