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Published in:
Physics of Plasmas

DOI:
[10.1063/1.5051815](https://doi.org/10.1063/1.5051815)

Published: 01/11/2018

Document Version
Publisher's PDF, also known as Version of record

Please cite the original version:

Altukhov, A. B., Gurchenko, A. D., Gusakov, E. Z., Irzak, M. A., Niskala, P., Esipov, L. A., Kiviniemi, T. P., Krutkin, O. L., & Leerink, S. (2018). Influence of the probing wave phase modulation on the X-mode radial correlation Doppler reflectometry performance in the FT-2 tokamak. *Physics of Plasmas*, 25(11), 1-8. [112503]. <https://doi.org/10.1063/1.5051815>

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Citation: [Physics of Plasmas](#) **25**, 112503 (2018); doi: 10.1063/1.5051815

View online: <https://doi.org/10.1063/1.5051815>

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Influence of the probing wave phase modulation on the X-mode radial correlation Doppler reflectometry performance in the FT-2 tokamak

A. B. Altukhov,¹ A. D. Gurchenko,¹ E. Z. Gusakov,¹ M. A. Irzak,¹ P. Niskala,² L. A. Esipov,¹ T. P. Kiviniemi,² O. L. Krutkin,¹ and S. Leerink²

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(Received 13 August 2018; accepted 21 October 2018; published online 8 November 2018)

The cross-correlation function of high field side radial correlation X-mode Doppler reflectometry (DR) measured in the FT-2 tokamak experiment is shown to be a factor of three narrower than that computed using the fast linear (Born approximation) version of the X-mode DR synthetic diagnostics developed in the framework of the ELMFIRE global gyrokinetic modeling of the FT-2 ohmic discharge. This difference is observed in spite of the fact that the computed DR signal frequency spectra are shown to be similar to those measured. A modest phase modulation of the probing and backscattering waves by the long-scale turbulent density fluctuations is shown, both experimentally and in computation, to be responsible for the observed effect. *Published by AIP Publishing.*
<https://doi.org/10.1063/1.5051815>

I. INTRODUCTION

Anomalous energy transport remains the main unresolved issue related to the implementation of magnetic confinement fusion using the tokamak concept. According to the present day understanding,^{1–4} anomalous transport is determined by drift-wave turbulence that requires the investigation of its characteristics. One of the frequently used and convenient diagnostics providing information on the turbulence density fluctuations and associated plasma flows is Doppler reflectometry or Doppler backscattering in different modifications. Doppler reflectometry (DR) is the microwave diagnostic widely used nowadays in toroidal fusion experiments to get information on the anomalous transport phenomena, namely, on plasma rotation velocity (both the mean value and its oscillations)⁵ and turbulence poloidal wavenumber spectrum.⁶ Recently, the radial correlation modification of this technique,⁷ providing information on the turbulence radial structure, was justified.^{8–10} Unfortunately, the interpretation of the radial correlation reflectometry data is complicated by the contribution from the poorly localized small-angle scattering along the wave trajectory, which can lead to the overestimation of the turbulence radial correlation length in the linear scattering regime^{11,12} and to its underestimation in the nonlinear regime (multiple scattering or strong phase modulation).^{13–15} In the former case, the small-angle scattering contribution could be suppressed by choosing a large enough incidence angle, satisfying the condition proposed in Ref. 8. However, in the latter (non-linear) case, the turbulence wavenumber spectrum measurements are questionable, and only the long-scale plasma turbulence mean velocity could be determined (but with poor spatial resolution).¹⁶ Moreover, in the case of X-mode DR, the probing wave turning point, providing the enhanced contribution to the DR signal in the slab plasma model, cannot be determined unambiguously due to the 2D propagation effects essential in a tokamak, which additionally complicates the

experimental data interpretation and its comparison with the gyrokinetic (GK) theory predictions. The above-mentioned circumstances make questionable the results of the direct comparison of measured cross-correlation functions (CCFs) of the radial correlation DR and two-point cross-correlation functions of density fluctuations provided by the GK modeling, in particular those performed recently at the FT-2 tokamak.^{17,18} The development of a synthetic diagnostics that allows the computation of DR signals based on GK simulations is useful for both comprehensive interpreting of the experimental results and the code benchmarking.^{17,19–26}

In the present paper, the linear version of the X-mode high magnetic field side (HFS) DR synthetic diagnostics developed recently²⁷ in the framework of the ELMFIRE global GK modeling of the FT-2 tokamak ohmic discharge is applied for the interpretation of the radial correlation Doppler reflectometry (RCDR) data obtained in Ref. 18. The X-mode DR signal is computed within the framework of the theory linear in the density fluctuation amplitude (Born approximation) using the reciprocity theorem technique and utilizing the probing wave field pattern provided by the full-wave computation that takes into account the 2D plasma inhomogeneity effects. The computed DR signal frequency spectrum, its frequency shift, and shape are compared with those measured in the experiment at the FT-2 tokamak HFS, where the decay of the trapped electron mode (TEM) turbulence takes place according to the GK code predictions. For the first time in the case of multi-frequency probing, the radial correlation DR cross-correlation function is computed based on the GK data, and its absolute value (coherence) is compared to that obtained in the experiment and shown to be substantially different. The multiple small-angle scattering or the phase modulation of the probing wave is shown to be responsible for this effect in spite of the small size of the tokamak. It is taken into account by multiplying the probing field computed in the linear approximation by the fluctuating phase factor calculated along the unperturbed ray trajectory

based on the GK code output. It is shown that the calculated phase standard deviation (SD) matches the SD of the phase measured in the specially performed experiment.

II. EXPERIMENTAL APPROACH

The experimental data interpreted in this paper were obtained¹⁸ at the FT-2 tokamak with major radius $R = 55$ cm and minor limiter radius $a = 7.9$ cm in a hydrogen ohmic discharge with a total duration of 40 ms in its stationary flat-top, which lasts for 5 ms. This discharge with plasma current $I_p = 19$ kA (and the safety factor 6.4 at the plasma boundary), central density $n_e(0) = 4 \times 10^{19} \text{ m}^{-3}$, and central electron and ion temperatures $T_e(0) = 370$ eV and $T_i(0) = 124$ eV is similar to that utilized for successful comprehensive benchmarking of the ELMFIRE GK code in Refs. 17 and 19; however, the toroidal magnetic field at the discharge axis $B_t(0) = 1.7$ T is 19% smaller. The ASTRA-code fits of the actual electron density [recovered from the combined measurements by an microwave interferometer with 7 vertical probing chords and multi-pass intra-cavity laser Thomson scattering (TS) utilizing 20 diagnostic pulses with a repetition rate 5 kHz] and temperature (provided by TS) profiles for the discharge used in the GK modeling and the ion temperature profile (determined with a 5-channel radially scanning charge-exchange neutral particle analyser in the core plasma and impurity spectroscopy at the periphery) are shown in Fig. 1 together with the corresponding error bars. The double antenna vertically movable by ± 2 cm relative to the equatorial plane (shown in scale in Fig. 2) installed at HFS allows both X-mode plasma probing at a variable incidence angle with frequencies $f_i = 50\text{--}75$ GHz and the reception of the scattered signal. The distance from the limiter to the mouth of the horn is 0.2 cm, and the microwave beam

FIG. 1. The discharge parameter radial profiles.

FIG. 2. The DR experiment geometry (horn antennae, X-mode cut-off, and the upper hybrid resonance surfaces) and the spatial structure of the weighting function $\frac{E_a(\vec{r}, \omega)}{n_e(\vec{r})} [\hat{\sigma}(\vec{r}, \omega) \vec{E}_a(\vec{r}, \omega)]$ for the probing antenna.

diameter there is 2 cm. In the past, this antenna, described in more detail in Ref. 28, was used for the electron temperature gradient (ETG) mode studies using the enhanced scattering in the upper hybrid resonance.^{29,30} In the present experiment, it was used both in standard DR measurements utilizing the in-phase and quadrature (IQ) detection scheme providing a complex DR signal and in the radial correlation DR technique based on the two-channel homodyne detection described in Ref. 18. In the former case, the DR spectrum of the complex DR signal was obtained via averaging over 155 random DR power spectra provided by using the Fourier transform of 12.8 μs samples of the complex DR signal. The mean DR signal component determined by averaging over all the 155 samples was subtracted from the DR signal. In the latter case, the reference channel generator was tuned to the master frequency $f_0 = 70$ GHz, determining the measurement position in the vicinity of $r = 5$ cm, whereas another microwave generator producing the probing signal at the slave frequency was used in the second (signal) channel to determine the turbulence two-point cross-correlation function at a set of turbulence frequencies. The slave frequency was varied by 100 MHz steps in the interval ± 1.2 GHz around the master frequency on the discharge to discharge basis.

III. GK COMPUTATIONAL APPROACH AND SYNTHETIC DIAGNOSTICS

The simulations are performed with the global electrostatic particle-in-cell code ELMFIRE³¹ to model the evolution of the full distribution function (full-f) of drift kinetic electrons, gyrokinetic protons, and O^{6+} ions, which dominate in the FT-2 discharges according to spectroscopic measurements.³² The simulation geometry has a circular cross-section and covers the radial range of $\rho \equiv r/a = [0 - 1]$. In the present paper, the time step is set to $\delta t = 30$ ns and the spatial grid to 110×8 cells in radial (r) and toroidal (φ) directions accordingly, whereas the maximal number of poloidal (θ) cells depending on the radius reaches 515. This grid limits the largest resolvable density fluctuation wave-number at $r = 5.5$ cm to $k_{\theta\text{max}} = 47 \text{ cm}^{-1}$ corresponding to $k_{\theta\text{max}} \rho_s = k_{\theta\text{max}} \sqrt{T_e / (2T_i)} \rho_i = 2.63$ (where ρ_i is the ion

Larmor radius). As discussed in Ref. 33, the radial and poloidal resolutions have been set such that the magnetic shear should be accurately modeled for all the poloidal modes.

The particles are initialized according to prescribed temperature and density profiles taken close to the experimentally measured ones (see Fig. 1) at the low field side of the tokamak, where the TEM typical for FT-2³⁴ is driven.

The normalized root mean square (rms) density fluctuation level is approximately 2.3% at $r = 5$ cm and increases to a maximum of 8% in the edge of the simulation volume at $r = 8$ cm.

The fast synthetic X-mode DR diagnostics developed in Ref. 28 is based on the reciprocity theorem of electrodynamics,³⁵ relating the high-frequency current in the plasma volume and the signal radiated by it. This approach, providing directly the scattering signal received by an antenna without computation of the scattered field in the plasma volume, is equivalent to the Born approximation treating the density fluctuations as a small perturbation. In the microwave backscattering case, the reciprocity theorem results in the following expression for the signal $A_{s\omega}(t)$:

$$A_{s\omega}(t) = \frac{\sqrt{P}}{4} \int_V \frac{\tilde{n}(\vec{r}, t)}{n_e} [\hat{\sigma}(\vec{r}, \omega) \vec{E}_a(\vec{r}, \omega)] \vec{E}_a^+(\vec{r}, \omega) d\vec{r}, \quad (1)$$

where P is the probing wave power, the DR signal power is given by $|A_{s\omega}(t)|^2$, $\tilde{n}(\vec{r}, t)/n_e$ is the relative density fluctuation distribution provided by the GK computation, $\hat{\sigma}(\vec{r}, \omega)$ is the high-frequency conductivity tensor of unperturbed plasma calculated accounting for the magnetic field poloidal component, $\sqrt{P} \vec{E}_a(\vec{r}, \omega)$ stands for the probing wave electric field distribution, and $\vec{E}_a^+(\vec{r}, \omega)$ stands for the wave electric field distribution produced by the probing antenna at the unit power and opposite sign of the plasma magnetic field. The integration in Eq. (1) was performed over the whole discharge poloidal cross-section. The electric field distribution was determined by solving the Maxwell equations in the realistic toroidal geometry and for the density profile shown in Fig. 1. The equations describing the probing wave

$$\vec{\nabla} \times [\vec{\nabla} \times \vec{E}_a] = \frac{\omega^2}{c^2} \left(1 + \frac{4\pi i}{\omega} \hat{\sigma} \right) \vec{E}_a \quad (2)$$

were solved numerically by full-wave code WaveTOP2D.³⁶ The boundary conditions at the antenna were modeling the Gaussian wave beam (experimentally measured in the range of not less than 12 dB for the probing power), focusing on air at the distance of 14 cm from the antenna where its diameter was equal to 1.5 cm in the poloidal direction. The infinite size of the beam in the toroidal direction (toroidal mode $n = 0$) was assumed. The reflection from the tokamak metallic wall was suppressed by introducing an artificial absorbing layer in the wall vicinity. The corresponding distribution of the weighting function $\frac{\vec{E}_a(\vec{r}, \omega)}{n_e(\vec{r})} [\hat{\sigma}(\vec{r}, \omega) \vec{E}_a(\vec{r}, \omega)]$ in the presence of plasma is shown in Fig. 2 for the incident wave frequency $f_i = 70$ GHz. The plasma density distribution temporal dependence provided by the GK code was used to extract the density fluctuations entering expression (1). Density fluctuations were obtained in 150 μ s interval as

$\tilde{n}(t_i) = n(t_i) - n_{avr}(t_i, T_{avr})$, where the mean plasma density n_{avr} was obtained by averaging n over the time interval $[t_i - 17.5 \mu\text{s}, t_i + 17.5 \mu\text{s}]$, i.e., $T_{avr} = 35 \mu\text{s}$ (at the start and at the end of the whole 150 μs interval, this averaging window was truncated). The mean plasma density obtained in each interval served to extract the fluctuating density component. This procedure helped to reduce the influence of density profile slow relaxation on the low-frequency part of the DR signal spectrum. The turbulent density fluctuations computed in eight toroidal cross-sections were used to obtain the synthetic complex DR signal lasting $8 \times 150 \mu\text{s} = 1200 \mu\text{s}$. This realization was divided into 12.45 μs samples in which the random Fourier spectra were calculated and used later to obtain the average DR power spectrum.

It should be mentioned, however, that since the unperturbed wave field is substituted in Eq. (1), the approach based on the linearized reciprocity theorem neglects the perturbations of the probing wave propagation caused by the density fluctuations. In the applied method, nonlinear effects, in particular, multiple small-angle scattering or the probing wave phase strong modulation, are not taken into account. In this sense, as was said above, the approach is equivalent to the so-called Born approximation linear in the fluctuation amplitude. The main merit of this method is the possibility of fast computations, whereas the main drawback is related to the neglect of the nonlinear effects, which, at the first glance, should not be that strong a limitation in a small-size tokamak.

IV. COMPARISON OF DOPPLER BACKSCATTERING SIGNALS AND RADIAL CORRELATION DOPPLER REFLECTOMETRY DATA

The DR power spectra, obtained in the experiment and provided by the synthetic diagnostics, were normalized to unity and then compared in Fig. 3 for the $y_a = +2$ cm vertical antenna shift.

As is seen in Fig. 3, the spectrum of the synthetic complex DR signal is asymmetric and fits reasonably well the experimental one. The spectral asymmetry is related to the Doppler effect at the backscattering off the poloidally

FIG. 3. Comparison of normalized DR spectra. Circles, experiment; and triangles, synthetic DR. The turbulence frequency $f = f_s - f_i$ is the difference of the scattered and incident frequencies. The negative Doppler frequency shift of both spectra corresponds to the electron diamagnetic drift direction.

rotating turbulent fluctuations. The estimation of the mean fluctuation poloidal velocity based on this asymmetry results in experiment in $v_\theta = \pi f_D / k_\theta \approx 2.1 \pm 0.2$ km/s, whereas in the synthetic DR, it provides $v_\theta \approx 2.0 \pm 0.1$ km/s. Not only are the velocities determined from the Doppler frequency shifts of both spectra close but also the spectral shapes look alike. The obtained agreement between the measured and computed DR spectra,²⁷ which are determined by the plasma flows, provides evidence in favor of correct modeling of plasma poloidal rotation and a correct reproduction of the radial electric field behavior in the FT-2 tokamak by the ELMFIRE code.

Inspired by a reasonable agreement of the experimental and synthetic DR spectral data presented in Fig. 3, we performed the comparison of the computed and measured radial correlation DR cross-correlation functions using the X-mode DR antenna displaced by 2 cm from the equatorial plane of the discharge at the HFS of the FT-2 tokamak. The measurements were performed on the discharge-to-discharge basis (up to 40 highly reproducible discharges) during 2 ms at the flat-top of the discharge with global parameters corresponding to the interval when the density and the temperature profiles shown in Fig. 1 were obtained. The corresponding cross-correlation functions of the homodyne signals were integrated over all the turbulence frequency range (± 1 MHz) and normalized. Their absolute value (coherence) is shown in Fig. 4(a) as functions of the frequency difference in the master and slave channels and in Fig. 4(b) as a function of the cut-off separation. As is seen in Fig. 4, the measured and computed coherences appear to be very different, whereas the computed density fluctuations' two-point CCF for the turbulence frequency $f \equiv f_s - f_i = 300$ kHz shown in Fig. 4(b) is close to the measured radial correlation DR coherence, as it was mentioned in Ref. 18. The slow decay of the synthetic radial correlation DR cross-correlation function computed within the Born approximation is determined by the small-angle scattering contribution to the DR signal,¹¹ which, in contradiction with the experimentalist's expectations,¹⁸ appears to be not suppressed at the probing wave incidence angle corresponding to $y_a = 2$ cm. Unlike Ref. 37,

in this case, the incidence angle appears to be just at the critical value determined by the criterion derived in Refs. 8 and 38 for the slab and cylinder plasma geometries, accordingly. At the larger incidence angle corresponding to $y_a = 4.0$ cm (which is not possible in the experiment but doable in the GK computation), the synthetic coherence decay with the growing frequency difference between the channels is much faster, as it is demonstrated by squares in Fig. 4(a). So, only at this large vertical displacement, the direct measurement of the turbulence radial correlation length could be possible if the DR regime is linear. At the smaller displacements, the application of the reconstruction procedure¹⁰ is needed.

There are at least two possible reasons for the drastic difference between the experimental and synthetic RCDR cross-correlation functions. The first one is related to the GK code spatial grid limitations, leading to underestimation of the turbulence wavenumber spectrum width²⁷ and thus to overestimation of the density fluctuation two-point radial cross-correlation functions width. The dependence of the absolute value of the density fluctuation CCF on point separation is shown in Fig. 4(b). As it is seen there, the width of the GK turbulence CCF is notably smaller than that of the measured RCDR CCF. The difference could be attributed to the effect of broadening of the RCDR CCF due to the small-angle scattering contribution to the fluctuation reflectometry signal.^{11,12}

The second reason is related to nonlinear effects, in particular, to the strong probing wave phase modulation by long-scale fluctuations or, in other words, to the multiple small-angle scattering, according to the theory,^{13–16} coming into play with the growth of the turbulence level. The importance of the nonlinear effect for the DR diagnostics performance and, in particular, for the turbulence poloidal wavenumber spectra measurements was shown recently at the ASDEX-upgrade.²³

The first possibility is illustrated in Fig. 5 where the radial correlation DR cross-correlation functions (absolute value) computed in the experimental geometry are presented for artificial turbulent density fluctuation two-point CCFs possessing different correlation lengths

FIG. 4. The radial correlation DR coherence against (a) the channel frequency separation: stars, synthetic DR for $y_a = 2$ cm; squares, synthetic DR for $y_a = 4$ cm; triangles, synthetic DR for $y_a = 2$ cm with phase modulation; and circles, experiment; (b) the cut-off radial separation: stars, synthetic DR for $y_a = 2$ cm; circles, experiment; and double crosses, turbulence coherence by GK code. (The level $1/e$ is shown by a horizontal solid line, and the level of noise coherence corresponding to the inverse square root of the number of averages is 0.08.).

FIG. 5. The measured radial correlation DR coherence (dots) and DR coherences computed for a model turbulence with $m_p = 75$ and $L_c = 4$ mm (dash), 2 mm (dash dot dot), 1 mm (dash dot), and 0.5 mm (solid).

$$\langle \tilde{n}(r, \theta, t) \tilde{n}(r', \theta', t') \rangle = \tilde{n}^2(\theta) \cos \left[\frac{m_p}{r} (r\theta - r'\theta') \right] \times \exp \left[-\frac{(r - r')^2 + r^2(\theta - \theta')^2}{L_c^2} \right], \quad (3)$$

where $m_p = 75$ stands for the poloidal mode number dominant in the DR signal, $\tilde{n}^2(\theta)$ describes the poloidal dependence of the turbulence level provided by the GK computation, and v stands for the fluctuation phase velocity. In the computation, we used the analytical relationship between the measured signal CCF and the density fluctuation two-point CCF derived in Ref. 8 and assumed $v = 0$. As is seen in Fig. 5, in the case of coherence calculated for the radial correlation length $L_c \leq 0.1$ cm, which is comparable to the code radial grid size ($\delta r \approx 0.068$ cm), the computed radial correlation DR signal coherence dependence on the channel frequency difference is not that far from the experimental one. However, this value of L_c is too different from $L_c = 0.2$ cm obtained in the GK computation [see Fig. 4(b)]. Because of that, the underestimation of the small-scale turbulence level by the GK code cannot be the sole explanation of the drastic difference between the measured and synthetic RCDR cross-correlation functions.

V. PROBING WAVE PHASE MODULATION IN SIMULATIONS AND MEASUREMENTS

To check the role of nonlinear effects in the FT-2 X-mode radial correlation DR experiment, we have computed and measured the random phase perturbation of the backscattered wave and determined its rms. The computation was based on the expression for the WKB phase of the reflected wave valid, strictly speaking, only for the contribution from the long-scale turbulence, satisfying conditions $L_c \geq [c^2(a - r)/\omega^2]^{1/3}$ and $k_\theta L_c \geq 1$. The calculation was performed in the first order approximation accounting for the density fluctuations but assuming the ray trajectory to be unperturbed by them. The corresponding expression for the backscattered wave phase derived in Ref. 15 is given by the integral along the unperturbed ray trajectory

$$\tilde{\varphi}(\omega, t, l) = -\left(\frac{\omega}{c}\right)^2 \int_0^l \frac{h_X(l') \tilde{n}(l', t)}{k_X(l') n_c} dl', \quad (4)$$

where $\omega = 2\pi f_i$, and the probing X-mode wavenumber is given by the expression

$$k_X(\omega, l) = \sqrt{\frac{[(\omega + \omega_{ce}(l))\omega - \omega_{pe}^2(l)][(\omega - \omega_{ce}(l))\omega - \omega_{pe}^2(l)]}{[\omega^2 - \omega_{pe}^2(l) - \omega_{ce}^2(l)]c^2}}. \quad (5)$$

The density fluctuation $\tilde{n}(l, t)$ is provided by the GK computation, n_c stands for the mean electron density in the turning point, whereas the factor $h_X(l, \omega)$, according to Ref. 15, is given by the expression

$$h_X(l, \omega) = \frac{[\omega^2 - 2\omega_{pe}^2(l)][\omega^2 - \omega_{ce}^2(l)] + \omega_{pe}^4(l)}{[\omega^2 - \omega_{pe}^2(l) - \omega_{ce}^2(l)]^2}. \quad (6)$$

Dependencies of the random density fluctuation amplitude and factor $\omega h_X(l, \omega)/(k_X c)$ on the coordinate along the ray trajectory are shown in Fig. 6 for different moments of time. As it is seen in Fig. 6, the factor h_X normalised to the probing wave refractive index $N_X = k_X c / \omega$ entering expression (5) is substantially larger than unity all over the trajectory. It increases even further when approaching the cut-off surface. This enhancement is associated with a small separation between the cut-off and the upper hybrid resonance layers in our experiment and, especially, with a strong dispersion of the X-mode in this region [see the resonant denominator in expressions (5) and (6)]. Due to this effect, the phase integral (3) is steeply growing when approaching the turning point l_c , as it is seen in Fig. 7. The temporal behaviour of the computed backscattering signal phase $\tilde{\varphi}(\omega, t, l_c)$ shown in Fig. 8 by a dashed curve demonstrates the variation in the range approximately from $-\pi$ to $+\pi$, which is associated with a substantial phase modulation of the probing wave and the DR signal by the long-scale turbulence, indicating the approach of the nonlinear regime of the reflectometry

FIG. 6. Density fluctuation distribution along the ray normalized to the critical density (at three different moments of GK computation time) and h_X -factor behavior along the ray trajectory, normalized to the extraordinary wave refractive index $N_X = k_X c / \omega$.

FIG. 7. The phase perturbation variation along the ray trajectory at three different moments of time.

FIG. 8. The DR signal phase temporal variation. Dashed line, calculated according to (3) ($SD = 0.93$); and solid line, experiment ($SD = 1.23$).

diagnostics. The fast events seen in the behavior of the phase integral perturbation computed using the GK density fluctuation data are related to the contribution from the fast small-scale fluctuations, which were not filtered out when computing the phase and which do not contribute directly to the phase of the measured signal.

The actual temporal variation of the DR signal phase (also shown in Fig. 8) was measured in a special experiment, for which a modification of the quadrature detection scheme utilizing the intermediate frequency signal inversion [shown in Fig. 9(a)] was developed. The microwave source 1 was used for plasma probing at frequency f_1 through the equatorially placed X-mode antenna horn 3. In the super-heterodyne detection scheme, the local oscillator 2 at frequency $f_2 = f_1 - \zeta$ ($\zeta = 150$ MHz) and the microwave mixer 6 were used for the analysis of the scattering signal received through the antenna horn 5. The super-heterodyne output of the microwave mixer 6 was split into two channels and supplied to RF-mixers 11 and 12. There, it was homodyne converted to the low-frequency range f_Ω using two signals at the intermediate frequency ζ , obtained from the microwave mixer 7 with a 90° phase difference between them provided by the phase shifter 10. The described implementation of the

FIG. 9. (a) Quadrature scheme with the intermediate frequency signal inversion. 1, microwave source (55–74 GHz; 20–40 mW); 2, LO (producing the intermediate frequency $\zeta = 150$ MHz); 3 and 5, probing and receiving antenna horns (X-mode, vertically movable, focusing); 4, plasma, 6 and 7, microwave mixers; 8, phase inverter; 9, pulse generator (pulse rise time: 8 ns); 10, 90° phase shifter; 11 and 12, RF-mixers; and 13, ADC (50 MHz, 12 bit). Directional couplers, power splitters, low-noise amplifiers, low-frequency amplifiers, valves, and low pass filters utilized in the actual scheme are not shown in the figure. (b)–(e) Examples of phase traces in the beginning of discharge (started at $t \approx 25 \mu s$) when the plasma volume is still transparent for probing microwave.

quadrature scheme is effective for the investigation of the backscattering signal amplitude and phase but loses the information on the evolution of the phase shift at the probing frequency corresponding to $f_\Omega = 0$. This problem was fixed by utilising the phase inverter 8 supplied by the pulse generator 9, which leads to generating “sine” and “cosine” phase traces shown in Fig. 9(b) for the time window, including $25 \mu s$ before the beginning of the discharge and $220 \mu s$ after for plasma without the cut-off layer for the probing wave. The zoomed windows of these traces with clearly visible periodic oscillations (resulting from the inversion of the reference signal) are shown in Figs. 9(c)–9(e). The period of oscillations on the phase traces is determined by the pulse generator 9. The ratio of normalised amplitudes of oscillations in two channels indicates the phase level of the backscattering signal. Without the plasma, this ratio is not changed and the phase is constant [look at $t < 25 \mu s$ in Fig. 9(c)]. The growth

of the signal phase shift associated with the density growth leads to relative changes in oscillation amplitudes there. Each crossing of the $\pi/2$ value results in alternation of the in-phase or counter-phase mode of the oscillations [see $t \approx 222 \mu\text{s}$ in Fig. 9(e)], making possible the signal phase evolution reconstruction. The example of the reconstructed phase temporal behaviour is shown in Fig. 8 by the solid curve. The interval of the DR signal random phase variation appears to be close to that determined numerically. The standard deviation of the computed DR signal phase there is 0.93, whereas for the measured phase, it is 1.23. The experimentally obtained value of the phase perturbation practically coincides with the computed one at the level, which is close to the borderline for the substantial phase modulation of the probing wave by turbulent density fluctuations and, accordingly, for the DR diagnostic transition into the saturated nonlinear regime of operation. In this parameter domain, the Born approximation still describes the backscattering in the vicinity of the Bragg resonance point correctly, but, strictly speaking, the reciprocity theorem in the form (1), operating with probing waves unperturbed by the turbulence, is not applicable. The obtained agreement of the measured and computed DR signal random phase standard deviation and its relatively high value provides an argument in favour of the second, based upon nonlinear effects, explanation of the drastic disagreement between the experimental and synthetic radial correlation DR diagnostics results seen in Fig. 4. A comparatively low (in terms of $\tilde{n}/n_c$) threshold of the X-mode DR transition to the regime of strong probing wave phase modulation by the turbulence is explained by the probing X-mode strong dependence on the frequency and plasma density, especially enhanced, according to expression (4), in the vicinity of the upper hybrid resonance.

In the nonlinear wave propagation and scattering regime, the synthetic DR diagnostics should be based upon a full-wave code accounting for the turbulence influence on the probing and scattered wave propagation.³⁹ Nevertheless, taking into account the fact that the Born approximation correctly describes the backscattering phenomena in the vicinity of the Bragg resonance, we can estimate the multiple small-angle scattering effect on the radial correlation DR cross-correlation function just by multiplying the DR signal $A_{sc}(\omega, t)$, provided by the reciprocity theorem (1), by the phase factor $\exp[i\tilde{\varphi}(\omega, t, l_c)]$ and evaluate the CCF for the modified signals $A_{sc}(\omega, t) \exp[i\tilde{\varphi}(\omega, t, l_c)]$ at different probing frequencies. This approach is based on the natural assumption justified in theory that the backscattering in DR is well localised in the trajectory turning point vicinity,⁴⁰ whereas the phase modulation is performed by the long-scale fluctuations all over the trajectory¹⁶ and therefore is less localised. The corresponding modified synthetic radial correlation DR cross-correlation function shown in Fig. 4(a) by triangles appears to be in much better agreement with the experiment than the original one. The remaining difference between the computed and measured cross-correlation functions could be attributed to the underestimation of the small-scale turbulence component in the GK modeling mentioned in Ref. 27.

It should be underlined, however, that the used intuitive approach is based on a separate description of the probing

wave propagation and backscattering. The applicability domain of this approach should be determined by modeling within the framework of a full-wave code correctly accounting for the nonlinear effects in the probing wave propagation and scattering in the turbulent plasma. This investigation will be performed in the forthcoming paper.

VI. CONCLUSIONS

To summarise, we can conclude that the comparison of the ELMFIRE code with the X-mode radial correlation Doppler reflectometry experimental data, performed for the HFS of the FT-2 tokamak with the help of the DR fast linear synthetic diagnostics, has demonstrated a good agreement between the measured and computed DR spectra. Both the spectrum frequency shift and the width, as well as the spectrum shape, were similar, thus demonstrating a correct reproduction of the electric field behavior in the FT-2 tokamak by the ELMFIRE GK code.

However, the drastic (by a factor of 3) difference was found in the decay of the DR signal cross-correlation functions with growing frequency separation in the probing wave channels. The quick decrease in the radial correlation DR coherence observed in the experiment is attributed in a larger part to the phase modulation of the probing wave due to the long-scale density fluctuations, which is shown to be close to 1 rad by both the specially performed measurements and the WKB estimation based on the GK modeling results. In spite of the fact that this value of the phase perturbation indicates only the beginning of the transition to the fluctuation reflectometry nonlinear regime (which therefore is not influencing the DR spectra), it already has a strong impact on the radial correlation DR performance. The easy way to take into account the phase modulation and make the results closer has been demonstrated.

ACKNOWLEDGMENTS

This investigation was supported by Russian Science Foundation under Grant No. 17-12-01110. M. A. Irzak acknowledges the support from Ioffe Institute. This work was also supported by the Academy of Finland under Grant Nos. 278487, 296853, and 318314. CSC – IT Center for Science is thanked for generous allocation of computational resources for this work.

¹P. H. Diamond, S.-I. Itoh, K. Itoh, and T. S. Hahm, *Plasma Phys. Controlled Fusion* **47**, R35 (2005).

²P. W. Terry, *Rev. Mod. Phys.* **72**, 109 (2000).

³E. J. Doyle, W. A. Houlberg, Y. Kamada, V. Mukhovatov, T. H. Osborne, A. Polevoi, G. Bateman, J. W. Connor, J. G. Cordey, T. Fujita *et al.*, *Nucl. Fusion* **47**, S18 (2007).

⁴A. J. Brizard and T. S. Hahm, *Rev. Mod. Phys.* **79**, 421 (2007).

⁵G. D. Conway, B. Scott, J. Schirmer, M. Reich, A. Kendl, and ASDEX Upgrade Team, *Plasma Phys. Controlled Fusion* **47**, 1165 (2005).

⁶P. Hennequin, R. Sabot, C. Honore, G. T. Hoang, X. Garbet, A. Truc, C. Fenzi, and A. Quéméneur, *Plasma Phys. Controlled Fusion* **46**, B121 (2004).

⁷J. Schirmer, G. D. Conway, E. Holzhauser, W. Suttrop, H. Zohm, and ASDEX Upgrade Team, *Plasma Phys. Controlled Fusion* **49**, 1019 (2007).

⁸E. Gusakov, M. Irzak, and A. Popov, *Plasma Phys. Controlled Fusion* **56**, 025009 (2014).

⁹E. Blanco and T. Estrada, *Plasma Phys. Controlled Fusion* **55**, 125006 (2013).

- ¹⁰E. Z. Gusakov, M. A. Irzak, A. Y. Popov, S. A. Khitrov, and N. V. Teplova, *Phys. Plasmas* **24**, 022119 (2017).
- ¹¹E. Z. Gusakov and B. O. Yakovlev, *Plasma Phys. Controlled Fusion* **44**, 2525 (2002).
- ¹²E. Z. Gusakov and N. V. Kosolapova, *Plasma Phys. Controlled Fusion* **53**, 045012 (2011).
- ¹³E. Mazzucato, *Rev. Sci. Instrum.* **69**, 2201 (1998).
- ¹⁴E. Z. Gusakov and A. Y. Popov, *Plasma Phys. Controlled Fusion* **44**, 2327 (2002).
- ¹⁵E. Z. Gusakov and A. Yu. Popov, *Plasma Phys. Controlled Fusion* **46**, 1393 (2004).
- ¹⁶E. Z. Gusakov, A. V. Surkov, and A. Yu. Popov, *Plasma Phys. Controlled Fusion* **47**, 959 (2005).
- ¹⁷E. Z. Gusakov, A. B. Altukhov, V. V. Bulanin, A. D. Gurchenko, J. A. Heikkinen, S. J. Janhunen, S. Leerink, L. A. Esipov, M. Yu. Kantor, T. P. Kiviniemi, T. Korpilo, D. V. Kouprienko, S. I. Lashkul, A. V. Petrov, and N. V. Teplova, *Plasma Phys. Controlled Fusion* **55**, 124034 (2013).
- ¹⁸A. B. Altukhov, A. D. Gurchenko, E. Z. Gusakov, L. A. Esipov, M. A. Irzak, M. Y. Kantor, D. V. Kouprienko, S. I. Lashkul, S. Leerink, P. Niskala, A. Y. Stepanov, and N. V. Teplova, *Plasma Phys. Controlled Fusion* **58**, 105004 (2016).
- ¹⁹S. Leerink, V. V. Bulanin, A. D. Gurchenko, E. Z. Gusakov, J. A. Heikkinen, S. J. Janhunen, S. I. Lashkul, A. B. Altukhov, L. A. Esipov, M. Y. Kantor, T. P. Kiviniemi, T. Korpilo, D. V. Kouprienko, and A. V. Petrov, *Phys. Rev. Lett.* **109**, 165001 (2012).
- ²⁰S. Leerink, V. V. Bulanin, E. Z. Gusakov, J. A. Heikkinen, S. J. Janhunen, T. P. Kiviniemi, T. Korpilo, M. Nora, and F. Ogando, *Contrib. Plasma Phys.* **50**, 242 (2010).
- ²¹J. C. Hillesheim, C. Holland, L. Schmitz, S. Kubota, T. L. Rhodes, and T. A. Carter, *Rev. Sci. Instrum.* **83**, 10E331 (2012).
- ²²U. Stroth, A. Bañón Navarro, G. D. Conway, T. Görler, T. Happel, P. Hennequin, C. Lechte, P. Manz, P. Simon, A. Biancalani, E. Blanco, C. Bottereau, F. Clairet, S. Coda, T. Eibert, T. Estrada *et al.*, *Nucl. Fusion* **55**, 083027 (2015).
- ²³T. Happel, T. Görler, P. Hennequin, C. Lechte, M. Bernert, G. D. Conway, S. J. Freethy, C. Honoré, J. R. Pinzón, U. Stroth, and ASDEX Upgrade Team, *Plasma Phys. Controlled Fusion* **59**, 054009 (2017).
- ²⁴C. Lechte, G. D. Conway, T. Görler, C. Tröster-Schmid, and ASDEX Upgrade Team, *Plasma Phys. Controlled Fusion* **59**, 075006 (2017).
- ²⁵C. Holland, J. C. DeBoo, T. L. Rhodes, L. Schmitz, J. C. Hillesheim, G. Wang, A. E. White, M. E. Austin, E. J. Doyle, W. A. Peebles, C. C. Petty, L. Zeng, and J. Candy, *Nucl. Fusion* **52**, 063028 (2012).
- ²⁶D. R. Ernst, K. H. Burrell, W. Guttenfelder, T. L. Rhodes, A. M. Dimits, R. Bravenec, B. A. Grierson, C. Holland, J. Lohr, A. Marinoni, G. R. McKee, C. C. Petty, J. C. Rost, L. Schmitz, G. Wang, S. Zemedkun, L. Zeng, and DIII-D Team, *Phys. Plasmas* **23**, 056112 (2016).
- ²⁷A. B. Altukhov, A. D. Gurchenko, E. Z. Gusakov, M. A. Irzak, P. Niskala, L. A. Esipov, T. P. Kiviniemi, and S. Leerink, *Phys. Plasmas* **25**, 082305 (2018).
- ²⁸E. Z. Gusakov, A. D. Gurchenko, A. B. Altukhov, V. V. Bulanin, L. A. Esipov, M. Yu. Kantor, D. V. Kouprienko, S. I. Lashkul, A. V. Petrov, and A. Y. Stepanov, *Plasma Phys. Controlled Fusion* **48**, B443 (2006).
- ²⁹A. D. Gurchenko, E. Z. Gusakov, A. B. Altukhov, A. Yu. Stepanov, L. A. Esipov, M. Yu. Kantor, D. V. Kouprienko, V. V. Dyachenko, and S. I. Lashkul, *Nucl. Fusion* **47**, 245 (2007).
- ³⁰A. D. Gurchenko and E. Z. Gusakov, *Plasma Phys. Controlled Fusion* **52**, 124035 (2010).
- ³¹J. A. Heikkinen, S. J. Janhunen, T. P. Kiviniemi, and F. Ogando, *J. Comput. Phys.* **227**, 5582 (2008).
- ³²S. I. Lashkul, A. B. Altukhov, V. V. D'yachenko, L. A. Esipov, M. Y. Kantor, D. V. Kouprienko, A. D. Lebedev, Y. A. Nikerman, and A. Y. Popov, *Plasma Phys. Rep.* **38**(11), 851–862 (2012).
- ³³S. Leerink, Ph.D. thesis, Aalto University, 2012.
- ³⁴P. Niskala, A. D. Gurchenko, E. Z. Gusakov, A. B. Altukhov, L. A. Esipov, M. Yu. Kantor, T. P. Kiviniemi, D. V. Kouprienko, T. Korpilo, S. I. Lashkul, S. Leerink, A. A. Perevalov, and R. Rochford, *Plasma Phys. Controlled Fusion* **59**, 044010 (2017).
- ³⁵A. D. Piliya and A. Y. Popov, *Plasma Phys. Controlled Fusion* **44**, 467 (2002).
- ³⁶E. Z. Gusakov, V. V. Dyachenko, M. A. Irzak, S. A. Khitrov, A. N. Saveliev, and O. N. Shcherbinin, *Plasma Phys. Controlled Fusion* **52**, 075018 (2010).
- ³⁷F. Fernandez-Marina, T. Estrada, and E. Blanco, *Nucl. Fusion* **54**, 072001 (2014).
- ³⁸E. Z. Gusakov and O. L. Krutkin, *Plasma Phys. Rep.* **43**(6), 605–613 (2017).
- ³⁹C. Lechte, *IEEE Trans. Plasma Sci.* **37**, 1099 (2009).
- ⁴⁰E. Z. Gusakov and A. V. Surkov, *Plasma Phys. Controlled Fusion* **46**, 1143 (2004).