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**Modelling anisotropy in non-oriented electrical steel sheet
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Modelling anisotropy in non-oriented electrical steel sheet using vector Jiles-Atherton model

Abstract

Purpose – Non-oriented electrical steel presents anisotropic behaviour. Modelling such anisotropic behaviour has become a necessity for accurate design of electrical machines. The main aim of the study is to model the magnetic anisotropy in the non-oriented electrical steel sheet of grade M400-50A using a phenomenological hysteresis model.

Design/methodology/approach – The well-known phenomenological vector Jiles-Atherton hysteresis model is modified in order to correctly model the typical anisotropic behaviour of the non-oriented steel sheet, which is not described correctly by the original vector Jiles-Atherton model. The modification to the vector model is implemented through the anhysteretic magnetization. Instead of the commonly used classical Langevin function, we introduced 2-D bi-cubic spline to represent the anhysteretic magnetization for modelling the magnetic anisotropy.

Findings – The proposed model is found to yield good agreement with the measurement data. Comparisons are done between the original vector model and the proposed model. Another comparison is also made between the results obtained considering two different modification to the anhysteretic magnetization.

Originality/value – The paper presents an original method to model the anhysteretic magnetization based on projections of the anhysteretic magnetization in principal axis, and apply such modification to the vector Jiles-Atherton model to account for the magnetic anisotropy. The replacement of the classical Langevin function with the spline resulted in better fitting. The proposed model could be used in the numerical analysis of magnetic field in an electrical application.

Keywords Magnetic anisotropy, Rotational magnetic field, Vector hysteresis model

Paper type Research paper

1. Introduction

Non-oriented (NO) electrical steel sheets have been found to be magnetically anisotropic. Although their anisotropic behaviour is mitigated compared with the grain oriented (GO) electrical steel sheets, accurate model for the local magnetic quantities improve the prediction of the global features for electrical applications. Among diverse methods for predicting locally the isotropic hysteresis, the mathematical Preisach model and the phenomenological Jiles-Atherton (J-A) model received a growing attention these last decades. In order to account for the anisotropic behaviour of electrical steel sheets, the improvement of the Preisach model can be performed either with a phenomenological bistroide model (Vernescu-Spornic *et al.*, 2000) or with a generalized Mayergoyz model (Mayergoyz, 2003; Dlala, 2011; Handgruber *et al.*, 2015). Although the former presents some discrepancies at low amplitude of the flux density under rotational field, the diverse improvements of the latter can accurately represent both alternating and rotating dissipative behaviour, including the excess field, of the ferromagnetic materials. In comparison with the Preisach model, the J-A model (Jiles and Atherton, 1983; Jiles and Atherton, 1984) seems interesting in terms of simplicity and computational efficiency (Benabou *et al.*, 2003). Moreover, both the alternating and rotating aspects could be modelled (Ivanyi, 2000). The implementation of the J-A model in finite element method (Gyselinck *et al.*, 2004; Benabou *et al.*, 2008) can also account for the excess field (Sadowski *et al.*, 2002).

In order to account for the anisotropic behaviour, Bergqvist (1996) suggests to replace the scalar parameters of the J-A model by tensor parameters in the differential equation representing the displacement of the walls. In spite of improving the prediction in the rolling and the transverse direction of the lamination, discrepancies are observed for the rotational loci. Leite *et al.* (2004b) extend this improvement by considering anisotropic susceptibility for the anhysteretic magnetization defined by two Langevin functions. In spite of neglecting the cross-coupling terms in the differential anhysteretic susceptibility tensor, this improved vector J-A model can correctly reproduce both the alternating loops and the rotating loci. Furthermore, this model will be denoted the original anisotropic J-A model. Albeit Li *et al.* (2011) extend this improvement by including the off-diagonal terms in the differential anhysteretic susceptibility tensor, every parameter of the model is tabulated with respect to the amplitude and the orientation of the magnetic flux density which significantly affects the simplicity of this anisotropic hysteretic model.

Although, the Langevin function holds a physical meaning for representing the anhysteretic behaviour of an idealized ferromagnetic material, the cavities and impurities within the magnetic domains can affect its accuracy especially if the alloy contains some porosity (Néel, 1948). For GO material, Ramesh and Jiles (1996) accounts for the uniaxial anisotropy with a Boltzmann distribution. Their model requires a numerical integration which significantly affect the computational effort. Modelling the anisotropic behaviour for the anhysteretic magnetic curve could be performed with the coenergy or the energy density. Depending on the formulation of the magnetic problem, the coenergy density could be employed with magnetic scalar formulations (Péra *et al.*, 1993) whereas the energy density is more adapted for magnetic vector formulations (Bíró *et al.*, 2010; Chwastek, 2013; Martin *et al.*, 2015). The contours of

the coenergy/energy density can be modelled with a modified ellipse for improving the accuracy of the anisotropic anhysteretic model. Although, this approach can be easy to implement, the interpolation of every components of the field with respect to all the components of the magnetization can significantly improve the accuracy of the anhysteretic curve (Enokizono and Soda, 1995; Martin *et al.*, 2016).

The aim of this paper is to present a further improvement of the original anisotropic vector J-A model for modelling NO electrical steel sheets. The modification consists in replacing the Langevin anhysteretic curves by the interpolation of every components of the anhysteretic magnetization with respect to all the components of the effective field. Hence, only three diagonal tensors are identified from alternating flux density measurements at 50 Hz in principal directions (Goričan *et al.*, 2000; Handgruber *et al.*, 2015). The proposed improvement is then compared with both the anisotropic J-A model and the rotating flux density measurements.

2. Methodology

2.1 Vector Jiles-Atherton Model

The vector generalization of the original scalar J-A model has been introduced by Bergqvist (1996). The change in magnetization ($\mathbf{M}$) following any change in the magnetic flux density ($\mathbf{B}$) is given by the differential equation (Leite *et al.*, 2004b):

i) If $(\chi_f \cdot d\mathbf{H}_e) > 0$

$$d\mathbf{M} = \frac{1}{\mu_0} \left[\mathbf{I} + \chi_f |\chi_f|^{-1} \cdot \chi_f \cdot (\mathbf{I} - \underline{\underline{\alpha}}) + \underline{\underline{c}} \cdot \underline{\underline{\xi}} \cdot (\mathbf{I} - \underline{\underline{\alpha}}) \right]^{-1} \cdot \left[\chi_f |\chi_f|^{-1} \cdot \chi_f + \underline{\underline{c}} \cdot \underline{\underline{\xi}} \right] d\mathbf{B} \quad (1)$$

ii) If $(\chi_f \cdot d\mathbf{H}_e) \leq 0$

$$d\mathbf{M} = \frac{1}{\mu_0} \left[\mathbf{I} + \underline{\underline{c}} \cdot \underline{\underline{\xi}} \cdot (\mathbf{I} - \underline{\underline{\alpha}}) \right]^{-1} \cdot \left[\underline{\underline{c}} \cdot \underline{\underline{\xi}} \right] d\mathbf{B} \quad (2)$$

where $\chi_f = k^{-1} \cdot (\mathbf{M}_{an} - \mathbf{M})$ is an auxiliary vector quantity. $\underline{\underline{c}}$, k and $\underline{\underline{\alpha}}$ are model parameters, $\underline{\underline{\xi}}$ is the differential anhysteretic susceptibility, $\mathbf{I}$ represents the unit matrix, and $\mathbf{M}_{an}$ represents the anhysteretic magnetization.

In the vector J-A model the anhysteretic magnetization vector is a function of the effective field, $(\mathbf{H}_e = \mathbf{H} + \underline{\underline{\alpha}} \cdot \mathbf{M})$, and is assumed to be parallel with it (Gyselinck *et al.*, 2004; Leite *et al.*, 2004b).

$$\mathbf{M}_{an} = M_{anx} \mathbf{i} + M_{any} \mathbf{j} = M_{an} \left(\frac{\mathbf{H}_e}{|\mathbf{H}_e|} \right) \quad (3)$$

$$\underline{\underline{\xi}} = \begin{pmatrix} \frac{\partial M_{anx}}{\partial H_{ex}} & \frac{\partial M_{anx}}{\partial H_{ey}} \\ \frac{\partial M_{any}}{\partial H_{ex}} & \frac{\partial M_{any}}{\partial H_{ey}} \end{pmatrix} \quad (4)$$

Given any sigmoid function to express the anhysteretic magnetization, the elements of (4) can be expressed as:

$$\frac{\partial M_{anx}}{\partial H_{ex}} = \dot{M}_{anx} \left(\frac{\mathbf{H}_e}{|\mathbf{H}_e|} \right) \frac{H_{ex}^2}{|\mathbf{H}_e|^2} + M_{anx} \left(\frac{\mathbf{H}_e}{|\mathbf{H}_e|} \right) \left[\frac{1}{|\mathbf{H}_e|} - \frac{H_{ex}^2}{|\mathbf{H}_e|^3} \right] \quad (5)$$

$$\frac{\partial M_{any}}{\partial H_{ey}} = \dot{M}_{any}(|H_e|) \frac{H_{ey}^2}{|H_e|^2} + M_{any}(|H_e|) \left[\frac{1}{|H_e|} - \frac{H_{ey}^2}{|H_e|^3} \right] \quad (6)$$

$$\frac{\partial M_{anx}}{\partial H_{ey}} = \dot{M}_{anx}(|H_e|) \frac{H_{ex} H_{ey}}{|H_e|^2} - M_{anx}(|H_e|) \frac{H_{ex} H_{ey}}{|H_e|^3} \quad (7)$$

$$\frac{\partial M_{any}}{\partial H_{ex}} = \dot{M}_{any}(|H_e|) \frac{H_{ex} H_{ey}}{|H_e|^2} - M_{any}(|H_e|) \frac{H_{ex} H_{ey}}{|H_e|^3} \quad (8)$$

The output of (1)-(2) can be changed to the differential reluctivity using following expression:

$$\frac{\partial \mathbf{H}}{\partial \mathbf{B}} = \nu_0 \mathbf{I} - \frac{\partial \mathbf{M}}{\partial \mathbf{B}} \quad (9)$$

where, $\mathbf{I}$ is the unit matrix, ν_0 is the air reluctivity.

Equation (9) is solved using (1)-(2), and hence the next time step value of the magnetic field can be computed as:

$$\mathbf{H}(t + \Delta t) = \mathbf{H}(t) + \frac{\partial \mathbf{H}}{\partial \mathbf{B}} \cdot [\mathbf{B}(t + \Delta t) - \mathbf{B}(t)] \quad (10)$$

where Δt is the time step value.

2.2 Modification of the anhysteretic magnetization

The method proposed in this study introduces two modifications to the anhysteretic magnetization. The first modification considers only the amplitude of the anhysteretic magnetization for the interpolation, and in the second modification, both the components of anhysteretic magnetization are taken into account.

2.2.1 Modification-1

The curve fitting done with the classical Langevin function in the vector J-A model does not always produce good results for certain electrical steel having steep knee in the measured $\mathbf{B}$ - $\mathbf{H}$ loop (Kokornaczyk and Gutowski, 2015). Jiles and Atherton (1983) provided a freedom for the free choice of the sigmoid function in place of the classical Langevin function, to model the anhysteretic magnetization. Kokornaczyk and Gutowski (2015) compared different sigmoid functions in a scalar J-A model. One of the possible choice is to use cubic splines for representing the anhysteretic magnetization. The 1-D spline is constructed considering the average values of the magnetic field strength for the same value of the magnetic flux density in ascending and descending $\mathbf{B}$ - $\mathbf{H}$ loop (Chwastek, 2011). The 2-D spline for the anhysteretic magnetization is then obtained by using the amplitude of the effective field, and the direction of the measurement (Figure 1 and Figure 4). The formulations presented in (4) remain the same except for the use of the Langevin function, and its derivative, that are replaced by the bi-cubic spline and its derivative. The modified anhysteretic magnetization function can be expressed as:

$$\mathbf{M}_{an} = M_{an}(|H_e|, \theta_{H_e}) \frac{\mathbf{H}_e}{|H_e|} \quad (11)$$

where θ_{H_e} is the angle of the effective field with respect to the rolling direction.

In (11), the assumption of the collinearity between the anhysteretic magnetization and the effective field is still retained. Therefore, partial differentiation of (11) is taken only with respect to the components of the effective field but not with the angle of it.

2.2.2 Modification-2

The anhysteretic magnetization is now considered for each of the projection of the vectors $\mathbf{B}$ and $\mathbf{H}$ in the principal axis (rolling and transverse). This gives both the components of the anhysteretic magnetization vector as a function of the amplitude of the effective field and the argument of the effective field as follows:

$$M_{\text{anx}} = M_{\text{anx}}(|\mathbf{H}_e|, \theta_{\mathbf{H}_e}), \quad M_{\text{any}} = M_{\text{any}}(|\mathbf{H}_e|, \theta_{\mathbf{H}_e}) \quad (12)$$

Care must be taken when evaluating elements of the differential anhysteretic susceptibility tensor using (12) in (4). Thus, a small change in one of the component of the effective field brings simultaneously a change in argument of the effective field, and in the partial derivative terms associated with the elements in (4). While building the 2-D bi-cubic spline, $\mathbf{M}_{\text{an}}$ obtained in the range $-\pi/2$ to $+\pi/2$ (positive half plane, see Figure 1) is rotated (1st quadrant by $-\pi$ and 4th quadrant by $+\pi$) to cover the full range of the rotation. This assumption of rotation is based on the fact that the effect of applied magnetization is π periodic.

2.4 Measurement

The measurement of the magnetic field strength is done using a rotational single sheet tester (RSST). Figure 1 depicts the schematic view of the circular silicon steel sample of grade M400-50A and 0.5 mm thickness. The sample is magnetized using the stator of an induction machine. The details of the measurement setup is explained in Goričan *et al.* (2000), and Handgruber *et al.* (2015). The measurement system is a $\mathbf{B}$ controlled and performed at 50 Hz. One full cycle of the $\mathbf{B}$ - $\mathbf{H}$ curve consists of 4000 measurement points (2000 each in ascending and descending curves). In the case of unidirectional alternating magnetic field, measurements are done for every 15° ranging from -90° to $+90^\circ$ (see Figure 1). The unidirectional alternating magnetic field measurements are shown in Figure 2, and Figure 6 depict the rotational measurements. Thus, the silicon steel sample of grade M400-50A reveals weak magnetic anisotropy (Figure 6) as compared to the measurement done on a NO electrical steel sample at 5 Hz by Martin *et al.* (2016).

Figure 1. M400-50A steel sample and arrow representing the direction of the measurements (left), and anhysteretic curves obtained from the unidirectional alternating measurements (right)

Figure 2. Measured and simulated b - h curves in the transverse direction (left), and in the rolling direction (right) using splines

Figure 3. Measured and simulated b - h curve using Langevin function in the rolling direction

2.5 Parameter identification

The J-A model parameter identification has been a topic of debate even in the present day research. Different optimization algorithms have been proposed in last few decades (Jiles and Atherton, 1992; Lederer *et al.*, 1999; Leite *et al.*, 2004a; Marion *et al.*, 2008). Genetic algorithm and particle swarm optimization are the two most widely used optimization technique for the J-A parameter optimization (Leite *et al.*, 2004a; Marion *et al.*, 2008). The fitting done using cubic spline for the anhysteretic magnetization function excludes the use of two model parameters (M_s and $\underline{a}$). The remaining model parameters are determined with the pattern search optimization method (Vaseghi *et al.*, 2013) from the unidirectional alternating measurements in rolling and transverse directions at 1.6 T. One more consideration is to evaluate the original anisotropic J-A model using (3), and with the tensor model parameters, whose components in the rolling ($k_x = 269.45$; $\alpha_x = 4.8443 \times 10^{-5}$; $c_x = 0.7509$) and in the transverse ($k_y = 171.37$; $\alpha_y = 2.7898 \times 10^{-5}$; $c_y = 0.4434$) directions are obtained from the measurements using the above mentioned optimization technique.

3. Results and Discussion

The comparison between the results obtained from the modification-1 and the original vector J-A model is presented in Figure 4. The original vector J-A model results in an elliptical waveform, and clearly, the loci match measurement results only in the extremes (rolling and transverse directions). On the other hand, the modification-1 brings immediate improvements by following the curvature in intermediate directions (Figure 4). However, the discrepancies between the measurement and the simulation result is quite visible. In the next approach, modification-2 is applied, that uses projections in principal directions of the unidirectional alternating magnetic field measurements. The results are shown in Figure 5 and Figure 6. The simulation result obtained using modification-2 present some improvement in the symmetry of the curvature.

Figure 4. Anhyseretic magnetization from modification-1 (left), and the simulated and measured hysteretic h_x - h_y loci for circular B input ($|B| = 1.6$ T) (right)

Figure 5. Components of the anhyseretic magnetization from modification-2: in the rolling direction M_{anx} (left), and in the transverse direction M_{any} (right)

Figure 6. Comparison between the simulated results from two modifications (left), and the simulated and measured h_x - h_y loci obtained using modification-2 only for higher magnetization level ($|B| = 1.6$ T, 1.8 T, and 2 T) (right)

4. Conclusion

The paper presents a phenomenological model to predict the anisotropic hysteretic behaviour of the M400-50A NO silicon steel sheet. Instead of representing the anhysteretic magnetization with the classical Langevin function in the J-A model, the authors suggest to develop two bi-cubic splines to account for the imperfection of the ferromagnetic material. Both components and the cross-coupling terms in the differential anhysteretic susceptibility tensor are investigated and implemented in the anhysteretic J-A model. The anisotropic parametric tensor for representing the domain interaction, the reversibility of the wall displacement and the coercive field effect on the losses are identified from the alternating measurements at rolling and transverse directions. The suggested improvement of the anisotropic J-A model is compared with the rotational flux density measurements. The original anisotropic J-A model presents some noticeable discrepancies which become mitigated with the presented modification for modelling the anhysteretic characteristic. Moreover, the consideration of only the amplitude of the magnetization instead of both its components significantly decrease the veracity of the anisotropic model. Hence, the cross-coupling terms in the differential anhysteretic susceptibility tensor affect the accurate prediction of the anisotropic hysteretic feature of non-oriented steel material.

The presented model gives better result for the higher amplitude of the rotating magnetic field excitations. Nevertheless, larger discrepancies are seen at low excitation levels. Further investigation must be done to understand the influence of each of the model parameters in such deviations. A more accurate analysis of the model at low frequency measurements should help in better modelling of the magnetic anisotropy. In future, the study of anisotropic parameters and its influence in the rotational loss, model response to the elliptical flux density, and implementation in the finite element subroutine will be considered.

References

Bergqvist, A. (1996), "A simple vector generalization of the Jiles-Atherton model of hysteresis," *IEEE Transactions on Magnetics*, Vol. 32, No. 5, pp. 4213-4215.

Benabou, A., Clénet, S. and Piriou, F. (2003), "Comparison of Preisach and Jiles-Atherton models to take into account hysteresis phenomenon for finite element analysis," *Journal of Magnetism and Magnetic Materials*, Vol. 261, pp. 305-310.

Benabou, A., Leite, J.V., Clénet, S., Simão, C. and Sadowski, N. (2008), "Minor loops modelling with a modified Jiles-Atherton model and comparison with the Preisach model," *Journal of Magnetism and Magnetic Materials*, Vol. 320, pp. 1034-1038.

Bíró, O., Außerhofer, S., Preis, K. and Chen, Y. (2010), "A modified elliptic model of anisotropy in nonlinear magnetic materials," *COMPEL*, Vol. 29, No. 6, pp. 1482-1492.

Chwastek, K., (2013), "Anisotropic properties of non-oriented steel sheets," *IET Electric Power Applications*, Vol. 7, No. 7, pp. 575-579.

- 1
- 2
- 3
- 4
- 5
- 6 Dłala, E. (2011), "Efficient algorithms for the inclusion of the Preisach hysteresis model in nonlinear finite-element methods," *IEEE Transactions on*
- 7 *Magnetics*, Vol. 47, No. 2, pp. 395–408.
- 8 Enokizono, M. and Soda, N. (1995), "Magnetic field analysis by finite element method using effective anisotropic field," *IEEE Transactions on*
- 9 *Magnetics*, Vol. 31, No. 3, pp. 1793-1796.
- 10 Goričan V., Hamler, A., Hribernik, Jesenik, M., and Trlep, M. (2000), "2-D measurements of magnetic properties using a round RSST," *Proc. 6th Int.*
- 11 *Workshop 1&2-Dimensional Magn. Meas. Test.*, pp. 66-75.
- 12 Gyselinck, J., Dular, P., Sadowski, N., Leite, J., and Bastos, J. P. A. (2004), "Incorporation of a Jiles-Atherton vector hysteresis model in 2D FE
- 13 magnetic field computations," *COMPEL*, Vol. 23, No. 3, pp. 685-693.
- 14 Handgruber, P., Stermecki, A., Bíró, O., Goričan, V., Dłala, E. and Ofner, G. (2015), "Anisotropic generalization of vector preisach hysteresis models
- 15 for nonoriented steels," *IEEE Transactions on Magnetics*, Vol. 51, No. 3, pp. 1-4.
- 16 Ivanyi, A. (2000), "Hysteresis in rotation magnetic field," *Phys. B*, Vol. 275, pp. 107–113.
- 17 Jiles, D. C. and Atherton, D. L. (1983), "Ferromagnetic hysteresis," *IEEE Transactions on Magnetics*, Vol. 19, No. 5, pp. 2183-2185.
- 18 Jiles, D. C., Atherton, D. L. (1984), "Theory of ferromagnetic hysteresis (invited)," *Journal of Applied Physics*, Vol. 55, No. 6, pp. 2115-2120.
- 19 Jiles, D., Thoeke, B. and Devine, M. (1992), "Numerical determination of hysteresis parameters for the modelling of magnetic properties using the
- 20 theory of ferromagnetic hysteresis," *IEEE Transactions on Magnetics*, Vol. 28, pp. 27-35.
- 21 Kokornaczyk, E. and Gutowski, M. W. (2015), "Anhysteretic Functions for the Jiles–Atherton Model," *IEEE Transactions on Magnetics*, Vol. 51,
- 22 No. 2, pp. 1-5.
- 23 Lederer, D., Igarashi, H., Kost, A., and Honma, T., (1999), "On the parameter identification and application of the Jiles–Atherton hysteresis model for
- 24 numerical modelling of measured characteristics," *IEEE Transactions on Magnetics*, Vol. 35, pp. 1211–1214.
- 25 Leite, J. V., Avila, S. L., Batistela, N. J., Carpes, W. P., Sadowski, N., Kuo-Peng, P., and Bastos, J. P. A., (2004a) "Real coded genetic algorithm for
- 26 Jiles–Atherton model parameters identification," *IEEE Transactions on Magnetics*, Vol. 40, pp. 888–891.
- 27 Leite, J. V., Sadowski, N., Kuo-Peng, P., Batistela, N. J. and Bastos, J. P. A. (2004b), "Inverse Jiles-Atherton vector hysteresis model," *IEEE*
- 28 *Transactions on Magnetics*, Vol. 40, No. 4, pp. 1769-1775.
- 29 Li, W., Kim, I. H., Jang, S. M., and Koh, C. S. (2011), "Hysteresis Modeling for Electrical Steel Sheets Using Improved Vector Jiles-Atherton
- 30 Hysteresis Model," *IEEE Transactions on Magnetics*, Vol. 47, No. 10, pp. 3821-3824.
- 31 Marion, R., Scorretti, R., Siauve, N., Raulet, M. A., and Krahenbuhl, L. (2008), "Identification of Jiles–Atherton model parameters using particle
- 32 swarm optimization," *IEEE Transactions on Magnetics*, Vol. 44, No. 6, pp. 894-897.
- 33 Martin, F., Singh, D., Belahcen, A., Rasilo, P., Haavisto, A. and Arkkio, A. (2015), "Analytical model for magnetic anisotropy of non-oriented steel
- 34 sheets," *COMPEL*, Vol. 34, No. 5, pp. 1475-1488.
- 35 Martin, F., Singh, D., Rasilo, P., Belahcen, A., and Arkkio, A. (2016), "Model of magnetic anisotropy of non-oriented steel sheets for finite-element
- 36 method," *IEEE Transactions on Magnetics*, Vol. 52, No. 3, pp. 1-4.
- 37 Mayergoyz, I. D. (2003), *Mathematical Models of Hysteresis and Their Applications*, Elsevier, Amsterdam.
- 38 Néel, L. (1948), "La loi d'approche en a: H et une nouvelle théorie de la dureté magnétique," *J. Phys. Radium*, Vol. 9, No. 5, pp. 184-192
- 39 Péra, T., Ossart, F. and Waeckerle, T. (1993), "Numerical representation for anisotropic materials based on coenergy modelling," *Journal of applied*
- 40 *physics*, Vol. 73, No. 10, pp. 6784-6786.
- 41 Ramesh, A., and Jiles, D. C. (1996), "A model of anisotropic anhysteretic magnetization," *IEEE Transactions on Magnetics*, Vol. 32, No. 5, pp. 4234-
- 42 4236.
- 43 Sadowski, N., Batistela, N. J., Bastos, J. P. A. and Lajoie-Mazenc, M. (2002), "An inverse Jiles-Atherton model to take into account hysteresis in
- 44 time-stepping finite-element calculations," *IEEE Transactions on Magnetics*, Vol. 38, No. 2, pp. 797-800.
- 45 Vaseghi, B., Mathekg, D., Rahman, S., and Knight, A. (2013), "Parameter optimization and study of inverse J-A hysteresis model," *IEEE*
- 46 *Transactions on Magnetics*, Vol. 49, No. 5.
- 47 Vernescu-Spornic, C., Kedous-Lebouc, A., Spornic, S. A. and Ossart, F. (2000), "Anisotropic and vector hysteresis model for magnetic materials
- 48 application to a cubic textured NiFe sheet," *Physica B: Condensed Matter*, Vol. 275, No. 1, pp. 99-102.
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