The effect of low stress triaxialities and deformation paths on ductile fracture simulations of large shell structures

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\textbf{ABSTRACT}

In accidental limit state analysis of ship structures and their components, the common assumption is that failure takes place in the plate field under multi-axial tension, thus most advancements in developing fracture criteria have focused on that region. In contrast, failure in low stress triaxialities is relatively unexplored territory in the context of large-scale crash analysis. The probability of this failure mode increases with the decreasing ductility that is characteristic of high and extra high strength steels. Therefore, ductile fracture simulations are performed with large thin-walled steel structures employing four different fracture criteria that differently account low stress triaxialities and deformation history. Criteria are compared as to their capabilities to reproduce and predict experimentally measured behaviour. Analyses demonstrate that failure under lower stress triaxialities affects the response significantly especially when complex deformation history is considered. Suggestions are made for the further enhancement of fracture criteria as well as experimental configurations employed for benchmarking failure criteria.

1. Introduction

The crashworthiness analyses of large complex steel structures such as ships are currently performed with the non-linear finite element simulations. Since these simulations involve large deformations and ductile fracture, the accuracy of simulations depends on the fracture criteria used to remove elements and thereby simulate fracture propagation. When experimental results are available, accuracy of simulation is evaluated by comparing measured and simulated force-displacement curves. However, measured results are rarely available, but simulations are still used to validate analytical models \cite{1,2} and design novel collision-resistant structures with improved crashworthiness \cite{3}. Therefore, accuracy, reliability and consistency of the simulation approach, i.e., fracture criterion, play a crucial role. Comparative studies, reviews and benchmark evaluations of different fracture criteria employed in collision and grounding analysis of ship structures have been previously reported \cite{4–12}. The main challenges addressed in these studies were the uncertainties in large crash analysis, fracture strain sensitivity to element size, and how and whether to include stress (or strain) state dependence.

In engineering applications, structures are often complex and subject to different loading conditions, meaning that stress state, quantified here through the stress triaxiality parameter (or simply triaxiality) $\eta = \sigma_{0}/\sigma$, also changes depending on the loading. Accordingly, triaxiality gives the ratio of hydrostatic mean stress (relates to volume change) to equivalent von Mises stress (relates to shape change) that allows efficient distinction between loading states. Furthermore, material fracture train is commonly related with triaxiality as shown in Fig. 1 (a) for marine structural steel in Fig. 1 (b). For three-dimensional stress states the fracture locus depends...
also on the Lode angle parameter, but it is not considered here as we limit the attention to plane stress states.

Novel fracture models tailored for analysis of smaller structural components give explicit analytical expression for loci shown in Fig. 1 covering the whole stress space, e.g. see Ref. [14]. However, fracture modelling approaches employed in crash analysis of large structures tend to incorporate latest research findings with delay because the upscaling of the material behaviour from small-scale test coupons to large thin-walled structures is rarely straightforward because of the mesh size dependence, e.g., see Refs. [15–20]. Consequently, modelling approaches for large structures have been mostly concerned with fracture under multi-axial tension regime (triaxialities between uni-axial to equi-biaxial tension, 1/3 < η ≤ 2/3) or by ignoring the stress state effect completely with the utilisation of equivalent plastic failure strain criterion, see the review by Calle and Alves [5]. Moreover, omitting some characteristics of the ductile fracture process is often justified on a basis of negligible influence on the large-scale analysis results, particularly on force-displacement curve, although the effect on the fracture path and opening shape can be considerable, e.g. see Ref. [21]. The failure modes associated with the lower triaxialities (η < 1/3) such as shear and compression (see Fig. 1) have been disregarded mostly because of the latter assumption. Furthermore, failure under these modes is relegated to a secondary role by the assumption that thin-walled structures typically accommodate compression and shear loading by buckling leading to a locally tensile-dominated problem. While some criteria incorporate failure under shear dominated modes [22,23] as well as over the whole triaxiality range [24,25], they neither demonstrate the emergence of shear dominated failure modes (η < 1/3) nor show the importance of incorporating such modes. First of these issues was addressed by Atli-Veltin et al. [26] who showed that in ship collision analysis shear and compression arise in beams and stiffeners of the struck ship, but disregarded the quantitative significance of this finding.

The second issue affecting the degradation of metallic materials during large plastic deformations is the deformation history or the deformation paths. Experimental observations in the field of metal forming, e.g. Ref. [27], have shown that non-proportional deformations significantly affect the in-plane principal strains at incipient plastic instability, the major strain ε1 and minor strain ε2. Recent experimental-numerical works also suggest that fracture strain depends on the strain history [28–30]. According to Hooputra et al. [31] consideration of deformation paths is particularly important for crash applications where highly non-linear paths are expected. For these reasons, recent approaches have focused on the development of non-linear damage accumulation rules accounting deformation history in the context of necking [32,33] or fracture [19,34]. From the failure criteria tailored for coarsely meshed shell structures, only the stress based BWH criterion by Alkos et al. [22] addresses the issue explicitly, as its presentation in stress coordinates is claimed to reduce the path dependence. While in collision and grounding analysis of ship structures the path dependency has not been explicitly considered by criteria that use equivalent plastic strain Φ and triaxiality dependent failure strain, the effect of deformation history is still partially considered as the equivalent plastic strain is ever increasing function of plastic straining as noted in Ref. [33]. Moreover, the equivalent plastic strain-based criteria are commonly formulated in terms of damage accumulation rule, which as will be shown is a simple, but effective means to consider the deformation history.

The aim of this paper is twofold: firstly, we attempt to elucidate the effect of lower triaxialities in large crash analysis. To this end, the fracture is simulated in large thin-walled structures with four different fracture criteria that are distinguished by the way they handle the fracture at low triaxialities and how they account the history of deformation. The analyses are performed with three carefully selected structures for which the experimental results in terms of force-displacement are available. The sensitivity of the numerical simulations to low triaxiality is presented and analyzed. Secondly, these analyses provide insight into the deformation paths in large crash analysis and thus, we provide some clarity for the notion how various criteria handle complex paths in the context of large-scale crash analysis. The hallmark of the present investigation is the insight it provides to different fracture criteria used in coarsely meshed shell structures and to the way they handle shear deformations and deformation paths up to the point of fracture. This insight is believed to be valuable in making conscious decisions regarding future developments of fracture criteria and experimental techniques that are used in accidental limit state design. The limitation of the study is that all used fracture criteria have been developed for membrane type of loading, thus bending dominated effects are not considered explicitly.
2. Preliminaries

2.1. Damage accumulation and fracture

In crash simulations of large thin-walled structures such as ship collision and grounding, the local material rupture is modelled by element erosion technique once the fracture criterion is satisfied. The most common fracture criterion in the context of large crash simulations is that of constant equivalent fracture strain $\varepsilon_f$, whereby element is removed once accumulated equivalent plastic strain $\varepsilon$ reaches critical value, see Ref. [5].

Alternatively, the fracture strain can be stress state dependent in which case a linear damage $D$ accumulation rule is commonly introduced to model the fracture for any combination of proportional and non-proportional deformation path. Most criteria employed in this study belong to this category. Thereby, damage during deformation is defined by the weighted integral with respect to the equivalent plastic strain

$$D = \int_0^\varepsilon \frac{d\varepsilon}{\varepsilon_f(\eta)}$$

where $\varepsilon_f(\eta)$ is the fracture locus in the space of equivalent plastic strain and triaxiality. Fracture occurs when $D = 1$. In a sense, integral damage formulation is a phenomenological way of considering the history of damage, which is necessitated by the way fracture loci have been defined using experiments. Since in experiments it is difficult to attain constant triaxiality locally while the specimen is loaded all the way to fracture [35], fracture loci are constructed using the triaxiality averaged over the loading history as in Refs. [36,37]:

$$\eta_f = \frac{1}{\varepsilon_f} \int_0^\varepsilon \eta d\varepsilon$$

that is denoted as the failure averaged triaxiality in the following. For example, Fig. 2 shows the fracture locus and deformation paths for marine structural steel determined using the combined experimental-numerical approach. Non-proportional deformation is obvious. The fracture point is identified in terms of the exact $\eta$ (□ marker) and the failure averaged triaxiality $\eta$ (○ marker) whereas latter is used for the construction of fracture locus. The path for point ○ could be constructed by drawing a straight vertical line until the fracture point (dash-dot line). However, this vertical path fails to convey any non-proportionality experienced by the material point during deformation. Thus, a complimentary average triaxiality definition is adopted that is more intuitive when considering both, the fracture point and the deformation path. The integral in eq. (1) is weighted by the accumulated plastic strain $\varepsilon$ instead of the fracture strain $\varepsilon_f$ to define the cumulative average

$$\eta_c = \frac{1}{\varepsilon} \int_0^\varepsilon \eta d\varepsilon$$

This is plotted in Fig. 2 (a) with a blue dotted line. Two advantages arise from this definition: (i) it provides the same triaxiality at failure as eq. (2) and thus is consistent with the way fracture loci are constructed, and (ii) it is partially able to convey the non-proportionality of the deformation.

The non-proportionality in Fig. 2(a) arises from the through thickness stress component developing during deformation as the loading imposed on the specimen remains proportional. During an actual crash event, the loading can also be non-proportional, leading to even higher non-proportionality in deformations. An illustrative non-proportional deformation path is shown in Fig. 2(b) with dashed line. The figure shows the effect of damage accumulation rule in eq. (1) on the final fracture point. Because we also

![Fig. 2](image-url)
employ criteria without the damage accumulation rule, the corresponding fracture point is identified with △ marker. In that case, the deformation history is not considered and fracture initiates when the path intersects with the fracture locus. However, at this point the corresponding damage value in the material point is \( D = 0.6 \) implying additional deformation capacity with respect to the normalized maximum value of \( D = 1 \). The offset, or the overshoot, compared with the initial fracture locus seems significant when considering the exact triaxiality \( \epsilon \) but is less so when the average triaxiality \( \eta \) is used as in experiments. Moreover, the amount of offset could be controlled by a non-linear damage accumulation rule, e.g. as in Refs. [34,38], but is not considered in this study.

This detailed explanation of the path effects on the fracture point facilitates later explanations of the analysis results. Particularly, two types of fracture criteria are employed in this study: with and without damage accumulation. Finally, it is worthwhile to mention that in present implementation damage \( D \) does not affect the plastic deformation, or in other words, it is uncoupled from the flow rule or yield criterion and thus, acts purely as a normalized indicator of plastic deformation.

### 2.2. Constitutive relation and plane stress transformation formulation

Common to all fracture criteria and simulations in this study is that material is assumed to be isotropic and follow the \( J_2 \) von Mises plasticity and associated flow rule. The yield function is given as

\[
f = \sigma - \sigma_f(\epsilon)
\]

where \( \sigma \) is the von Mises equivalent stress under plane stress condition \( \sigma_{33} = \sigma_{13} = \sigma_{23} = \epsilon_{13} = \epsilon_{23} = 0 \). The current flow stress \( \sigma_f \) in eq. (4) depends on the equivalent plastic strain \( \epsilon \) according to the modified Swift hardening rule

\[
\sigma_f(\epsilon) = \begin{cases} \sigma_0, & \text{if } \epsilon \leq \epsilon_t, \\ K(\epsilon_t + \epsilon^n), & \text{if } \epsilon > \epsilon_t, \end{cases}
\]

where \( K \) and \( n \) are the parameters governing the work hardening, and \( \sigma_0 \) is the initial yield stress. To account for the existence of the Lüders plateau, the hardening is delayed until the plastic strain reaches the plateau strain \( \epsilon_p \), while the parameter \( \epsilon_0 = (\sigma_0/K)^{1/n} - \epsilon_t \) enforces continuity of the stress strain curve at \( \epsilon = \epsilon_0 \).

Furthermore, following quantities and the plane stress transformation formulas will be useful in this study. By assuming the incompressibility at the plastic flow and proportional deformations, it can be shown that there is a one-to-one mapping from strain to stress space. Under the proportional loading the increments of principal plastic strain follow the equation:

\[
d\epsilon_2: d\epsilon_3: d\epsilon_1 = 1: \alpha = -(1 + \alpha)
\]

Whereas principal plastic strain ratio \( \alpha = d\epsilon_2/d\epsilon_3 = \epsilon_2/\epsilon_3 \) can be further related to principal stress ratio \( \beta = \sigma_2/\sigma_1 \) by

\[
\beta = \frac{2\alpha + 1}{2 + \alpha}
\]

And inversely, \( \alpha \) could be found from \( \beta \)

\[
\alpha = \frac{2\beta - 1}{2 - \beta}
\]

Thereby, the triaxiality \( \eta \) and equivalent plastic strain \( \epsilon \), can be expressed in terms of strain or stress ratio

\[
\eta = \frac{\sigma_0}{\sigma} = \frac{1 + \beta}{3\sqrt{1 - \beta + \beta^2}} = \frac{1 + \alpha}{\sqrt{31 + \alpha + \alpha^2}}
\]

\[
\epsilon = \frac{2\epsilon_0}{\sqrt{31 + \alpha + \alpha^2}}
\]

### 2.3. Shear failure

The objective is to capture the failure with large plane-stress shell elements under low triaxialities (\( \eta < 1/3 \)). The purpose of this Section is to explain how the state of shear can develop in shells. Consequently, macroscopic viewpoint on the shear failure mechanism is adopted, which relates to the way the forces are applied to the material. Notably, in sheet metal forming shear is dominant when forming forces are parallel to the surface of the material. Therefore, two shear failure mechanisms arise that are relevant to sheet metals: in-plane shear and out-of-plane shear. These mechanisms are shown on the schematic specimen in Fig. 3. In case of in-plane shear through thickness strain remains zero and thickness remains constant. The loading mode has a direct corollary with the loading by stretch as shown in Fig. 3 (c), where difference between two cases arises only due to the chosen coordinate system. Because of the compression in the element in one direction, practical levels of deformations are limited by the possible buckling and wrinkling, Emmens [39]. Fig. 3 shows that the out-of-plane shear stress is associated with the bending deformation. As noted in the Introduction, bending dominated effects and the through thickness stress distribution are not considered explicitly meaning that the out-of-plane shear is also not further discussed.
3. Fracture criteria

3.1. RTCL damage criterion

The RTCL criterion proposed in Refs. [41] and [25] distinguishes between shear- and tension-dominated stress states using triaxiality $\eta$. The RTCL function reads

$$f_{RTCL} = \begin{cases} 
0 & \text{if } \eta < -1/3 \text{ (compression)} \\
2 + \frac{\eta}{\sqrt{12 - 27\eta}} & \text{if } -1/3 \leq \eta < 1/3 \text{ (shear)} \\
\frac{1}{1.65}\exp(1.5\eta) & \text{if } \eta \geq 1/3 \text{ (tension)} 
\end{cases}$$

(11)

The damage indicator is calculated as follows

$$D = \frac{1}{\varepsilon_{cr}} \int f_{RTCL} \, d\varepsilon \geq 1$$

(12)

where the critical strain $\varepsilon_{cr}$ is the calibration coefficient to adjust the RTCL criterion for different element size

$$\varepsilon_{cr} = n + (\varepsilon_{f,cal} - n) \frac{L_e}{L_e}$$

(13)

where $\varepsilon_{f,cal}$ is the fracture strain in uniaxial tension for a mesh size $L_e = \ell_e$, and $n$ is the hardening law exponent. Here the exponent $n$ characterises the uniform strain outside of the neck, which is assumed to be mesh size independent. To handle the mesh size sensitivity, only the local strains with high gradients inside the neck are considered element size dependent. The criterion is calibrated based on the values reported in Ref. [10] since the same experiment is employed here as a case study: $\varepsilon_{f,cal} = 0.67$ for $L_e = \ell_e = 5$ mm and $n = 0.195$. For these set of parameters the criterion is plotted in Fig. 6 for $L_e/\ell_e = 4$ (e.g., $\ell_e = 5$ mm $\rightarrow L_e = 20$ mm).

3.2. Bressan-William-Hill or BWH fracture criterion

This criterion was presented by Alsos et al. [22] who combined the theoretical necking instability model of Hill [42] with Bressan-Williams [43] shear stress criterion and transformed it into the stress space to estimate the point of instability in thin sheets. Conversion from strain to stress space was motivated by the earlier developments of Stoughton [44], who showed that for different amounts of pre-strains, path dependent effects on the forming limit reduce when viewed in stress coordinates. Though, in a more
recent study by Stoughton and Yoon [45] strain-based diagrams were used to overcome the path dependence.

Although BWH criterion provides the point of necking instability as opposed to fracture, its utilisation in crash analysis has been justified by the small amount of energy dissipated between these two stages. More compelling argument however, is the ease of calibration that can be performed by only knowing the material hardening characteristics. Therefore, BWH criterion is formulated by

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**Fig. 4.** (a) Classical Marciniak-Kuczynski (MK) model where loading is 90° with respect to imperfection band. (b) MK-model setup with periodic boundary conditions adopted from Pack and Mohr [54]. The direction of principal stress is variable in simulations while direction of imperfection (localization band) is always along the horizontal 1-direction of the unit cell. Normal (un) and shear (ut) displacement component are assigned separately to two phantom nodes (not connected to model), which govern the behaviour of all the edge nodes to impose periodicity.

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**Fig. 5.** MK-analyses results with two plate thicknesses.

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**Fig. 6.** Fracture criteria used in this study visualized in the stress and strain space. Additionally, the MMC fracture locus that is the upper bound for the 2FS and 2FS-ex criterion is shown.
assuming incompressibility at the plastic flow and proportional strain and stress paths. Thus, BWH criterion is expressed in stress space in terms of principal strain ratio \( \alpha \) as follows

\[
\sigma_1 = \begin{cases} 
\frac{2K}{\sqrt{3}} \left( \frac{1 + 0.5\alpha}{1 + \alpha} \right)^n \left( \frac{2}{\sqrt{3} + \alpha} \right)^n \sqrt{1 + \alpha + \alpha^2} & \text{if } -1 < \alpha \leq 0 \\
\frac{2K}{\sqrt{3}} \left( \frac{1}{\sqrt{1 - (\alpha/2)^2}} \right)^n & \text{if } \alpha > 0
\end{cases}
\]

(14)

It is worth noting that in present implementation of BWH criterion the strain-ratio \( \alpha \) is calculated from stress-ratio \( \beta \) according to eq. (8) similarly as in Ref. [18]. Furthermore, like Alsos et al. [46] we make calibration parameter \( \xi \) mesh size dependent using eq. (13) with \( \xi_{\text{cal}} = 2n \) due to the HillFs analysis of the equivalent plastic strain at the onset of local necking, thus \( \xi = n(1 + t/L_t) \). The BWH model is restricted to the strain ratio of \( -1 < \alpha \leq 1 \), corresponding to stress states between pure shear (\( \eta = 0 \)) and equi-biaxial tension (\( \eta = 2/3 \)). Since BWH criterion is assumed path independent, linear damage accumulation rule is not used in the implementation and failure takes place once the critical first principal stress \( \sigma_{1,cr} \) is reached.

### 3.3. Fracture strain scaling based on stress state and mesh size – 2FS criterion

Kõrgesaar et al. [47] recently showed that the mesh size dependence of the fracture strain is different at different triaxialities. The formulation which adjusts fracture strain based on the triaxiality and mesh size was provided by Walters [16]. Details of the criterion are presented in Refs. [16,17]. Herein we denote the approach as two-factor-scaling and abbreviate it as the 2FS criterion. As an input the approach requires a lower and upper bound. The lower bound is applicable to large shell elements and the upper bound applicable to small elements in local scale. In between the fracture strain is adjusted according to eq.(13) with

\[
\begin{align*}
\tilde{\eta} &= \left\{ \frac{K}{C_2f_1}\left[ \frac{1 + C_1^2}{3}f_1 + C_1\left( \hat{\eta} + \frac{f_2}{3} \right) \right] \right\}^{-1/MMC} \\
f_1 &= \cos\left( \frac{1}{3} \arcsin\left[ \frac{-27}{2} \eta^3 - \frac{1}{3} \right] \right) \\
f_2 &= \sin\left( \frac{1}{3} \arcsin\left[ \frac{-27}{2} \eta^3 - \frac{1}{3} \right] \right) \\
f_3 &= C_3 + \frac{1}{2 - \sqrt{3}}(1 - C_3) + \left( \frac{1}{f_1} - 1 \right)
\end{align*}
\]

(15a-15d)

where \( K_{\text{MMC}}, n_{\text{MMC}}, C_1, C_2 \) and \( C_3 \) are the material constants that must be calibrated. Ideally, these should be calibrated with three fracture tests. Alternatively, one could calibrate this curve by recognizing the most important experimental trends, which indicate that the fracture ductility is lowest in plane strain tension and approximately equal in uniaxial and equi-biaxial tension. The latter approach is advocated by Woelke and Abboud [50] and Voormeeren et al. [51] as a cost-efficient and viable alternative to expensive and time consuming experimental testing, thus this approach is employed herein. For one type of marine structural steel fracture locus in \( \tilde{\eta} - \eta \) space was calibrated in Ref. [17] based on the uniaxial tension test data. Since the case study structures described in the next Section are built from similar steel, we calibrate the MMC criterion (\( K_{\text{MMC}}, n_{\text{MMC}}, C_1, C_2 \) and \( C_3 \)) using the same approach: fracture strain under plane strain (\( \eta = 1/\sqrt{3} \)) is 40% lower than under uniaxial tension (\( \eta = 1/3 \)). Furthermore, to be consistent with the RTCL criterion described in the previous section, we constrain the MMC locus to pass the point \( \xi_{\text{cal}} = 0.67 \) that characterized the fracture strain of elements with \( L_e = L_t = 5 \text{ mm} \) and was used for the RTCL criterion calibration. Based on these constraints, the following material constants were selected: \( C_1 = 0.205, C_2 = 385 \) and \( C_3 = 0.972, K_{\text{MMC}} = 680 \text{ MPa}, n_{\text{MMC}} = 0.205 \). The fracture strain is scaled by distinguishing between uniform and localized strains as in eq. (13), yet the terms are made triaxiality dependent:

\[
\tilde{\eta}(\eta, t/L_t) = \eta_n(\eta) + (\xi_{\text{cal}}(\eta) - \eta_n(\eta)) \frac{L_e}{L_t}
\]

(16)

where \( \xi_{\text{cal}}(\eta) \) is the calibration function depending on the upper bound \( \xi_{\text{MMC}} \) and lower bound \( \eta_n \) as follows

\[
\xi_{\text{cal}}(\eta) = \eta_n(\eta) + (\xi_{\text{MMC}}(\eta) - \eta_n(\eta)) \frac{L_e}{L_t}
\]

(17)

Since we have \( L_e/L_t = 5/5 = 1 \), calibration function reduces to
\[ \varepsilon_{\text{f,cal}}(\eta) = \varepsilon_{f,\text{MMC}}(\eta) \]

and eq. (19) becomes:

\[ \varepsilon_f(\eta, t_i/L_o) = \varepsilon_f(\eta) + (\varepsilon_{f,\text{MMC}}(\eta) - \varepsilon_f(\eta)) \frac{t_i}{L_o} \]  \hspace{1cm} (18)

The lower bound \( \varepsilon_f(\eta) \) is the equivalent necking strain calculated from the Swift principal major and minor strain according to eq. (10). It is worth noting that \( \varepsilon_f(\eta) \) is only defined for the range \( 1/3 \leq \eta \leq 2/3 \), i.e., multi-axial tension, that limits the applicability of the 2FS criterion to these stress states. Therefore, for the 2FS criterion the triaxiality cut-off value equal to 1/3 is applied, below which the fracture does not occur. The fracture is modelled using damage rule by inserting eq. (18) into eq. (1).

### 3.4. Extension of the 2FS criterion for lower triaxialities — 2FS-ex

The objective of this section is to extend the applicability of the 2FS criterion to lower triaxialities. This requires definition of fracture strain (upper bound) as well as instability strain (lower bound) as a function of triaxiality. Fracture strain for local material elements is conveniently given by MMC fracture locus for the whole triaxiality range. However, Swift instability is defined only for the region \( \eta \geq 1/3 \). Therefore, we employ the theoretical Marciniak-Kuczynski [52] imperfection analyses to provide qualitative insight into the localization instability under lower triaxialities.

#### 3.4.1. MK-analyses

In the classical MK-model an initial thickness imperfection is introduced in the form of a narrow groove and loading is imposed so that principal stress is oriented 90° to the groove orientation, see Fig. 4 (a). According to Aretz [53] however, this does not allow realistic predictions of instability for region of negative minor strain or alternatively \( \alpha < 0 \). To overcome this deficiency of the original MK-model and facilitate variations in the \( \sigma_1 \) direction with respect to groove, we adopt the MK-model setup presented in Ref. [54]. Therein, proportional loading is imposed to unit cell via periodic boundary conditions. The MK-analysis will be performed using finite element simulation with plane stress shell elements (S4R in Abaqus), see Fig. 4 (b). A square unit cell (20 x 20 mm², approximately the same size as employed in the crash analysis presented in the next Section) representing a large shell element is discretized using S4R shell elements with an edge length of 0.5 mm. An initial through-thickness imperfection of \( f_0 = 2\% \) is introduced to the central row of elements

\[ f_0 = 1 - \frac{t_{\text{groove}}}{t_{\text{out}}} \]  \hspace{1cm} (19)

with \( t_{\text{groove}} \) and \( t_{\text{out}} \) denoting the initial thicknesses of the sheet inside and outside of the central groove, respectively. Periodic boundary conditions are imposed on the edges of the unit cell employing macroscopic deformation gradient tensor as in Ref. [54].

\[
\mathbf{F} = \begin{bmatrix} u_{ij}/L + 1 & u_{ij}/L \\ u_{ij}/L & u_{ij}/L + 1 \end{bmatrix} = \begin{bmatrix} \lambda_1 \cos^2 \phi + \lambda_2 \sin^2 \phi & (\lambda_1 - \lambda_2) \sin \phi \cos \phi \\ (\lambda_1 - \lambda_2) \sin \phi \cos \phi & \lambda_1 \sin^2 \phi + \lambda_2 \cos^2 \phi \end{bmatrix}
\]  \hspace{1cm} (20)

where \( \phi \) is the inclination of the \( \sigma_1 \) with respect to groove, and \( \lambda_1 \) and \( \lambda_2 \) are the principal stretch histories in the direction 1 and 2, respectively. For a constant equivalent plastic strain rate \( \dot{\varepsilon} \), \( \lambda_1 \) is expressed from eq. (10) after substituting \( \varepsilon = \ln(\lambda_1) \), while \( \lambda_2 \) is expressed using strain ratio \( \alpha \):

\[
\lambda_1[i] = \exp\left( \frac{\dot{\varepsilon} i - \sqrt{3}}{2\sqrt{1 + \alpha + \alpha^2}} \right) \]  \hspace{1cm} (21a)

\[
\lambda_2[i] = \exp\left( \frac{\dot{\varepsilon} i - \alpha \sqrt{3}}{2\sqrt{1 + \alpha + \alpha^2}} \right) \]  \hspace{1cm} (21b)

where \( i \) denotes time. Thereby, the stress state in the unit cell is controlled by the strain ratio \( \alpha \) eq. (10) and direction of \( \sigma_1 \) is controlled by the inclination angle \( \phi \). These periodic boundary conditions were implemented through user defined loading functionality in Abaqus/Standard where components of \( \mathbf{F} \) were defined using \( \alpha \) and \( \phi \) as input variables (Supplementary material). Normal and shear displacement components defined in subroutine were separately imposed on two phantom nodes located in the origin, which were in turn controlling the displacement degrees of freedom of all the unit cell edge nodes by linear constraint equations.

The analyses are performed with Abaqus/Standard 6.133 for a given triaxiality \( \eta \) and principal orientations of \( \phi = 45°...90° \) with a step of 5°. Analyses are performed with two plate thicknesses to convey the effect of thickness on localization limit, namely 3 mm and 5 mm. In computations, the onset of localization is defined by the criterion

\[
\left| \frac{d\eta}{dt} \right| > 1 \]  \hspace{1cm} (22)

Material parameters used in the simulations are characteristic to normal strength shipbuilding steel taken from Ref. [13], see Table 1.
3.4.2. Results of the MK-analyses

Fig. 5 shows the history of deformation for one shell element outside of the groove for each unit cell analysis conducted. Constant triaxiality until the limit defined in eq. (22) confirms that loading is proportional. Nearly identical response was obtained when different element was selected outside of the groove, e.g. element closer to the band or neighbouring the groove. Comparison of Fig. 5 (a) and (b) shows that in the multi-axial tensile regime ($\eta > 1/3$) thickness has negligible effect on the response. Moreover, for these triaxiality results are consistent with analysis of Pack and Mohr [54], who also performed analysis in the range of $\eta > 1/3$.

We proceed to analyze the localization under lower triaxialities ($\eta < 1/3$). Surprisingly, with initial decrease of triaxiality from 1/3 localization strain increases. When triaxiality is further decreased, localization strain shows notable thickness dependence. For example, increase of sheet thickness considerably postpones localization (Fig. 6 b) compared with thinner 3 mm sheet (Fig. 6 a). Nevertheless, both cases exhibit distinctive drop in stability in the region of pure shear ($\eta \approx 0$). These results show that the shear-dominated deformation mechanisms are particularly sensitive to the $L_t/t_e$ ratio, i.e. the mesh size. This increased mesh sensitivity is not explicitly considered in the current study, but should be considered in future investigations.

3.4.3. Extension of 2FS criterion – 2FS-ex criterion

Fig. 5 provides qualitative insight into the necking instability, as opposed to fracture, under low triaxialities. Nevertheless, the necking strain provides useful information about macroscopic fracture behaviour of sheet metal because, once the necking instability sets in, essentially no further strains occur outside of the localization zone, [55]. On that basis, we introduce a simple ad-hoc extension to 2FS criterion to enable fracture under low triaxialities ($\eta < 1/3$). Namely, fracture strain in the range of uniaxial tension ($\eta = 1/3$) to compression ($\eta = -1/3$) is assumed to be equal with the fracture strain in uniaxial tension. This approximation complies well with results of 5 mm sheet thickness (Fig. 5 b), while is slightly non-conservative for 3 mm sheet (Fig. 5 a). On the other hand, it is considerably more conservative than provided by the rest of the criteria as shown in Fig. 6. A cut-off triaxiality of $\eta = -1/3$ was selected based on the results of Bai et al. [56], who showed that fracture never occurs below $\eta = -1/3$. Consequently, this criterion gives highest probability of failure for lower triaxialities. Moreover, criteria can be distinguished by the way they handle fracture under lower triaxialities, by which we can demonstrate the significance of different stress ranges on the analysis results. Fracture is again modelled through damage indicator.

All failure criteria used in this study are illustrated in Fig. 6, where the curves are calibrated for $L_t/t_e = 4$ (e.g., $t_e = 5 \text{ mm} \rightarrow L_e = 20 \text{ mm}$). For purpose of illustration, the MMC criterion is shown. Notice how RTCL, 2FS and 2FS-ex, provide the same fracture strain at uniaxial tension ($\eta = 1/3$). This is because strain at uniaxial tension was equal in calibration of both criteria ($\epsilon_m = 0.67$), underlying scaling equation is the same, eq. (13), and the necking strain $n$ in eq. (13) is the same for both cases at uniaxial tension. It is worth noting that all criteria consider only the material mesh scaling that accounts the strain concentration due to neck development, but neglect geometric mesh scaling that accounts the strain concentration from local bending near stiffeners and frames. To the best of author knowledge the geometric mesh scaling is accounted only by the modified BW criterion presented in Ref. [18].

4. Numerical simulations with case study structures

The explicit time integration solver Abaqus/Explicit v.6.133 has been used to model fracture in case study structures. In all models, quadrilateral four-node reduced integration S4R (Abaqus library) shell elements with 3 integration point and default hourglass control were used. Contact between different objects was modelled with general contact algorithm by defining rigid objects as masters. Contact definition included model for tangential and normal behaviour as well as contact damping. Contact damping was defined through unitless damping coefficient in terms of the fraction of critical damping associated with the contact stiffness; the value of 0.5 was used. Tangential behaviour between surfaces was modelled with penalty type friction formulation. Friction coefficient was de-pendent on the case study and is reported below. Drilling stiffness was turned off in all simulations using keyword *Section controls.

4.1. Numerical simulations with double hull side structures

4.1.1. Large-scale collision experiments

The first case study reliably mimics the actual collision scenario by two vessels; FE model set-up is shown in Fig. 7 and details can be found in Ref. [57]. The experiments with the large-scale conventional double hull ship structure were performed in Netherlands by TNO during the period of 1997–1998 as part of the larger experimental program. During the experiment a 762 tons inland waterway tanker struck another 1442 tons inland waterway tanker at an approximate collision angle of 90° with a speed around 2.5 m/s. The striking vessel was fitted with a rigid bulb filled with concrete, and the struck vessel had a total of four different deformable side panels representing the conventional double hull arrangement. The test section was dimensioned in an approximate scale of 1:3 to a typical full-scale ship structure. The test section fitted with the load measuring devices was mounted with a set of clamps to the target
vessel. The loads and deformations measured during the experiment will be used to validate the simulations.

Because the collision experiment was performed under floating conditions without laboratory control of the boundary conditions, the exact impact location and impact angle are uncertain. The approximate incident location shown in Fig. 7(a) is chosen according to description given in Ref. [57]. The details of the rigid indenter itself are shown in Fig. 7(b). The modelled test section was fixed by constraining all the nodal degrees of freedom in the minimum and maximum x-coordinate. The incident location was meshed with 20 mm elements, while rest of the model had 40 mm elements. The simulation was performed for a constant velocity of the rigid bulb of 6 m/s. Material properties used in simulation are shown in Table 2. The static friction coefficient in simulations was 0.3.

4.1.2. Indentation experiments with double hull side structure – TUHH

These experiments were also performed with double hull side structure with approximate scale of 1–3 with respect to actual ship structure, Tautz et al. [58]. In contrast to TNO experiments, tests were performed in the controlled laboratory environment meaning that response includes only inner mechanics, excluding the ship motions. The experimental set-up is shown in Fig. 8(a) and details of the indenter are given in Fig. 8(b). At the boundary, outside of the modelled test section steel plates with a thickness of 20 mm were utilized, creating a nearly rigid support frame around the panel. Therefore, support structure was not modelled, and all nodal degrees of freedom were constrained on the periphery. Mesh size in the model was 25 mm. The static friction coefficient was assumed equal to 0.23, similar to the assumptions in Ref. [58]. The simulation was performed for a constant velocity of the rigid bulb of 7 m/s. To reduce the computational time, mass of the entire model was scaled in the beginning of the analysis by a factor of 2. With these settings kinetic energy was less than 5% of the total internal energy. Material properties used in simulation are shown in Table 2. The static friction coefficient in simulations was 0.3.

4.1.3. Indentation experiments with double hull side structure – CHALMERS

These experiments with bulb impacting the scaled double side are reported by Karlsson et al. [59]. Therefore, the experimental set-up is like in TUHH, but the structural layout is slightly different. Besides the L-profiles stiffening the top and bottom plate, top plate additionally includes a T-beam running across stiffeners, see Fig. 9(b). In real structure L-shape holes were cut to vertical plates and T-beam webs, around which L-profiles were welded. In FE model, these holes were not modelled and the rest of the FE model set-up is like in Ref. [59]. The FE model included the reinforcing frame as shown in Fig. 9(a). This frame was made of the same material and thickness as the side-shell structure and thus, was modelled using the same material properties. The reinforcing frame was welded around the test object along its edges to create clamped boundary conditions. The lower part of the frame was welded to rigid fixture that was represented in FE model by fixing the outer edges of the support frame lower beams as shown in Fig. 9(a). Model was discretized with mesh of 30 mm, which was further refined at the impact location to 10 mm. The test object was manufactured from

Table 2
Material parameters used in the TNO simulation.

<table>
<thead>
<tr>
<th>$\sigma_0$ (MPa)</th>
<th>K (MPa)</th>
<th>n</th>
<th>$\varepsilon_L$</th>
<th>E (GPa)</th>
<th>Poisson's ratio</th>
<th>Density (kg/m$^3$)</th>
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<td>284</td>
<td>680</td>
<td>0.205</td>
<td>0.0025</td>
<td>206</td>
<td>0.3</td>
<td>7850</td>
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</table>

Fig. 7. Finite element model set-up showing the large-scale experiment. Half of the outer shell removed to show the framing system. Double bottom side girder is also removed in the figure to show the flat bar stiffeners in the bottom structure. (b) Details of the indenter as adapted from Ref. [57].
Fig. 8. (a) General view of the FE model of the TUHH indentation experiments. Elements not shown on top and bottom shell. (b) Dimensions of the indenter.

Table 3
Material parameters for TUHH test as given by Tautz et al. [58].

<table>
<thead>
<tr>
<th>Material</th>
<th>( \sigma_0 ) (MPa)</th>
<th>( K ) (MPa)</th>
<th>( n )</th>
<th>( \varepsilon_L )</th>
<th>( E ) (GPa)</th>
<th>Poisson’s ratio</th>
<th>Density (kg/m(^3))</th>
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<td>0.167</td>
<td>0.015</td>
<td>204</td>
<td>0.3</td>
<td>7850</td>
</tr>
<tr>
<td>Web plate</td>
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<td>728</td>
<td>0.158</td>
<td>0.01</td>
<td>196.5</td>
<td>0.3</td>
<td>7850</td>
</tr>
<tr>
<td>HP profile</td>
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<td>755</td>
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<td>0.01</td>
<td>211</td>
<td>0.3</td>
<td>7850</td>
</tr>
</tbody>
</table>

Fig. 9. FE model set-up of the Chalmers experiments. (a) Reinforcing frame with fixed boundary highlighted. (b) Test structure. (c) Stress-strain relationship for the NVA shipbuilding mild steel and the fitted curve used in simulations.

Table 4
Material parameters used in the Chalmers simulations.

<table>
<thead>
<tr>
<th>( \sigma_0 ) (MPa)</th>
<th>( K ) (MPa)</th>
<th>( n )</th>
<th>( \varepsilon_L )</th>
<th>( E ) (GPa)</th>
<th>Poisson’s ratio</th>
<th>Density (kg/m(^3))</th>
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<td>0.015</td>
<td>206</td>
<td>0.3</td>
<td>7850</td>
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</tbody>
</table>
the K240-Z ship-building mild steel for which the material parameters shown in Table 4 were fitted based on the true stress-strain curve given by Ringsberg et al. [12], see Fig. 9 (c). The solid half sphere indenter from SS2541 steel with radius of 135 mm was assumed rigid in simulations. A constant velocity of 1 m/s was assigned to the rigid bulb. To reduce the computational time, mass of the entire model was scaled in the beginning of the analysis by a factor of 14. Despite this large factor, comparison with non-mass scaled solution indicated that changes in the mass and consequent increases in the inertial forces did not alter the solution accuracy nor did it increase the kinetic energy over the suggested limit value of 5% of total internal energy.

5. Results and discussion

5.1. Analysis of force versus indenter displacement

We begin the analysis of results by presenting and comparing simulated and measured force versus indenter displacement ($F-d$) along with the relative force ratio between simulated and measured force ($F_{sim}/F_{test}$), see Fig. 10. When data was available, the
distinctive points in the experimental curves are indicated. The largest scatter in F-d results is obtained in simulations of large-scale collision experiment shown in Fig. 10 (a), abbreviated in the sequel as TNO experiments. The first distinctive point in the experimental curve is the load drop at 0.2 m bulb displacement that is associated with the reduced resistance due to fracture propagation. In simulations, this load drop is well captured except in the analysis with the 2FS criterion that grossly overestimates the peak load. In the 2FS simulation fracture initiates simultaneously in the outer shell and cruciform composed of the stiffener and deck, while there is a considerable delay in those two events in all other simulations. This is further discussed in the next Section. The second load drop at 0.87 m bulb displacement is associated with the inner shell rupture. Compared with the outer shell fracture, different criteria show larger scatter in capturing this point.

In contrast, the scatter in results between different criteria is considerably reduced in simulations of TUHH experiments as shown in Fig. 10 (b). All criteria capture the experimental trend with good accuracy. The overall accuracy in capturing the CHALMERS experimental results is also good as indicated in Fig. 10 (c), but there is some scatter in capturing the initial load drop. To quantify these differences in results we proceed to analyze the point of fracture initiation obtained with different fracture criteria.

5.2. Analysis of deformations and failure modes

5.2.1. Deformation history and failure in TNO simulation

It is first instructive to describe how different criteria handle deformation under non-proportional paths. Therefore, for each analysis the deformation paths to fracture are extracted from the numerical simulations for each element that failed during the simulation. Recall that these paths describes the evolution of the equivalent plastic strain as a function of the triaxiality. Fig. 11 shows the corresponding plots for TNO simulations with each criterion. To demonstrate the difference between simulation results with and without damage accumulation, Fig. 11 (a) and (b) show the plastic strain at fracture for RTCL and BWH criterion as a function of exact \( \eta \) (□ marker) and cumulative average triaxiality \( \bar{\eta}_c \) (○ marker). Fig. 11 (a) shows that the fracture points corresponding to average triaxiality \( \bar{\eta}_c \) are located around the fracture locus. Conversely, for BWH criterion the points corresponding to the exact triaxiality \( \eta \) are near the fracture locus. This subtle, but important difference arises because the RTCL criterion uses damage accumulation while the BWH criterion does not.

Fig. 11 (a) and (b) additionally show the complete deformation path for two elements. From the curves corresponding to the exact triaxiality measure (Ω, blue curves) it is evident that deformation is highly non-proportional. This non-proportionality leads to stark contrast how fracture initiation is interpreted between approaches where damage accumulation is active (Fig. 11 a) and where it is not (Fig. 11 b). For example, the two load paths plotted in Fig. 11 (b) show that while the average triaxiality (black lines) is \( \bar{\eta}_c < 0 \),
which characterises a nearly compressive stress state, the fracture is still predicted since instantaneous triaxiality (blue curves & □ markers) is $\eta > 0.4$. This sudden change of the stress state might be caused by various reasons and can happen under minimal increase of plastic deformation as indicated by the results of elements 1&2 in Fig. 11 (b). The possible causes can be the change of contact conditions (at all levels of plastic deformation) as well as reaching the instability condition in the element, after which strains localize and the strain state thus changes drastically (at the early stage of plastic deformation). The integral damage formulation which accounts the history of deformation, although in linear manner, is clearly less sensitive to such sudden changes in loading conditions (black curves & ○ markers). Consequently, the $\varepsilon_f(\eta_f)$ points (○ markers) suggest that when damage accumulation had been used in BWH analysis, fracture would not have been predicted in many of the elements. Although the results are not shown here for the sake of brevity, the latter notion was confirmed with FE calculations where BWH criterion was formulated via damage concept, which led to a similar overestimation of the load peaks as obtained with the 2FS criterion in Fig. 10 (a). Furthermore, the non-proportional paths observed in all simulations suggest that the assumption of proportionality made in the construction of the BWH criterion using the strain ratio $\alpha$ (eqs. (6)–(8)) is highly questionable.

Comparison of Fig. 11 (c) and (d) together with F-d curve shown in Fig. 10 (a) highlights the significance of allowing elements to fail under low triaxialities. When cut-off value is set to $\eta = 1/3$ (2FS criterion) the peak load at outer and inner shell failure is overestimated in Fig. 10 (a). To explain the delay in load drop observed in 2FS F-d curve, the analysis results in Fig. 11 are distinguished by the structural component: points with lighter fill correspond to outer and inner side shell elements while points with darker fill are for the transverse webs, decks and stiffeners. Results obtained with different criteria in Fig. 11 suggest that failure of outer and inner shell structure takes place predominantly under higher triaxialities ($\eta > 0$). In contrast, the elements of webs, decks and stiffeners fail under wider range of triaxialities and compared with the shell plating, demonstrate a clearly increased tendency to fracture under the lower triaxialities. This finding is consistent with results of Atli-Veltin et al. [26].

Postprocess of TNO analysis results in Fig. 12 at 0.168 m bulb displacement shows that fracture in all simulations except 2FS initiated from the cruciform composed of the stiffener and deck. By shutting off fracture at lower triaxialities as with 2FS criterion the fracture is considerably delayed in the cruciform and occurs simultaneously with the side shell as late as 0.3 m bulb displacement, see Fig. 12 (a). In other words, the cruciform that is not allowed to fracture under shear and compression provides additional support to the outer shell as well. Notably, this leads to steep load drop in the F-d curve.

5.2.2. Deformation and failure in TUHH and CHALMERS simulations

By observing the plot of plastic strain at fracture as a function of triaxiality in Fig. 13, the contrasting feature of TUHH and CHALMERS simulations compared with TNO simulations is that overall number of elements that fail outside of the multi-axial tension range ($\eta < 1/3$) is considerably reduced. This is particularly true in the case of TUHH simulations (red markers) where indenter hits exactly between the stiffeners. As shown in Fig. 10 (b), in TUHH simulations fracture initiates in the top plate and then in the stiffening members. Fig. 13 shows that plate failure occurs under multi-axial tension. At these triaxialities differences between fracture criteria are small, which also explains the considerably reduced scatter in F-d response in Fig. 10 (b) compared with the TNO simulations.
Compared with the TUHH simulations, simulated CHALMERS F-d curves in Fig. 10 (c) display larger deviation at the first load peak, while at second peak these deviations are considerably reduced. The contrasting response in first load peak prediction between the criteria arise because of the shift in fracture initiation from the plate to the stiffening member. In the CHALMERS model there is a T-beam directly under the indenter that provides stiffness to the structure. The T-beam and the connected stiffeners are prone to failure under lower triaxialities as suggested by Fig. 13 (light & dark blue markers). To further confirm this, we took snapshots from the CHALMERS simulation with the 2FS-ex criterion shown in Fig. 15 (a), which reveals that the fracture initiates in the T-beam web at the intersection with the stiffener, leading to eventual rupture of the T-beam. The stress state prior to fracture corresponds to shear. However, the early rupture of stiffening member in case of 2FS-ex does not lead to an immediate load drop as the membrane forces

Fig. 13. Equivalent plastic strain plotted as a function of triaxiality for all elements that failed during Chalmers and TUHH simulations. Markers with lighter fill correspond to top and bottom plate. Notice again that for BWH analysis in (b) we have used the exact triaxiality ($\eta$), while in rest of the plots average triaxiality ($\eta_c$) is used.

Fig. 14. Mesh size effect in Chalmers simulations. (a) 2FS and RTCL criterion. (b) 2FS-ex and BWH criterion.
dominate the response. Moreover, the failure in a cruciform formed by the T-beam and stiffener lends itself to softer F-d response after rupturing of the top plate.

To confirm that these differences in the first load peak between criteria are not only because of the different mesh resolution used in CHALMERS model (10 mm mesh) compared with TUHH model (25 mm), additional simulation runs were performed with CHALMERS models with 20 mm mesh. Both F-d curves (10 and 20 mm) are shown in Fig. 14. The differences between criteria are independent of the mesh resolution.

The through thickness stress triaxiality history in Fig. 15(b) and (c) additionally indicates to the challenge associated with the fracture prediction based only on the membrane considerations. In ABAQUS/Explicit the element is removed once all the through thickness integration points satisfy the fracture criterion. The element in the middle of the folding T-beam web exhibits failure in the tensile side, but element is still retained in the calculation as on the compressive side fracture criterion is not satisfied. In contrast, close to the stiffener through thickness behaviour is more uniform and the average triaxiality corresponds to shear prior to the fracture initiation (Fig. 15a). Future studies should address the issue of stress gradients through thickness to reveal potential implications on the response prediction.

Fig. 16 shows how the mesh size affected the fracture initiation in CHALMERS simulations. The important point emphasized is the location of fracture initiation depending on the mesh resolution. In 10 mm simulations with 2FS and RTCL criteria initiation takes place in stiffening members, while with 20 mm mesh fracture initiates in the plate. In BWH simulation with 10 mm mesh fracture initiated simultaneously in plate and stiffeners, while in 20 mm model in the plate. Only criteria that consistently predicts fracture initiation in stiffening members is 2FS-ex criterion. Furthermore, this example shows the importance of fracture strain definition in lower stress triaxiality range and how mesh resolution can affect the stress state experienced by structural members. To illustrate the last point, MatLab image processing toolbox was employed to obtain normalized difference in stress triaxiality between coarse and fine mesh, see Fig. 17. Figure exposes how variations in stress state are larger in regions that exhibit more complex deformation process like bending and folding. Notice also that the differences are insignificant in top plate that is under bi-axial tension. In other words, coarse mesh constrains the deformations and thus also limits the developing triaxiality differently depending on the deformation mode.

6. Concluding summary

A numerical study has been conducted to show the effect of low stress triaxialities and complex deformation paths on the fracture prediction in large scale crash analysis. Three different structural configurations reminiscent of actual ship structures were used in the
simulations. The structural models were discretized to cover the element sizes ranging from 2-5 times the plate thickness. Fracture was modelled using four fracture criteria, which differ by the way they handle fracture initiation under low triaxialities and how they account the deformation history: RTCL, BWH, 2FS and 2FS-ex. The latter is the 2FS criterion extended to accommodate triaxialities in the range of $-1/3$ to $1/3$.

After coarse calibration, all criteria were capable of handling fracture successfully under tensile dominated loading, but led to mixed results when fracture initiated in the stiffening members. Analyses of the stress state in the stiffening members revealed that

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**Fig. 16.** Fracture initiation in Chalmers simulations with 10 and 20 mm mesh. Contours show equivalent plastic strain.

**Fig. 17.** Stress triaxiality in CHALMERS simulation with 2FS-ex criterion depending on the mesh size. Top figures show the simulated deformations (stress triaxiality contours). Bottom figures show the normalized difference between the two top figures and thus convey the variations in triaxiality between 10 and 20 mesh solutions.
these members showed increased tendency to fracture under low stress triaxialities (shear stress) like in Ref. [26]. Evidently, the scatter in simulated force-displacement predictions between different criteria increased when fracture in stiffening members preceded fracture in the plate, or the plate fracture was accompanied by the stiffening member fracture. Based on this, an important suggestion for future experimental studies in the field of large-scale crash analysis can be made. The structural configurations should be designed so that the plate fracture is accompanied by the significant fracture in the stiffening members. This will challenge the corresponding numerical simulations to predict the fracture over wide range of triaxialities. This is important considering that the maritime industry is facing similar legal regulations regarding emission requirements as the car industry [60], and as noted by Lillemäe et al. [61] consequential inevitable shift towards the light-weight high strength steel structures. Thin-walled high strength steel structures however are more prone to fracture under shear. Furthermore, unit cell instability analyses showed that instability point under shear is particularly sensitive to the element size variations. As a result, elements are more prone to buckling and bending making the prediction of fracture point more difficult. This sensitivity to element size was exemplified with the grillage simulations (Chalmers). RTCL, BWH and 2FS criteria exhibited shift in fracture initiation point from stiffening member to plate member when element size increased from 10 to 20 mm.

The analyses performed further confirmed the existence of highly non-proportional deformation paths present in the crash analyses. The way deformation history is accounted in the fracture prediction significantly affects the outcome of the simulation, especially when the fracture strain strongly varies with the stress state and stress states experienced by the material point cover the full spectrum of compression to multi-axial tension. This was shown by simulations with two criteria that provided very similar fracture locus, but one accounted damage history (2FS) and the other did not (BWH). Furthermore, neither criteria gave failure under low stress triaxiality (around zero), although these states were shown to prevail in stiffening members. Despite the inability to predict fracture under shear, BWH accurately predicted the force-displacement response. The 2FS criterion on the other hand significantly overestimated the load and displacement to fracture initiation. Without the insight to the actual damage history, BWH results would give a false indication on the criterion abilities to predict fracture in stiffening members that experienced shear and compression. The underlying reason why fracture was predicted earlier compared with the 2FS criterion was that the deformation paths were highly non-proportional. For instance, it was shown that while the average triaxiality was low (outside of the limits where fracture is given by BWH) the sudden jump in stress triaxiality towards the tensile region caused the immediate compliance with the BWH criterion and failure of the element. With the 2FS criterion such scenario was impossible as history of the damage was accounted.

Therefore, the use of stress state dependent fracture criteria coupled with the damage accumulation rule is advocated. This damage accumulation rule accounts the deformation history, although in a simplified manner. Future investigations should strive for explaining the effects of highly non-proportional loading on the fracture initiation in the setting of crash analysis with large thin-walled structures. This entails also considerations of the through thickness strain gradients when loading is bending rather than membrane dominated. Another challenge related to bending is related to mesh size. In particular, analysis of the same structure with different mesh size showed that elements under biaxial tension experienced similar stress triaxiality independent of mesh size, while elements that belonged to stiffening members and experienced bending as well as shear showed larger differences in triaxiality when mesh size was varied. These effects should be considered in next generation criteria.

Acknowledgements

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Appendix A. Supplementary data

Supplementary data related to this article can be found at https://doi.org/10.1016/j.marstruc.2018.08.004.

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[8] Hogström P, Ringsberg JW, Johnson E. Survivability analysis of a struck ship with damage opening through Optimization of Strain Paths is gratefully acknowledge. Author would also like to thank CSC-JT Centre for Science Ltd. for providing Abaqus package license. Appreciation goes to Prof. Jani Romanoff, Prof. Heikki Remes and Pekka Palokangas for valuable comments and suggestions. The author also gratefully acknowledges Dr. Keunhwan Pack regarding MK analysis setup in Abaqus.

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References


