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Analysis of Bridge Currents and UMP of an Induction Machine with Bridge Configured Winding using Coupled Field and Circuit Modeling

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Unbalanced magnetic pull (UMP) reduction is an important aspect of high-speed electrical machines and it can be reduced using a specialized winding scheme called bridge configured winding (BCW). A bridge configuration winding has two parallel paths in each phase. The mid points of these two parallel paths can be short-circuited to achieve passive control of the unbalanced magnetic pull (UMP) or a power supply can be added in between these points to achieve active control of UMP. Moreover, it is important to understand the behavior of UMP in different eccentricity conditions for its active suppression. A finite element modeling of bridge configured induction machine has been developed to study the effects of different eccentricity conditions on UMP and bridge currents. A generalized circuit equation has been coupled with the field equation for the implementation of bridge configuration winding. All of the three kinds of eccentricity conditions have been simulated. In addition, two experimental setups have been developed to study the UMP induced vibration and to study the effect of eccentricity on bridge currents.

Index Terms—Bridge configured winding (BCW), finite element method (FEM), induction machine, unbalanced magnetic pull (UMP).

I. INTRODUCTION

The Unbalanced magnetic pull (UMP) in an electrical machine has been studied by various authors since the beginning of 20th century. The eccentric position of a rotor makes magnetic field distribution asymmetric and as a result, a force acts on the rotor in the direction of the shortest air gap. The transverse force generated due to the asymmetric distribution of the magnetic field is called UMP. The asymmetry in the air gap magnetic field can occur due to the faults such as eccentric rotor positions and it can also be created purposely to generate a controllable transverse force on the rotor. These generated transverse force can be utilized for the suppression of vibration as well as for the development of bearingless machines. The UMP can be suppressed either passively or actively. Generally parallel winding [1] and damper winding [2] help in passive suppression of UMP. The dual set of winding scheme can be used for passive [3] as well as active suppression of UMP [4]. In the dual set of winding scheme the main winding is used for the torque production and the auxiliary winding is used to create asymmetry in the air gap magnetic field to generate transverse force on the rotor. The ultimate utilization of dual set of winding scheme is the development of bearingless machines [10]. Khoo [5] developed a single set of winding scheme called bridge configured winding scheme which has the capability to suppress the unbalanced magnetic pull actively as well as passively. For the generation of transverse force in an electrical machine of \( p \) pole pair, \( p \pm 1 \) pole pair magnetic field is needed. In a bridge configured winding, each phase of the winding is divided into 4 groups which are connected as a Wheatstone bridge in such a way that an additional supply source across the bridge points of the winding would generate magnetic fields of \( p \pm 1 \) pole pairs. The advantage of BCW winding scheme is that the same winding can be used for the torque as well as transverse force generation. Khoo, Kalita, and Garvey [6] practically implemented BCW scheme on a brushless permanent magnet motor and showed its capability to generate controllable forces. Kalita, Natesan, Kumar, and Tammi [7] demonstrated partial active vibration control capability of BCW scheme in a 4-pole induction motor. Natesan, Kumar, Kalita and Rahman [17] developed a 37 kW bridge configured induction machine and showed experimentally that an eccentric rotor creates a potential difference across the bridge points of the bridge configured winding. When these bridge points are short circuited a current flows in the winding which passively controls UMP. To understand the behavior of bridge configured winding it is necessary to analyse the bridge currents in different eccentricity conditions.

There are no studies in the literature which demonstrate the effect of the different eccentricity conditions on the bridge currents and generated UMP in an electric machine with bridge configured winding. The finite element (FE) modeling is an efficient modeling technique for the investigation of machine behavior [14], [15]. Recently few authors have used finite element modeling for the investigation of unbalanced magnetic pull in different eccentricity conditions and the effect of parallel paths on unbalanced magnetic pull [11] - [13]. A few work has been carried out on FE modeling of bridge configured winding to analyse the bridge currents in static eccentricity condition [8]. The developed model does not predict the correct behavior of the bridge currents as well as UMP in the static eccentricity condition. It has been shown that in static eccentricity condition bridge currents have \( f_r \pm f_s \) frequency components, where \( f_s \) is the frequency of the main supply voltage and \( f_r \) is rotor rotational frequency. However
in the static eccentricity condition the bridge current should have only \( f_z \) frequency component. It happens because in \( [8] \) a transformation matrix has been used for the implementation of bridge configuration winding, which enforces equal bridge currents in all the four legs of a particular phase. This is not a proper modeling of the bridge configuration winding scheme.

The aim of the author is to develop a simulation model which can be be used to study bridge configured winding for the development of a bearingless machine. Hence, to understand and investigate the actual behavior of the bridge currents and UMP in different eccentricity conditions in bridge configured winding, a FE model of 3.7 kW bridge configured induction machine has been developed. A generalized circuit equation has been used to implement the bridge configuration winding in the FE model. The term generalized circuit means that the same numerical model can be used for the active as well as passive vibration control analysis, i.e. when bridge points are short circuited or bridge points are connected with an external isolated supply source. However the present analysis is based on two conditions, (i) when the bridge points are open (bridge OFF condition) and (ii) when the bridge points are short circuited (bridge ON condition). The analysis for the external supply through the bridge points has not been considered in this paper. The material nonlinearity has not been considered in the developed FE model. Air gap stitching method has been used for the implementation of rotor rotation, the static, the dynamic and the mixed eccentricity conditions. Considering the complexity in combining rotor transverse motion in commercial softwares, this FE model has been developed in MATLAB environment. However in this work transverse vibration of the rotor has not been coupled with the magnetic field and circuit equation. Two experimental setups have been developed to analyse the bridge currents and vibration generated due to eccentric rotor. The type of eccentricity present in the experimental setup is not known because no known external eccentricity has been provided and only the eccentricity due to the mechanical assembly faults is present in the system. The measured bridge currents and vibration behavior obtained from these experimental setups have been compared with the numerical investigation to establish the type of eccentricity present in the system.

The rest of this paper is organized as follows. In section II, fundamental equations of an electrical machine have been discussed. In section III, finite element modeling of BCW machine has been presented in detail. In section IV, numerical investigation of BCW machine in different eccentricity conditions have been presented. In section V, experimental investigation has been carried out to have a comparison with the numerical investigation.

II. FUNDAMENTAL EQUATIONS

This work is restricted to 2D field analysis, where the current density \( \vec{J} \) and the magnetic vector potential \( \vec{A} \) are \( z \)-directed (parallel to the axial length of the machine) and independent of \( z \).

\[ \vec{J} = J(x,y,t) \vec{k} \quad (1) \]
\[ \vec{A} = A(x,y,t) \vec{k} \quad (2) \]

where \( x \) and \( y \) are the cartesian coordinates, \( \vec{k} \) is the unit vector parallel to the \( z \)-axis and \( t \) is the time. Since, vectors \( \vec{J} \) and \( \vec{A} \) have only one component, they can be treated as scalars.

The 2D approximation of field equation in \( x-y \) plane can be written as

\[ \frac{\partial}{\partial x} \left[ \frac{1}{\mu} \frac{\partial A}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \frac{1}{\mu} \frac{\partial A}{\partial y} \right] - \sigma \frac{\partial A}{\partial t} - \sigma \text{grad}V = 0 \quad (3) \]

where \( \sigma \text{grad}V \) is the source of the excitation. This formulation is based on a squirrel cage induction machine, where two different types of conductors are used. The thick conductors (single conducting area) are used in the rotor and the thin conductors (multi turn coils) are used in the stator winding. The application of electrical continuity equation for the thick and the thin conductors gives \( \text{grad}V = -U/l \), where \( U \) is the applied voltage. Hence, the 2D approximation of the field equation can be written as

\[ \frac{\partial}{\partial x} \left[ \frac{1}{\mu} \frac{\partial A}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \frac{1}{\mu} \frac{\partial A}{\partial y} \right] - \sigma \frac{\partial A}{\partial t} + \sigma \frac{U}{l} = 0 \quad (4) \]

we get the circuit equations for the thick and thin conductors by using \( E + \frac{\partial A}{\partial t} = -\text{grad}V \) and Ohm’s law, \( J = \sigma E \), as (5) and (6) respectively.

\[ U_r = R_s I_r + R_s \int_{S_r} \frac{\partial A}{\partial t} \sigma \cdot ds \quad (5) \]
\[ U_s = R_s I_s + \frac{n l}{S_s} \int_{S_s} \frac{\partial A}{\partial t} \sigma \cdot ds \quad (6) \]

The subscripts \( r \) and \( s \) represent the rotor side and the stator side variables respectively and it has been used consistently in the entire article. The combined field equation considering the thick and the thin conductor excitation can be written as [9]

\[ \frac{\partial}{\partial x} \left[ \frac{1}{\mu} \frac{\partial A}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \frac{1}{\mu} \frac{\partial A}{\partial y} \right] - \sigma \frac{\partial A}{\partial t} + \frac{n_0}{S_s} I_s + \sigma \frac{U_r}{l} = 0 \quad (7) \]

where \( \sigma_r \) is the electrical conductivity of the rotor bar, \( n_0 \) is the no. of turns in the stator winding, \( S_s \) is the c/s area of the stator coil, \( l \) is the length of the rotor bar, \( U_r \) is the voltage across the rotor bar and \( \mu \) is the permeability.

III. FINITE ELEMENT MODELING OF BCW ELECTRICAL MACHINE

In general any electrical machine can be divided into five domains for the purpose of numerical modeling i.e the stator core domain, the stator winding domain, the air gap domain, the rotor core domain and the rotor conductor domain. The Galerkin method has been used for the application of FE discretization in (5), (6) and (7). The elemental matrices have been obtained for the field equation (7) for different domains of the machine. The elemental stiffness matrix for the entire domain, the elemental matrix for the contribution of the induced emf in the field equation due to rotor bar, the elemental matrix for the contribution of the excitation at the stator winding, and the elemental matrix for the contribution of the excitation at the rotor conductor can be represented as (8), (9), (10), and (11) respectively.
k^e_n = \iint_s \nu \{N_x\} [N_x] + \{N_y\} [N_y] \, dx \, dy \quad (8)

c^e_r = \iint_s \sigma_r \{N\} [N] \, dx \, dy \quad (9)

p^e_s = \frac{n_0}{\nu_k} \iint_s \{N\} \, dx \, dy \quad (10)

p^e_r = \iint_s \frac{\sigma_r}{T} \{N\} \, dx \, dy \quad (11)

The elemental matrix for the rotor circuit equation and the elemental matrix for the stator circuit equation can be represented as (13) and (12) respectively.

\[ q_r = R_\sigma r \iint_s \{N\} \, dx \, dy \quad (12) \]

\[ q_s = \frac{n_0 l}{\nu_k} \iint_s \{N\} \, dx \, dy \quad (13) \]

where \( \nu \) is the magnetic reluctivity \( (\nu = \frac{1}{\mu}) \) and \( \{N\} \) is a vector of the shape functions. Two types of elements have been used for the FE discretization. Eight nodded quadrilateral elements have been used in the stator core, the stator coil, the rotor core and the rotor conductors, whereas six nodded triangular elements have been used for the air gap modeling. For eight nodded quadrilateral elements, the elemental stiffness matrix (8) can be written in natural coordinate system \((\xi - \eta)\) as

\[ k^e_n = \int_{-1}^{1} \int_{-1}^{1} \nu \{\text{grad}N\} [\text{grad}N] [J] \, d\xi \, d\eta \quad (14) \]

However for six nodded quadrilateral elements, the elemental stiffness matrix (8) can be written in natural coordinate system as

\[ k^e_n = \int_{0}^{1} \int_{0}^{1} \nu \{\text{grad}N\} [\text{grad}N] [J] \, d\xi \, d\eta \quad (15) \]

Similarly for eight nodded quadrilateral element, the elemental stiffness matrix (9) to (12) can be written in natural coordinate system as

\[ c_r = \int_{0}^{1} \int_{0}^{1} \sigma_r \{N\} [N] [J] \, d\xi \, d\eta \quad (16) \]

\[ p_s = \frac{n_0}{\nu_k} \int_{0}^{1} \int_{0}^{1} \{N\} [J] \, d\xi \, d\eta \quad (17) \]

\[ p_r = \int_{0}^{1} \int_{0}^{1} \frac{\sigma_r}{T} \{N\} [J] \, d\xi \, d\eta \quad (18) \]

\[ q_s = \frac{n_0 l}{\nu_k} \int_{0}^{1} \int_{0}^{1} \{N\} [J] \, d\xi \, d\eta \quad (19) \]

\[ q_r = R_\sigma r \int_{0}^{1} \int_{0}^{1} \{N\} [J] \, d\xi \, d\eta \quad (20) \]

After assembling all the elemental matrices the final assembled field equation can be written as (21). The assembled rotor and stator circuit equations can be written as (22) and (23) respectively. The stiffness matrix has the contribution from the entire domain, hence \( k^e \) has to be assembled for all the elements in the domain. However, \( c_r, p_r \) and \( q_r \) are applied to the rotor conductor domains only and \( p_s \) and \( q_s \) are applied in the stator coil domains.

\[ [K]_{n\times n} [A]_{n\times 1} + [C]_{n\times n} \frac{\partial}{\partial t} [A]_{n\times 1} + [P]_{n\times n\times 1} [T]_{n\times 1} = 0 \quad (21) \]

\[ [R]_{v\times v} [I]_{v\times 1} + [Q]_{v\times n} \frac{d}{dt} [A]_{n\times 1} = [U]_{v\times 1} \quad (22) \]

\[ [R]_{u\times u} [I]_{u\times 1} + [Q]_{u\times n} \frac{d}{dt} [A]_{n\times 1} = [U]_{u\times 1} \quad (23) \]

where \( n \) represents the number of nodes in the domain, \( u \) is the number of stator circuit currents, \( v \) is the number of rotor conductors and \( z \) represents the number of coil side currents. In our analysis the typical value of \( n \) is 29,572. The transformation matrix used for the transformation of phase currents to the coil side currents is \([T]_{v\times u}\).

The BCW scheme has a double layer winding with two parallel paths and one additional source of power supply in each phase of the stator winding. The simulated motor is a 4-pole squirrel cage induction motor which has 36 stator slots and 26 rotor bars. In a conventional 3-phase machine the value of the stator circuit currents would be three. However in our analysis the value of the stator circuit current would depend on whether we use a simple parallel winding connection or a short circuited parallel winding connection with or without a power source. Hence for the inclusion of parallel paths and bridge connections, the field equation (21) and the stator circuit equation (23) have to be modified. To incorporate the parallel paths of the rotor bars of the squirrel cage along with the end rings, the rotor bar equation (22) has to be modified. This has been explained in details in next subsections.
A. Introduction to BCW winding

Fig. 1 represents the winding diagram of an induction machine with 36 stator slots and Fig. 2 represents the coil connection diagram of the winding. Coils \((a1-a3), (a4-a6)\) and \((aa1-aa3), (a4-a6)\) form two parallel paths in phase \(A\) of the winding. Coils \((b1-b3), (bb4-bb6)\) and \((bb1-bbb3), (b4-b6)\) form two parallel paths in phase \(B\) of the winding. Coils \((c1-c3), (cc4-cc6)\) and \((cc1-cc3), (c4-c6)\) form two parallel paths in phase \(C\) of the winding. For the explanation of the modeling of the circuit, twelve coil groups (W1-W12) have been formed. Coils \((a1-a3), (a4-a6), (aa1-aa3), (a4-a6), (b1-b3), (b4-b6), (bb1-bb3), (bb4-bb6), (c1-c3), (c4-c6), (cc1-cc3), and (cc4-cc6)\) has been termed as coil group W1, W2, W3, W4, W5, W6, W7, W8, W9, W10, W11, and W12 respectively.

![Winding diagram](image)

**Fig. 1: Winding diagram of double layer bridge configuration winding**

B. Stator winding circuit equation when bridges are open (bridge OFF condition)

Fig. 3 represents the star connected circuit diagram of a bridge configuration wound machine when bridge points are not connected and machine is supplied with a three phase star connected power source. We will refer this connection of the bridge as bridge OFF condition. The two bridge points of phase \(A\) are \(A1\) and \(A2\), the two bridge points of phase \(B\) are \(B1\) and \(B2\) and the two bridge points of phase \(C\) are \(C1\) and \(C2\) respectively. The neutral points of the stator winding and the supply source are \(O\) and \(O'\) respectively. The three phase supply voltages are \(V_A, V_B,\) and \(V_C\) and the three phase currents of phase \(A\), phase \(B\), and phase \(C\) are \(i_A, i_B,\) and \(i_C\) respectively. The two parallel paths of phase \(A\) are \(A−A1−O\) and \(A−A2−O\) and \(i_1\) and \(i_1−i_1\) are the currents flowing in these parallel paths respectively. The two parallel paths of phase \(B\) are \(B−B1−O\) and \(B−B2−O\) and \(i_2\) and \(i_2−i_2\) are the currents flowing in these parallel paths respectively. The two parallel paths of phase \(C\) are \(C−C1−O\) and \(C−C2−O\) and \(i_3\) and \(i_C−i_3\) are the currents flowing in these parallel paths respectively.

The DC resistance of each coil group including the end winding resistance is \(R\) and the end winding inductance of the each coil group is \(L_{es}\). The Kirchhoff’s voltage law (KVL) has been used to write the circuit equations. There are five loops in the circuit shown in Fig. 3. Equations for these five loops can be written as (24) to (28). In addition to these equations, (29) can be written by imposing the Kirchhoff’s current law (KCL) about the star point \(O\) of the winding. By
assembling (19) for coil group domains W1, W2, ..., W12, matrices $Q_1, Q_2, \ldots, Q_{12}$ have been obtained respectively.

KVL for loop 3 ($O'\{A\}_1OB\{B\}$) can be written as

$$[
Q_1 + Q_4 - Q_5 - Q_8]_{1 \times n} \frac{d\{A\}_{n \times 1}}{dt} + 2Ri_1 - 2Ri_2 \tag{24}
$$

$$+ 2L_{es} \frac{di_1}{dt} - 2L_{es} \frac{di_2}{dt} = V_B - V_A$$

KVL for loop 2 ($O'\{B\}_2OC\{C\}$) can be written as

$$[Q_6 + Q_7 - Q_9 - Q_{12}]_{1 \times n} \frac{d\{A\}_{n \times 1}}{dt} + 2R(i_B - i_2)
$$

$$- 2Ri_3 + 2L_{es} \frac{d(i_B - i_2)}{dt} - 2L_{es} \frac{di_3}{dt} = V_C - V_B \tag{25}$$

KVL for loop 3 ($A\{A\}_2OA\{A\}$) can be written as

$$[Q_2 + Q_3 - Q_4]_{1 \times n} \frac{d\{A\}_{n \times 1}}{dt} + 2R(i_A - i_1)
$$

$$- 2Ri_1 + 2L_{es} \frac{d(i_A - i_1)}{dt} - 2L_{es} \frac{di_1}{dt} = 0 \tag{26}$$

KVL for loop 4 ($B\{B\}_1OB\{B\}$) can be written as

$$[Q_5 + Q_8 - Q_6 - Q_7]_{1 \times n} \frac{d\{A\}_{n \times 1}}{dt} - 2R(i_B - i_2)
$$

$$+ 2Ri_2 - 2L_{es} \frac{d(i_B - i_2)}{dt} + 2L_{es} \frac{di_2}{dt} = 0 \tag{27}$$

KVL for loop 5 ($C\{C\}_1OC\{C\}$) can be written as

$$[Q_9 + Q_{12} - Q_{10} - Q_1]_{1 \times n} \frac{d\{A\}_{n \times 1}}{dt} - 2R(i_C - i_3)
$$

$$+ 2Ri_3 - 2L_{es} \frac{d(i_C - i_3)}{dt} + 2L_{es} \frac{di_3}{dt} = 0 \tag{28}$$

And KCL about point O can be applied as

$$i_A + i_B + i_C = 0 \tag{29}$$

Final circuit equation can be represented as

$$[Q] \frac{d\{A\}}{dt} + [R] \{I\} + [L_{es}] \left\{ \frac{di_1}{dt} \right\} = \{U_s\} \tag{30}$$

where $I = [i_1, i_A, i_2, i_B, i_3, i_C]^T$ and $U_s = [V_A - V_B, V_B - V_C, 0, 0, 0, 0]^T$.

C. Stator winding circuit equation when bridges are closed (bridge ON condition)

Fig. 4 represents the star connected circuit diagram of a bridge configuration wound machine when the bridge points are closed or connected to a voltage source and the machine is supplied with a three phase star connected power source. We will refer this connection of bridge as bridge ON condition. An eccentric rotor generates a potential difference across the bridge points of the winding due to which, upon connecting these bridge points, a current flows across the bridge points which changes the dynamics of the machine [7]. An isolated controlled voltage source in current control mode can also be connected to these bridge points by which current can be injected in the winding to generate the desired force for bearingless operation or for the purpose of active vibration control. The aim of the present work is to study the effect of eccentric rotor when bridges are open and closed. However a generalized model has been developed considering a voltage supply across the bridge points. The DC resistance and the inductance of the coil connecting across the bridge points are $R_b$ and $L_b$ respectively. The voltage source connected between the bridge points $A_1$ and $A_2$ of phase $A$ of the winding is $V_{ba}$. The voltage source connected between the bridge points $B_1$ and $B_2$ in phase $B$ of the winding is $V_{bb}$. The voltage source connected between the bridge points $C_1$ and $C_2$ in phase $C$ of the winding is $V_{bc}$. The bridge currents in phase $A$, phase $B$, and phase $C$ of the winding are $i_{ba}, i_{bb},$ and $i_{bc}$ respectively.

The bridge currents in the coil groups are shown in Fig. 4. Similarly to the bridge open condition the Kirchhoff’s voltage law has been used to write the circuit equations for eight loops as shown in (31) to (38) and the Kirchhoff’s current law has been applied about the star point $O$ as shown in (39).

KVL for loop 1 ($O'\{A\}_1OB\{B\}$) can be written as

$$[Q_1 + Q_4 - Q_5 - Q_8]_{1 \times n} \frac{d\{A\}_{n \times 1}}{dt} + 2Ri_1 - 2Ri_2
$$

$$- Ri_3 + 2L_{es} \frac{di_1}{dt} - 2L_{es} \frac{di_2}{dt} = V_B - V_A \tag{31}$$

KVL for loop 2 ($O'\{B\}_2OC\{C\}$) can be written as

$$[Q_6 + Q_7 - Q_9 - Q_{12}]_{1 \times n} \frac{d\{A\}_{n \times 1}}{dt} - 2Ri_2 - 2Ri_3
$$

$$+ 2Ri_3 + 2L_{es} \frac{d(i_B - i_2)}{dt} - 2L_{es} \frac{di_3}{dt} = V_C - V_B \tag{32}$$

KVL for loop 3 ($A\{A\}_2OA\{A\}$) can be written as

$$[Q_3 - Q_1]_{1 \times n} \frac{d\{A\}_{n \times 1}}{dt} - 2Ri_1 + Ri_b - Ri_{bb}
$$

$$- 2L_{es} \frac{di_1}{dt} + L_{es} \frac{di_b}{dt} = V_{ba} \tag{33}$$
KVL for loop 4 (OA1A2O) can be written as
\[ [Q_2 - Q_4]_{1 \times n} \frac{d\{A\}}{dt} + 2Ri_1 + Ri_A + 2Ri_B + 2Ri_B = \frac{dI_{bA}}{dt} + L_{end} \frac{di_A}{dt} + L_{es} \frac{di_B}{dt} = V_{bA} \] (34)

KVL for loop 5 (B1OB2B1) can be written as
\[ [Q_5 - Q_7]_{1 \times n} \frac{d\{A\}}{dt} + 2Ri_2 - Ri_B - 2Ri_B = \frac{dI_{bB}}{dt} + L_{end} \frac{di_B}{dt} - L_{es} \frac{di_B}{dt} = V_{bB} \] (35)

KVL for loop 6 (BB1B2B) can be written as
\[ [Q_8 - Q_6]_{1 \times n} \frac{d\{A\}}{dt} + 2Ri_3 - Ri_C - 2Ri_B = \frac{dI_{bC}}{dt} + L_{end} \frac{di_C}{dt} - L_{es} \frac{di_C}{dt} = V_{bC} \] (37)

KVL for loop 7 (C1OC2C1) can be written as
\[ [Q_9 - Q_1]_{1 \times n} \frac{d\{A\}}{dt} + 2Ri_4 - Ri_C - 2Ri_B = \frac{dI_{bC}}{dt} + L_{end} \frac{di_C}{dt} - L_{es} \frac{di_C}{dt} = V_{bC} \] (38)

And Kirchhoff’s current law at point O can be applied as
\[ i_A + i_B + i_C = 0 \] (39)

Final circuit equation for stator winding can be written as
\[ \{Q\} \frac{d\{A\}}{dt} + [R] \{I_s\} + [L_{es}] \{dI_s\} = \{U_s\} \] (40)

where \( I_s = [i_1 \; i_A \; i_2 \; i_B \; i_3 \; i_C \; i_B \; i_B \; i_B]^{\top} \) and \( U_s = [V_A - V_B \; V_B - V_C \; V_B - V_A \; -V_B - V_B - V_C - V_B - V_C - V_C]^\top \).

If there is no external supply across the bridge points the supply voltage would be represented as \( U_s = [V_A - V_B \; V_B - V_C \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \]^{\top}. \)

D. Rotor bar circuit equation

The Squirrel cage rotor conductors are connected in parallel and short-circuited with each other using the end rings. Fig. 5 represents the circuit connection of a squirrel cage rotor of an induction machine.

![Fig. 5: Parallel connection of the rotor bar conductors](image)

The rotor bar currents for the rotor bar 1 to rotor bar 26 are \( I_{b1}, \ldots, I_{b26} \) respectively. The end ring effect has been modeled with 26 end resistances \((R_{er})\) and 26 end inductances \((L_{er})\). The end ring currents for loop 1 to loop 26 are \( I_{er1}, \ldots, I_{er26} \) respectively. For each individual rotor bar the circuit equation can be written as
\[ \frac{dI_{b1}}{dt} + R_i I_{b1} - U_{b1} = 0 \] (41)
\[ \vdots \]
\[ \frac{dI_{b26}}{dt} + R_i I_{b26} - U_{b26} = 0 \]

where \( Q_{r1}, \ldots, Q_{r26} \) have been obtained by assembling (20) for rotor bar 1 to rotor bar 26 respectively. The DC resistance of each rotor bar is \( R_i \) and \( U_{b1}, \ldots, U_{b26} \) are the voltage drop across rotor bar 1 to rotor bar 26 respectively. The rotor bar circuit equation (41) can be written in matrix form as
\[ [Q_r] \frac{d\{A\}}{dt} + [R_i] \{I_s\} - \{U_s\} = 0 \] (42)

The Kirchhoff’s voltage law can be used to obtain the circuit equation for each loop as
\[ 2R_{er}I_{er1} + 2I_{er1} \frac{dI_{er1}}{dx} - U_{b1} + U_{b26} = 0 \]
\[ \vdots \]
\[ 2R_{er}I_{er26} + 2I_{er26} \frac{dI_{er26}}{dx} - U_{b26} + U_{b26} = 0 \]

It can be written in matrix form as
\[ 2\{R_{er}\} \{I_{er}\} + 2\{L_{er}\} \frac{d\{I_{er}\}}{dt} + \{M_2\} \{U_s\} = 0 \] (44)

where \( \{R_{er}\} = \begin{bmatrix} R_{er} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & R_{er} \end{bmatrix} \)
\( \{L_{er}\} = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & L_{er} \end{bmatrix} \)
\[ \{I_{er}\} = \begin{bmatrix} I_{er1} \\ \vdots \\ I_{er26} \end{bmatrix} \]

The connection between the rotor bar currents and the end ring currents can be written as
\[ \begin{bmatrix} I_{b1} \\ I_{b2} \\ \vdots \\ I_{b26} \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 & \cdots & 0 \\ 0 & -1 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & -1 \end{bmatrix} \begin{bmatrix} I_{er1} \\ I_{er2} \\ \vdots \\ I_{er26} \end{bmatrix} \] (45)

Which can be written in matrix form as
\[ \{I_s\} = \{M_2\}^T \{I_{er}\} \] (46)

The final connection equation of the rotor bar and the end ring has been obtained by combining (44) and (46) as
\[ \{M_2\}^T \{M_2\} \{I_{er}\} + 2\{R_{er}\} \{I_{er}\} + 2\{L_{er}\} \frac{d\{I_{er}\}}{dt} = 0 \] (47)

E. Transformation matrix

It can be observed from (30) that in the case of bridge open condition there are six current variables available from
the circuit equation which includes 3 phase currents and 3 branch currents. In bridge closed condition there are nine current variables available which includes 3 phase currents, 3 branch currents and 3 bridge currents as written in (40). A transformation matrix is needed for the coupling of the circuit current in the field equation as the size of the source distribution matrix, \([P_s]\), in the field equation (21) is \(n \times z\). The value of \(z\) is 72 as we have 72 coil sides. The size of source current, \(i\), in bridge open and bridge closed conditions are \(6 \times 1\) and \(9 \times 1\) respectively. Hence transformation matrices of sizes \(72 \times 6\) and \(72 \times 9\) are required to couple the circuit currents in the field equation for bridge open and bridge closed condition respectively.

In modeling of the thin coils, three terms have been used, coil sides, full coil and coil group. Each slot of the machine has two layers and consist of two different coil sides, as shown in Fig. 1. The two-coil sides of a coil are +\(a1\) and -\(a1\). The +ve sign indicates that the current is coming out of the plane and -ve sign indicates that the current is going into the plane. The coil groups W1, W2, W3 and W4 form phase A of the winding, the coil groups W5, W6, W7 and W8 form phase B of the winding and the coil groups W9, W10, W11 and W12 form the phase C of the winding as shown in Fig. 2. There are 72 coil side currents, 36 full coil currents and 12 coil group currents. There are 6 circuit currents in bridge open condition and 9 circuit currents in bridge closed or bridge supply condition. Fig. 6 represents the flow chart for conversion of circuit currents to coil side currents. Circuit currents are transformed to coil group currents and coil group currents are transformed to full coil currents and full coil currents are transformed to coil side currents. Conversion of group coil currents to full coil currents and coil side currents has same transformation matrix in bridge open and bridge supply or bridge closed condition. For the transformation from full coil currents to coil side currents, the following transformation has been used,

\[
\begin{bmatrix}
+i_{a1} \\
-i_{a1} \\
\vdots \\
+i_{cc6} \\
-i_{cc6}
\end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ -1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & 1 \\ 0 & 0 & 0 & \cdots & -1 \end{bmatrix} \begin{bmatrix} i_{a1} \\ \vdots \\ i_{cc6} \\ \vdots \\ i_{cc6} \end{bmatrix}
\]

(48)

which can be written as

\[
\{i_{coil, side}\}_{72 \times 1} = \begin{bmatrix} T_{o} \end{bmatrix}_{72 \times 36} \{i_{coil}\}_{36 \times 1}
\]

(49)

Since each coil of the same coil group are connected in series so they must have the same current. For the transformation of phase A coil group currents to phase A full coil currents the following transformation has been used,

\[
\begin{bmatrix} i_{a1} \\ i_{a2} \\ \vdots \\ i_{aa4} \\ i_{aa5} \\ i_{aa6} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \begin{bmatrix} i_{w1} \\ \vdots \\ i_{w12} \end{bmatrix}
\]

(50)

Similarly, the transformation matrices for other two phases can also be written and can be combined as

\[
\{i_{coil}\}_{36 \times 1} = \begin{bmatrix} T_{c} \end{bmatrix}_{36 \times 12} \{i_{coil, group}\}_{12 \times 1}
\]

(51)

For the case of bridge open condition the transformation from circuit currents to coil group current can be achieved as written in (52).

\[
\begin{bmatrix} i_{w1} \\ i_{w2} \\ i_{w3} \\ i_{w4} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ -1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} i_{1} \\ i_{4} \end{bmatrix}
\]

(52)

Similar transformation matrices can be obtained for phase B and phase C. The combined transformation matrix can be written as

\[
\{i_{coil, group}\}_{12 \times 1} = \begin{bmatrix} T_{o} \end{bmatrix}_{12 \times 6} \{i_{circuit}\}_{6 \times 1}
\]

(53)

In the case of bridge closed and bridge supply condition, the bridge currents pass through the bridge points in each phase. The transformation of circuit currents to coil group currents for phase A can be written as

\[
\begin{bmatrix} i_{w1} \\ i_{w2} \\ i_{w3} \\ i_{w4} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} i_{1} \\ i_{4} \end{bmatrix}
\]

(54)

Similarly transformation matrices can be obtained for phase B and phase C. The combined transformation matrix can be written as

\[
\{i_{coil, group}\}_{12 \times 1} = \begin{bmatrix} T_{c} \end{bmatrix}_{12 \times 9} \{i_{circuit}\}_{9 \times 1}
\]

(55)

So, the final transformation matrix for the bridge open condition can be written as

\[
\{i_{coil, side}\}_{72 \times 1} = \begin{bmatrix} T_{open} \end{bmatrix}_{72 \times 6} \{i_{circuit, current}\}_{6 \times 1}
\]

(56)

where \([T_{open}]_{72 \times 6} = [T_{1}]_{72 \times 36}[T_{2}]_{36 \times 12}[T_{o}]_{12 \times 6}\)

Similarly, the final transformation matrix for the bridge closed and bridge supply condition can be written as

\[
\{i_{coil, side}\}_{72 \times 1} = \begin{bmatrix} T_{supply} \end{bmatrix}_{72 \times 9} \{i_{circuit, current}\}_{9 \times 1}
\]

The combined transformation matrix can be written as

\[
\{i_{coil, side}\}_{72 \times 1} = \begin{bmatrix} T_{supply} \end{bmatrix}_{72 \times 9} \begin{bmatrix} T_{o} \end{bmatrix}_{9 \times 12} \begin{bmatrix} T_{c} \end{bmatrix}_{12 \times 6} \{i_{circuit}\}_{6 \times 1}
\]

TABLE I: Current Transformation Group

<table>
<thead>
<tr>
<th>Bridge Conditions</th>
<th>Coil Side Currents</th>
<th>Coil Currents</th>
<th>Group Coil Currents</th>
<th>Circuit Currents</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open</td>
<td>72</td>
<td>36</td>
<td>12</td>
<td>6</td>
</tr>
<tr>
<td>Closed</td>
<td>72</td>
<td>36</td>
<td>12</td>
<td>9</td>
</tr>
</tbody>
</table>

Fig. 6: Winding arrangement of BCW machine
closed or bridge supply condition can be written as
\[
\{i_{\text{circ}, \text{side}}\}_{72 \times 9} = [T_{\text{closed}}]_{72 \times 9} \{i_{\text{circ}, \text{current}}\}_{9 \times 1}
\] (57)
where \([T_{\text{closed}}]_{72 \times 9} = [T]_{72 \times 36}[T]_{36 \times 12}[T]_{12 \times 9}\)

The combined discretized final field and circuit equations can be represented as
\[
[K]_{n \times n}[A]_{n \times 1} + [C_T][A]_{n \times 1} \frac{\partial [A]_{n \times 1}}{\partial t} - [P]_{n \times 72}[T]_{72 \times m}[i_{t}]_{m \times 1}
\]
\[+ [P]_{n \times 26}[U_r]_{26 \times 1} = 0
\] (58)
\[
[Q]_{1 \times n} \frac{d[A]_{n \times 1}}{dt} + [R]_{1 \times x \times 1}[i_{s}]_{x \times 1} + [L_{es}]_{x \times x} \left\{ \frac{dl_{s}}{dt}\right\}_{x \times 1} = [U_s]_{x \times 1}
\] (59)
\[
[Q_r]_{26 \times n} \frac{d[A]_{n \times 1}}{dt} + [R_r]_{26 \times 26}[i_{r}]_{26 \times 1} - [U_r]_{26 \times 1} = 0
\] (60)
\[
[M_2]_{26 \times 26} [M_2]_{26 \times 26} [U_r]_{26 \times 1} + [2][R_{er}]_{26 \times 26}[I_{r}]_{26 \times 1}
\]
\[+ 2[L_{er}]_{26 \times 26} \frac{d[i_{r}]}{dt} = 0
\] (61)

where \(m\) is 6 in the case of bridge open condition and \(m\) is 9 in bridge closed or bridge supply condition.

**F. Time-discretization**

The combined field and circuit equations for thick and thin conductors are shown in (58) to (61). This system of differential equations has been solved using Crank-Nicolson method. The Crank-Nicolson method is a second-order method, which is implicit in time and numerically stable. The time-discretization of (58) to (61) in matrix form can be written as shown in (62). The boundary condition has been applied by forcing the magnetic vector potential (\(A\)) to be zero at the outer boundary of the machine.

\[
\left[
\begin{array}{ccc}
\left( \frac{K^{t+\Delta t} + \Delta t}{2} + \frac{C_T}{2} \right) & [P]_r & [T] & 0 & 0 & [P]_r \\
\frac{Q_f}{2} & \left( \frac{R_{es} + \frac{\Delta t}{2} \Delta L_{es}}{2} \right) & 0 & 0 & 0 & 0 \\
\frac{Q_f}{2} & \left( \frac{R_{es} + \frac{\Delta t}{2} \Delta L_{es}}{2} \right) & 0 & 0 & 0 & 0 \\
0 & 0 & \left( \frac{R_{es} + \frac{\Delta t}{2} \Delta L_{es}}{2} \right) & 0 & 0 & 0 \\
\end{array}
\right] \{D\}^{t+\Delta t} =
\left[
\begin{array}{c}
\{U_s\}_{x \times 1} \\
\{U_s\}_{x \times 1} \\
\{U_s\}_{x \times 1} \\
\{U_s\}_{x \times 1} \\
\end{array}
\right]
\]

\[
= - \left[
\begin{array}{ccc}
\left( \frac{K^{t+\Delta t}}{2} + \frac{C_T}{2} \right) & [P]_r & [T] & 0 & 0 & [P]_r \\
\frac{Q_f}{2} & \left( \frac{R_{es} + \frac{\Delta t}{2} \Delta L_{es}}{2} \right) & 0 & 0 & 0 & 0 \\
\frac{Q_f}{2} & \left( \frac{R_{es} + \frac{\Delta t}{2} \Delta L_{es}}{2} \right) & 0 & 0 & 0 & 0 \\
0 & 0 & \left( \frac{R_{es} + \frac{\Delta t}{2} \Delta L_{es}}{2} \right) & 0 & 0 & 0 \\
\end{array}
\right] \{D\}^t + \{S\}^{t+\Delta t} + \{S\}^t
\] (62)

where \(\{S\} = \left\{ \begin{array}{c} \{0\}_{1 \times n} \\
\{U_s\}_{m \times 1} \\
\{0\}_{1 \times 26} \\
\{0\}_{26 \times 1} \\
\end{array} \right\} \) and \(\{D\} = \left\{ \begin{array}{c} \{A\}_{n \times 1} \\
\{U_s\}_{m \times 1} \\
\{0\}_{1 \times 26} \\
\{0\}_{26 \times 1} \\
\end{array} \right\} \)

**G. Modelling of rotor eccentricity**

To incorporate the rotor movement and the rotor eccentricity, the air gap stitching method has been applied. Fig. 7 represents the meshed region of the stator and the rotor domains.

**Fig. 7: Meshed stator and rotor with unmeshed air gap**

**Fig. 8: Stitching in air gap region**

**Fig. 9: Algorithm used for air gap stitching**

It can be observed from Fig. 7 that the air gap region has been divided in four parts. Two parts of the air gap have been modeled with the stator region and one part of the air gap has been modeled with the rotor region. To incorporate the rotation and the eccentricity condition the remaining part of the air gap is stitched using six-noded triangular elements as shown in Fig. 8. The following algorithm has been used for the stitching of the air gap region. Suppose A, C, E, G, ... are the stator side nodes and B, D, F, ... are its mid-side nodes. Similarly, a, c, e, ... are the rotor side nodes and b, d, ... are its mid-side nodes as shown in Fig. 9. At first any two nodes from the stator side and the rotor side, for example Aa is connected arbitrarily. Now the shorter distance between Ac and aC is chosen to form the first triangle. Considering aC<Ac, \(\triangle\) aAC can be formed. The mid-point of the aC is chosen for the mid-side node of aC. The next triangle is formed by choosing the shorter distance between aE and Cc. In the similar manner the whole domain is stitched. For the incorporation of rotation and eccentricity condition, one portion of the air gap is restitched at each time step. This process is carried out in three steps.
First of all rotor portion is rotated about the center point and then its location is shifted as per the eccentricity condition and after that stitching between the rotor side nodes and the stator side nodes is carried out. A rotor is said to be statically eccentric if the rotor centre does not coincide with the stator centre and the rotor rotates about the rotor centre. The rotor is said to be dynamically eccentric when the rotor centre does not coincide with the stator centre and the rotor rotates about its centre as well as whirls about the stator centre. A rotor is said to have mixed eccentricity if the rotor centre does not coincide with the stator centre and the rotor rotates about its centre as well as whirls about any other point in between the stator centre and the rotor centre.

IV. Numerical Investigation

The coupled field and circuit equation shown in (62) has been simulated for two sec to calculate the magnetic vector potential at each node, the circuit currents, the rotor bar currents and the voltage across each rotor bar. The magnetic flux density in the air gap has been calculated from the obtained magnetic vector potential using the relationship, \( B = \nabla \times A \) and subsequently the transverse forces \( (F_x, F_y) \) acting on the rotor have been calculated using the Maxwell stress tensor method as explained in [8]. A self-developed preprocessor has been used to generate the mesh for the numerical simulation where the stator, rotor and air gap domain has 5760, 3536 and 1025 elements respectively. Time stepping scheme, Crank Nicolson method and MATLAB’s inbuilt solver has been used for the simulation.

The results have been generated for two conditions, when (i) bridge connections of the winding are open or bridge OFF condition, (ii) bridge connections of the winding are closed or bridge ON condition. Table II shows the important parameters of the induction machine used for the simulation. The machine parameters have been calculated using the expressions explained in [18]. The simulation has been carried out for all three types of eccentricity conditions. For the investigation of the static eccentricity condition, the rotor has been shifted by a distance of 1e-4 m along the \( x \)-axis which is 10 % of the air gap and the rotor rotates about this point. In case of the dynamic eccentricity the rotor center rotates about the center point of the stator with a whirling radius of 1e-4 m and the rotor also rotates about its own center. Whereas, in the mixed eccentricity condition a static eccentricity of 0.5e-4 m, which is 5% of the air gap, has been given along the \( x \)-axis, the rotor center rotates about this point with a whirling radius of 0.5e-4 m and the rotor rotates about its own center as well. The bridge currents and the transverse forces acting on the rotor have been analyzed to study the effect of different eccentricity conditions. The bridge currents, and transverse forces acting on the rotor have been converted to the complex domain to investigate the various modes of the bridge currents and forces. Three phase bridge currents have been converted from \( a, b, c \) to \( d, q, 0 \) sequence using (63), where \( d, q \) represent the orthogonal components of current and 0 is the zero sequence component of the current. The space vector of the bridge currents can be written as \( d + jq \). Similarly, transverse forces can be written in complex form as \( F_x + jF_y \), where \( F_x \) is the force acting along \( x \) axis and \( F_y \) is the force acting along \( y \) axis. A full spectrum Fast Fourier Transform of space vector of bridge currents and transverse forces have been carried out to investigate the relationship between the bridge current modes and force modes. Generally in balanced system zero sequence component of current is negligible, however a substantial amount of zero sequence component of bridge current is present in bridge configured winding. Fast Fourier Transform of zero sequence component of bridge current has also been carried out to know its frequency components.

\[
\begin{align*}
\begin{bmatrix}
  d \\
  q \\
  0
\end{bmatrix}
&= \begin{bmatrix}
  \cos(0) & \cos(-\frac{2\pi}{3}) & \cos(\frac{2\pi}{3}) \\
  -\sin(0) & -\sin(-\frac{2\pi}{3}) & -\sin(\frac{2\pi}{3}) \\
  0.5 & 0.5 & 0.5
\end{bmatrix}
\begin{bmatrix}
  a \\
  b \\
  c
\end{bmatrix}
\end{align*}
\]

(63)

Fig. 10 and Fig. 11 represent the bridge currents and full spectrum Fast Fourier Transform (FFT) of bridge currents space vector in static eccentricity condition respectively. Fig. 12 and Fig. 13 represent zero sequence component of bridge currents in static eccentricity condition and its FFT respectively. Fig. 14 and Fig. 15 represent the bridge currents and full spectrum FFT of space vector of bridge currents in

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of poles</td>
<td>4</td>
</tr>
<tr>
<td>No. of stator slots</td>
<td>36</td>
</tr>
<tr>
<td>No. of rotor bars</td>
<td>26</td>
</tr>
<tr>
<td>Outer dia. of stator</td>
<td>160 mm</td>
</tr>
<tr>
<td>Outer dia. of rotor</td>
<td>103 mm</td>
</tr>
<tr>
<td>Air gap length</td>
<td>1 mm</td>
</tr>
<tr>
<td>Axial length of the motor</td>
<td>135 mm</td>
</tr>
<tr>
<td>Resistance per phase</td>
<td>6.8 Ω</td>
</tr>
<tr>
<td>End winding inductance</td>
<td>3.94 e-4 A/m</td>
</tr>
<tr>
<td>Resistance of each rotor bar</td>
<td>1 e-5 Ω</td>
</tr>
<tr>
<td>Rotor end ring resistance</td>
<td>2.49e-6 Ω</td>
</tr>
<tr>
<td>Rotor end ring inductance</td>
<td>1.54 E-8 A/m</td>
</tr>
<tr>
<td>Supply voltage</td>
<td>37 V, 50 Hz</td>
</tr>
<tr>
<td>Slip</td>
<td>0.02</td>
</tr>
</tbody>
</table>

TABLE II: Machine Parameters
It is a known fact that the eccentric position of a rotor generates $p \pm 1$ pole pair magnetic field in the air gap, where $p$ is the fundamental pole pair of the winding. However, these magnetic field components have different angular frequency in different eccentricity conditions. The angular frequency of these components in static and dynamic eccentricity are $f_s$ and $f_s \pm f_r$ respectively [16]. The rotor frequency is given by $f_s(1 - s)/p$, where $s$ is the slip. The simulated bridge configured winding is a 4-pole winding, hence eccentric position of the rotor generates 2-pole and 6-pole magnetic fields in the air gap. Based on the above mentioned relationships it can be found that the angular frequency of the 2-pole and 6-pole magnetic fields in the static eccentricity condition is...
50 Hz, whereas in dynamic eccentricity condition the angular frequency of 2-pole magnetic field is 25.5 Hz and the angular frequency of 6-pole magnetic field is 74.5 Hz. These magnetic field components create potential difference across the bridge points of the bridge configured winding. When these bridge points are short circuited or in bridge ON condition current flows in the winding which has same frequency components as that of the eccentricity generated magnetic field.

In static eccentricity condition, the space vector of bridge currents have only supply frequency component \( f_s \) i.e. 50 Hz. In dynamic eccentricity condition the space vector of bridge currents have \( f_s \pm f_r \) components i.e. 25.5 Hz and 74.5 Hz. In mixed eccentricity condition the space vector of the bridge currents have combination of the static and dynamic eccentricity components and they are 25.5 Hz, 50 Hz and 74.5 Hz. In all eccentricity conditions the space vector of the bridge currents have both negative and positive mode. A substantial amount of zero sequence component of bridge currents is also present in all the eccentricity conditions, which has the same frequency components as that of the respective space vector of the bridge currents.
Fig. 23: Full spectrum FFT of force in dynamic eccentricity condition.

Fig. 24: Full spectrum FFT of force in mixed eccentricity condition.

Fig. 22, Fig. 23 and Fig. 24 represent the full spectrum FFT of force acting on the rotor in static eccentricity condition, dynamic eccentricity condition and mixed eccentricity condition respectively. The relationship between different bridge current modes and transverse force modes can be written as $\pm f_s \pm f_{bridge}$. Here, $f_{bridge}$ is the frequency component of space vector of bridge currents. It can be observed in Fig. 22 that in the static eccentricity condition, when bridges are in OFF condition, the forces acting on the rotor has three components, $0 \text{ Hz and } \pm 100 \text{ Hz (} \pm 50 \pm 50)$, whereas in bridge ON condition the double supply frequency component $(\pm 2f_s)$ becomes almost negligible and only the DC component $(0 \text{ Hz})$ of the force acts on the rotor. It can be seen in Fig. 23 that in the dynamic eccentricity condition, when bridges are in OFF condition the force acting on the rotor has six components, $\pm 24.5 \text{ Hz, } \pm 75.5 \text{ Hz and } \pm 124.5 \text{ Hz (} \pm 50 \pm 25.5 \text{ and } \pm 50 \pm 74.5)$, whereas in bridge ON condition most of the force component becomes negligibly small and only $+f_r$ (24.5 Hz) component of the force acts on the rotor. In the mixed eccentricity condition all the frequency components of the forces of static and dynamic eccentricity conditions are present. It can be observed in Fig. 24 that in bridge OFF condition the forces acting on the rotor have $0 \text{ Hz, } \pm 24.5 \text{ Hz, } \pm 100 \text{ Hz, } \pm 75.5 \text{ Hz and } \pm 124.5 \text{ Hz component whereas in bridge ON condition the forces acting on the rotor has 0 Hz component and } +f_r (24.5 \text{ Hz}) \text{ component.}$

V. EXPERIMENTAL INVESTIGATION

Two experimental setups with two different types of rotor shafts have been used for the investigation of bridge currents and UMP generated vibration. For this purpose a 36-slots 3.7 kW and a 60-slots 37 kW 4 - pole induction machines have been re-wound with bridge configured winding scheme. The winding patterns of these two machines are same as that of the winding pattern of the machine used for the numerical investigation. Fig. 25 and Fig. 26 show the two developed experimental setups.

Fig. 25: Experimental setup with original rotor.

Fig. 26: Experimental setup with a long shaft, (1,2) - bearing housing at ND and D end, 3 - test machine, 4 - perforated disc, (5,6,7) - location for the rotor responses, 8 - panel board.

Fig. 25 represents a 3.7 kW 4-pole induction machine which is rewound using bridge configuration winding scheme where the original shaft of the machine has been used for the experiment. Since only a small portion of the shaft is out of the motor so an accelerometer has been placed on the body of the motor to measure the vibration signal of the motor. The rating of this experimental setup is same as that of the numerically simulated machine. Fig. 26 represents a 37 kW induction machine which is rewound using bridge configuration winding.
scheme and the original shaft of the rotor has been replaced with a 1.5 meter long shaft to make the rotor system flexible so that all the frequency components of the force acting on the rotor can be observed. The eddy current sensors have been used to measure the displacement signals along x and y axes which have been placed at location 6 in the experimental setup as shown in Fig. 26. The bridge currents flowing through the bridge points of the motor have been measured using three current transducers. All the measurements have been recorded using NI-DAQ system and both motors have been operated in no load condition.

Similar to the numerical investigation, the measured bridge currents have been converted to space vector as explained in equation (63) and vibration measurements have been converted in complex domain as, \( d_x + jd_y \) where \( d_x \) and \( d_y \) are the rotor vibration along x and y axis respectively. In the case of short rotor experimental setup, the machine has been supplied using a three phase auto transformer with a rated voltage of 415 V at 50 Hz (\( f_r \)). The motor has been operated at no load so the rotor rotational frequency (\( f_r \)) is very close to 25 Hz. Since the rotor is a rigid rotor, the bridge OFF and the bridge ON conditions do not have any effect on the accelerometer signals. So, the accelerometer data has been presented only for the bridge ON condition. It should be noted that in bridge OFF condition there would not be any flow of current through the bridge points so the bridge currents can only be recorded in bridge ON condition. Fig. 27 shows the bridge currents flowing through the bridge points of the machine and Fig. 28 shows the full spectrum FFT of space vector of measured bridge currents. The presence of \( \pm 25 \) Hz (\( f_r \)) and \( \pm 75 \) Hz (\( f_s + f_r \)) components in full spectrum FFT of space vector of bridge currents signifies the presence of dynamic eccentricity in the machine whereas the presence of 50 Hz (\( f_s \)) component represent the presence of static eccentricity condition in the machine. The presence of all three component of the bridge currents indicates that the machine is being operated in mixed eccentricity condition as shown in numerical investigation. Similar to numerical investigation zero sequence component is also present as shown in Fig. 29. Fig. 30 represents FFT of zero sequence component of the bridge currents which also signifies the presence of mixed eccentricity condition as all three frequency components are present. The acceleration data shows the presence of 0 Hz and 100 Hz component due to static eccentricity. Whereas 25 Hz, 75 Hz and 125 Hz components show the presence of dynamic eccentricity. The presence of these frequency components in combination as shown in Fig. 31 signifies the presence of mixed eccentricity condition in short rotor motor. In numerical investigation it has been observed that the bridge currents suppress almost all the frequency components of the UMP other than DC component and \( f_s - f_r \) component. Whereas in this experimental setup bridge currents are not able to suppress motor vibration. It might be happening due to the rigid rotor.

So, to investigate further a rotor with a flexible shaft has been used for the experiment as shown in Fig. 26. Also to have a generalized effect the rotor vibration has been analyzed at two different supply frequencies i.e. 20 Hz and 30 Hz using a variable frequency drive. 20 Hz and 30 Hz have been chosen as main supply frequencies because shaft of the motor is flexible and it can not be operated at higher speeds. Again, the motor has been operated at no load condition so for supply frequencies (\( f_s \)) 20 Hz and 30 Hz, the rotor rotational frequencies (\( f_r \)) are very close to 10 Hz and 15 Hz respectively. Fig. 32 and Fig. 33 represent the bridge currents and full spectrum FFT of space vector of measured bridge currents in longer shaft rotor experimental setup when the main supply frequency is 20 Hz. All three components \( \pm 10 \) Hz (\( f_s - f_r \)), \( \pm 20 \) Hz(\( f_s \)) and \( \pm 30 \) Hz(\( f_s + f_r \)) are present which indicates the presence of mixed eccentricity condition in the machine.
Fig. 30: FFT of zero sequence bridge currents when motor has its original shaft.

Fig. 31: FFT of accelerometer data when motor has its original shaft.

Fig. 32: Measured bridge currents in experimental setup when motor has a longer shaft.

Fig. 33: Full spectrum FFT of space vector of measured bridge currents when motor has a longer shaft.

Fig. 34 and Fig. 35 show the zero sequence current and its FFT respectively. FFT plot of zero sequence component of the bridge currents show the presence of all three frequency components. Similar behavior of the bridge currents have been observed when frequency of the main supply is 30 Hz.

Fig. 34: Zero sequence bridge currents when motor has a longer shaft.

Fig. 35: FFT of zero sequence bridge currents when motor has a longer shaft.

components as per the relation \( \pm f_s \pm f_{\text{bridge}} \). \( f_{\text{bridge}} \) is \( \pm 10 \) Hz, \( 20 \) Hz and \( \pm 30 \) Hz when \( f_s \) is \( 20 \) Hz and \( f_{\text{bridge}} \) is \( \pm 15 \) Hz, \( 30 \) Hz and \( \pm 45 \) Hz when \( f_s \) is \( 30 \) Hz. The rotor response shows the presence of 0 Hz, \( \pm 10 \) Hz, \( \pm 30 \) Hz, \( \pm 40 \) Hz, and \( \pm 50 \) Hz frequency components when the bridge are in OFF condition and main supply frequency is 20 Hz. The rotor response shows the presence of 0 Hz, \( \pm 15 \) Hz, \( \pm 45 \) Hz, \( \pm 60 \) Hz, and \( \pm 75 \) Hz frequency components when bridges are in OFF condition and the main supply frequency is 30 Hz.

Fig. 36 and Fig. 37 represent the full spectrum FFT of the rotor vibration measured at location 6 of the experimental setup as shown in Fig. 26 when the main supply frequencies are 20 Hz and 30 Hz respectively. It can be observed from Fig. 36 and Fig. 37 that the rotor response has all the frequency components when the main supply frequency is 20 Hz. The rotor response shows the presence of 0 Hz, \( \pm 10 \) Hz, \( \pm 30 \) Hz, \( \pm 40 \) Hz, and \( \pm 50 \) Hz frequency components when the bridge are in OFF condition and the main supply frequency is 30 Hz. The rotor response shows the presence of 0 Hz, \( \pm 15 \) Hz, \( \pm 45 \) Hz, \( \pm 60 \) Hz, and \( \pm 75 \) Hz frequency components when bridges are in OFF condition and the main supply frequency is 30 Hz.
TABLE III: Summary of the investigation

<table>
<thead>
<tr>
<th>Type of Eccentricity</th>
<th>Supply Freq.</th>
<th>Slip</th>
<th>Space Vector Component</th>
<th>Zero Sequence Component</th>
<th>Bridge ON</th>
<th>UMP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static Eccentricity</td>
<td>50 Hz</td>
<td>0.02</td>
<td>±24.5 Hz, ±74.5 Hz</td>
<td>±24.5 Hz, ±74.5 Hz</td>
<td>±24.5 Hz</td>
<td>24.5 Hz</td>
</tr>
<tr>
<td>Dynamic Eccentricity</td>
<td>50 Hz</td>
<td>0.02</td>
<td>±24.5 Hz, ±74.5 Hz</td>
<td>±24.5 Hz, ±74.5 Hz</td>
<td>±24.5 Hz</td>
<td>24.5 Hz</td>
</tr>
<tr>
<td>Short Rotor</td>
<td>50 Hz</td>
<td>0.02</td>
<td>±24.5 Hz, ±74.5 Hz</td>
<td>±24.5 Hz, ±74.5 Hz</td>
<td>±24.5 Hz</td>
<td>24.5 Hz</td>
</tr>
<tr>
<td>Long Rotor</td>
<td>50 Hz</td>
<td>-0.2</td>
<td>±24.5 Hz, ±74.5 Hz</td>
<td>±24.5 Hz, ±74.5 Hz</td>
<td>±24.5 Hz</td>
<td>24.5 Hz</td>
</tr>
</tbody>
</table>

Experimental Measurements

<table>
<thead>
<tr>
<th>Type of Eccentricity</th>
<th>Supply Freq.</th>
<th>Slip</th>
<th>Type of Rotor</th>
<th>Space Vector Component</th>
<th>Zero Sequence Component</th>
<th>Bridge ON</th>
<th>Bridge OFF</th>
<th>Motor Body/Rotor Vibration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not Known</td>
<td>50 Hz</td>
<td>No load</td>
<td>Short Rotor</td>
<td>±25 Hz, ±50 Hz, ±75 Hz</td>
<td>±25 Hz, ±75 Hz, 100 Hz, 125 Hz</td>
<td>±25 Hz, ±75 Hz, 100 Hz, 125 Hz</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Not Known</td>
<td>50 Hz</td>
<td>No load</td>
<td>Long Rotor</td>
<td>±25 Hz, ±75 Hz, 100 Hz</td>
<td>±25 Hz, ±50 Hz, ±75 Hz</td>
<td>±25 Hz, ±75 Hz, 100 Hz, 125 Hz</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Not Known</td>
<td>50 Hz</td>
<td>No load</td>
<td>Long Rotor</td>
<td>±25 Hz, ±75 Hz, 100 Hz</td>
<td>±25 Hz, ±50 Hz, ±75 Hz</td>
<td>±25 Hz, ±75 Hz, 100 Hz, 125 Hz</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

VI. Conclusion

A FE modeling of a 36 slots 4-pole induction machine with bridge configured winding has been carried out to study the effect of eccentric rotor on the bridge currents and the forces acting on the rotor. This model is able to provide correct behavior of bridge currents and UMP generated due to an eccentric rotor in all types of eccentricity conditions. Various modes of the bridge currents have been analysed by transforming it from $a, b, c$ sequence to $d, q, 0$ sequence. A relationship of bridge current modes with respect to different eccentricity conditions have been established. It has been shown that the zero sequence component of the bridge currents can not be ignored in bridge configured winding. The space vector of bridge currents show the presence of only one frequency component i.e. $\pm f_s$, two frequency components i.e. $\pm (f_s - f_r)$, $\pm (f_s + f_r)$ and three frequency components i.e. $\pm f_s$, $\pm (f_s - f_r)$ and $\pm (f_s + f_r)$ of the bridge currents for the case of the static, dynamic and mixed eccentricity conditions respectively. A relationship with respect to the main supply frequency component ($f_s$) and bridge currents frequency component ($f_{bridge}$) has been established. It has been shown that the developed UMP due to the eccentric rotor in bridge OFF condition has $\pm f_s \pm f_{bridge}$ frequency components. It has been also established that bridge currents suppresses all the UMP components other than DC component and $f_s - f_r$ component. It happens because eccentric rotor generates pulsating component of magnetic field which gets suppressed when bridges are in ON condition. Two experimental setups have been developed where bridge currents and UMP generated vibration have been analysed. In comparison with the numerical simulation, experimental behavior of bridge currents and UMP generated vibration show the presence of mixed eccentricity condition in both of the experimental setup. This behavior of experimental measurements in one way also verifies the capability of the developed code. The present study will be helpful in understanding the behavior of the bridge currents in different eccentricity conditions of the rotor and this would eventually be useful for the development of a bearingless machine using bridge configuration winding scheme.

**APPENDIX A**

**List of symbols**

Fig. 36: Full spectrum FFT of rotor response when main supply frequency is 20 Hz.

Fig. 37: Full spectrum FFT of rotor response when main supply frequency is 30 Hz.
\[ x, y \quad \text{Cartesian coordinates} \]
\[ t \quad \text{Time} \]
\[ \vec{J} \quad \text{Current density} \]
\[ \vec{A} \quad \text{Magnetic vector potential} \]
\[ \vec{k} \quad \text{Unit vector parallel to z axis} \]
\[ \mu \quad \text{Permeability} \]
\[ \nu \quad \text{Magnetic reluctivity} \]
\[ V \quad \text{Scalar potential} \]
\[ E \quad \text{Electric field intensity} \]
\[ B \quad \text{Magnetic field density} \]
\[ \sigma \quad \text{Electrical conductivity of the rotor bar} \]
\[ U_r \quad \text{Voltage across the rotor bar} \]
\[ U_s \quad \text{Voltage across stator coil} \]
\[ n_0 \quad \text{No. of turns in the stator winding} \]
\[ S_s \quad \text{Cross sectional area of the stator coil} \]
\[ l \quad \text{Length of the rotor bar} \]
\[ l_s \quad \text{Stator coil current} \]
\[ \xi, \eta \quad \text{Natural coordinate system} \]
\[ k_n, c_r, p_s, p_r, q_r, q_s \quad \text{Elemental matrix in cartesian coordinate system} \]
\[ \{N\} \quad \text{Shape function} \]
\[ |J| \quad \text{Jacobian} \]
\[ n \quad \text{Number of nodes} \]
\[ u \quad \text{Number of stator circuit current} \]
\[ z \quad \text{Number of coil sides} \]
\[ v \quad \text{Number of rotor conductors} \]
\[ [K]_{n \times n} \quad \text{Global stiffness matrix} \]
\[ [C_r]_{n \times n} \quad \text{Global matrix for induced emf due to rotor bars} \]
\[ [P_s]_{n \times z} \quad \text{Global matrix for the stator coil excitations} \]
\[ [P_r]_{n \times y} \quad \text{Global matrix for the rotor conductor excitations} \]
\[ [T]_{z \times u} \quad \text{Transformation matrix for the transformation of phase currents to the coil side currents} \]
\[ \{I_s\}_{u \times 1} \quad \text{Stator circuit current vector} \]
\[ \{U_r\}_{v \times 1} \quad \text{Rotor bar voltage vector} \]
\[ \{I_r\}_{v \times 1} \quad \text{Rotor bar current vector} \]
\[ \{U_s\}_{u \times 1} \quad \text{Stator coil voltage vector} \]
\[ [R_s]_{v \times u} \quad \text{Stator coil resistance matrix} \]
\[ [R_r]_{u \times u} \quad \text{Rotor bar resistance matrix} \]
\[ [Q_s]_{u \times n} \quad \text{Global matrix of the induced emf for the stator circuit equation} \]
\[ [Q_r]_{v \times n} \quad \text{Global matrix of the induced emf for the rotor circuit equation} \]
\[ Q_1 \ldots Q_{12} \quad \text{for the stator circuit equation for coil groups} \]
\[ R \quad \text{Stator coil resistance} \]
\[ L_{er} \quad \text{End winding inductance of the stator coil} \]
\[ V_A, V_B, V_C \quad \text{Three phase voltages of the stator} \]
\[ i_A, i_B, i_C \quad \text{Three phase currents of the stator} \]
\[ i_1, i_2, i_3 \quad \text{Coil currents in the parallel paths of the stator winding} \]
\[ L_b \quad \text{Coil inductance of the bridge connection} \]
\[ R_b \quad \text{Coil resistance of the bridge connection} \]
\[ i_{bA}, i_{bB}, i_{bC} \quad \text{Bridge currents} \]
\[ R_{er} \quad \text{Rotor end ring resistance} \]
\[ L_{er} \quad \text{Rotor end ring inductance} \]
\[ U_{b1} \ldots U_{b26} \quad \text{Rotor bar currents} \]
\[ r \quad \text{Rotor bar resistance} \]
\[ U_{b1} \ldots U_{b26} \quad \text{Voltage across each rotor bar} \]
\[ \{M_2\} \quad \text{Transformation matrix for conversion of rotor end ring currents to rotor bar currents} \]
\[ F_x, F_y \quad \text{UMP along x and y axes} \]
\[ f_s \quad \text{Main supply frequency} \]
\[ f_r \quad \text{Mechanical frequency of the rotor} \]
\[ f_{bridge} \quad \text{Frequency of the bridge currents} \]
\[ d_x, d_y \quad \text{Displacement along x and y axes} \]

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**REFERENCES**


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