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The Answer Set Programming Paradigm

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Abstract
In this paper, we give an overview of the answer set programming paradigm, explain its strengths, and illustrate its main features in terms of examples and an application problem.

Introduction
Answer set programming (ASP, for short) is a declarative programming paradigm for solving search problems and their optimization variants. In ASP a search problem is modeled as a set of statements (a program) in a logic programming type of a language in such a way that the answer sets (models) of the program correspond to the solutions of the problem. The paradigm was first formulated in these terms by Marek and Truszczyński (1999) and Niemelä (1999). The ASP paradigm has its roots in knowledge representation and nonmonotonic logics research as described by Marek et al. (2011) in a historic account on the development of the paradigm. A more recent and more technical overview of ASP has been contributed by Brewka et al. (2011).

The ASP paradigm is most widely used with the formalism of logic programming under the semantics given by answer sets (Gelfond and Lifschitz 1988; 1990). The term answer sets was proposed by Gelfond and Lifschitz (1991) for sets of literals, by which programs in an extended syntax are to be interpreted where the classical negation operator and disjunctions of literals are allowed in the heads of program rules. Lifschitz’ article (2016) in this special issue gives an introduction to the notion of an answer set and the language of ASP, as well as a comparison to Prolog systems. An alternative approach to ASP has been to use directly first-order logic as the basis and extend it with inductive definitions. The details can be found in the articles by Denecker and Vennekens (2014), Denecker and Ternovska (2008), East and Truszczynski (2006), and the one by Bruynooghe et al. (2016) in this special issue.

A main reason for the increasing interest in ASP is the availability of fast software tools that makes it possible to tackle problems of practical importance. Most of the current software tools employ two steps commonly referred to as grounding and solving, reflecting the definition of answer sets for programs with variables (Lifschitz 2016). The idea is to separate concerns so that the grounding phase takes care of the evaluation of more complicated data structures and variable instantiations using logic programming and deductive database techniques, and then the solving phase focuses on search for answer sets for a much simpler type of programs by employing advanced search methods. The papers by Kaufmann et al. (2016) and by Gebser and Schaub (2016) in this special issue provide more information on the solving and grounding techniques.

There is a growing number of successful applications of ASP including molecular biology (Gebser et al. 2010a; 2010b), decision support system for space shuttle controllers (Balduccini, Gelfond, and Nogueira 2006), phylogenetic inference (Erdem 2011; Koponen et al. 2015), product configuration (Soininen and Niemelä 1998; Finkel and O’Sullivan 2011) and repair of web-service work flows (Friedrich et al. 2010), Erdem et al. (2016) give an account of the applications of ASP in this special issue.

On the one hand, ASP is closely related to logic programming and Prolog and, on the other hand, to constraint programming (CP), propositional satisfiability (SAT), and linear/integer programming (LP/IP). Unlike Prolog-like logic programming ASP is fully declarative and neither the order of rules in a program nor the order of literals in the rules matter. Moreover, Prolog systems are tailored to find proofs or answer substitutions to individual queries whereas ASP systems are finding answer sets corresponding to complete solutions to a problem instance. The basic idea in ASP is very close to the paradigm of CP, SAT, or LP/IP where problems are represented by constraints and where systems are tailored to find satisfying variable assignments corresponding to complete solutions.

However, there are significant differences. The ASP paradigm allows for a very systematic approach to problem representation through uniform encodings where the problem statement can be developed independently of data on a particular instance. This leads to a large degree of elaboration tolerance. The ASP approach enables structured representation of problems where more complicated constraints are composed of simpler ones using rules. On the other hand, rules enable one to encode conditions that are challenging (like representing disjunctive constraints or other basic relational operations on constraints) or not available at all (like recursive constraints) when comparing to CP or...
LP/IP paradigms. Because of these properties ASP allows for incremental development of an application and supports well rapid prototyping.

The goal of this paper is to provide an up-to-date overview of the ASP paradigm and illustrate its usage with examples as well as a more comprehensive application problem. We proceed as follows. In the next section, we explain the fundamental ideas of the ASP paradigm. The use of the paradigm and its main features are then illustrated by developing ASP encodings for an application problem step by step. The application considered in this paper is about designing a locking scheme for a building so that certain safety requirements are met. Having introduced the basic paradigm, we briefly address main ways to implement ASP—either using native answer-set solvers or translators enabling the use of solver technology from neighboring disciplines. The paper ends with a summary and discussion of future prospects. In addition, we illustrate the potential computational hardness of our application problem by explaining its connection to the NP-complete decision problem Exact-3-SAT.

**Basic ASP Paradigm**

The conceptual model of the ASP paradigm is depicted in Figure 1. We start by explaining how to understand search problems at an abstract level and then illustrate how ASP is typically employed to solve such problems using the approach illustrated in the figure. Finally, we address a number of features and attractive properties of the paradigm.

**Problem Solving.** The ASP paradigm provides a general purpose methodology for solving search and optimization problems encountered in many real world applications. To get started, the key step is to identify and formalize the problem to be solved, i.e., to work out a problem statement. Typically this consists of clarifying what the potential solutions of the problem are like and then setting the conditions that solutions should satisfy. Solving the problem means that given the data on an instance of the problem we should find one or more solutions satisfying the given conditions (see the topmost arrow in Figure 1). For illustration, we use the task of finding a seating arrangement for a dinner as the first simple example. The respective problem statement could read as formulated below.

**Example 1 (Seating Arrangement Problem)**

A certain group of people, say persons \( p_1, \ldots, p_n \), are invited for dinner. There are tables \( t_1, \ldots, t_k \) with the respective capacities \( c_1, \ldots, c_k \) available for seating such that \( c_1 + \cdots + c_k \geq n \). The host has some prior knowledge about the relationships of the guests: there are both friends and enemies among the invitees. This information should be taken into account when designing the arrangement. A solution to this problem is a mapping \( s(p_i) = t_j \) of persons \( p_i \) to tables \( t_j \) so that the mutual relationships are respected.

The problem statement above uses mathematical symbols to abstract the details of the problem such as the number and the identity of persons involved and the collection of tables available for seating. This reflects an important methodological feature, namely the separation of instance data from the actual problem statement. The point is that the problem can be stated without listing all details for a particular instance of the problem. In case of the seating arrangement problem, the instance data would consist of the names of invitees together with lists of tables and their capacities, and the pairs of persons who are known to be either friends or enemies. More concretely put, suppose that we have a group of 20 people: Alice, Bob, John, etc. There are four tables, seating 7, 6, 5, and 4 people, respectively. Moreover, we know that Alice likes Bob, Bob likes John and so on. Given all such pieces of information, the goal is

- to find at least one solution that fulfills the criteria set in the problem statement of Example 1, or
- to show that no solution exists.

Given what we know so far, we can expect solutions where Alice, Bob, and John are seated together at one of the four tables available. However, if we state additionally that Alice and John dislike each other, for instance, the seating problem instance under consideration has no solutions.

**ASP Encoding.** But how do we achieve the goal stated above using ASP and get the problem solved? As suggested by Figure 1, we should formalize the problem statement by writing down a (logic) program. Before we can really do this, we should have a basic understanding of syntax, also introduced in the article by Lifschitz (2016) in this issue. In ASP, programs consist of rules, i.e., statements of the form

\[
\text{head} :- \text{body}_1, \text{body}_2, \ldots, \text{body}_m.
\]

The intuitive reading of the rule above is that the head can be inferred if (and only if) the body conditions \( \text{body}_1, \text{body}_2, \ldots, \text{body}_m \) have been inferred by any other rules in the program. The conditions in the rule are either atomic statements (a.k.a. atoms) like \( \text{seat}(a_1, 1) \) for Alice being seated at Table 1, or count-bounded sets of atoms

\[
\text{count}\{\text{atom}_1; \ldots; \text{atom}_k\} \leq u
\]

where at least \( \leq 1 \) but at most \( u \) atoms among \( \text{atom}_1, \ldots, \text{atom}_k \) should be inferable. The cardinality constraint above can also be expressed in terms of a counting aggregate

\[
\text{#count}\{\text{atom}_1; \ldots; \text{atom}_k\}
\]

where appropriate bounds can be incorporated using relation symbols \( <, \leq, >, \geq, \) and =. Atoms can also be negated using the operator \( \neg \) for default negation. A rule with an empty body (n=0) stands for a fact whose head holds unconditionally. As a further special case, a rule without a head stands for a constraint whose body \( \text{body}_1, \text{body}_2, \ldots, \text{body}_n \) must not be satisfied. In this article, we do not consider extensions of rules by classical negation nor disjunctions in rule heads (Gelfond and Lifschitz 1991).

We are now ready to describe typical steps in writing down a program in ASP, resulting in an encoding1 given as

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1The encodings presented in this paper are directly executable using contemporary ASP grounders and solvers compatible with the ASP-core-2 language specification (Calimeri et al. 2012).

The second step concerns the actual program formalizing the problem statement. Writing down the rules is of course a creative activity, which one learns best by doing, but in ASP one can concentrate on defining the relevant concepts (relations) in terms of rules, as well as thinking about conditions on which certain relations should hold. To understand the outcome of the formalization in Listing 1, let us give the intuitive readings for the rules involved. The rule in line 8 stipulates that every person \( P \) must be seated at exactly one table \( T \). A few constraints follow. The capacities of tables are enforced in line 9: it is unacceptable if more than \( C \) persons are seated at table \( T \) which seats at most \( C \) persons. Moreover, if person \( P_1 \) likes person \( P_2 \), they should not be seated at different tables \( T_1 \) and \( T_2 \). This constraint is expressed in lines 10–12. The other way around, if \( P_1 \) does not like \( P_2 \), then they should not be seated at the same table \( T \). The respective rule is given in lines 13–14. The rules and constraints in lines 8–14 explained so far form a uniform encoding of the seating problem, as the representation is independent of any problem instance described by facts of the type in lines 2–5.

So far, we have demonstrated the modeling philosophy of ASP in terms of a simple application. The later section on locking design provides further insights into modeling and typical design decisions made. Yet further information is available in the articles of Bruynooghe et al. (2016) and Gebser and Schaub (2016) in this special issue.

**ASP Solving.** It remains to explain how the encoding from Listing 1 solves the problem instance in practice. First, the rules of the program have to be instantiated and evaluated with respect to the present facts. This means, e.g., that the rule in line 8 yields an instance

\[
\begin{array}{l}
\text{\{ seat(a,1); seat(a,2); } \\
\text{seats(a,3); seat(a,4) } \text{\}} 1.
\end{array}
\]

when \( P \) is replaced by \( a \) and \( T \) ranges over the available tables 1, 2, 3, and 4. This particular instance concerns the seating of Alice. While instantiating the rules also some evaluations take place. For example, when handling the rule in line 9 for table 1 with capacity 7 the lower bound \( C \) of the constraint is substituted by the value 7. The ground program, also indicated in Figure 1, is typically generated by running a dedicated tool, i.e., a grounder, on the input. After that the search for answer sets can be performed by invoking an answer set solver. Finally, the solution(s) of the original problem instance are obtained by extracting relevant part(s) from the answer set(s) found. For the encoding under consideration, this means that whenever an occurrence of \( \text{seat}(P,T) \) is contained in an answer set, then person \( P \)

---

2 However, ASP builds on the closed world assumption (CWA): the given information is treated as complete information and the problem is solved under this assumption.
incremental solving

Niemelä 1998), this could be the number of variants that a

count

applications. In product configuration (see, e.g., (Soininen and

their number. For certain applications, the number

constraint or as a special reasoning mode.

Brave reasoning is the dual of cautious reasoning and then

to search for answer sets. A solver is for some reason

counter-example indicating that we would like to find any

is supposed to be seated at table T. Using the notions from

Example 1, we would have the required mapping s : P → T

from persons to tables. If no answer set can be found, then a

problem instance has no solutions. This is actually the case

for the instance described by lines 2–5 in Listing 1, since

it is impossible to place Alice, Bob, and John at the same
table due to their relations. However, if the facts in line 4

are removed, obtaining answer sets is still feasible—the re-

lationships of other guests permitting.

Beyond Basic ASP. The basic paradigm illustrated in Fig-

ure 1 solves the problem at hand by eventually finding one

or more solutions to the problem, or by showing that no so-

lution exists. If there are multiple solutions to the problem,

then it may be desirable to select the best solution among

the alternatives using some criterion such as price, capacity,

e.g. This turns the problem into an optimization problem. In

ASP, objective functions for such problems can be defined

in terms of optimization statements like

"#minimize \{ w_1 \cdot \text{atom}_1; \ldots; w_n \cdot \text{atom}_n \}.

The statement above assigns weights \( w_1, \ldots, w_n \) to atoms

\text{atom}_1, \ldots, \text{atom}_n \), respectively, and the goal is to mini-

mize the sum of weights for atoms contained in an an-

swer set—when evaluated over all answer sets. As regards

the seating arrangement problem, the respective optimiza-

tion problem could deal with obviously inconsistent settings

like the one described above. Rather than satisfying all con-

straints resulting from the mutual relations of persons,

the goal would be to satisfy as many as possible. In the prece-

ding example, this would mean that either Alice is seated at

the same table as Bob, or Bob is seated with John, but Alice

and John are placed at different tables.

Besides the optimization of solutions, there are also other

reasoning modes of interest. It is sometimes interesting to

see how much the solutions are alike. In cautious reasoning,

the idea is to check whether a certain atom is present in all or

absent from some answer set. For instance, if \text{seat}(a,1)

is for some reason contained in all answer sets, then Alice

will be unconditionally seated at the first table and no op-

tions remain to this end. Cautious reasoning corresponds to

basic query evaluation over answer sets and it can be imple-

mented by adding a constraint to the program. In the case of

our example, the constraint would read \( \neg \text{seat}(a,1) \).

indicating that we would like to find any counter-example,

i.e., an answer set \( \neg \text{seat}(a,1) \). Alternati-

vely, cautious reasoning can be implemented by solvers as

a special reasoning mode while searching for answer sets.

Brave reasoning is the dual of cautious reasoning and then

the presence in some or absence from all answer sets is

required. Again, this can be implemented by adding a con-

straint or as a special reasoning mode.

It is also possible to enumerate answer sets and, hence,
count their number. For certain applications, the number

of solutions could actually be an interesting piece of infor-

mation. In product configuration (see, e.g., (Soininen and

Niemelä 1998)), this could be the number of variants that a

production line should be able to produce. There are also

complex use cases of ASP. In incremental solving, the idea

is to compute partial solutions to a problem (or show their

non-existence) by calling an ASP solver several times and

by extending the instance data on the fly. Various kinds of

planning problems (with an increasing plan length) typically

fall into this category. The latest developments even suggest

multi-shot solving (Gebser et al. 2014) where solver calls

are freely mixed and the ground programs used upon solver

calls may evolve in more complex ways.

Constraints over Infinite Domains. Since grounding is

an inherent part of ASP work flow, the basic paradigm is

based on Boolean or finite-domain variables only. How-

ever, certain applications call for variables over infinite do-

mains such as integers and reals. For instance, there have

been proposals to extend ASP rules by linear inequalities

(Gebser, Ostrowski, and Schaub 2009; Liu, Janhunen, and

Niemelä 2012; Mellarkod, Gelfond, and Zhang 2008) as

difficulty constraints (Janhunen, Liu, and Niemelä 2011).

From the modeling perspective, the goal of such ex-
tensions is to increase the expressive power of ASP suitably

so that new kinds of applications become feasible. For in-

stance, referring back to the seating problem in Listing 1,

we could refine the specification for each person \( P \) by in-

troducing integer variables \( e(P) \) and \( l(P) \) denoting the

points of time when \( P \) enters and leaves the table in ques-
tion. Using difference constraints, we could state a specifi-
cation given as Listing 2. Intuitively, the rules in lines 1 and

2 insist that person \( P \) stays at the table from 5 to 90 min-
utes. The constraint in lines 3–5 refines the last one from

Listing 1. It is not allowed that any two persons \( P1 \) and \( P2 \)

who dislike each other are seated at the same table at the

same time. It is important to notice that when the constraint

in line 1 is instantiated for Alice, the resulting constraint

\( \neg l(a) - e(a) < 5 \). Thus, the infinity of the underlying

domain is not refelected to the size of the resulting ground

program. Naturally, the interpretation of \( l(a) \) and \( e(a) \)

as integer variables must be dealt with by the implementa-

tion of such constraints.

Application: Locking Design

Having introduced the ASP paradigm on a general level, we

now illustrate its main features in terms of an application

problem where the goal is to design a locking scheme for a

building. This is to be understood comprehensively, i.e., we

are not just interested in locks but also anything else that can

affect accessibility in a building. For simplicity, we consider

a single floor. A sample floor plan of such a building is de-

picted in Figure 2. There are 12 rooms altogether, numbered

from 1 to 12 in the figure. Given this domain, our objectives

Listing 2: Examples of difference constraints

:- l(P)-e(P)<5, person(P).

:- l(P)-e(P)>90, person(P).

:- l(P1)-e(P2)\geq0, l(P2)-e(P1)\geq0,

\text{dislikes}(P1,P2), \text{person}(P1), \text{person}(P2),

\text{seat}(P1,T), \text{seat}(P2,T), \text{tbl}(T_\infty).
are as follows. First, we describe the domain in a uniform way by selecting adequate predicates for the representation of domain information. Second, we take one concrete design goal from this domain into consideration. To this end, we concentrate on the configuration of locks installed on (potential) doors between the rooms in such a way that certain accessibility criteria are met. A particular safety requirement is that the floor can be effectively evacuated in case of an emergency. The idea is to develop ASP encodings for a design problem like this and, at the same time, illuminate the basic line of thinking and typical primitives used when modeling in ASP.

Uniform Encoding. The goal is to choose predicate symbols and the respective relations that are needed to represent an instance of the application problem at hand. To abstract the physical coordinates of the rooms, we rather represent the adjacency relation of rooms in terms of a predicate $\text{adj}/2$. For simplicity, we also assume that this relation captures the potential of installing doors between any adjacent rooms. The floor plan of Figure 2 can be represented by constants 1..12 for the rooms and the following facts:

$$
\text{adj}(1,2), \quad \text{adj}(1,3), \quad \text{adj}(2,3), \\
\text{adj}(2,4), \quad \ldots \quad \text{adj}(11,12).
$$

In total, there are 21 such facts and they are sufficient for the purposes of our examples to describe the interconnections of the rooms. For space efficiency, the adjacency information is represented asymmetrically, i.e., $\text{adj}(X,Y)$ is reported only if $X < Y$. In addition, the rooms having exits are reported using a unary predicate $\text{exit}/1$. For the running example in Figure 2, this is captured by the fact $\text{exit}(5)$. Now, if the given floor plan were changed in one way or another, or a completely different floor plan were taken into consideration, this should be reflected in the facts describing the problem instance. The other rules describing the application problem are based on these two predicates, hence making the encoding uniform. As typical in ASP encodings, some subsidiary domain predicates are defined in order to make the description of the actual problem easier. Some domain rules for the locking design problem are collected in Listing 3 and explained below.

Relational Operations. The rules in lines 1–2 of Listing 3 are used to extract room information from the adjacency information by a simple projection operation. As a result $\text{room}(R)$ is true for only those values of $R$ that actually appear in the adjacency information. In principle, a door between two rooms provides symmetric access from a room to another. Thus, the adjacency relation is not well-suited as such for the description of accessibility and we form the union of the accessibility relation with its reverse relation using rules in lines 3–4. The relation $\text{pot}/2$ stands for potential access depending on instrumentation such as locks, handles, press buttons, etc.

Defaults. To illustrate the use of defaults in encodings, we have included the rules in lines 5–6 of Listing 3. The rule in line 5 defines the condition $\text{otherexit}/0$ meaning that some other room than the room 1 has an exit. The rule in line 6 ensures that, by default, there is an exit at room 1. This is to hold unless another exit has been declared for the particular problem instance. There can be multiple exits. For instance, if there are two exits at rooms 1 and 5, this can be stated explicitly using facts $\text{exit}(1)$ and $\text{exit}(5)$. Adding these facts overrules the default in line 6 because $\text{otherexit}$ can be inferred by the rule in line 5.

Defining the Search Space. Typical ASP encodings include a part where the solution candidates for the problem being formalized are generated. This can be achieved by expressing a number of choices that aim at capturing the varying aspects of solutions. As regards syntax, such choices can be expressed in terms of choice rules whose heads are count-bounded sets of atoms. Bounds can also be omitted if an arbitrary choice is of interest. As explained above, the access from a room to another can be asymmetric due to physical constructions. In particular, this is true for emergency situations where persons try to leave the building as soon as possible but might have no keys to unlock any door. For simplicity, we introduce a two-argument predicate $\text{evac}/2$ that is used to express the existence of an evacuation route from a room to another. Given adjacent rooms $R_1$ and $R_2$, such a design choice can be made in terms of a choice rule

$$
\{ \text{evac}(R_1, R_2) \} \leftarrow \text{pot}(R_1, R_2).
$$

The intuitive reading is that if $\text{pot}(R_1, R_2)$ is true, then the truth value of $\text{evac}(R_1, R_2)$ is subject to a choice. Hence, the selection of evacuation routes between rooms is formalized. Note that the analogous normal rule

---

**Listing 3: Domain rules for locking design**

```
1 room(R1) :- adj(R1,R2).
2 room(R2) :- adj(R1,R2).
3 pot(R1,R2) :- adj(R1,R2).
4 pot(R1,R2) :- adj(R2,R1).
5 otherexit :- exit(X), X>1.
6 exit(1) :- not otherexit.
```
The essential constraint on the evacuation plan is given in lines 6–7 of Listing 4. Any given room \( R \) is considered to be OK, if some exit \( X \) is reachable from it (line 6). The auxiliary predicate \( \text{ok}(R) \) is defined in order to detect this aspect for each room. The actual constraint (line 7) excludes scenarios where some of the rooms would not be OK. Last, we want to minimize the number of evacuation connections by the objective function given in line 9. Using the encoding devised so far and an ASP solver, it is possible to check for the floor plan of Figure 2 that the minimum number of connections is 11. This is clear since there are 12 rooms in total each of which (except room 5) must be connected to some other room for the purpose of evacuation. But ASP solvers can find out more for our running example. For instance, it is possible to enumerate and count all possible evacuation plans with 11 connections. In fact, there are 22,020 such plans and further constraints can be introduced to identify the most suitable ones. It is indeed the case that the current requirements allow for very long evacuation routes through the building of Figure 2 such as \( 7 \rightarrow 6 \rightarrow 11 \rightarrow 12 \rightarrow 10 \rightarrow 9 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1 \rightarrow 3 \rightarrow 5 \). Given this observation, the lengths of routes seem important. Thus, we now pay special attention to the number of evacuation steps, i.e., moves from a room to another, and from the room perspective. The number of steps ought to be limited.

### Elaboration Tolerance

It is straightforward to modify the recursive encoding so that the number of steps is reflected. The revised encoding is presented as Listing 5. The domain for steps is first declared by the rule in line 1 where the maximum number of steps \( s \) is determined from the command line of the grounder. The base case in line 3 simply states that each room \( R \) is reachable from itself in zero steps. The main modification in the recursive case (lines 4–5) concerns counting: the number of steps \( s \) is increased by one to \( s+1 \) whenever a further step is made. However, since both \( s \) and \( s+1 \) must be members of the domain of steps, the maximum value is effectively determined by the constant \( s \) in line 1. Given the floor plan of Figure 2 and \( s=2 \), no evacuation plans can be found. By increasing \( s \) by one, solutions with 11 connections are found again and there are only 152 plans where the number of evacuation steps is at most three.

In summary, we have now tackled one particular aspect of locking design, i.e., ensuring that an evacuation plan exists for a building. In reality further requirements are imposed on evacuation plans making the problem computationally more and more challenging. For instance, it can be shown that if we incorporate conditions which can make rooms along an evacuation route mutually exclusive, e.g., for certain security reasons, it is unlikely that we are able to find a polynomial-time algorithm for solving the problem (mathematically expressed the problem becomes NP-complete). This justifies well the use of powerful search methods like ASP for tackling the problem. For readers interested in computational complexity, we sketch the justifications of computational hardness in the sidebar.

### Computing Answer Sets

So far, we have concentrated on the conceptual model of Figure 1 with an emphasis on the modeling side. As regards the actual computation of answer sets, grounding and

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**Listing 4: ASP Encoding of the Evacuation Plan**

```
reach(R,R) :- room(R).
reach(R1,R2) :- reach(R1,R3), evac(R3,R2), room(R1), pot(R3,R2).
ok(R) :- room(R), reach(R,X), exit(X).
:- not ok(R), room(R).
#minimize{1,R1,R2: evac(R1,R2), pot(R1,R2)}.
evac(R1,R2) :- pot(R1,R2).
```

**Listing 5: Revised ASP Encoding of the Evacuation Plan**

```
step(0..s).
reach(R,R,0) :- room(R).
reach(R1,R2,S+1) :- reach(R1,R3,S), evac(R3,R2), room(R1), pot(R3,R2), step(S), step(S+1).
ok(R) :- room(R), reach(R,X,S), exit(X), step(S).
evac(R1,R2) :- pot(R1,R2).
```

would falsify \( \text{evac}(R1,R2) \) by default if \( \text{pot}(R1,R2) \) were false, e.g., rooms \( R1 \) and \( R2 \) were not adjacent. Since the relation \( \text{pot}/2 \) is symmetric, this gives rise to four different scenarios if \( \text{pot}(R1,R2) \) and thus also \( \text{pot}(R2,R1) \) is true. Evacuation in one direction is possible if either \( \text{evac}(R1,R2) \) or \( \text{evac}(R2,R1) \) holds. If they are both true, this allows for bidirectional evacuation between \( R1 \) and \( R2 \). If such an option is not considered safe, it is easy to introduce an integrity constraint to exclude such a possibility in general:

\[
\text{:- evac}(R1,R2), \text{evac}(R2,R1), \text{pot}(R1,R2). 
\]

If both \( \text{evac}(R1,R2) \) and \( \text{evac}(R1,R1) \) are false, then there is no connection between rooms \( R1 \) and \( R2 \) in case of an emergency. It remains to ensure that there exists an overall evacuation plan, i.e., it is possible to reach at least one exit of the building from every room.

### Recursive Definitions

The existence of an evacuation plan is governed by constraints that concern the mutual reachability of rooms, to be formalized using a predicate \( \text{reach}/2 \). The first two rules of Listing 4 give a recursive definition for this predicate. Every room \( R \) is reachable from itself: the corresponding base case is given in line 1. The recursive case is formulated in lines 2–4: the reachability of \( R2 \) from \( R1 \) builds on the reachability of an intermediate room \( R3 \) from \( R1 \) and the condition that \( R3 \) can be evacuated to \( R2 \) (cf. line 3).

### Constraining Solutions

The essential constraint on the evacuation plan is given in lines 6–7 of Listing 4. Any given room \( R \) is considered to be OK, if some exit \( X \) is reachable from it (line 6). The auxiliary predicate \( \text{ok}(R) \) is defined in order to detect this aspect for each room. The actual constraint (line 7) excludes scenarios where some of the rooms would not be OK. Last, we want to minimize the number of evacuation connections by the objective function given in line 9. Using the encoding devised so far and an ASP solver, it is possible to check for the floor plan of Figure 2 that the minimum number of connections is 11. This is clear since there are 12 rooms in total each of which (except room 5) must be connected to some other room for the purpose of evacuation. But ASP solvers can find out more for our running example. For instance, it is possible to enumerate and count all possible evacuation plans with 11 connections. In

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**Elaboration Tolerance.** It is straightforward to modify the recursive encoding so that the number of steps is reflected. The revised encoding is presented as Listing 5. The domain for steps is first declared by the rule in line 1 where the maximum number of steps \( s \) is determined from the command line of the grounder. The base case in line 3 simply states that each room \( R \) is reachable from itself in zero steps. The main modification in the recursive case (lines 4–5) concerns counting: the number of steps \( s \) is increased by one to \( s+1 \) whenever a further step is made. However, since both \( s \) and \( s+1 \) must be members of the domain of steps, the maximum value is effectively determined by the constant \( s \) in line 1. Given the floor plan of Figure 2 and \( s=2 \), no evacuation plans can be found. By increasing \( s \) by one, solutions with 11 connections are found again and there are only 152 plans where the number of evacuation steps is at most three.

In summary, we have now tackled one particular aspect of locking design, i.e., ensuring that an evacuation plan exists for a building. In reality further requirements are imposed on evacuation plans making the problem computationally more and more challenging. For instance, it can be shown that if we incorporate conditions which can make rooms along an evacuation route mutually exclusive, e.g., for certain security reasons, it is unlikely that we are able to find a polynomial-time algorithm for solving the problem (mathematically expressed the problem becomes NP-complete). This justifies well the use of powerful search methods like ASP for tackling the problem. For readers interested in computational complexity, we sketch the justifications of computational hardness in the sidebar.

**Computing Answer Sets**

So far, we have concentrated on the conceptual model of Figure 1 with an emphasis on the modeling side. As regards the actual computation of answer sets, grounding and
solving were also identified as the main steps involved. Grounders are implemented either as stand-alone tools, such as the state-of-the-art grounder GRINGO\(^3\), or integrated as a front-end of the solver. Native answer-set solvers are able to handle ground logic programs directly and, hence, truly implement the search step illustrated in the figure. Typically, this step is the most demanding one from the computational perspective. A number of answer-set solvers have been developed in the history of ASP and we mention here DLV\(^4\), CLASP\(^3\), and WASP\(^3\) since they are actively maintained and developed at the moment. The article by Kaufmann et al. (2016) in this special issue gives a more detailed account of grounding and solving. If ASP is extended by constraints which cannot be directly handled by the ASP solver being used, the typical solution is to isolate extensions from rules themselves and to treat them by appropriate solvers externally. This leads to an architecture where two or more solvers are cooperating and interacting in analogy to SAT modulo theories (SMT) solvers. Then each sort of constraints can be handled by native algorithms.

**Translation-Based ASP.** The other constraint-based disciplines discussed in the introduction offer similar solver technology at the user’s disposal for handling, in particular, the search phase. However, they cannot be used straightforwardly, as ground programs are not directly understood by such solvers and certain kinds of transformations become indispensable. The idea of translation-based ASP is to translate (ground) logic programs into other formalisms so that a variety of solvers can be harnessed to the task of computing answer sets. Such an approach can be understood as a refinement of the search step in Figure 1. There are existing translations from ASP, e.g., to SAT (Janhunen 2004), and its extension as SMT (Niemelä 2008), and mixed integer programming (MIP) (Liu, Janhunen, and Niemelä 2012). These translations indicate the realizability of ASP in other formalisms and they have all been implemented by translators in the ASPTOOLS\(^6\) collection. They offer another way of implementing the search phase in ASP using off-the-shelf solvers as black boxes. This approach is already competitive in certain application problems and it can be seen as an effort to combine the expressive power of the modeling language offered by ASP with the high performance of existing solvers. Translations are also useful when implementing language extensions in a single target language. For instance, the idea of (Janhunen, Liu, and Niemelä 2011) is to translate programs enriched by difference constraints into difference logic altogether. The strength is that a single solver is sufficient for the search phase, but on the other hand, the original structure of constraints may be lost.

**Cross Translation.** The translations mentioned above are based on very similar technical ideas but yield representations of the ground program in completely different formats.

Since the development of several translators brings about extra programming work, it would be highly desirable to integrate the variety of translators in a single tool—having options for different back-end formats. This is not as simple as that due to the wide variety of formats under consideration. However, this issue is partly solved by a recent translation from ASP to SAT modulo acyclicity (Gebser, Janhunen, and Rintanen 2014) where graph-based constraints are interconnected with ordinary logical constraints (i.e., clauses). The translation can be implemented by instrumenting a ground logic program with certain additional rules and meta information formalizing the underlying recursion mechanism in terms of the acyclicity constraint. This leads to a new implementation strategy for translation-based ASP: the choice of the target formalism can be postponed until the last step of translation where the constraints are output in a particular solver format. This idea is analogous to cross compilation in the context of compiling conventional programming languages and hence we coin the term cross translation for ASP. In the current implementation of this idea, a back-end translator transforms the instrumented program into other kinds of constraints understood by SMT, MIP, and pseudo-Boolean (PB) solvers, for instance. Interestingly, by implementing an additional acyclicity check inside a native ASP solver, the instrumented program can also be processed directly by the solver (Bomanson et al. 2015), which offers yet another approach to answer set computation.

**Summary and Future Prospects**

This paper provides an introduction to the ASP paradigm as well as explains its main features—first generally, but also in terms of examples. We also discuss the two mainstream approaches to implementing the search for answer sets using either native solvers, or translators combined with solver technology offered by neighboring disciplines.

**Towards Universal Modeling.** There is a clear trend in the area of constraint-based modeling where methods and techniques are being transferred from one discipline to another. Various ideas from knowledge representation, logic programming, databases, and Boolean satisfiability served as a starting point for the ASP paradigm. But there are signs of knowledge transfer in the other direction as well. For instance, ASP solvers have been integrated into logic programming systems such as XSB (Rao et al. 1997). Advanced query evaluation mechanisms of ASP (Faber, Greco, and Leone 2007) are also relevant for deductive databases. The very idea of answer sets has been brought to the context of CP by introducing so-called bound-founded variables (Aziz, Chu, and Stuckey 2013). Quite recently, the algorithms for projected answer set enumeration have been exported for model counting in the context of SAT (Aziz et al. 2015).

We foresee that the exchange and incorporation of ideas and technologies in this way is gradually leading towards a universal approach where the user may rather freely pick the right language for expressing constraints of his or her interest. The underlying reasoning system is then supposed to (i) take care of required translations transparently and (ii) forward the resulting constraints for a solver architec-
ture that can realize the search for answers. The first attempts to define a modular framework for multi-language modeling have already been made (Järvisalo et al. 2009; Lierler and Truszczynski 2014; Tasharrofi and Ternovska 2011). However, a lot of work remains to be done in order to realize the universal modeling scenario. Our experience from integrating various kinds of tools suggests that finding a universal format for the constraints of interest is one of the key issues for tool interoperability. There are existing formats such as the DIMACS format in SAT, the Smodels format in ASP, and the FlatZinc format in CP; that can be used as starting points for designing the universal format.

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**Sidebar: Locking Design Can Be Computationally Challenging**

It is not surprising that finding a locking scheme satisfying given conditions can become computationally challenging when more involved conditions need to be satisfied. Here we consider the problem of finding a locking scheme that allows an evacuation plan such that for each room there is exactly one evacuation direction and the evacuation routes respect a given set of room conflicts, i.e., a set of pairs of rooms \((R_1, R_2)\) such that when following the evacuation routes if you enter room \(R_1\), then you cannot enter room \(R_2\). We show that this locking design problem is NP-complete indicating that it is unlikely that a polynomial time algorithm for solving this problem can be found. See, for example, (Papadimitriou 1994) for an introduction to computational complexity and the required concepts used below.

![Figure 3: Floor Plan and Evacuation Routes for the NP Completeness Proof](image-url)

Technically, the NP-completeness of a problem can be shown by establishing a reduction computable in polynomial time from a known NP-complete problem to the problem and showing that it can be checked in polynomial time that a potential solution satisfies the required conditions for the problem. As such a known NP-complete problem we use the Exact-3-SAT problem where we are given a conjunction of 3-literal clauses and the problem is to find a truth assignment that satisfies exactly one literal in each of the clauses.

**Reduction from Exact-3-SAT.** Any given 3-SAT instance \(C_1 \land \ldots \land C_n\) can be transformed into a floor plan illustrated in Figure 3. For each 3-literal clause \(C_i = l_{i,1} \lor l_{i,2} \lor l_{i,3}\), we introduce a corridor \(C_i\) connected to rooms \(R_{i,1}, R_{i,2}, R_{i,3}\) that are connected to corridor \(C_{i+1}\). Moreover, rooms \(R_{i,1}, R_{i,2}, R_{i,3}\) do not have doors in-between. The (only) exit is located next to corridor \(C_{n+1}\) which means that all corridors and rooms must be eventually evacuated through it. Moreover, each room \(R_{i,j}\) is labeled by the respective literal \(l_{i,j}\), the idea being that \(l_{i,j}\) is satisfied if \(C_i\) is evacuated via the room \(R_{i,j}\). Consequently, if there are two rooms labeled by complementary literals (i.e., a Boolean variable \(x\) and its negation \(\neg x\)), then those rooms are in conflict. This means that evacuation routes involving any pair of conflicting rooms are not feasible. It is also easy to see that the floor plan in Figure 3 and the associated set of conflicts can be computed in polynomial time.

It can be shown that a 3-SAT instance \(C_1 \land \ldots \land C_n\) has...
a satisfying truth assignment such that each clause has exactly one literal satisfied if and only if for the corresponding floor plan there is a locking scheme that allows an evacuation plan such that (i) for each room there is exactly one evacuation direction and (ii) the evacuation routes respect the set of room conflicts arising from the complementary literals. The key observation is that for the corresponding floor plan evacuation is possible only if there is a route from $C_1$ to $C_{n+1}$ such that for each $i = 1, \ldots, n$ the route visits exactly one of the rooms $R_{i,1}$, $R_{i,2}$, and $R_{i,3}$ and all room conflicts are respected. A satisfying truth assignment such that each clause has exactly one literal satisfied gives directly such a route and if such a route is available, it gives directly an appropriate truth assignment where literals corresponding to the visited rooms in the route are satisfied.

Moreover, it is clear that given a locking scheme with exactly one evacuation direction for each room, it can be checked in polynomial time that evacuation is possible and that all room conflicts are respected.